WEAPONS SYSTEMS FUNDAMENTALS

Analysis of Weapons

PUBLISHED BY DIRECTION OF
THE CHIEF OF THE BUREAU OF NAVAL WEAP

NAVWEPS OP 3000 VOLUM

NAVWEPS OP 3000 (VOLUME 2)

(ordnance pamphlet)

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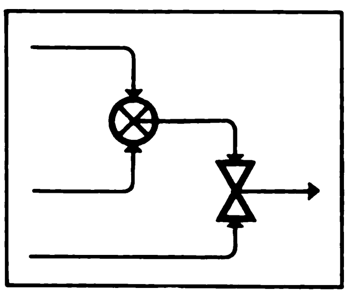
CHAPTER 1

ANALYSIS OF WEAPONS

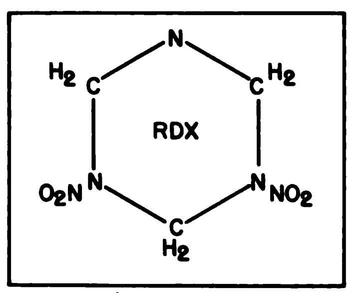
- warheads
- propulsion
- flight paths
- vehicles
- launching
- weapon control

INTRODUCTION

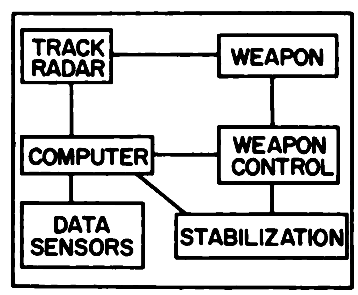
Military weapons systems have improved greatly in speed, accuracy and potency through the years, and at an accelerated pace since World War II. The superiority of the modern weapons system over its antecedants is derived primarily from three sources: mechanization of the weapon control problem, application of technical advances to the basic components, and integration of the weapons into advanced weapons systems.



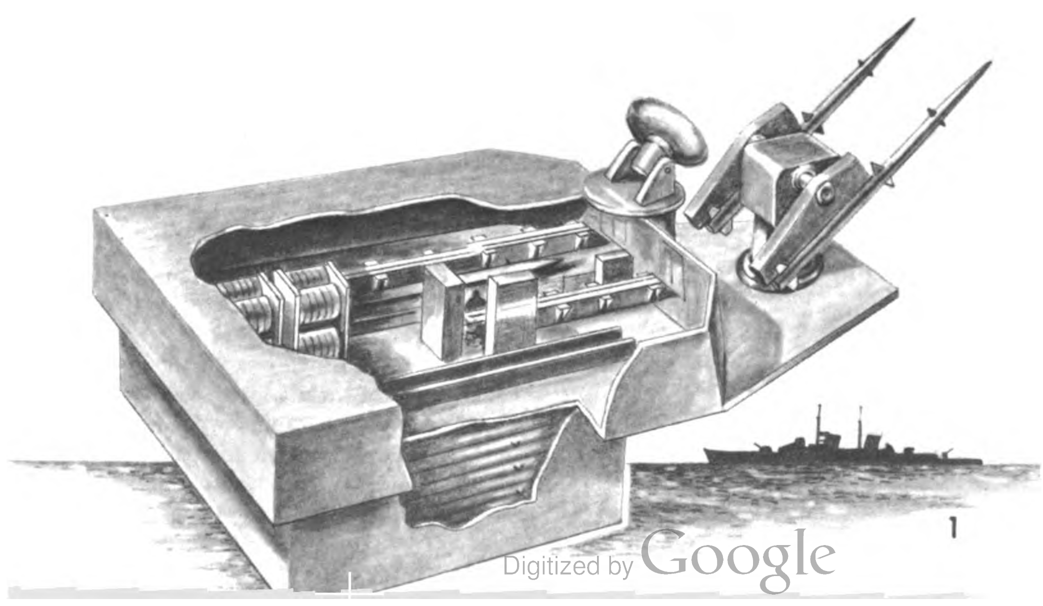
mechanization



technical advance



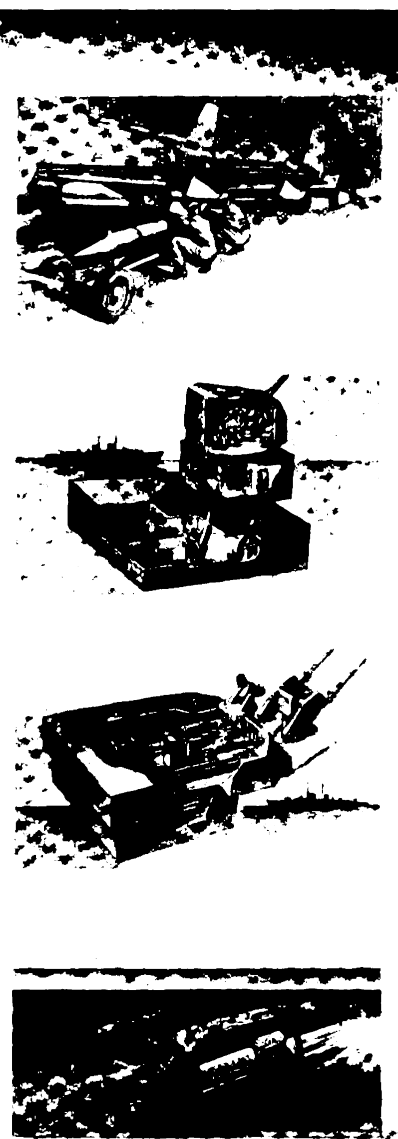
systems



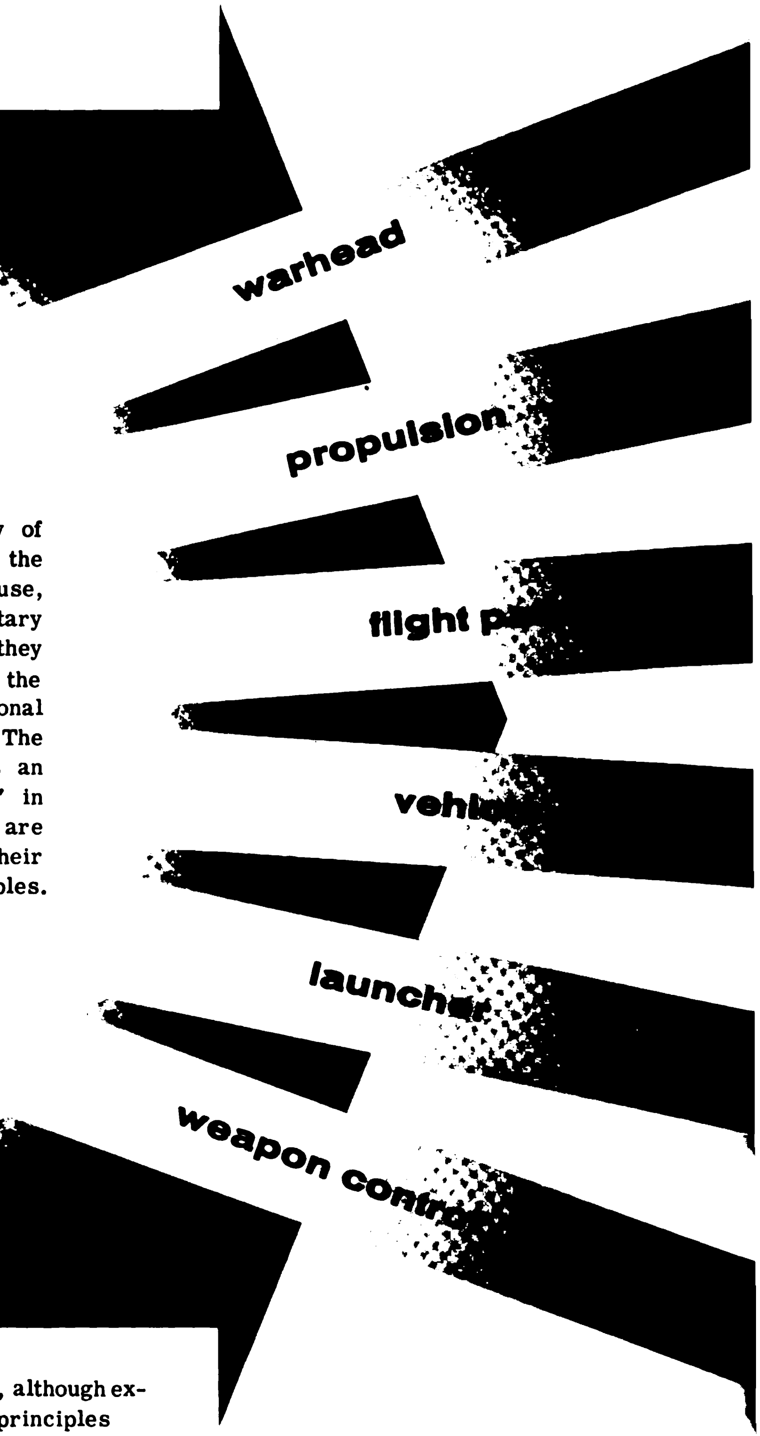
ANALYSIS OF WEAPONS

- warhead
- propulsion
- flight path
- vehicle
- launcher
- weapon control

analysis . . . separation of anything into constituent parts or elements; also, an examination of anything to distinguish its component parts or elements, separately or in their relation to the whole.

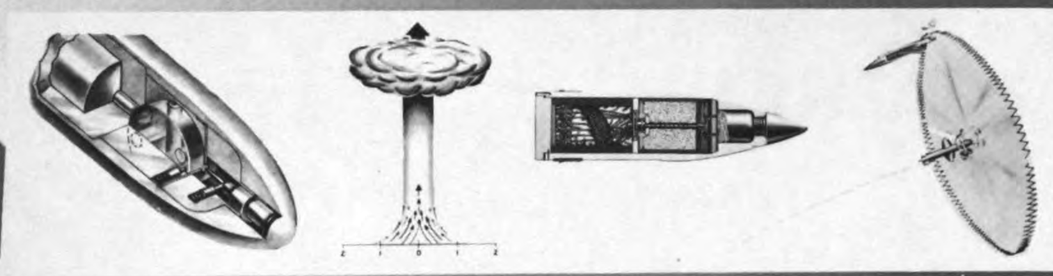


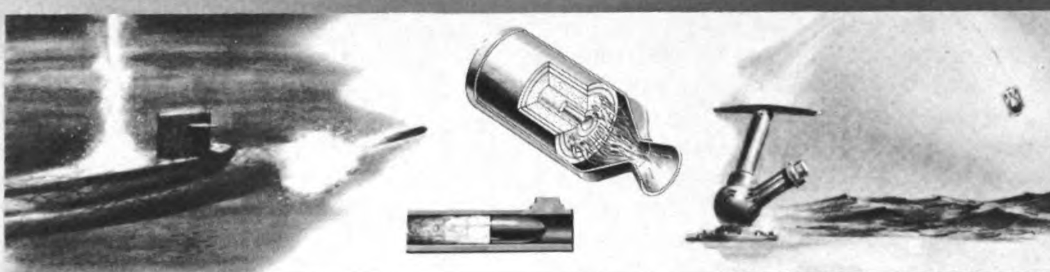
Despite the complexity of modern weapons and the multiplicity of types in use, an examination of military weapons reveals that they can all be divided into the six fundamental functional elements illustrated. The format of volume 2 is an "analysis of weapons" in which these elements are examined in terms of their basic underlying principles.

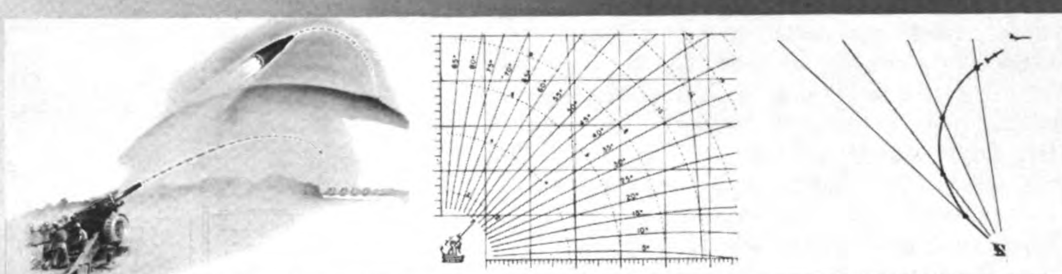


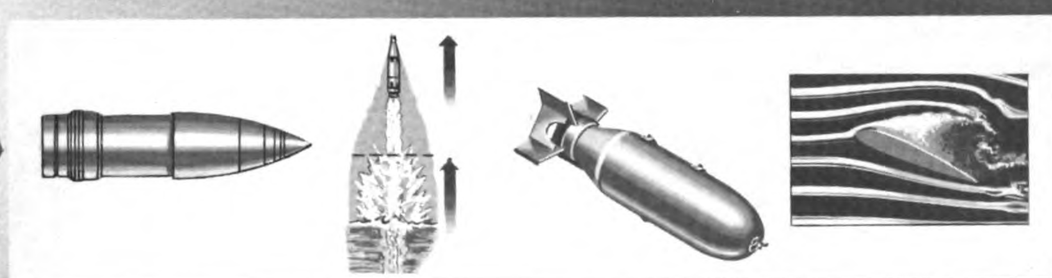
No specific models or systems are covered, although existing weapons are cited as examples of the principles or techniques discussed. The intent is to introduce the principles and requirements from which weapon designs are derived. An understanding of these topics should enable the reader to grasp the details of any weapons system from descriptive material written for the particular equipment with minimum time and effort.

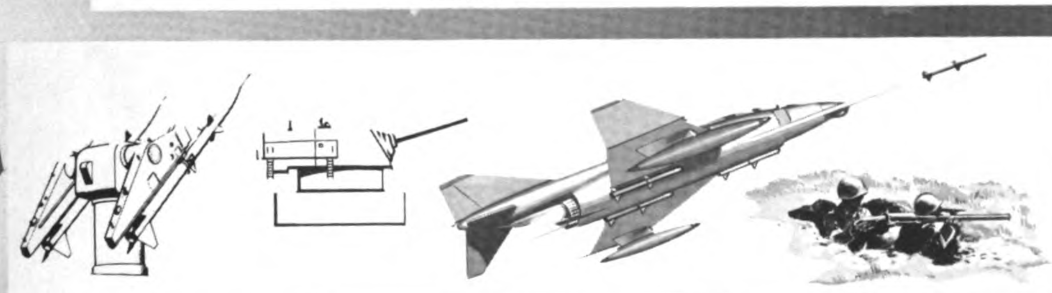
weapon . . . an instrument of offensive or defensive combat; something to fight with . . . any means by which one contends against another . . .

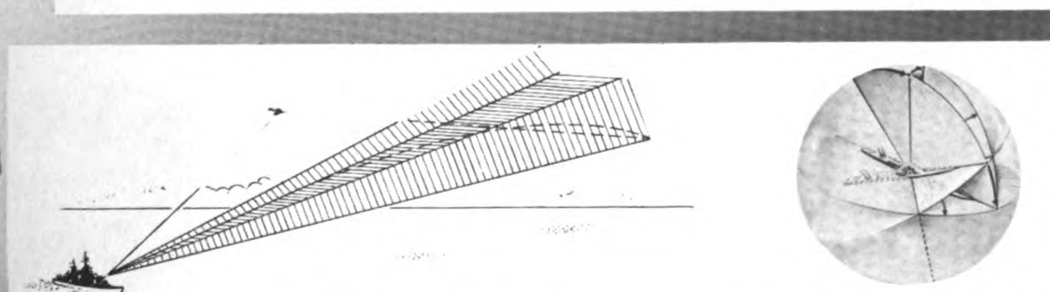












CHAPTER 2

introduction to **WARHEADS**

Man, in his desire for a quicker means of destroying an enemy's capacity for waging war, has continually increased the complexity and destructive force of the weapons in his arsenal. From the era of the rock-throwing caveman, weaponry has now evolved to the era of the guided missile. Yet today, as in the era of the caveman, the basic function of the weapon is to deliver some kind of a destructive force to an enemy area. The warhead of a missile is, or at least contains, the destructive agent. Modern warfare, therefore, demands continued research and development to improve warhead design and consequent performance.

Targets of today include military bases, factories, missile launching sites, airfields, troop concentrations, railroads, bridges, ships, etc., and are coming to include submarines capable of launching missiles while submerged, satellites orbiting the earth, and computer complexes guiding fleets of unmanned missiles. Since each type of target presents different tactical problems, a vast spectrum of warheads has been developed so that each target may be attacked with maximum effectiveness.

WARHEAD COMPONENTS

The basic warhead consists of three functional parts: a payload, a fuze, and a safety and arming (S & A) device. Variations in warhead type are obtained by altering any or even all three components. It should be noted that these components are not necessarily separate physical entities. For example, the warhead of a guided missile generally has a separate payload, fuze, and safety and arming device. A gun projectile, on the other hand, often combines S & A and the fuze into a single device, although the separate functions are still performed. For the projectile, the combination of S & A and the fuze is generally referred to as the "fuze".

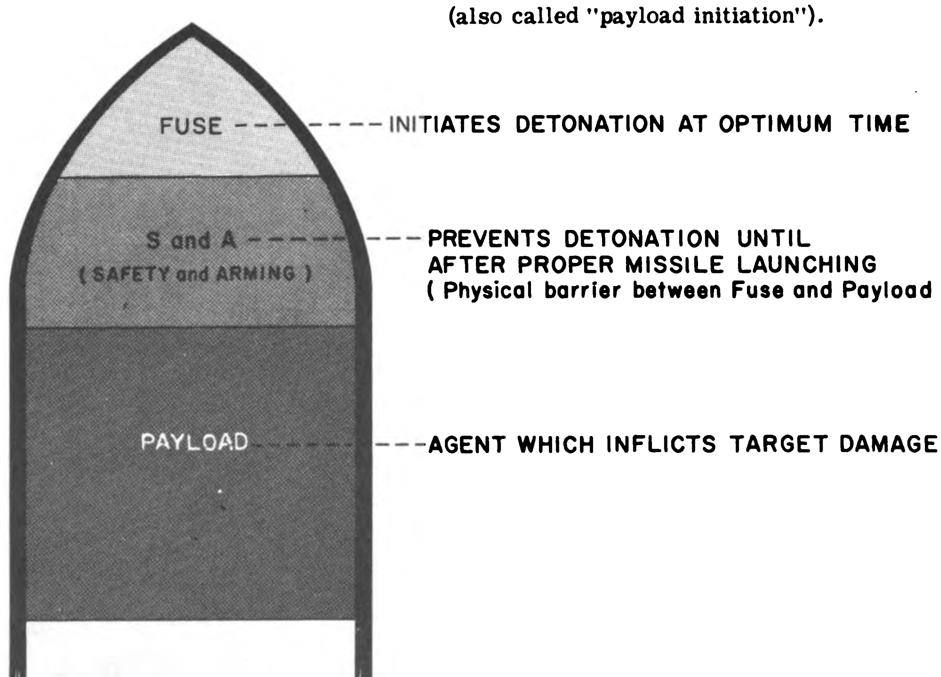
payload

The payload of the warhead is the destructive agent that directly causes damage to the enemy. When referring to a missile, the term payload is sometimes used for that portion of the total weight carried by the missile for destructive purposes, i.e. the entire warhead. Damage caused by the payload is usually considered as physical only, although such items as illuminating flares which expose the enemy to visual observation, and propaganda leaflets which damage morale are sometimes included. Physical damage to a target may be caused by converting stored chemical, mechanical, or nuclear energy into a destructive force; or by releasing chemical, biological, or radiological agents which destroy all forms of life and corrode inanimate matter. The type of payload used in a specific instance is determined by the characteristics of the target that is subject to attack.

fuze

The fuze is that part of the warhead which initiates detonation of the payload. In guided missiles the fuze is referred to as the Target Detection Device (TDD). For an attack to be effective, detonation must occur at the time during the missile's trajectory that will cause maximum damage to the target. Called the "optimum time of detonation", it is determined by the nature of the target and the attack geometry involved. If effectively designed, the fuze will always recognize and initiate detonation at this optimum time. Some method of obtaining the required data is necessary. The data may be derived from energy which is either generated or influenced by the target, or from the motion or acceleration of the missile. The fuze may perform this data-gathering function independently or may be supplied with the required data by a central data-gathering system which also supplies data to other components of the missile, such as the guidance system. The complexity of a fuze may vary from a simple contact-sensing device to one which solves an entire fire control problem to determine the correct time for payload detonation (also called "payload initiation").

warhead components



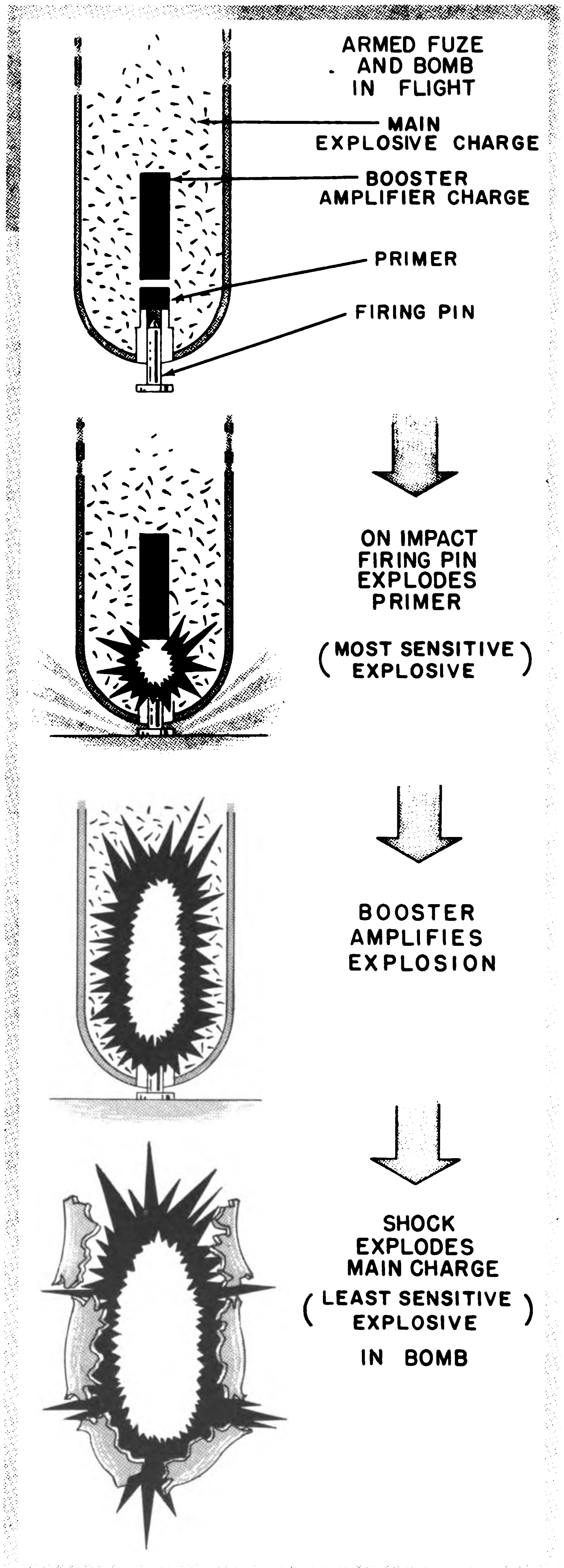
safety and arming (S and A) devices

The safety and arming (S & A) device performs two sequential functions within the warhead. First, it prevents accidental detonation of the warhead (safety feature) by interrupting the path between the fuze and the payload until it is ascertained that detonation will not be dangerous to friendly forces. Secondly, it provides a detonation path (arming feature) between the fuze and the payload, after safety has been assured, by removing the interruptor. The S & A device thus acts as an open switch until safe detonation can be accomplished, and is then closed thereafter. Normally the method employed to accomplish the safety function is to insert a physical barrier between the fuze and the payload.

In some cases the S & A device is used to destroy the missile by initiating the payload or another destructive charge if the target is not met and there is a chance that the missile may become a danger to friendly forces. As a secondary consideration, it is often desirable to delay arming until just before optimum detonation time in order to prevent accidental detonation due to incorrect fuze action caused by enemy countermeasures, etc.

EXPLOSIVE TRAIN

In most conventional warheads, the payload is expelled by a bursting charge. This bursting charge, for reasons of safety, must be relatively insensitive to initiation. An explosive train, which is an arrangement of a series of explosives, is first used to initiate the bursting charge. A small sensitive explosive acts as the detonator to set up the initial detonation wave. If this detonation is not sufficient to initiate the bursting charge properly, a booster charge is placed between the two explosives. The booster charge will be detonated by the explosive wave of the detonator, and in turn detonates the burster charge. The initiating intermediate explosive elements of the explosive train are usually integral parts of the fuze and/or safety and arming device.



operation of bomb explosive train

TARGET KILL

energy coupling

The transference of destructive energy from the warhead to the target is known as "energy coupling". The energy forms liberated when a payload detonates include thermal, kinetic, chemical, and nuclear energy. In addition, pressure or shock waves are a by-product of the blast. The effectiveness of the energy transfer depends on many factors: the target location and defenses; whether the energy source is chemical or nuclear; the strength, power, and efficiency of the explosive agent; the attenuation and propagation constants of the energy forms released; and the loss of efficiency due to environmental conditions. The problem of energy coupling requires the consideration of two important energy transfer factors: dispersion and damage volume.

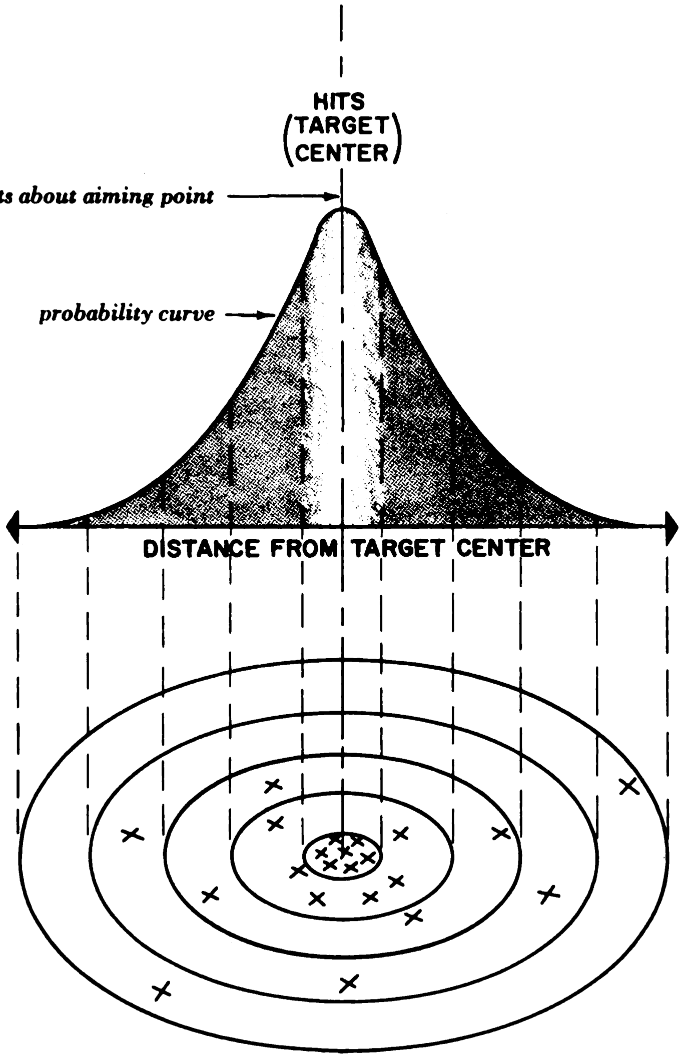
dispersion

When a series of identical missiles is fired at a target under identical conditions, a distribution of hits about the aiming point will occur. This distribution is referred to as "dispersion". When environmental conditions change between successive missile firings or the target changes course suddenly (evasive maneuvers), the dispersion will cover a larger area. Dispersion occurs whether or not the missile is an unguided gun projectile or a guided missile. However, since it is possible to control a guided missile over its entire flight path, compensations may be made for changes in environmental conditions and evasive maneuvers by the target. The dispersion of a series of guided missiles fired at a target thus covers a lesser area than for a

damage volume

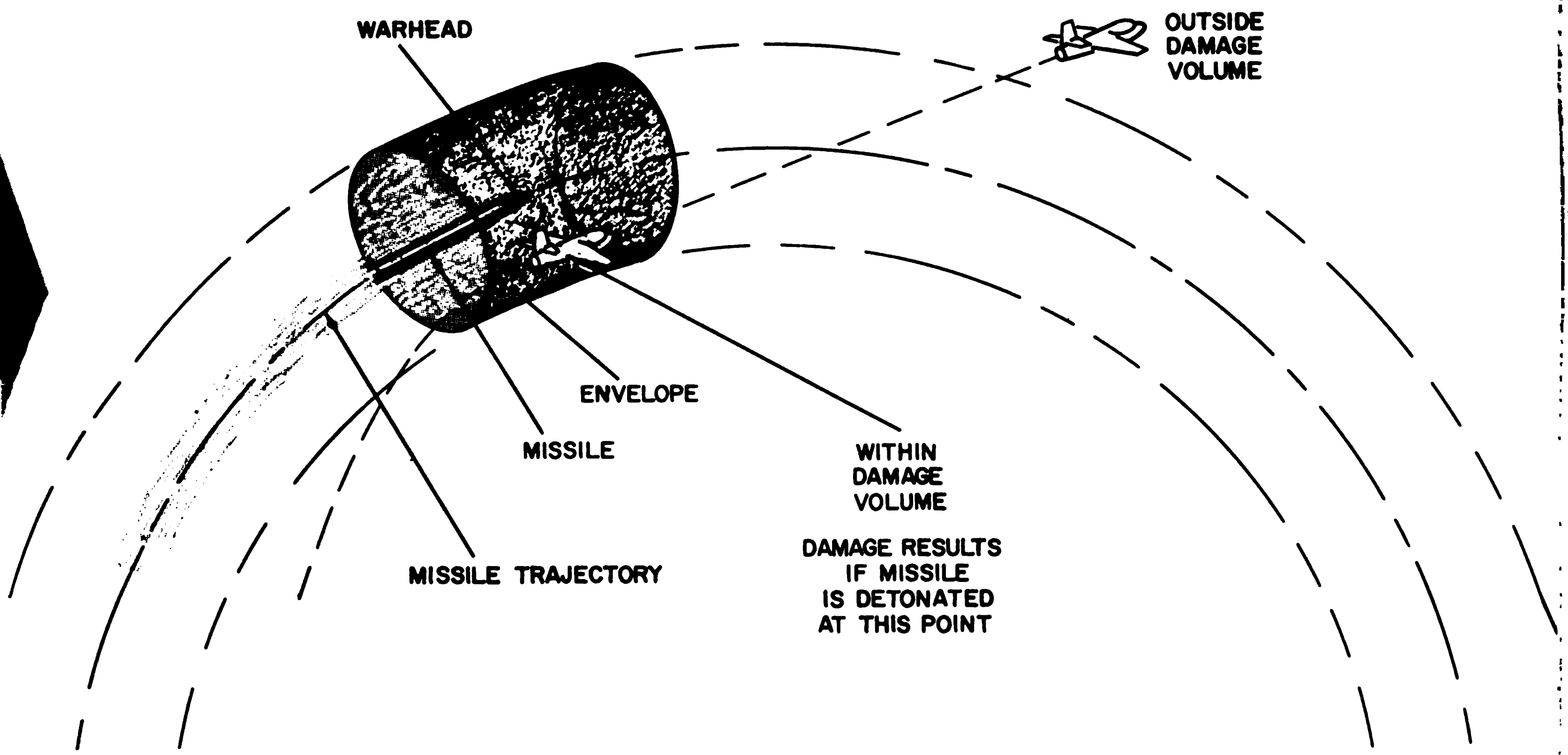
The warhead may be considered as the focal point of a destructive agent. The warhead, in addition, may be thought of as being enclosed by an envelope which sweeps along the trajectory of the missile and defines the limits of effectiveness of the destructive agent. That is, if the warhead is detonated when the target is within this envelope, damage to the target will occur. The volume enclosed by this envelope is called the "damage volume". The damage to a target caused by a missile without a payload can only be a result of the kinetic energy of the missile with respect to the target, or a result of the missile's excess fuel burning in the area of the target. However, since explosives are better payloads than fuel, pound for pound, excess fuel is very seldom carried for destructive purposes. In addition, the target may be light and the missile traveling at a high velocity. Although the kinetic energy of the missile may be a very destructive force, under such circumstances the missile might well pass through the target, punching a hole equal to its own cross section, yet without transferring sufficient energy to the target to produce the desired kill.

Suppose that several pounds of explosive are added to the warhead, and the destructive energy of the explosive is released as the missile passes through the target. The released energy is then radiated in all directions, thus increasing the damage volume. This increase in damage volume results in a more efficient coupling of energy to the target. The damage volume will increase with an increasing payload until a point is reached where damage to the target will occur even when the missile misses the target. The size of the damage volume is limited, in the case of chemical explosives, by the prohibitively heavy payloads that are required. However, because of the gap which exists in the explosive between chemical and nuclear explosives, the damage volume necessary to make up for missile dispersion can be achieved usually by switching to nuclear payloads.

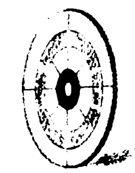


series of unguided missiles fired at the same target under the same conditions. A large number of missiles fired at a target under similar conditions will form a family of flight paths in a cluster about a median flight path and distributed in some statistical manner. A plane passed through these flight paths perpendicular to the median and immediately in front of the target will form a family of intersections. The area over which these intersections is distributed is the dispersion area, and should center on the projected target area.

When a direct hit on a small target is required, a large number of missiles must be fired. This fact demonstrates one of the dilemmas involved in weapons system design, namely: Should the size and weight of the missile guidance system be increased to decrease dispersion, or should the size and weight of the payload be increased to ensure damage to the target even if the miss distance is large?



TARGET TYPE AND KILL PROBABILITY



Two of the most important factors in missile design are the type of target to be destroyed and the required probability of kill. Most weapons systems are not general purpose in nature but rather are designed to combat a class of targets with essentially similar characteristics. By classifying targets as to type, problems common to many individual targets may be handled at the same time. A target can be classified by location, such as air, surface or subsurface; or by strength of structure, such as personnel, building, fortified installation, etc. The required warhead damage volume and the method of attacking each target is similar within a particular class but varies from class to class. Aircraft, for example, form one class of targets with similar vulnerability. Since the targets in a particular class have similar vulnerability characteristics, the kill probability for an entire class can be calculated mathematically.

Probability is defined empirically as the limit of the number of times during an operation that an event occurs, divided by the number of times that the operation is tried as the number of tries increases to infinity. For example, if an event has been observed to occur S times out of N trials, then, until further knowledge of the event is obtained, S/N may be taken as an estimate of the probability that the event will occur at any future time. Confidence in this estimate is increased as the number, (N) of observed cases increases, and the estimate becomes more and more of a mathematical certainty as N approaches infinity. To determine probability in a practical situation, where an infinite number of tries is impossible, a large number of tries is sufficiently accurate for most purposes. Kill probability is a specific form of probability in which the event of interest is the destruction or "killing" of a target.

If an event, such as the killing of a target, can occur in s different ways and can fail in f different ways where all these ways are equally likely, probability p of the event occurring at any trial is defined as the number of ways of occurrence divided by the total number of ways of occurrence and failure, or:

$$p = \frac{s}{s+f}$$

The probability "q" of the event failing to occur is:

$$q = \frac{f}{s+f}$$

From this it can be seen that $p+q = 1$, or that $p = 1 - q$ and $q = 1 - p$. If a number of mutually independent operations occur simultaneously or consecutively, the probability that each of the desired events will occur is the product of the probabilities of all desired events occurring separately.

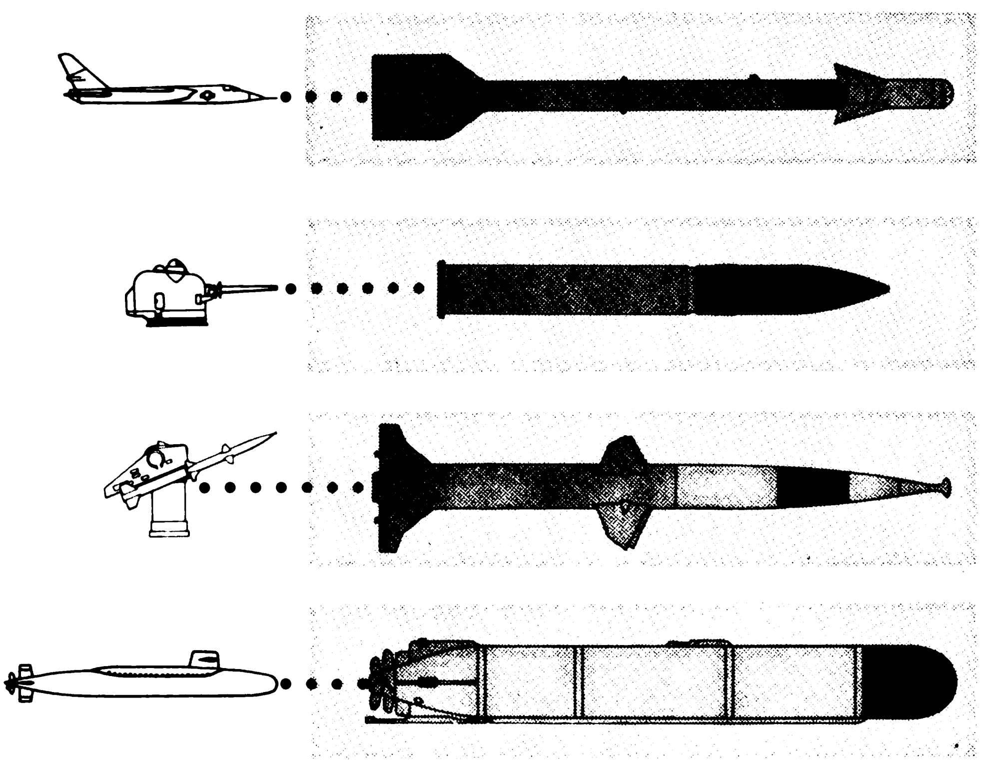
The probability of killing a target by using a salvo of m missiles, each having kill probability pk, can be determined from these facts. The kill probability for a salvo is one minus the probability that all m missiles will fail, which is $(1-pk)^m = qm$. The cumulative kill probability is then $1 - (1-pk)^m$. For example, a missile with a single shot kill probability of 0.6, has a probability of failure of 1 - 0.6 or 0.4. If a two-round salvo of these missiles were fired, the probability of both missiles failing would be $(0.4)^2$ or 0.16. The probability of at least one missile hitting the target then is 1 - 0.16 or 0.84. This could have been found by using the formula, $1 - (1 - pk)^m$, directly and resulting in $1 - (1 - 0.6)^2$ or 0.84. A three-round salvo would have a cumulative kill probability of 0.936.

In actuality, the cumulative kill probability is more difficult to compute since each shot of a salvo is not mutually independent, but depends on the same tracking information, weapon line drive equipment, launcher operation, etc. Computing cumulative kill probability based solely on individual kill probability does not take into account the factor that, although a single missile might not damage the target sufficiently to kill and is thus classified a failure, a series of these "failures" may succeed in killing the target. The kill probability of a specific missile is a function of many parameters: the reliability of the missile (does it function as intended?), the accuracy of the missile guidance system, the fuzing operation (is the warhead detonated at the correct time?), and the effectiveness of energy coupling to the target.

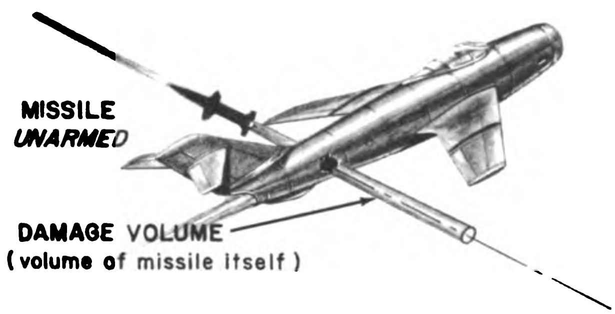
PAYLOAD

The primary component of the warhead is the payload, defined earlier as the destructive agent of the warhead, or that portion of the warhead which accomplishes the desired end result. It should be noted that the term "payload" is also used to describe satellites being carried into orbit, reconnaissance instruments and similar items. This section is restricted to weapons systems payloads.

The major topics discussed are explosives (chemical, nuclear, etc.), methods of coupling energy to a target (blast, fragmentation, etc.) and nonexplosive payloads (e.g., incindiary or leaflet).

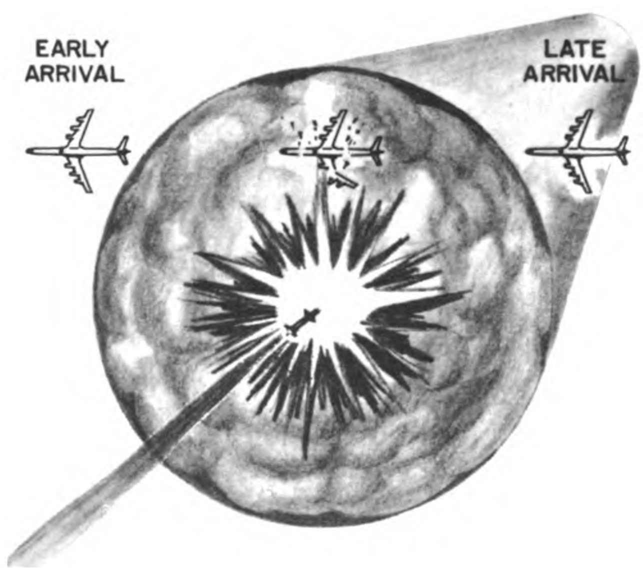


DAMAGE PARAMETERS



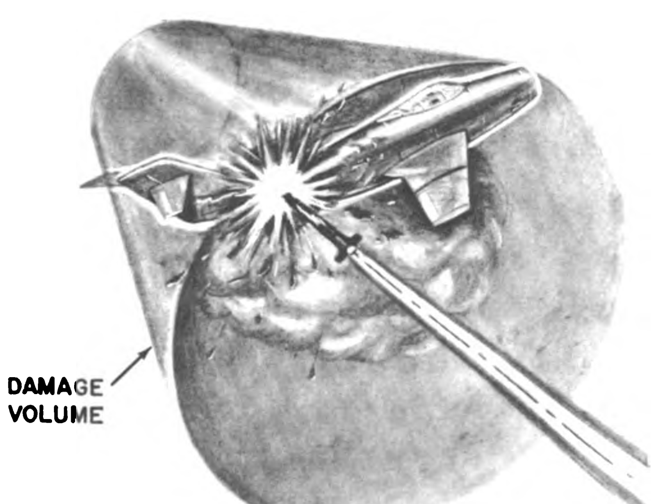
damage volume

The damage volume, as previously defined, is the limit of destructive effectiveness of the missile payload. The appearance of the target within the damage volume then is a necessary condition for target damage. It must be understood, however, that the damage volume is a theoretical entity which does not actually exist until the payload is initiated. Once the payload is initiated, the damage volume remains in existence for only a finite time. Therefore, if a payload is initiated too early or too late, the target may appear within the damage volume either before or after it exists. Thus, the condition for damage to the target is that it pass through the damage volume of the warhead while the destructive parameter of the payload is active.



attenuation

As shock and bomb fragments leave the point of origin, a reduction in their destructive potential per unit area takes place. This effect is known as "attenuation". The environmental conditions that must be encountered: air density, water pressure, temperature, etc., affect both the velocity and the direction of flight of the fragment bursts. The pressure or shock waves from the blast encounter resistance which is caused by the characteristic of the media being traversed. In addition, the energy transformation can be likened to an expanding sphere in which the energy available per unit area constantly decreases until it is completely harmless. This effect of lowering the energy level per unit area of the energy source is also called "attenuation". The attenuation characteristic of an energy form influences the limit of distance from the point of detonation at which effective target damage can result. The actual distance limit, which corresponds to the outer limits of the damage volume, is a function of the explosive efficiency, the target armor, and the environment as well as the attenuation.

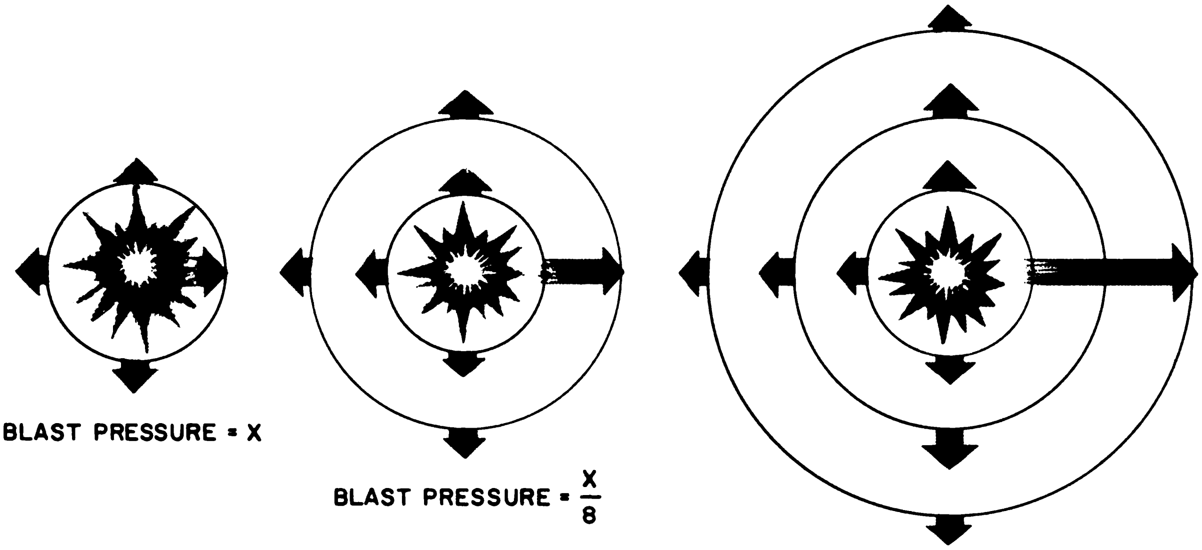


two types of damage volume

blast

If the blast pressure expands spherically outward from its point of origin, it will decrease as the volume of the sphere increases. Since the amount of energy, or density level, decreases inversely as the volume in which it is contained, the blast pressure must decrease approximately as the inverse cube of the sphere radius.

Under actual conditions, however, cooling effects and non-ideal expansion cause the attenuation factor to be greater than is indicated by theory.



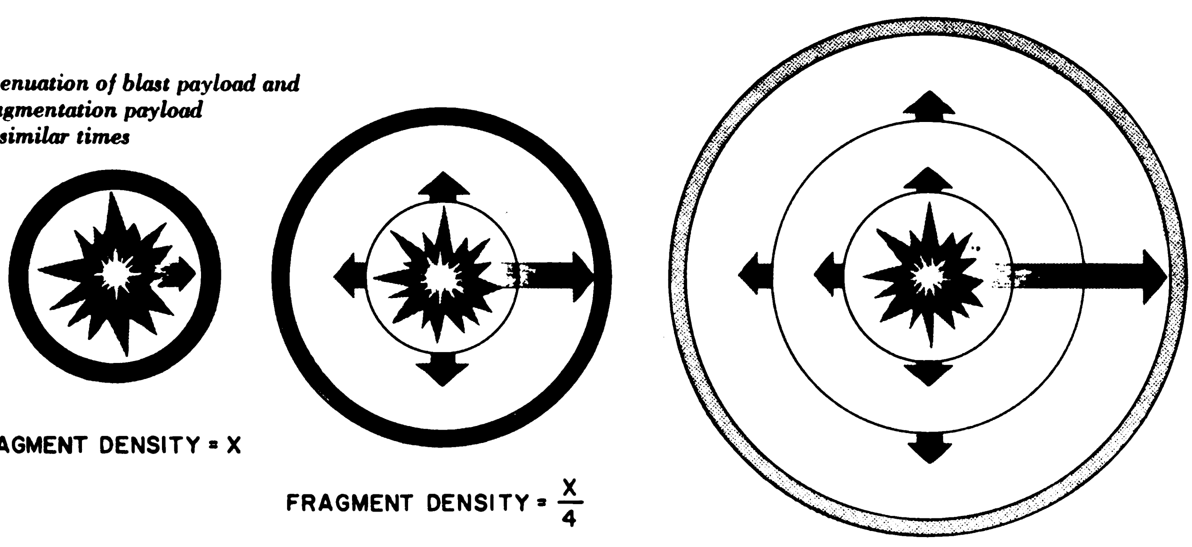
attenuation of blast

fragmentation

Since most warheads require a metal container, fragments are emitted when detonation of the payload occurs. A fragmentation warhead is specifically designed to emit a maximum number of fragments at a specific velocity. The decrease in fragment velocity due to air resistance is much less than the decrease in the velocity of the expanding spherical blast volume. This difference is due to a fragment having a constant cross-sectional area, while the surface of the spherical blast increases as the square of the distance. As a result, the advance of the blast volume lags behind the movement of the fragments.

A major consideration in the damage effect caused by fragments is fragment density. An isotropic fragmentation warhead may be considered as producing an expanding spherical shell of uniform thickness and composed of small fragments. Since the surface area of this shell increases in direct proportion to its radius, the fragment density decreases approximately inversely with the square of the distance. In practice, drag forces and variations in fragment size cause greater attenuation than described. When comparing the attenuation effects of a fragmentation warhead with a blast warhead, it is seen that a particular missile system with a fragmentation warhead can have a greater miss distance and still remain effective. However, the type of warhead incorporated into a missile system depends upon many other factors, aside from attenuation.

attenuation of blast payload and fragmentation payload at similar times



propagation

Propagation is defined as the manner in which energy and material, emitted by the warhead at detonation, travel through the medium in which the blast occurs. The characteristics of propagation include the velocity, direction, duration, etc., of the energy and material emitted. Propagation and attenuation are related in that propagation determines the basis under which attenuation takes place. Propagation, however, must be taken into consideration in the gunnery or fire control problem which describes how the destructive emissions of the payload will travel to the target. Any description of propagation must specify the conditions for which it is given, since propagation is a variable quantity. As an example, the propagation of a payload at rest differs from the propagation of the same payload traveling at some finite velocity.

When the propagation of a payload is uniform in all directions, it is called an "isotropic payload". A payload may be designed so that more energy or fragments are released in one direction than another, thus increasing the damage effect in that direction. A payload so designed is referred to as a nonisotropic payload, and is more effective than an isotropic payload of comparable type and weight when its emission is properly aimed.

For an isotropic blast warhead, the pressure falls off inversely with the volume of the expanding sphere, or $\frac{1}{4/3\pi R^3}$ where R is the radius of the sphere.

The pressure, P, therefore also diminishes inversely as the cube of the distance R, or $P = 1/K_1R^3$. If some overpressure is needed to cause damage, and the pressure generated by the warhead is initially proportional to the weight, W, of the explosive, or $P = K_2 W$, then the maximum range of damage is inversely proportional to the cube root of the weight.

to the weight, W, of the explosive, or $P = K_2 W$, then

$$\text{Since} \quad P = \frac{1}{K_1R^3} = K_2W$$

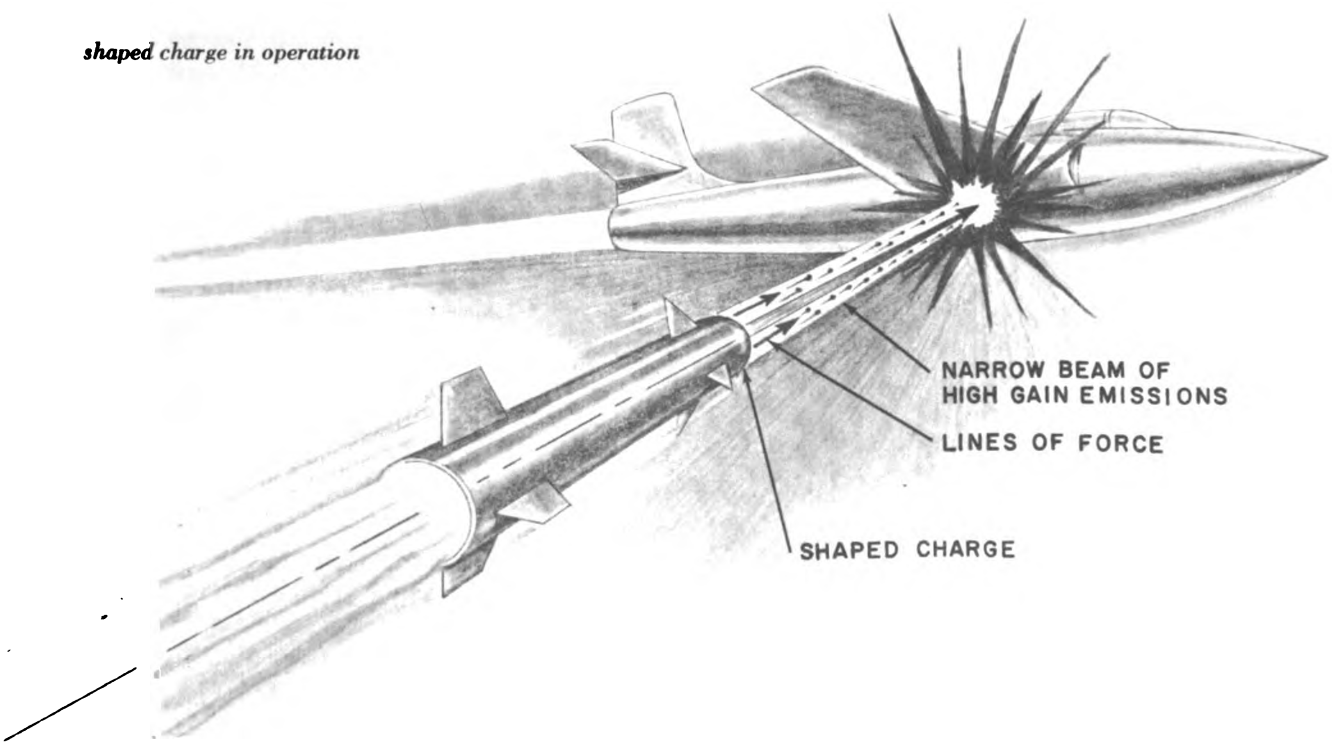
$$\text{then} \quad R = \frac{1}{\sqrt[3]{K_1K_2W}} = \frac{K}{\sqrt[3]{W}}$$

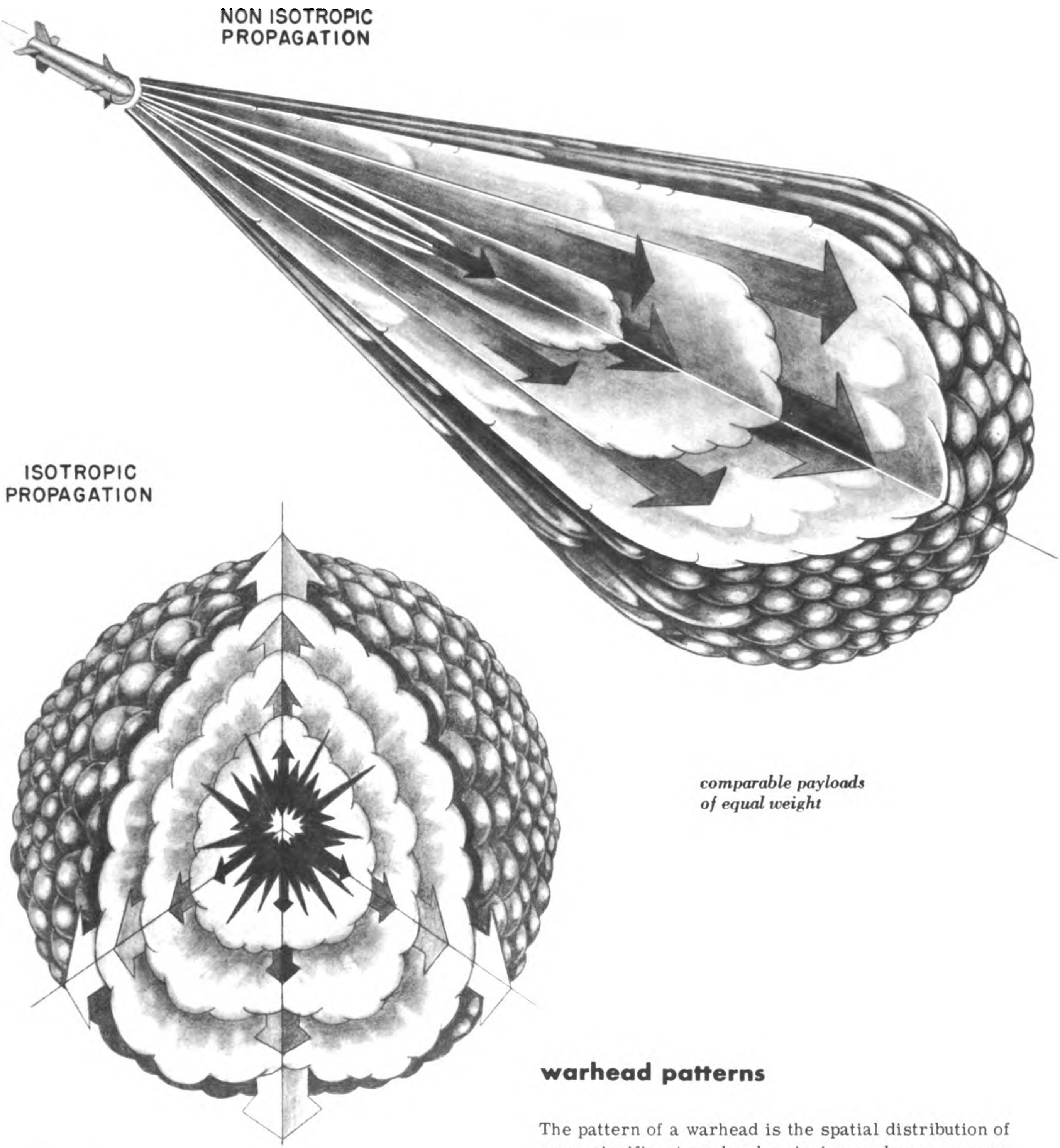
Thus, if the pressure from a charge of W_1 pounds is equal to P_1 lbs/in^2 at a distance R_1, then for a charge weighing W_2 pounds, the pressure will be equal to P_1 lbs/in^2 at some other distance, let us say R_2, which can then be calculated as:

$$R_2 = R_1 \sqrt[3]{\frac{W_2}{W_1}}$$

An isotropic payload is required for a missile system in which the only available target information is the time at which the point of closest approach to the warhead is reached, since any target direction relative to the warhead has equal probability. On the other hand, the use of a nonisotropic payload requires that specific target information be available so that the fuze may determine the optimum time of payload initiation. An example of nonisotropic propagation, in which all warhead emission is directed to the target in a very narrow beam, is produced by a shaped charge.

shaped charge in operation





comparable payloads of equal weight

warhead gain

To describe the advantage of the non-isotropic warhead over the isotropic warhead of equal weight of payload, the concept of warhead gain is employed. Gain can be defined as the ratio of the maximum value of a warhead emission parameter to the value which the parameter would have if the warhead were isotropic. The parameter to be compared may be created by the pressure of an explosion, the density of the metal fragments that are emitted, the damage effect on certain types of targets, etc.

warhead patterns

The pattern of a warhead is the spatial distribution of some significant warhead emission or damage parameter such as energy-density, damage-effect, or material-density distribution. Patterns are most useful in describing the emission of fragmenting payloads, but can be used for any type of payload. The warhead pattern may be described in a coordinate system based upon a stationary warhead, a warhead traveling through air or empty space at supersonic speed, or a warhead traveling in water at a much slower speed. The warhead pattern helps to describe the zone in which the target must lie when the fuze initiates detonation of the payload.

EXPLOSIVES

Warheads, whether conventional or nuclear, use the mechanism of an explosion to creat the pressures necessary to achieve damage. An explosion is the result of a chemical or nuclear change which causes a rapid and violent release of energy. The explosion liberates large amounts of heat and produces a great deal of gas. Due to the formation of these gases and their rapid expansion caused by the heat generated, a violent bursting of the explosive container, or warhead, occurs. Explosions may be divided into three types: mechanical, chemical, and nuclear. Mechanical explosions, such as the disruption of a steam boiler, are of no concern in military applications, and will not be discussed here.

chemical explosive reaction

A chemical explosive is a compound or mixture which, upon the application of heat or shock, decomposes or rearranges with extreme rapidity, yielding much gas and heat. Many substances not ordinarily classed as explosives may do one, or even two, of these things. For example, a mixture of nitrogen and oxygen can be made to react with great rapidity and yield the gaseous product nitric oxide, yet the mixture is not an explosive since it does not evolve heat, but rather absorbs heat, during the reaction. To be an explosive, a substance must exhibit all of the phenomena mentioned, that is: 1) formation of gases, 2) evolution of heat, 3) rapidity of reaction, and 4) initiation of reaction by shock or heat. A military explosive must also be suitable for, and be used for, military purposes.

FORMATION OF GAS.

When wood or coal is burned in the atmosphere, the carbon and hydrogen in the fuel combine with the oxygen in the atmosphere to form carbon dioxide and steam together with flame and smoke. When the wood or coal is pulverized so that the total surface in contact with the oxygen is increased, and burned in a furnace or forge where more air can be supplied, the burning can be made more rapid and the combustion more complete. When the wood or coal is immersed in liquid oxygen or suspended in air in the form of dust, the burning takes place with explosive violence. In each case, the same action occurs, i.e., a burning combustible forms a gas. The only difference is the speed at which the reaction takes place. Thus, materials which will burn can be made to explode if sufficient oxygen is readily available. Most explosives utilize this principle, but usually contain their own oxygen and thus are independent of oxygen supplied from the air. The high-pressure characteristic of explosive reactions is due to the gaseous products of combustion being expanded very rapidly by the heat liberated in the reaction. The maximum pressure developed and the way in which the energy of the explosion is applied depend not only upon the volume of the gases and the amount of heat liberated, but also the speed of the reaction.

detonation

Explosive compounds or mixtures decompose at different rates. When the decomposition rate (rate at which molecules split and rearrange their atoms into other molecules-mostly gaseous) is almost instantaneous, detonation occurs. Explosive compounds consist of unstable molecules which tend to revert to a more stable state. For example, in its oxidized form nitrogen combines with other elements, primarily carbon, hydrogen, and oxygen, to produce some chemical compounds which are unstable and explode violently. When the explosion occurs, the carbon and hydrogen tend to unite with the oxygen while the nitrogen tends to return to its elemental form. The unstable compound has thus reverted to a more stable state.

sample reactions:

reactants = products ⟶ high pressure
(GASES AND HEAT)

EVOLUTION OF HEAT.

When an explosive chemical reaction occurs, a large amount of heat is rapidly liberated. It is this rapid liberation of heat which causes the gaseous products of reaction to expand and generate high pressures. This rapid generation of high pressures of the released gas constitutes the explosion. It should be noted that the liberation of heat with insufficient rapidity will not cause an explosion. For example, although a pound of coal yields five times as much heat as a pound of nitroglycerin, the coal cannot be used as an explosive because the rate at which it yields this heat is quite slow.

RAPIDITY OF REACTION.

An explosive reaction differs from an ordinary combustion reaction in the speed with which it takes place. Unless the reaction occurs rapidly, the thermally expanded gases will be dissipated in the medium, and there will be no explosion. Again consider a wood or coal fire; in the fire there are the evolution of heat and the formation of gases, but not liberated rapidly enough to cause an explosion.

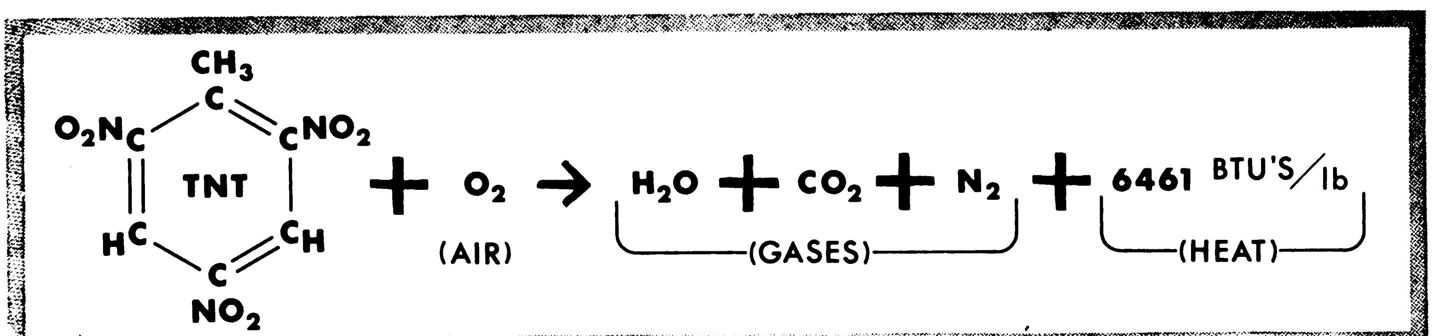
The rates of decomposition of explosives vary greatly. The slower forms of decomposition take place in storage and are of interest only from a stability standpoint. Of interest are the two rapid forms of decomposition, burning and detonation. Detonation is the term used to describe an explosive phenomenon of almost instantaneous decomposition. Explosives are classified as low or high explosives according to their rates of decomposition.

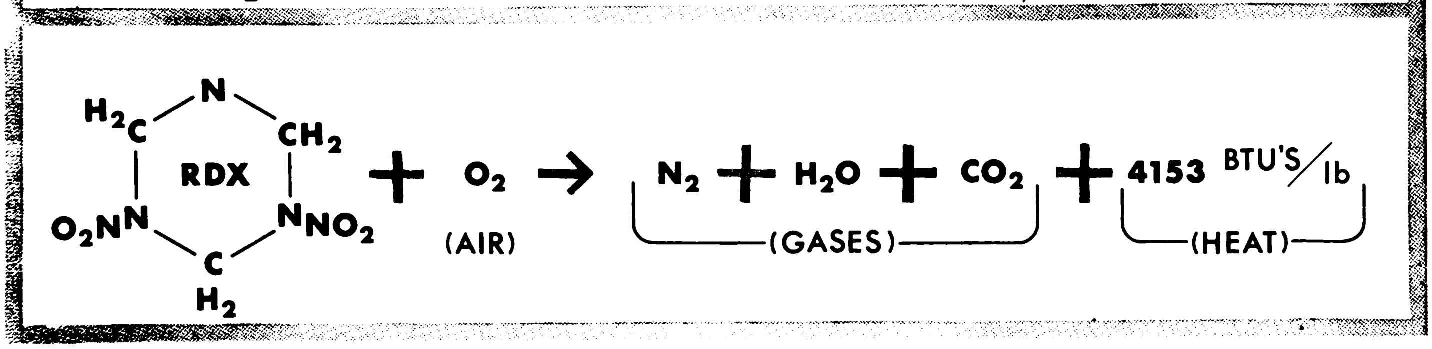
The major use of military high explosives in warheads is to form the destructive emission. Explosives are used in blast warheads to obtain the blast effect, and they are employed in fragmentation warheads to produce high velocity fragment by bursting the specially designed cases containing them.

Low explosives are used primarily as propelling charges. A propelling charge may be defined as a powder charge used in a weapon which, when ignited, produces large amounts of gas and heat which are employed to impart motion to a missile.

INITIATION OF REACTION.

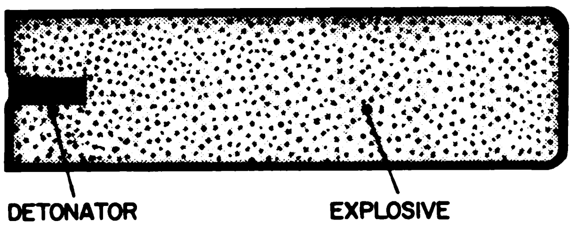
In order to qualify as an explosive the material must readily undergo a rapid reaction upon the application of a certain amount of shock or heat to a small portion of its mass. A material in which the first three factors exist can not be considered suitable as an explosive unless the reaction can be made to occur when desired. High explosives, such as the payloads of mines or torpedoes, in general require the sudden application of a strong shock or detonation to initiate the explosive reaction. This detonation is usually obtained by the action of an explosive train. This amounts to exploding a smaller charge of a more sensitive high explosive that is in contact with or in close proximity to the payload. The initiating effect is a result of the passage of a shock wave from one mass to the other.



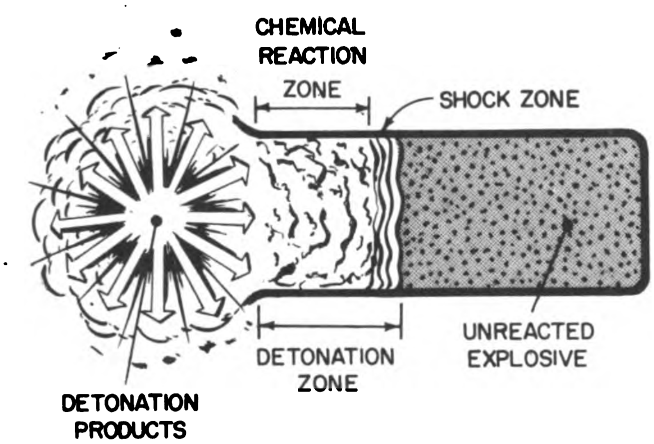


The following is one of several theories explaining

THE MECHANICS OF DETONATION



before detonation



after detonation

Initiation of the first phase of detonation is accomplished by applying sufficient energy, usually in the form of heat or shock, to an explosive. This energy sets up disruptive forces within the molecule, which exceed the attractive forces between some of its atoms, and decomposition commences. It should be noted that the tendency toward disruption is due to the instability of the molecules in question, as maintained above.
When the bond of the molecular and atomic attractive forces which tie the molecule together is broken, the atoms of the molecule rearrange into a more stable state and, in so doing release energy. The released energy then disrupts adjacent molecules and, if the initial applied energy is sufficient, the entire explosive mass will be decomposed in chain reaction fashion.
The one condition necessary for successful detonation is that the energy released by one molecule is sufficient to detonate more than one adjacent molecule.
The second phase of detonation is the formation and expansion of gas molecules. When the first molecule is decomposed, the carbon and hydrogen atoms are oxidized by the heat of the detonating wave, and their volume becomes greatly expanded. This process takes place almost instantaneously.
The process of detonation may be visualized more clearly by referring to the illustration. The detonator consists of a small but sensitive amount of explosive characterized by easy initiation. The explosive force of the detonator, upon initiation, sets off the main explosive charge as follows:

A detonation zone, 0.04 to 0.4 in. thick, travels with great rapidity through the main explosive charge. The detonation zone is usually considered to include a shock zone or shock wave approximately 4×10^{-6} in. thick, followed by a zone of chemical reaction. Very little, if any, chemical reaction occurs in the shock zone, but it is in this zone that peak pressure occurs. The compression in the shock zone raises the material to a high temperature, and thus initiates the chemical reaction. This is at or near the beginning of the chemical reaction zone. Maximum density and pressure are found to occur at the start of the chemical reaction, while temperature and velocity are found to reach their peak as the chemical reaction is completed. Directly behind the detonation zone lie the detonation products; in front of the shock zone is the unreacted explosive in its original state of density, pressure, and temperature. The detonation products flow through the undetonated explosive with a velocity which, although great, is still less than the velocity of the detonation zone. The velocity of advance of the detonation zone through the explosive is referred to as the detonation rate or detonation velocity.

properties of military chemical explosives

To determine the suitability of an explosive substance for military use, its physical properties must first be investigated. The usefulness of a military explosive can only be appreciated when these properties and the factors affecting them are fully understood.
Many explosives have been studied in past years to determine their suitability for military use - and most have been found wanting. Several of those found acceptable have displayed certain characteristics which are considered undesirable and, therefore, limit their usefulness in military applications.
The requirements of a military explosive are stringent and very few explosives display all of the characteristics necessary to make them acceptable for military standardization. The more important characteristics are:

Availability and Cost	Brisance
Sensitivity	Density
Stability	Volatility
Strength	Hygroscopicity
Power	Toxicity

These terms will now be defined as follows:

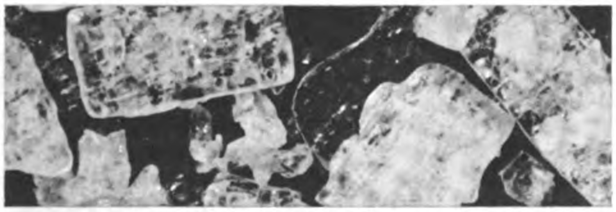
AVAILABILITY AND COST.

In view of the enormous quantity demands of modern warfare, explosives must be produced from cheap raw materials that are non-strategic and available in great quantity. In addition, manufacturing operations must be reasonably simple, cheap, and safe.

SENSITIVITY.

Sensitivity, as regards an explosive, refers to the ease with which it can be ignited or detonated, i.e., the amount and intensity of shock, friction or heat which is required. Those properties which contribute to the sensitivity of an explosive include: molecular structure, crystal size, distortion of crystalline structure, crystal coating, density of the explosive, moisture, and temperature. High density, increased moisture, and coatings of wax or similar substance on the crystals tend to decrease sensitivity. Sensitivity will be increased with increased temperatures or distortion of the crystalline structure.

ammonium nitrate crystals, 25x



tnt crystals, 30x

rdx crystals, 25x

The effect upon sensitivity of reducing or enlarging the size of the crystals is determined by the explosive considered. Depending upon the amount of internal strain within or between crystals the sensitivity may be increased or decreased.

The sensitivity of a military explosive is often determined through the use of an impact test. The conditions under which this test is to be performed must be rigidly defined and, even then, the absolute values obtained by different workers generally do not agree. However, sensitivity measured for a number of explosives will normally fall in the same sequence regardless of the test apparatus used. In one test, the sensitivity of the material is expressed in terms of the distance through which a standard weight must be dropped to cause the material to explode.

In general, the sensitivity of an explosive, as determined by the impact test, increases with temperature; and the molten material is much more sensitive than the hot, solid material. Test data for TNT, RDX, and ammonium nitrate explosives is given below:

Temperature, °C	Impact Test, Picatinny Apparatus, Minimum Height in Inches of Fall of 2KG Weight to Explode		
	RDX	TNT	Ammonium Nitrate
25	8	14	31
75			28
80		7	
90	8	*3	
100			27
105	5	2	
150			
			27
175			*12

*Material in molten condition

When the term sensitivity is used care must be taken to clarify what kind of sensitivity is under discussion. For example, the relative sensitivity of a given explosive to impact may vary greatly from its sensitivity to friction or to heat, and these sensitivity of a given explosive to a given method of initiation may also vary somewhat with the test method used to determine that sensitivity.

Sensitivity is an important consideration in selecting an explosive for a particular purpose. For example, the explosive in an armor piercing projectile must be relatively insensitive or the shock of impact would cause it to detonate before it penetrated to the point desired. The sensitivity to impact of some common explosives are listed below:

Explosive	Sensitivity Height in cm of fall of 2KG Weight to Explode
Mercury Fulminate (loose)	2
Nitroglycerine	4
Dynamite (75%) (nitroglycerine)	7
PETN	20
RDX	30
Tetryl	50
TNT (Granular)	90
TNT (Cast)	150
Explosive "D"	170

stability

Stability is often used as an antonym for sensitivity, but in military application it is taken to mean the ability of an explosive to be stored without deteriorating. Sensitivity does not imply instability of the explosive in storage, and insensitivity does not mean that the explosive will be stable in storage. A substance, although extremely reactive chemically, may be quite stable in the absence of another substance or form of energy with which it can react. As an example, a slight shock can cause lead azide to explode but, if stored properly, the substance is stable. The following factors affect the stability of an explosive:

chemical constitution

The very fact that some chemical compounds can undergo explosion when heated indicates that there is something unstable in their structures. While no precise explanation has been developed for this, it is generally recognized that certain groups, nitro dioxide (NO_2), nitrate (NO_3) and azide (N_3), are intrinsically in a condition of internal strain. Increased strain through heating causes a sudden disruption of the molecule and consequent explosion.

In some cases, this condition of molecular instability is so great that decomposition takes place at ordinary temperatures. As an example of an extreme case, mercuric azide sometimes explodes as rapidly as it crystallizes from solution.

EXPLOSIVE	FORMULA or PROPORTIONS	REL. SENS.	RELATIVE STRENGTH	RELATIVE BRISANCE	DETONATION VELOCITY	USE
Mercury Fulminate	$Hg(ONC)_2$	1	100	15.5-22.4	4700-5400	Primers and Detonators
Nitroglycerine	$C_3H_5(ONO_2)_3$	1-1.5	340-400	60	8400	Double Base Powder-Dynamite
Lead Styphnate	$C_6H(NO_2)_3(O_2Pb)$	1-1.5	80	9.5-21.4	4900-5200	Primers - Sensitizer for Primers and Detonators
Lead Azide	PbN_6	1.5-2	75	13.9-18	4000-5000	Detonators and Primers
PETN	$C(CH_2ONO_2)_4$	3	330-370	61.9	8300	Booster - Mixtures, Primacord
Cyclonite	$(CH_2)_3N_3(NO_2)_3$	3.5	350	61	8400	Booster - Mixtures
Tetryl	$C_6H_2(NO_2)_3(NCH_3NO_2)$	4	250	53.5	7500	Booster - Mixtures
Nitrostarch	40/37.7/20/.8/1.5, NS/SodNit/BarNit/Oil/Stabilizer	4	185	37.7	6100	Demolitions
Pentolite	50/50-PETN/TNT	4.5	230	53.0	7500	Bursting Charge
Torpex	42/40/18, RDX/TNT/AL	4.5	320	57.9	7300	Bursting Charge
Tetrytol	75/25, Tetryl/TNT	5	235	50.0	7300	Burster - Chemical Shell Demolition
Minol II	40/40/20, NH_4NO_3/TNT/AL	5-5.5	310	40-41	5400-5700	Bursting Charge
Cyclotol (Comp. B)	60/40, RDX/TNT	5.5	250	51.8	7800	Bursting Charge
Tritonal	80/20, TNT/AL	6	240	42.0	5500	Bursting Charge
Composition C2	80.1/4/10/4/1/.9, RDX/MNT/DNT/TNT/Collodion Cotton/Dimethyl Formamide	6.25	300	55.0	8000	Bursting Charge - Demolitions
Picric Acid	$C_6H_2OOH(NO_2)_3$	6-6.5	200	45.0	7200	Mixtures
TNT	$C_6H_2CH_3(NO_2)_3$	7	190	43.0	6900	Bursting Charge
Composition C	88/12, RDX/Oil	7	240	46.5	7400	Demolitions - Bursting Charges
Amatol 50/50	50/50 Ammonium Nitrate/TNT	7	220	38.0	6500	Bursting Charge
Composition A-3	91/9, RDX/wax	7	275	49.6	7500	Bursting Charge
Amatol 80/20	80/20, Ammonium Nitrate/TNT	7.5	240	32.0	5400	Bursting Charge
Picratol	52/48 ExD/TNT	9	185	43.0	6972	Bursting Charge
Explosive D	$C_6H_2(ONH_4)(NO_2)_3$	9	180	35.0	6500	Bursting Charge

Properties of Some Typical Military High Explosives

temperature of storage

The rate of decomposition of explosives increases at higher temperatures. All of the standard military explosives may be considered to be of a high order of stability at temperatures of 59° to 77°F, but each has a high temperature at which the rate of decomposition becomes rapidly accelerated and stability is reduced. Mercury fulminate undergoes unduly rapid deterioration at temperatures as low as 85° to 95°F. Nitroglycerin may withstand many years of storage at ordinary temperatures, but undergoes accelerated decomposition at 122°F and above. PETN and nitrocellulose begin to undergo rapid decomposition at less than 248°F. On the other hand, TNT and ammonium picrate will withstand heating at 290°F for 40 hours without significant decomposition and RDX is only slightly less stable at that temperature.

presence of impurities

Impurities may make some explosives unstable. For example, certain impurities such as dinitrotoluene (DNT) lower the melting point of TNT, trinitrotoluene, causing a sensitive mixture to form which may liquify at storage temperatures and exude from the solid TNT This exudate from TNT is very unstable.

exposure to sun

If exposed to the ultraviolet rays of the sun, many of the explosive compounds which contain nitro will rapidly decompose. This decomposition may also affect their stability.

STRENGTH.

The term strength, as applied to an explosive, refers to its ability to do work. Actually it is defined as the ability of the explosive to displace the medium in which it is confined or, from an engineering standpoint, it is the quantity of energy released when the explosive is detonated. The atoms which comprise the explosive's crystals and their arrangement determine the amount of energy which will be available at detonation. The rate of decomposition, the quantity of gas liberated, and the ensuing degree of rearrangement, determine the amount of work done by the explosive.

POWER.

The term power is used to define the rate at which an explosive does its work or releases energy, and is equal to the product of the explosive strength and the detonation velocity. Power is expressed at times as the ability of an explosive to do damage at a distance.

BRISANCE.

In addition to strength and power, explosives display a third characteristic which is their shattering effect or brisance (from the French meaning "to break"), which is distinguished from their total work capacity. This characteristic is of practical importance in determining the effectiveness of an explosion in fragmenting shells, bomb casings, grenades, etc.

The rapidity with which an explosive reaches its peak pressure is a measure of its brisance. An explosive with high brisance is one in which maximum pressure of detonation is reached so rapidly that the material surrounding or in contact with it will shatter. Since a rapid rise in pressure is dependent upon both the suddenness with which the gaseous products of the explosion are liberated and the amount of heat available to expand this gas, both the velocity of detonation and the strength of the explosive are major factors in determining brisance. It should be noted that these are the same factors which determine the power of the explosive. There exists in fact a linear relationship between velocity of detonation and brisance. It is small wonder then that for explosives of greater strength and velocity of detonation, both power and brisance increase.

In two explosives of equal velocity of reaction, the stronger will be more brisant, because there is more force back of the blow. The stronger explosive will also be more powerful, because more energy is delivered in the same time. It is possible to increase velocity and reduce strength, thereby increasing both brisance and power. It is also possible to slightly reduce velocity and greatly increase strength in order to increase both brisance and power. In the first case, brisance is increased by the sharpness of the blow, and power is increased by the increased speed with which the blow is delivered. In the second case, both power and brisance are increased by the weight of the blow, in spite of the fact that time of delivery is increased. Brisance is sometimes expressed as the ability of an explosive to do damage in the close vicinity, or its ability to shatter its confining medium.

DENSITY.

As one of the final steps in the manufacture of warheads, they are filled with high explosive. Several methods of loading are available and the one used is determined by the characteristics of the explosive. The methods available include: pellet loading, cast loading, or press loading.

By press loading, an average density of the loaded charge is obtained which is greater than the actual density of the explosive. High load density reduces sensitivity by making the mass more resistant to internal friction and to the creation of hot spots during firing. By increasing the continuity of the explosive mass the velocity of detonation is increased, the load is made more dense, and the tendency for cavities to form is decreased. Cavities may cause misfires or premature detonations. Increased load density also permits the use of more explosive in the space provided, thereby increasing the strength of the warhead. Loading density is an important characteristic of a military explosive, a maximum density being desirable because of the fixed volume of the space available for explosives in a round of ammunition. The greater the loading density at which a fixed weight of a given explosive is pressed or cast, the greater its effect when detonated.

VOLATILITY.

Volatility, or the readiness with which a substance vaporizes, is an undesirable characteristic in military explosives. Explosives must be no more than slightly volatile at the temperature at which they are loaded or at their highest storage temperature. Excessive volatility often results in the development of pressure within rounds of ammunition and separation of mixtures into their constituents.

Stability, as mentioned before, is the ability of an explosive to stand-up under storage conditions without deteriorating. Volatility affects the chemical composition of the explosive such that a marked reduction in stability may occur. This will result in an increase in the danger of handling.

HYGROSCOPITY.

The introduction of moisture into an explosive is highly undesirable since it reduces the sensitivity, strength, and velocity of detonation of the explosive. Hygroscopity is used as a measure of a material's moisture-absorbing tendencies. Moisture affects explosives adversely by acting as an inert material which absorbs heat when vaporized, and by acting as a solvent medium which can cause undesired chemical reactions. Sensitivity, strength, and velocity of detonation are reduced by inert materials which reduce the continuity of the explosive mass. When the moisture content evaporates during detonation, cooling occurs which reduces the temperature of reaction.

In the case of ammonium nitrate explosives, moisture may render the explosives so insensitive that detonation is made impossible. Stability is also affected by the presence of moisture since moisture promotes decomposition of the explosive and, in addition, causes corrosion of the explosive's metal container. For all of the above stated reasons hygroscopity must be negligible in military explosives.

TOXICITY.

Most explosives, due to their chemical structure, are toxic to some extent. Since the effect of toxicity may vary from a mild headache to serious damage of internal organs, care must be taken to limit toxicity in

FUNDAMENTALS OF

A blast warhead is one which is designed to achieve target damage primarily from the blast effect. The payload of a blast warhead consists of a high explosive or nuclear charge capable of producing a center of high pressure which propagates isotropically from the center of detonation. The excess pressure or overpressure (magnitude of pressure in excess of atmospheric) created by the blast warhead tends to decrease in proportion to the inverse cube of the distance from the source of energy. Prior to detonation, the payload must be confined within some sort of container or casing. This casing ruptures when the payload is detonated and forms highspeed fragments. These fragments constitute a secondary source of damage from a blast warhead detonation. For this reason, no pure blast warheads exist, since there will always be some fragments emitted from a blast warhead.

However, in the case of nuclear payloads, the effect of the fragments is insignificant compared to the vast amount of blast energy released. Ideally, the propagation from a blast warhead is isotropic. This ideal situation is closely approximated by nuclear blasts, but high explosive blasts will generally exhibit something less than isotropic expansion. This is due to such factors as the variation in resistance of the warhead case, the shape of the charge, and the manner in which it is detonated. These effects limit a conventional blast warhead to a modified spherical pattern. However, in spite of these initial effects, a blast warhead cannot be designed with any appreciable warhead gain or directionality, and therefore can be considered essentially isotropic in character.

A high explosive bomb is an example of a blast warhead. When the bomb payload detonates and forms pressures of the magnitude of 700 tons per square inch at 3000-4500°C, the metal casing of the bomb expands very rapidly to approximately 1.5 times its original diameter before breaking into fragments. Of the total energy liberated by the detonation of the charge, as much as half may be used for expanding the casing prior to rupture and imparting velocity to the fragments of the casing. The remainder of the available energy is expended in compression of the air surrounding the bomb and is responsible for the blast effect produced.

mechanisms of blast

When an explosive is detonated in air, expansion of the gases liberated during the explosion compress the surrounding air and thus initiate a shock wave. These expanding gases referred to as a flame front, have little inertia, cool rapidly, and lose most of their velocity at a distance equal to 40 to 50 times the diameter of the charge. Initially, the compressed air which comprises the shock wave has a high radial velocity and, except for its intensity, displays all of the characteristics of a sound wave in that it travels through the surrounding air without the transmitting medium moving along with it.

The shock wave is bounded by an extremely sharp front called the shock front which represents a discontinuity in density, pressure, and temperature of the medium through which it passes. Here the pressure rises change abruptly in this region from atmospheric pressure to peak pressure; the pressure then returns to atmospheric pressure. This phase is known as the positive or pressure phase of the shock wave. The pressure continues to decline to subatmospheric pressure and then returns to normal. The second phase is called the negative or suction phase.

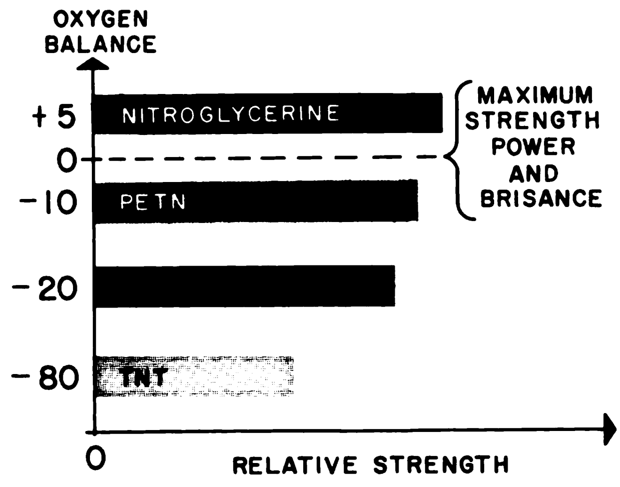
The negative phase of the shock wave is the result of the air and gases of detonation moving outward, behind the shock front as a strong wind. As the pressure in the core of gases decreases, the air and gas are prevented, by their own inertia, from slowing down quickly enough and the rarefaction formed extends outward behind the positive phase. As the negative phase passes, the wind reverses direction and blows toward the point of detonation until its velocity subsides as the pressure returns to normal.

military explosives to a minimum. Any explosive of high toxicity is completely unacceptable for military use.

OXYGEN BALANCE.

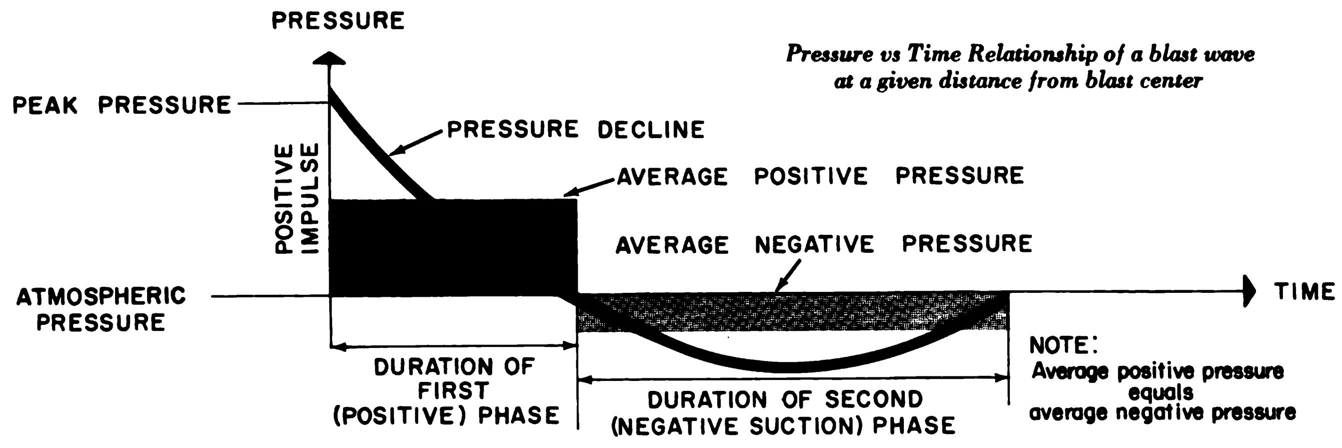
Oxygen balance is an expression which is used to indicate the degree to which an explosive has been oxidized and is defined as the ratio of oxygen surplus or lack, to that required for complete oxidation of all the carbon and hydrogen present in the explosive. If an explosive molecule contains just enough oxygen to convert all of its carbon to carbon dioxide and all of its hydrogen to water with no excess, the molecule is said to have a zero oxygen balance. If the molecule doesn't contain enough oxygen for conversion of the carbon and hydrogen it is said to have a negative oxygen balance. The molecule has a positive oxygen balance if it contains more oxygen than is needed for converting the carbon and hydrogen to carbon dioxide and water.

The strength, power, and brisance of an explosive are all dependent upon the oxygen balance and approach their maximums as the oxygen balance approaches zero.

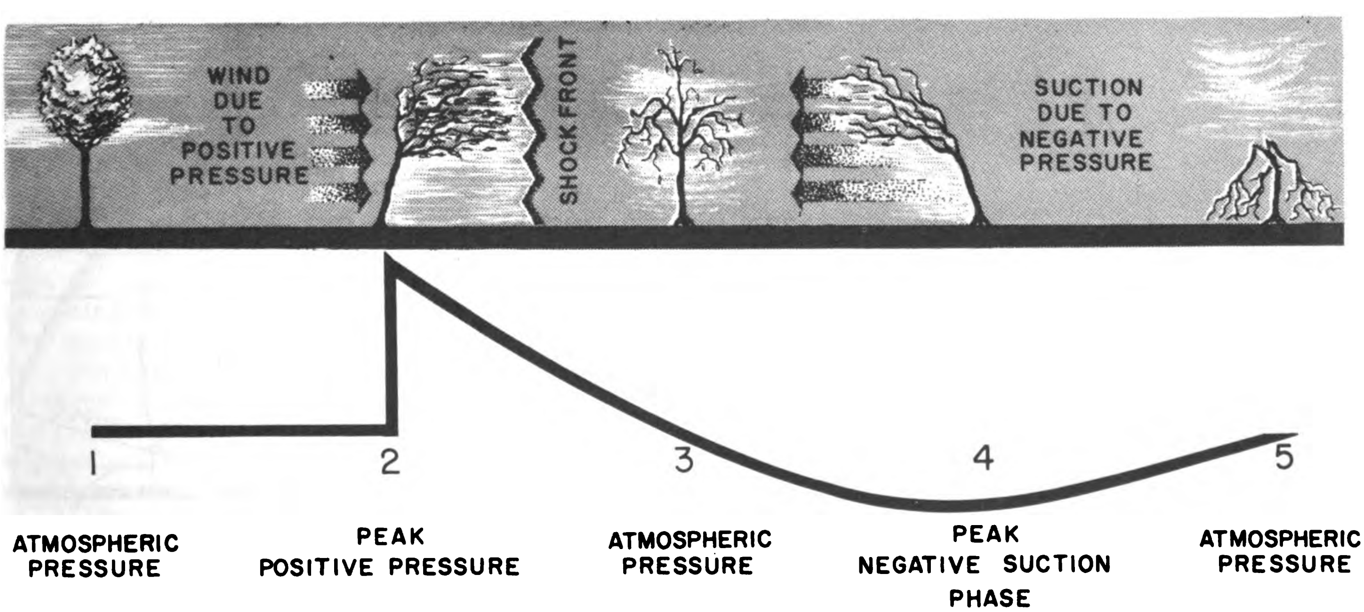


effect of oxygen balance on strength of explosion

BLAST WARHEADS



Pressure vs Time Relationship of a blast wave at a given distance from blast center



SHOCK WAVE.

As has been previously stated, the expanding gases released, when an explosive is detonated, compress the surrounding air and initiate a shock wave. The physical properties of this shock wave are generally measured by the peak overpressure and impulse of the positive phase at several distances from the point of detonation as explained below.

PEAK OVERPRESSURE.

The peak overpressure is the instantaneous rise in pressure at the shock front and is the highest pressure in the shock wave (measured in pounds per square inch above atmospheric pressure). The positive phase is generally of short duration, lasting about 0.0008 seconds at ten feet for a 100 pound general purpose (GP) bomb* and 0.05 seconds at 400 feet for a 4000 pound bomb. The negative phase, on the other hand, lasts 5 to 6 times longer than the positive phase. The magnitude of the maximum pressure developed during the negative phase is only a fraction of the maximum positive pressure developed during the positive phase.

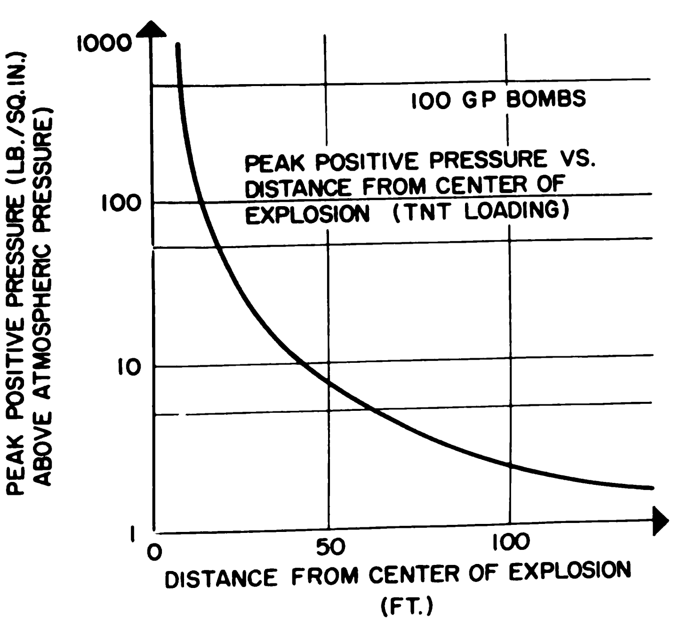
*A general purpose bomb has a case of medium thickness containing approximately 50% by weight of explosive fillers.

As the shock wave moves outward from the center of the explosion its peak pressure decreases quickly. The factors contributing to this rapid decrease are:

a) Air passed through the shock front is heated and thus extracts energy from the shock front.

b) As the shock front moves outward from the center of the explosion it becomes a larger and larger sphere and the energy per unit area of this expanding sphere decreases as its surface area increases.

A plot of peak positive pressure vs. distance from center of explosion for 100 and 1000 pound general purpose bombs is shown. Note that each bomb causes the same peak pressures but at different distances.

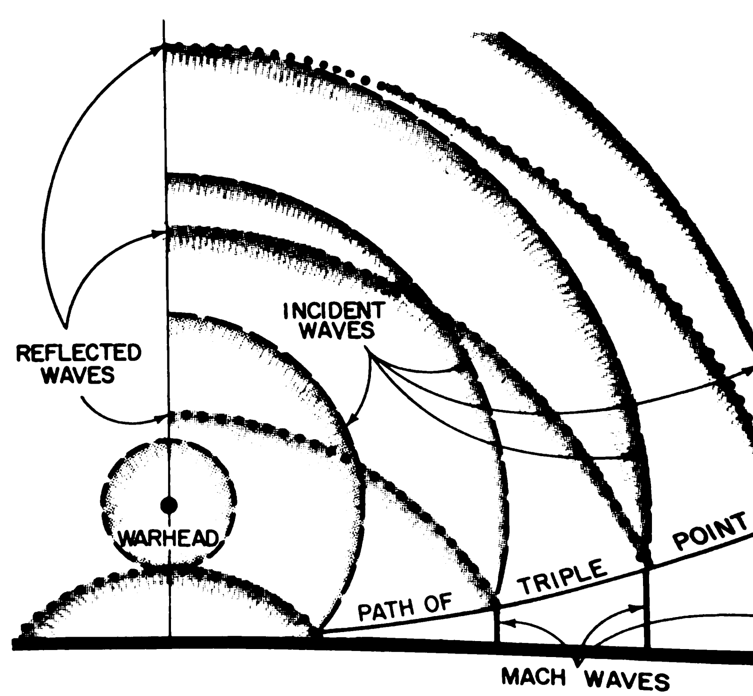


peak positive pressure vs distance from center of explosion (tnt loading)

A major consideration in the discussion of overpressure is the phenomenon of Mach reflections called the "Mach Effect". When a bomb is detonated at some distance above the ground, the reflected wave combines with the original shock wave, called the incident wave, to form a third wave which has a vertical front at ground level. The third wave is called a "Mach Wave" and the point at which the three waves intersect is called the "Triple Point". The Mach wave grows in height as it spreads laterally, and as the Mach wave grows, the triple point rises, describing a curve through the air. The point of origin of the triple point, and its path, depend upon the magnitude of the explosive charge and the height of detonation above ground. At the triple point, the incident wave is reinforced by the reflected wave, and both the peak pressure and impulse are at a maximum which is considerably higher than the peak pressure and impulse of the original shock wave at the same distance from the point of explosion.

As the Mach wave grows in height it absorbs the incident and reflected waves and, ultimately, at distances which are very large compared with the height at which the burst took place, the configuration of shocks becomes approximately sphere shaped and intersects the ground perpendicularly.

Utilizing the phenomenon of Mach reflections, it is possible to considerably increase the radius of effectiveness of a bomb. By detonating a warhead at the proper height above the ground, the maximum radius at which a given pressure or impulse is exerted can be increased, in some cases by almost 50% over that for the same bomb detonated at ground level. The area of effectiveness may thereby be increased by as much as 100%.



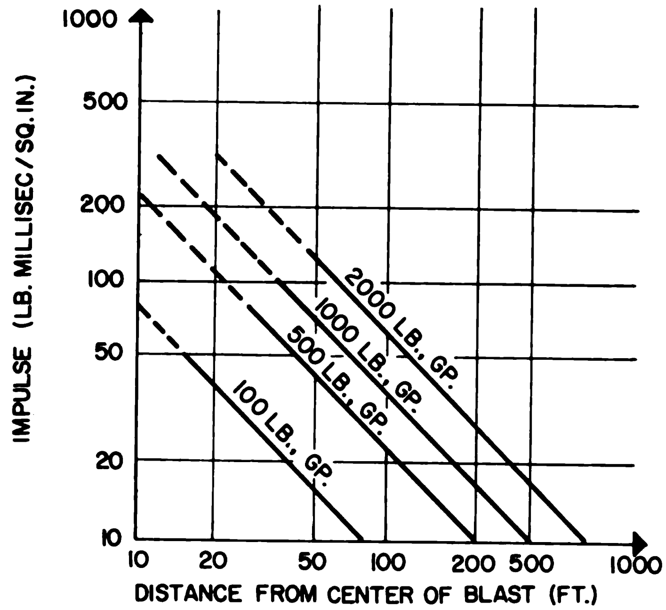
formation of mach wave and triple point

IMPULSE.

Impulse is defined as the area under the pressure-time curve of the positive phase, at a given location. This definition takes into account the duration of the positive phase and the variation in overpressure during that time period. Impulse is equal to one half the peak pressure (P_p) times the duration of the positive phase (T_{pp}), or: Impulse = $1/2\ P_p \times T_{pp}$.

As the shock front moves outward from the center of blast, the peak pressure of the shock front at any selected point will decrease with a subsequent increase in the duration of the positive phase at that point.

These changes of peak pressure and time duration with distance will vary in accordance with the magnitude of the peak pressure but the net result is a definite decrease in impulse with increased distance from the center of blast. Impulse decreases approximately as 1/R, where R is the radius of the expanding sphere. If the minimum values of impulse necessary to destroy a target are known, then the radii for which these values are satisfied may be calculated as a function of explosive weight.



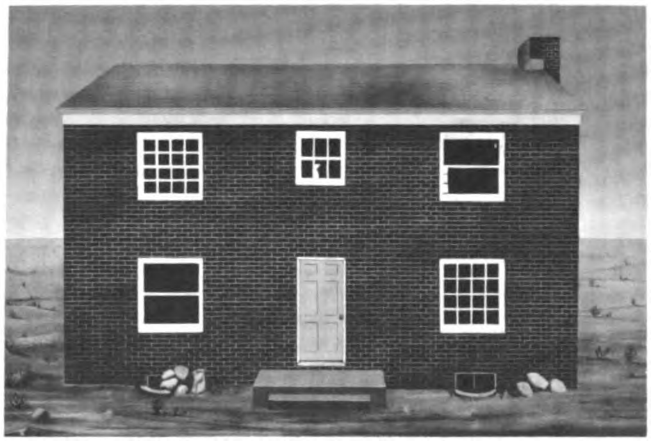
impulse vs distance from center of blast (tnt loading)
tnt loaded general purpose bombs

effects of air blast

The behavior of an object or structure exposed to the blast wave from an explosion may be characterized by two effects. The first, called loading is the force which results from the action of the blast pressure. The second is the response or distortion of the structure due to a particular loading. Generally, response is taken to be synonymous with damage since a sufficient amount of permanent distortion will impair the usefulness of a structure. Damage may also arise when a moving object strikes the ground or an object which is fixed. As an example, when a vehicle tumbles it is damaged primarily when it strikes the ground. In addition, when glass, splinters of wood, bricks, pieces of masonry, and various other objects are loosened by a blast wave, they are hurled through the air and become destructive missiles. Indirect types of damage such as are caused by these destructive missiles are, of course, greatly dependent upon the circumstances involved when the explosion occurs.

The direct damage caused to a structure by an air blast takes many forms, for example: structural steel frames may be bent or twisted, roofs and walls may collapse, and windows may shatter. Generally, direct damage is the result of either displacement or distortion. The manner in which such displacement may arise because of a nuclear explosion will be examined below.

For an air burst, the direction of propagation of the incident wave is perpendicular to the ground at ground zero. The forces exerted upon structures in the regular reflection region have a large vertical component prior to passage of the reflected wave; the loading has an appreciable initial downward crushing force instead of being lateral as is the case with Mach waves. This results, for example, in dished-in roofs, and in the distortion caused by translational motion of roof structures.

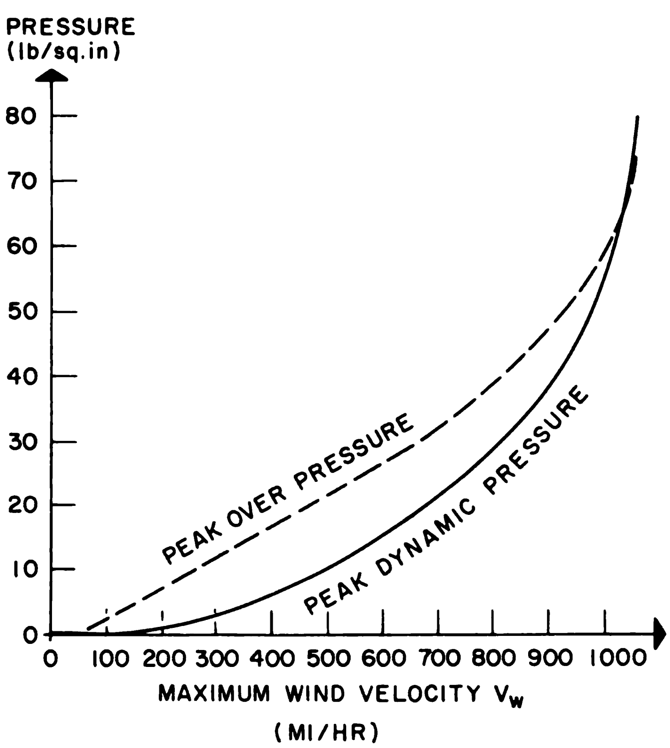


unreinforced brick house
before a nuclear explosion . . .

. . . same brick house
after a nuclear explosion

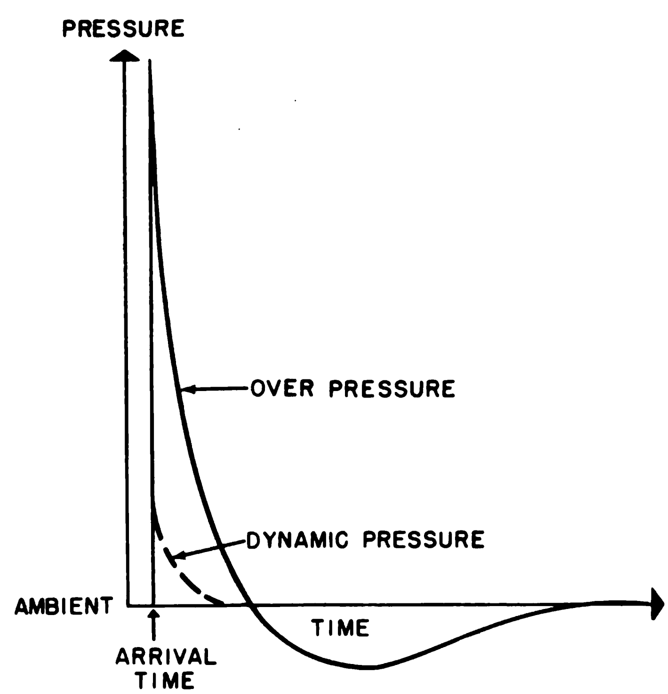
DYNAMIC PRESSURE.

The destructive effects of a blast wave are generally related to values of peak overpressure. However, another quantity of equal importance, dynamic pressure, may be used. For a great many types of buildings, the degree of damage caused by blast depends largely on the drag force associated with the strong (transient) winds accompanying passage of the blast wave. This drag force is influenced by certain physical characteristics (primarily the shape and size) of the structure. The drag force is, in general, dependent upon the magnitude of the impulse caused by the blast.
The dynamic pressure, Pq is a function of the wind velocity, Vw, and the density, ρ , of the air behind the shock front, or $Pq = \frac{1}{2}\rho Vw^2$.



peak pressure vs maximum wind velocity at sea level

As shown by the curves, the peak dynamic pressure may exceed the peak overpressure when the shock is so strong that high maximum wind velocities are obtained. As in the case of peak shock overpressure, the peak dynamic pressure decreases with increasing distance from the center of explosion but at a greater rate.
The dynamic pressure at a given location varies with time in a manner somewhat similar to the change in the overpressure, but the rates of pressure decrease behind the shock front differ. The accompanying curves indicate how the two pressures vary in the first second following the arrival of the shock front. Actually, the wind velocity and the dynamic pressure will drop to zero sometime later, due largely to the inertia of the moving air. For purposes of estimating damage, this time difference is not significant.
For the entire period of the positive phase, during which the air pressure wave is passing (and for a short time thereafter), a structure is subjected to dynamic pressure loading or drag loading caused by the strong transient winds behind the shock front. This loading is equivalent to a lateral force acting upon the structure or object exposed to the blast.
It is the effect of drag loading on structures which constitutes an important difference between nuclear and high-explosive detonations. For the same peak overpressure in the blast wave, a nuclear bomb is more destructive than a conventional bomb, especially in the case of buildings which respond to drag loading. This is so because the blast wave is of much shorter duration for a high-explosive bomb, than for a nuclear bomb. The duration of the positive phase of the blast wave is increased for warheads of high energy yield and such warheads cause more destruction than might be expected from the peak overpressure alone.



pressure vs time at a given distance from blast

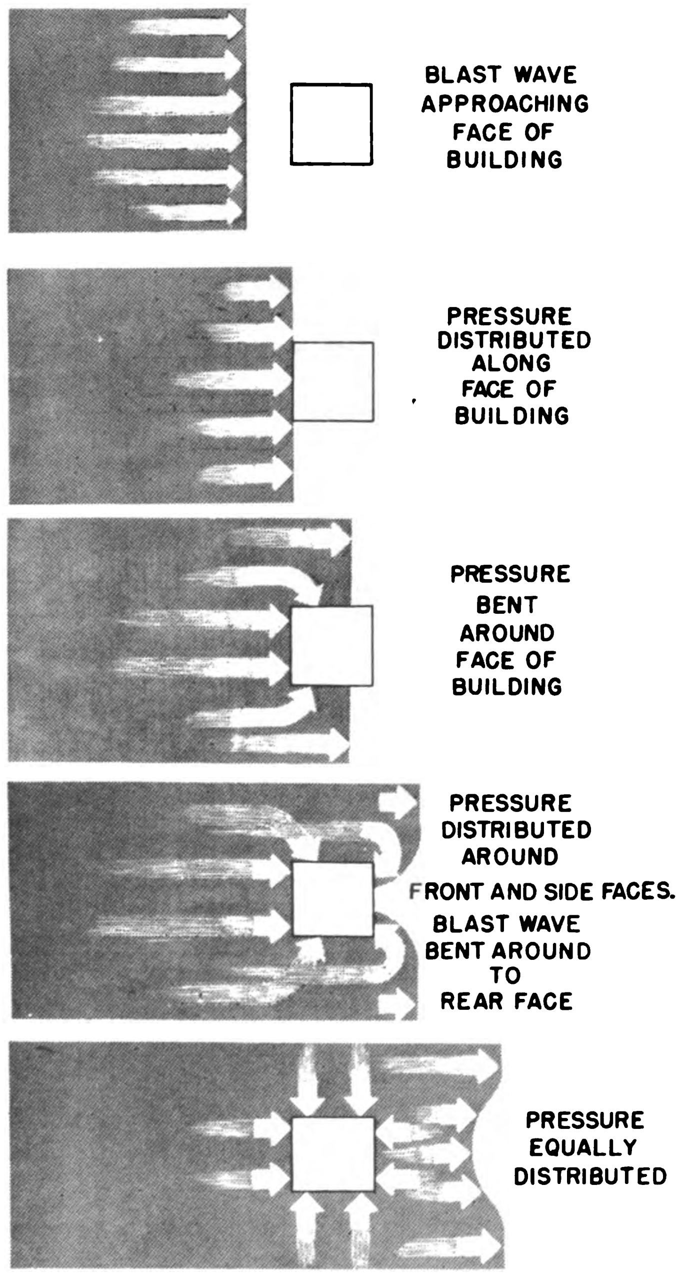
diffraction loading

Reflection occurs when the front of an air pressure wave strikes a building. A result of this reflection is the rapid build-up of overpressure to at least twice (but in most cases to several times) the overpressure in the incident shock front. The actual pressure attained is determined by such factors as the velocity of the shock wave and the configuration of the face of the building. The overpressure on the face drops rapidly as the shock wave moves forward to the value of overpressure produced by the blast wave without reflection. At the same time, the air pressure wave bends or diffracts around the structure, so that the structure is eventually engulfed by the blast. Approximately the same pressure is exerted on all the faces and the roof of the structure.

Before the blast wave has completely surrounded the structure, a considerable pressure differential exists between its front and back faces. This pressure differential causes a force which tends to move the structure bodily in the same direction as the blast wave. This force is called "diffraction loading", since it occurs while the blast wave is being diffracted around the structure. The extent and nature of the motion imparted to the structure depend upon the size, shape, and weight of the structure and how firmly it is attached to the ground.

After the blast wave has engulfed the structure, the pressure differential drops almost to zero because the actual pressure exerted on all faces becomes approximately the same. The pressure exerted on each of the faces, however, will remain in excess of the ambient atmospheric pressure until the positive phase of the shock wave has passed. Due to this period of excess pressure, the diffraction loading is replaced by a pressure directed inward, acting to compress or squeeze the structure. In a structure with no openings, this will cease only when the overpressure drops to zero. The damage caused during the diffraction stage is determined by the magnitude and duration of loading. The loading is directly related to the peak overpressure in the blast wave and this is consequently an important factor. If the structure under consideration has no openings, as has been tacitly assumed so far, the duration of loading is roughly equal to the time required for the shock front to move from the front to the rear of the building. Diffraction loading is thus affected by the size of the structure involved. For example the diffraction loading for a structure 75 feet long operates for a period on the order of one-tenth of a second. For thin structures, such as telegraph poles or smoke stacks, the diffraction period is so short that the corresponding loading is negligible.

If the building exposed to the blast wave has openings, windows, panels, light siding, or doors which fail in a very short time, a rapid equalization of pressure occurs between the inside and outside of the structure. This tends to reduce the pressure differential while diffraction is taking place. Diffraction loading on the structure as a whole is decreased, but the loading on interior walls and partitions is greater than for a structure with few openings. In addition, if the building has many openings after the diffraction stage, the subsequent crushing action, due to the pressure being higher outside than inside, will not occur.



diffraction of a blast by a structure

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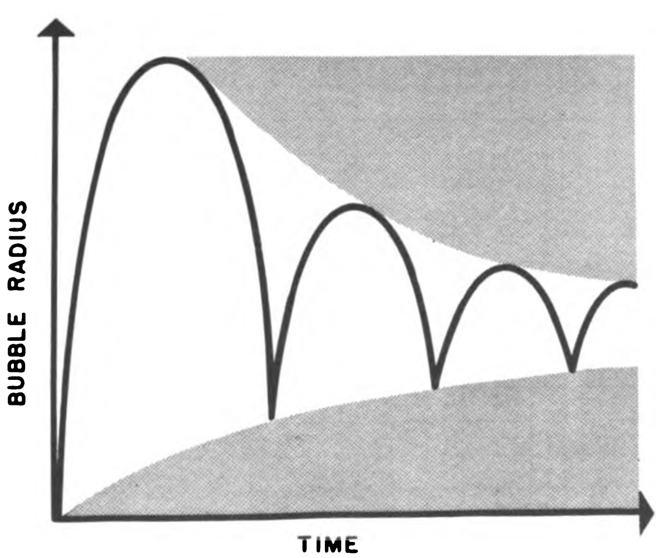
underwater blast

We have discussed the effects of air blasts upon various types of targets. We will now examine the effects of such blasts under water.

GAS BUBBLE FORMATION AND CHARACTERISTICS.

An underwater explosion creates a cavity filled with high-pressure gas which pushes the water out radially against the opposing external hydrostatic pressure.

At the instant of explosion, a certain amount of gas is instantaneously generated at high pressure and temperature creating a bubble. In addition, the heat causes a certain amount of water to vaporize, adding to the volume of the bubble. This action immediately begins to force the water in contact with the blast front in an outward direction. The potential energy initially possessed by the gas bubble by virtue of its pressure is thus gradually communicated to the water in the form of kinetic energy. The inertia of the water causes the bubble to overshoot the point at which its internal pressure is equal to the external pressure of the water. The bubble then becomes rarefied, and its radial motion is brought to rest. The external pressure now compresses the rarefied bubble. Again, the equilibrium configuration is overshot, and, since by hypothesis there has been no loss of energy, the bubble comes to rest at the same pressure and volume as at the moment of explosion (in practice, of course, energy is lost by acoustical and heat radiation).



bubble radius vs time relationship

The bubble of compressed gas then expands again and the cycle is repeated. The result is a pulsating bubble of gas slowly rising to the surface, with each expansion of the bubble creating a shock wave. This phenomena explains how an underwater explosion appears to be followed by other explosions. The time interval of the energy being returned to the bubble (the period of pulsations), varies with the intensity of the initial explosion.

The rapid expansion of the gas bubble formed by an explosion under water results in a shock wave being sent out through the water in all directions. The shock wave is similar in general form to that in air, although it differs in detail. Just as in air, there is a sharp rise in overpressure at the shock front. However, in water, the peak overpressure does not fall off as rapidly with distance as it does in air. Hence, the peak values in water are much higher than at the same distance from an equal explosion in air. For example, the peak overpressure at 3000 feet from a 100-kiloton burst in deep water is about 2700 pounds per square inch, compared with a few pounds per square inch for an air burst.

On the other hand the duration of the shock wave in water is shorter than in air. In water it is of the order of a few hundredths of a second, compared with something like a second or so in air.

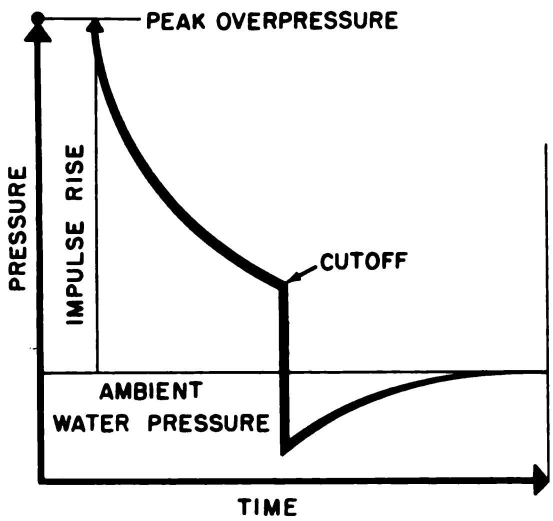
The velocity of sound in water under normal conditions is nearly a mile per second, almost five times as great as in air. When the peak pressure is high, the velocity of the shock wave is greater than the normal velocity of sound. The rate of motion of the shock front becomes less at lower overpressures and ultimately approaches that of sound, just as it does in air.

The close proximity of the upper and lower boundaries between which the shock wave is forced to travel (water surface and ocean floor) causes complex shock wave patterns to occur as a result of reflection and rarefaction. Also, in addition to the initial shock wave which results from the initial gas bubble expansion, subsequent shock waves are produced by bubble pulsation. The pulsating shock wave is of lower magnitude and of longer duration than the initial shock wave.

SURFACE CUTOFF

When a detonation occurs at a distance from the upper and lower boundaries, early in the formation of the gas bubble, a shock wave commences. This shock wave radiates outward spherically travelling at a rate of approximately 5000 ft/sec (the speed of sound in water). When this shock wave is formed, the initial peak overpressure rises instantaneously and then decays exponentially. When the pressure wave reaches the surface (upper boundary), it is reflected as a rarefication or tension wave, because of the difference in compressibility of the water and air. This reflected wave cancels the pressure of the tail of the primary shock wave, thereby decreasing the duration of the positive phase. This phenomenon is referred to as surface cutoff. This is shown in the illustration. This cutoff effect decreases rapidly with depth of target and depth of detonation. When the shock wave in water strikes a rigid, submerged surface, such as the hull of a ship or the sea bottom, reflection occurs as in air. The incident and reflected waves may even fuze in certain circumstances to produce a shock front of enhanced pressure. However, when the water shock wave reaches the upper (air) surface, an entirely different reflection phenomenon occurs.

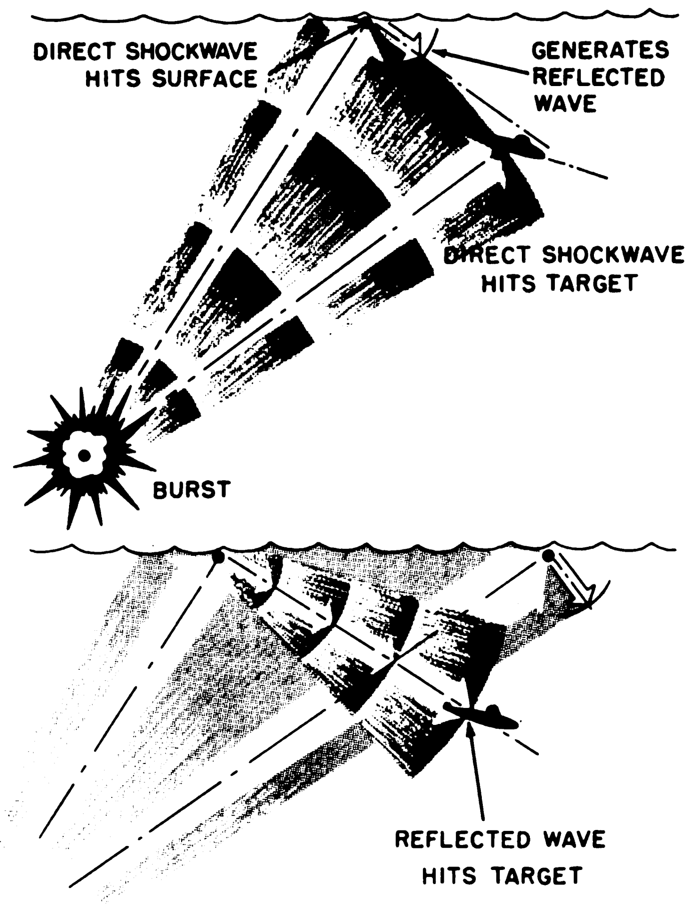
At the surface between the water and the air, the shock wave moving through the water meets a much less rigid medium, namely the air. As a result, a reflected wave is sent back into the water, but this is a rarefaction or suction wave. At a point below the surface, the combination of the reflected suction wave with the direct incident wave produces a sharp decrease in the water shock pressure. This is also surface cutoff.



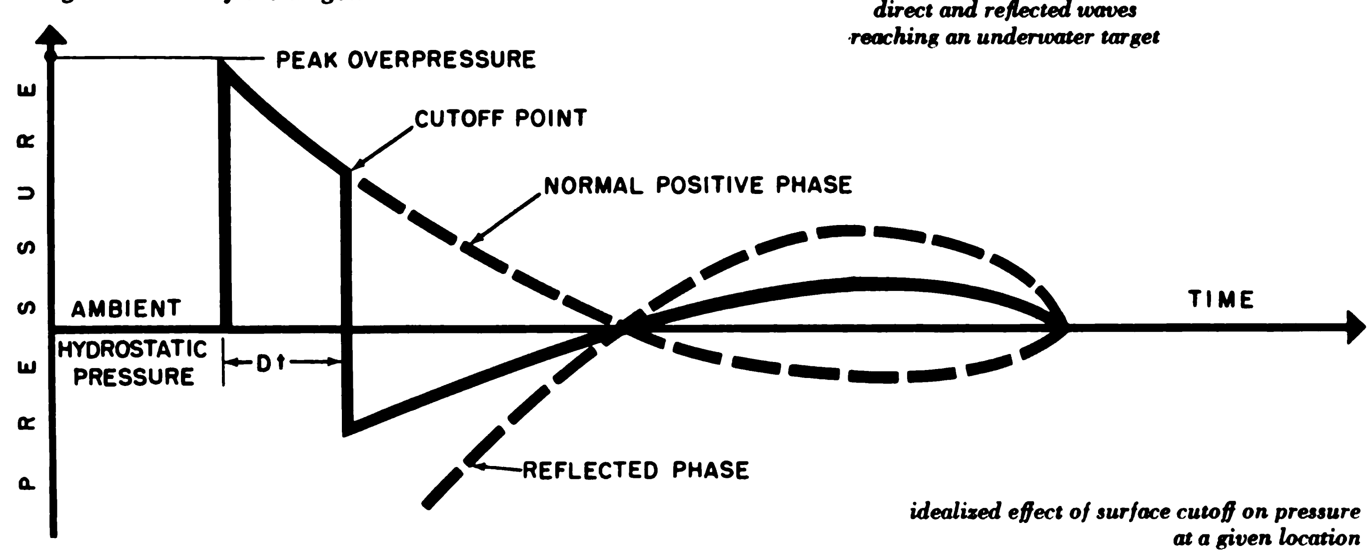
pressure vs time
for underwater explosion

The variation at a given location of the shock overpressure with time after the explosion at a point under water not too far from the air surface, is illustrated. After the lapse of a short interval, which is the time required for the shock wave to travel from the explosion to the given location, the overpressure rises suddenly due to the arrival of the shock front. Then, for a period of time, the pressure decreases steadily, as in air. Soon thereafter, the arrival of the reflected suction wave from the surface causes the pressure to drop sharply, even below the normal (hydrostatic) pressure of the water. This negative pressure phase is of short duration. This can result in a decrease in the extent of damage sustained by the target.

The time interval between the arrival of the direct shock wave at a particular location (or target) in the water and that of the cutoff, signaling the arrival of the reflected wave, depends upon the depth of burst, the depth of the target, and the distance from the burst point to the target. If the underwater target is close to the surface, for example, a ship bottom, then the time elapsing between the arrival of the two shock fronts will be small and the cutoff will occur soon after the arrival of the shock front.



direct and reflected waves
reaching an underwater target

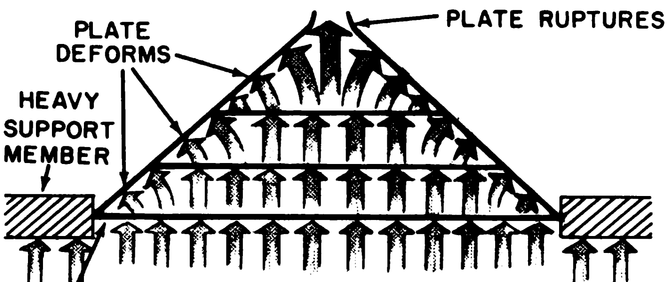


idealized effect of surface cutoff on pressure
at a given location

underwater blast effects

When detonated under water, a charge produces pressures which act upon structures submerged in the water. At the instant of burst, the pressure rises almost instantly (due to the expansion of gases released by the explosion) to a high value and then decreases exponentially. The overpressure phase of the shock wave is of such short duration that it takes the form of a distinct wave of pressure traveling through the water at finite speed.

The impact of a shock wave caused by underwater detonations of TNT and other chemical explosives on a ship or structure, such as a breakwater or dam, is a sudden blow. But, whereas the shock produced by such an explosion is localized, that resulting from a nuclear explosion acts over a large area, such as the hull of a ship, almost instantaneously. The effects of an underwater nuclear burst on a ship may be of two general types. First, there will be the direct effect of the shock on the vessel's hull and, second, the indirect effects due to components within the ship being set in motion by the shock.



exaggerated deformation pattern and final rupture of a thin plate

An underwater shock acting on the hull of a ship tends to cause distortion of the hull below the water line and rupture of the shell plating, thus producing leaks as well as severely stressing the ship's framing. The underwater shock also causes a rapid movement in both horizontal and vertical directions. This motion causes damage by shock to components and equipment within the ship. Main feed lines, main steam lines, shafting, and boiler brickwork within the ship are especially sensitive to shock. Due to the effect inertia, the supporting members or foundations of heavy components, such as engines and boilers, are likely to collapse or become distorted. Lighter or inadequately fastened articles will be thrown about with great violence, causing damage to themselves, to bulkheads, and to other equipment. Equipment which has been properly mounted against shock will probably not suffer as seriously.

The damage to the component plates of a ship is dependent mainly on the peak pressure of the underwater shock wave. The same is probably true for the gate structure of canal locks and drydock caissons. Within the range of very high pressures at the shock front, such structures may be expected to sustain appreciable damage.

On the other hand, damage to large, rigid subsurface structures, such as harbor installations, is more nearly dependent upon the shock wave impulse. The impulse is dependent upon the duration of the shock wave as well as its pressure.

A study of pressure acting against a steel plate in the hull of a ship or submarine can be divided into two separate and distinct phases: (1) primary shock phase and (2) tension phase. These phases are described in the following paragraphs.

PRELIMINARY SHOCK PHASE.

Before the explosive action begins, the elastic stresses on the plate wall will be at equilibrium with the difference between the hydrostatic pressure in front of the plate and air pressure on the back of the plate. When the shock wave hits, each element of the plate will therefore start moving as if it were part of an infinite plate, acted upon by a wave of infinite lateral extent. At first, the increment of pressure (P_1) due to the incident wave is doubled by reflection; then, the plate sets up an impulsive velocity in itself. The time that it takes for a plate to attain its maximum forward velocity under the action of the shock wave is known as the "compliance time" (Tm).

Diffraction time (Td) may be defined with sufficient precision as the time required for a sound wave to travel from the center of the plate to the edge of the plate. Diffraction is regarded as a process which acts to equalize the pressure in a lateral direction or in a direction perpendicular to the direction of propagation of the wave. Since pressure waves travel through water at the speed of sound, the time required for equalization is of short duration. Thus, during an interval much shorter than the diffraction time, after a shockwave has struck a plate, lateral equalization of pressure will not have time to progress very far. During this short interval, each part of the plate responds to the incident wave more or less independently and according to the laws that hold for the one-dimensional action of shock waves on plates. The approximate time of duration of a shock wave (Tw) depends upon the strength of the charge involved.

TENSION PHASE.

If the water remains in contact with the plate, tension develops in the plate and tends to arrest the plate motion. If the plate is limited in extent, however, forming part of some larger structure, the influence of diffraction will usually be such that the plate will retain part of the velocity which it acquired during the primary shock phase. If the shock wave is of very brief duration, the plate may come almost to rest, and then be accelerated again as the diffracted pressure is propagated on from the edge. The plate will then continue moving until it is arrested by forces due to other parts of the structure. The time required for the final arresting of the plate constitutes a fourth characteristic time factor, the "swing time" of the plate (Ts). In the case of ships, the swing time is usually much longer than the duration of the shock wave.

In the foregoing discussion of a typical sequence of events, the relative magnitudes of four characteristic time factors have played a determining role:

1. Duration of shock wave (Tw).
2. Compliance time (Tm) of the structure, or the time required for the shock wave to set the structure in motion at maximum velocity.
3. Diffraction time (Td), or the time required for a wave to travel from the center of a structure to its edge.
4. The swing time (Ts) of the structure, or the time required for it to undergo maximum deflection and come to rest.

explosive and implosive damage

A blast may burst or explode the target structure, or it may cause the target to collapse or implode, depending upon whether the explosive charge is initiated externally or internally with respect to the target. Each of these damage types will now be discussed.

EXPLOSIVE DAMAGE

Explosive damage may be achieved by forcing a blast warhead into a structure and detonating it. To accomplish this it is of course necessary to hit the target with the warhead. The warhead must then be of such construction that it survives the shock of impact and penetration and detonates properly within the target. These criteria require that blast warheads, designed for explosive damage, be protected by a heavy, impact-resistant case, and that they be delivered by an extremely accurate guidance system.

When a sufficiently accurate guidance system is not available, explosive blast damage can still be achieved through the use of blast cluster warheads. This type of warhead is composed of a number of small missiles or warheads clustered together to form one large warhead. When the main warhead is fired in the vicinity of the target the small missiles are launched in a predetermined pattern (usually isotropic). The small missiles each contain a blast warhead and operate only if they hit and penetrate the target.

This type of warhead can be employed very effectively against aircraft since it takes only 1 to 1 1/2 pounds of TNT to kill a large bomber by explosive damage.

IMPLOSIVE DAMAGE

Implosive damage can be caused by initiating blast charges in the vicinity of a target. High explosive or nuclear payloads are employed in blast warheads, depending upon the size or vulnerability of the intended target. Implosive damage will destroy, or at least incapacitate, a target if the blast warhead is initiated sufficiently close to the target. The attenuation factor (inverse cube function of distance) limits the maximum radius of effectiveness of blast warheads. This necessitates the use of target payloads for a given radius rather than other types of warheads. However, any increase in payload weight results in a disappointingly small increase in the effective radius of the blast. For a given peak pressure the effective radii R_1 and R_2 of two warheads of payload weights W_1 and W_2 are related as follows:

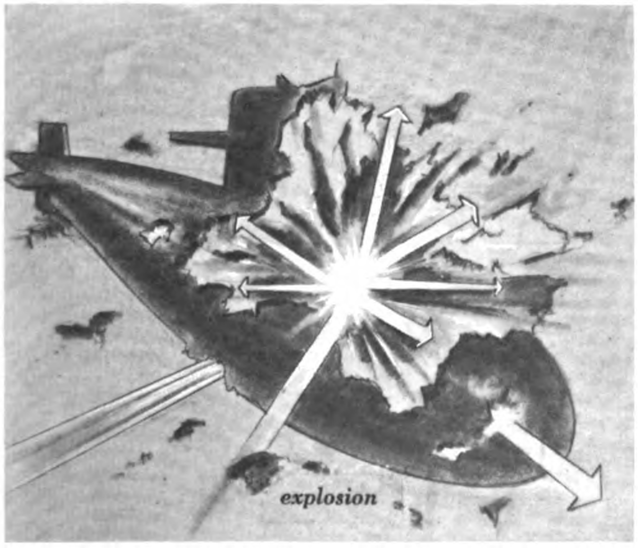
$$R_2 = R_1 \left(\frac{W_2}{W_1}\right)^{1/3}$$

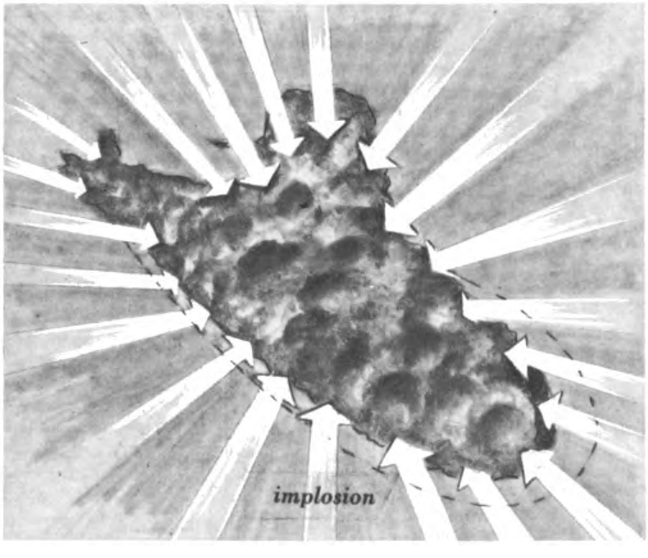
For example, increasing a high-explosive payload's weight from 100 pounds to 10,000 pounds only increases the effective radius of the warhead by a factor of about 4.6, as seen in the relationship:

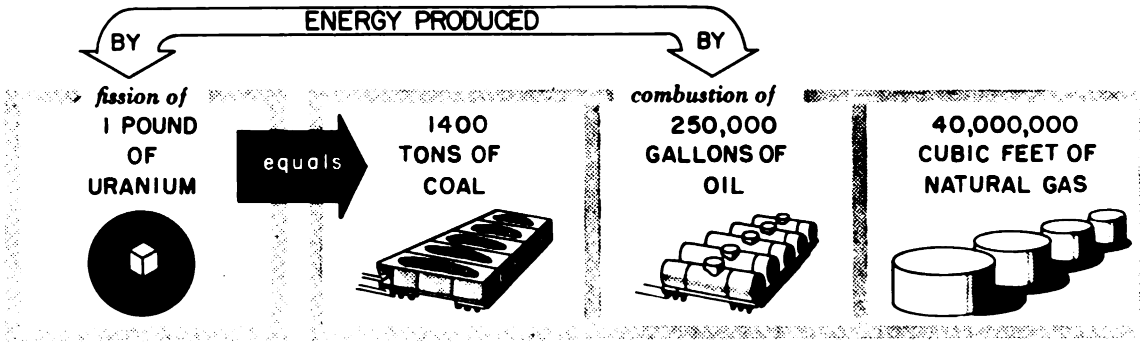
$$R_2 = R_1 \left(\frac{10{,}000}{100}\right)^{1/3} = R_1\,(100)^{1/3} = R_1\,4.6.$$

The effectiveness of blast warheads is also limited by the fact that they are essentially isotropic in character and have a propagation velocity of only about Mach. Thus they may not be suitable for use against high speed air targets, and, in any case, they must be detonated either in or very near to their targets.

From this discussion it is plain to see that conventional, or high-explosive, blast warheads are suitable for use against small targets and require very accurate delivery. Nuclear warheads, on the other hand, can be employed successfully without the stringent delivery requirements and may be used against large target areas such as cities. However, to destroy small, hard to get at targets, such as submarine pens or underground missile silos, a nuclear warhead must, as in the case of the conventional blast warhead, be detonated either in contact with or in close proximity to the target.



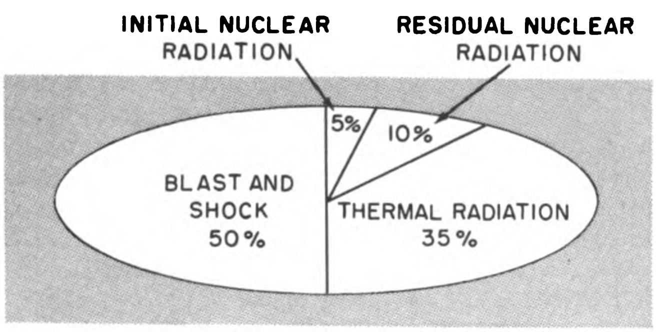




atomic energy compared to chemical energy

nuclear explosive reaction

A conventional explosion is a chemical reaction which, upon the application of heat or shock, decomposes its explosive with extreme rapidity followed by an immediate release of energy in the form of intense heat or gas. When a nuclear explosion occurs, it is characterized by a tremendous increase on the strength of its blast or shock wave pressures (per unit weight of explosive as compared with a chemical explosive).



typical atomic energy distribution

It also radiates an enormous amount of thermal energy (heat) into its damage volume area. Another more ominous aspect of the striking power of a nuclear blast is the amount of radiator (gamma radiation, alpha beta particles, etc.) that is released into the atmosphere with the resultant probability of contamination. The illustration shows how energy is distributed in a representation nuclear explosion.

About 86% of the total energy appears first as intense heat. Almost immediately a considerable part of this heat is converted to energy in the form of blast or shock waves; the remaining thermal energy moves radially outward as heat or visible light. Five percent of the total energy appears immediately as invisible but extremely powerful nuclear radiation - alpha & beta particles, gamma rays and neutrons. The residual nuclear radiation occurs over a long time and is produced by the decay of the numerous radioactive isotopes that are formed by fission reaction. It is not the purpose of this text to describe the chemical reactions that occur upon the detonation of a nuclear explosive but to analyze the reactions and effects of nuclear bombs when exploded in air, on the surface, under water, and under ground.

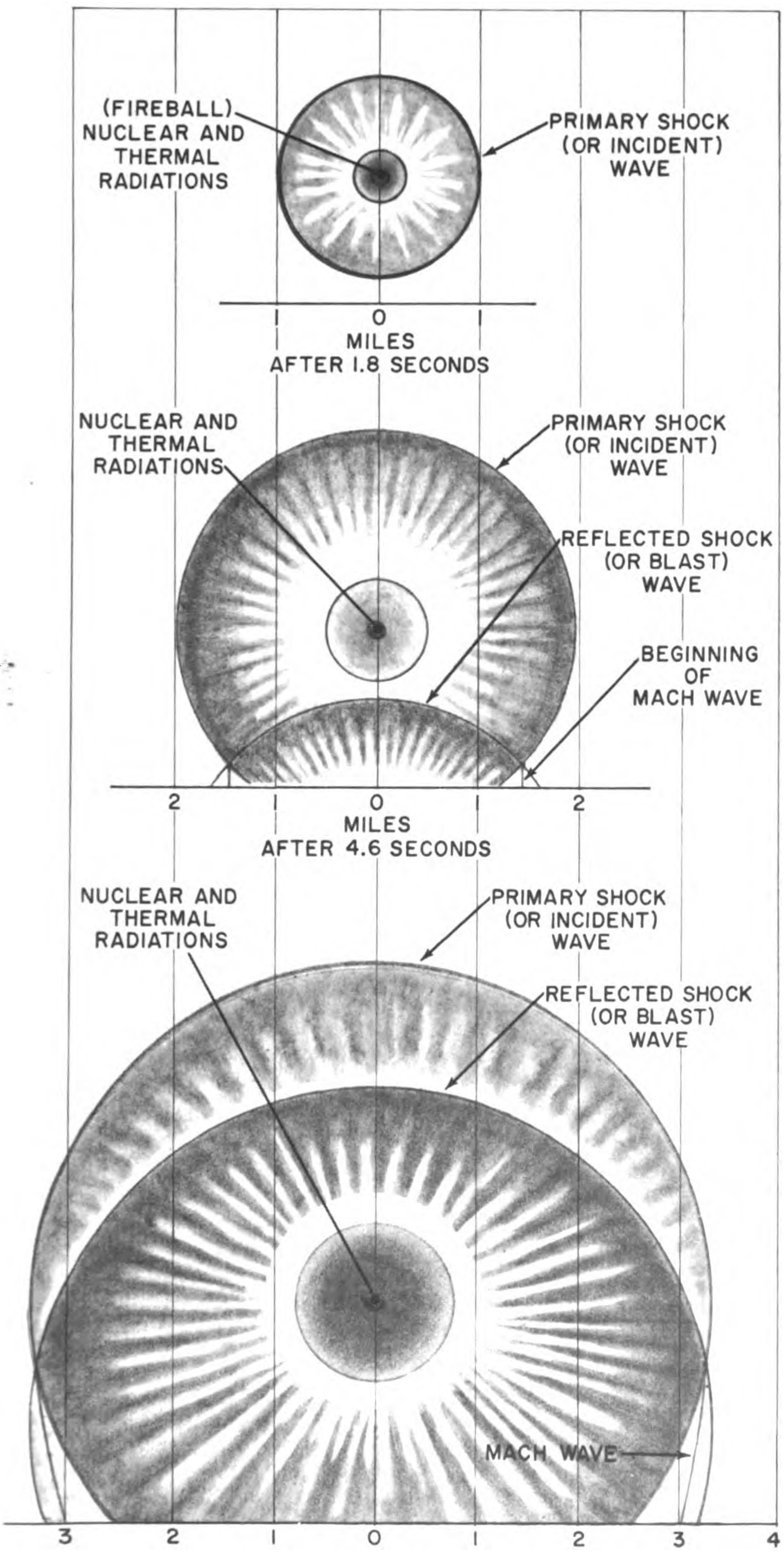
NUCLEAR AIR BLAST.

The overpressure has declined to 1 lb/sq. inch and the velocity of the wind is down to 40 miles/hour. As the fireball rises, it sucks the air inward and upward. This suction phase creates strong winds in an opposite direction to the Mach wind. Within the second minute after a 1 megaton detonation the top of the mushroom cloud is about 7 miles in the air. The after winds draw up the surface dirt and light debris. The nuclear detonation has released a large amount of radioactive fission products. In time this fallout may cause severe damage to any human being within the fallout area.

When an atomic or nuclear payload detonates in air, a rapidly multiplying reaction vaporizes all sections of the carrier and container. The resultant reaction is the formation of a large sphere of hot, luminous gases called a fireball and resembling the sun. The fireball radiates heat, light, and nuclear particles. Its intensity and size depend on the yield of the explosive. The three parts of the illustration show the stages in the development of a 1 megaton air burst.

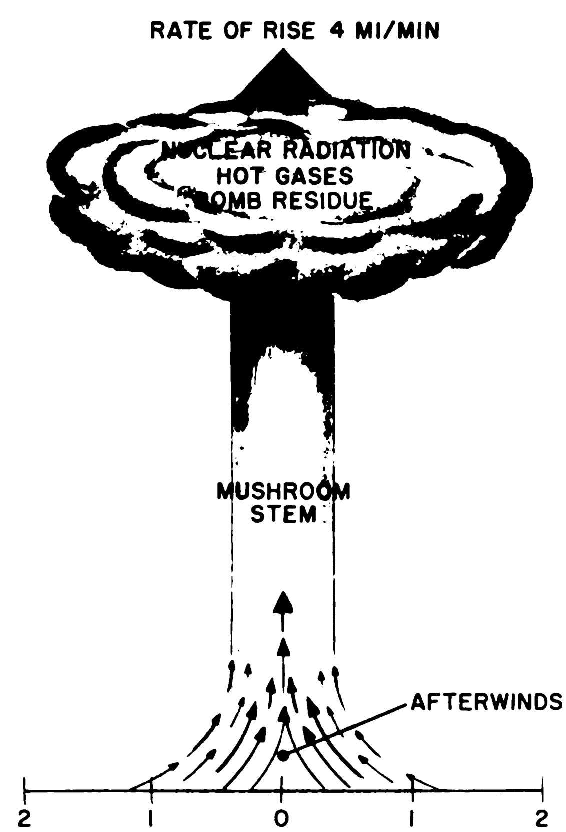
The reaction causes a pressure wave of tremendous proportions which propagates outward from the point of detonations. This outward rush generates a partial vacuum causing air to be drawn inward, creating an extremely high wind velocity which exerts dynamic pressure on the target. When the primary blast wave (incident wave) reaches the ground, a second or reflected wave begins to move outward and upward followed by the formation of a Mach front as previously explained. The Mach effect is one reason for the long range shattering power of a nuclear air burst. While the Mach front is being formed, the fireball is still radiating large amounts of thermal energy. At the end of 11 seconds the Mach pressure rating at this time is about 6 lbs/sq. in., and the wind velocity is about 180 miles/hour. By the end of about 30 seconds, however, significant changes have taken place, as shown.

three stages in the development of a 1-megaton air burst



CHARACTERISTICS OF THERMAL RADIATION.

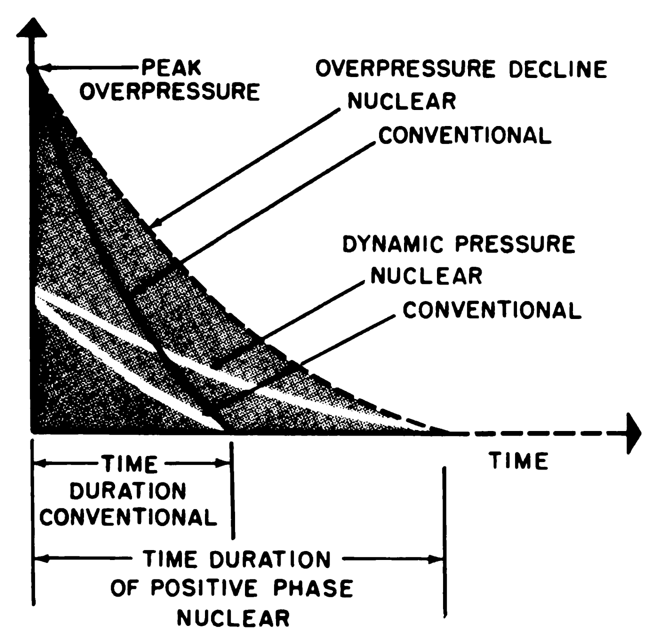
There are two pulses of emission from the fireball, and only about 1 percent of the thermal radiation appears in the initial phase. The second pulse containing 99 percent of the thermal energy is responsible for the skin burns suffered by exposed individuals up to 12 miles or more from the center of the explosion of a 1 megaton bomb. Bombs of higher energy have greater effective radii of damage.
An air burst produces intense heat radiation, primary radiation from the fireball, and secondary nuclear radiation resulting from fallout of fission products, high velocity suction winds away from the point of origin, and then towards it, and the formation of a Mach front which can exert tremendous blast pressure against targets. Because of its destructive capacities and minimized radioactive effects, an air burst is a likely method of nuclear attacks.



formation of mushroom cloud after a 1-megaton air burst

NUCLEAR SURFACE BLAST.

The characteristics of a contact surface burst are similar to those of an air burst. The instantaneous nuclear radiation emanating from the fireball is essentially the same as from an air burst of the same yield. The intense heat or thermal radiation vaporizes and irradiates a considerable portion of the earth in its damage area. Being closer to the earth in a surface blast, the suction phase of the explosion draws much more debris into the mushroom cloud. The heavier particles of rock and soil propagated by a surface blast fall back fairly near the point of burst; the lighter particles remain airborne for a longer time and may float in space or be carried by the wind so that the threat of hazardous radioactive fallout is great. If any significant portion of the fireball touches the earth some cratering will be accomplished. The crater area will depend on the size of the yield bomb. The thermal characteristics of a surface blast are essentially the same as for an air blast; the same amount of heat is radiated in each case. The thermal damage volume of a surface burst is less than that of an air burst, however. This is because thermal radiation at ground level is impeded by buildings, terrain features, and the effects of man made shelters.



conventional vs nuclear
dynamic pressure decline

NUCLEAR UNDERWATER BLAST.

When a nuclear weapon is detonated under the surface of a body of water, most of the energy of the burst appears as underwater blast. Because it is subject to hydrostatic pressure, the destructive effects are smaller than for a bomb of comparable yield detonated in the air. A true underwater burst is one in which the detonation and the formation of the complete fireball both occur below the surface of the water. As the fireball moves upward towards the surface, hot gases are violently expelled into the atmosphere, sucking up a hollow column of water.

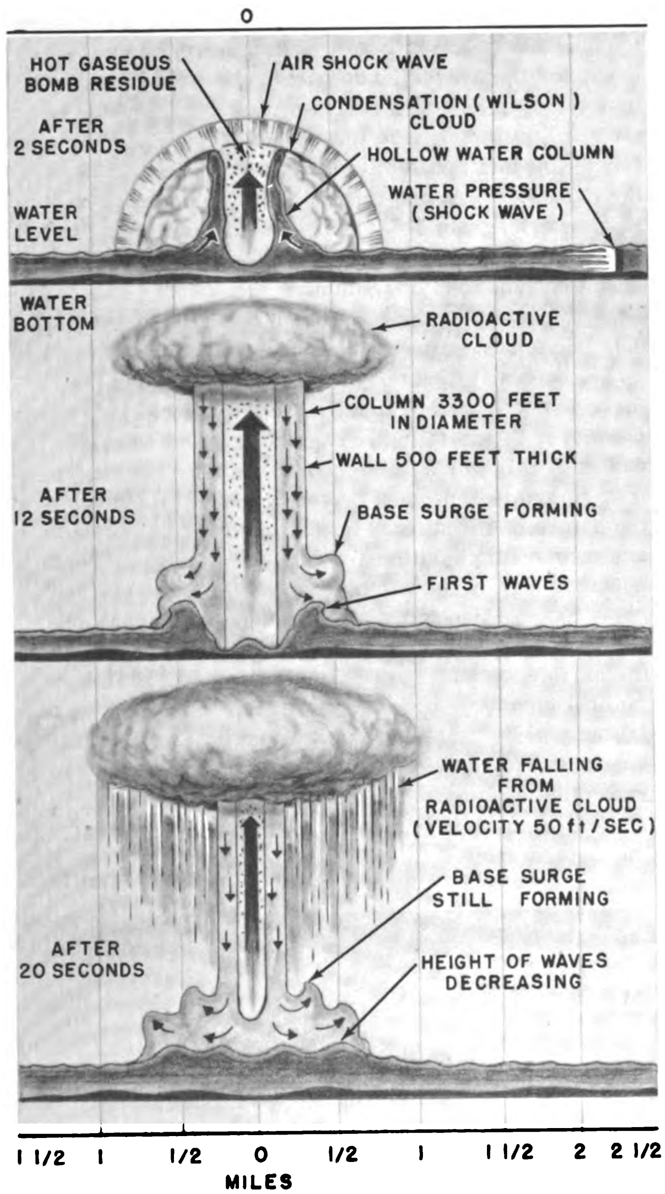
The various elements of pressure exerted upwards cause the formation of a Wilson condensation cloud to exist about the hollow column. The Wilson cloud remains only for a second or two and is not radioactive. The illustration shows three characteristic steps in a typical underwater burst. The upper part of the illustration shows the formation of the air shock front, the hollow water column; and the surrounding Wilson cloud. Special note should be taken of the water pressure shock wave traveling radially through the water. Large waves traveling at high speeds are the result of this base surge. Twelve seconds after detonation, the water column is 500 feet thick and 3300 feet in diameter.

The fission products venting through the center of the column have begun to condense into an atomic cauliflower cloud of highly radioactive composition. The base surge causes waves of 176 feet in height to propagate outwardly from the center of the explosive origin at velocities up to 120 knots. The water thrown upward in the columns is irradiated and when this water falls back to the surface it contaminates the area with residual radioactivity. At the end of 20 seconds conditions are as shown. Through diffusion the residual radioactive contamination is soon diluted and dissipated. During any type of underwater nuclear explosion, all or a great percentage of the radiant heat is absorbed by the water. The effects of blast or shock waves through the water and the danger of fallout of radioactive particles are the chief causes of damage due to an underwater burst.

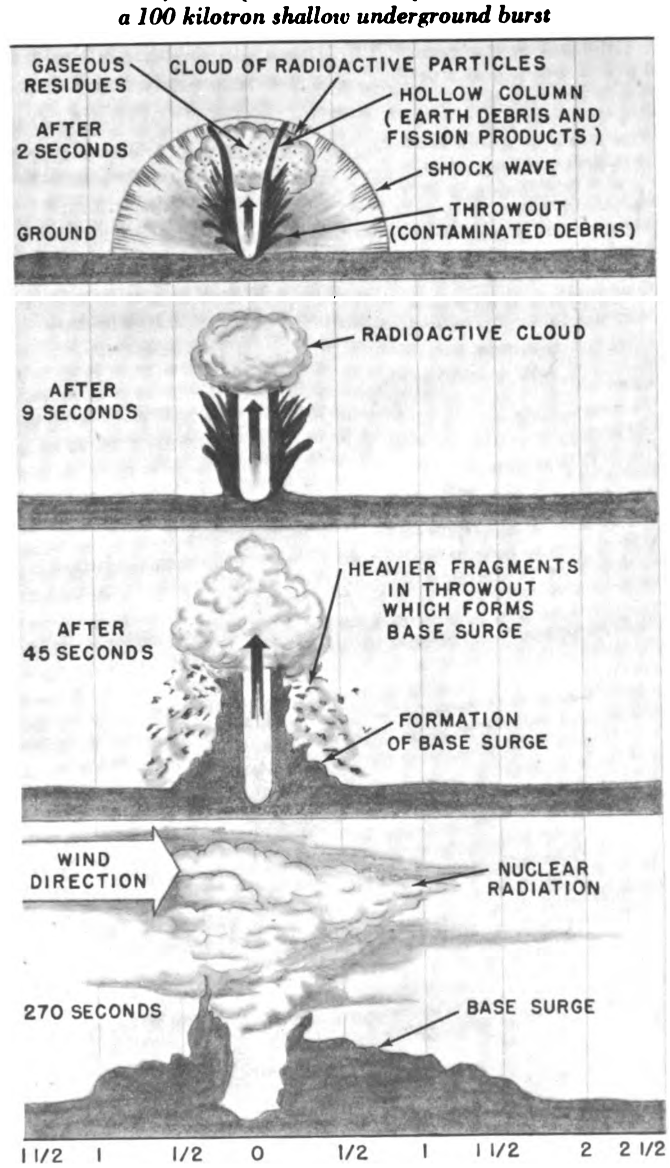
NUCLEAR UNDERGROUND BLAST.

When a nuclear payload is detonated below the surface of the earth, the formation of the fireball causes the hot pressurized gases within it to expand and force the surrounding earth upward and outward. Depending on the strength or yield of the bomb a crater is formed. The nuclear radiation products released by the detonation process are absorbed by the ground. Almost all of the thermal radiation is released into the ground and thermal effects above ground are almost negligible. The illustration shows the development of a 100-kiloton shallow underground burst. When the earth is thrown upwards it produces a hollow column of contaminated debris. The heavier particles in the column fall back to earth and produce a concentric cloud of earth which expands outward and is called a base surge. The dust particles are heavily contaminated with nuclear byproducts. The results of an underground burst are no significant thermal radiation, greatly reduced blast effects, cratering, and hardly any nuclear radiation.

three stages in the development of a 100 kilotron shallow underwater burst



four stages in the development of a 100 kilotron shallow underground burst



weight vs damage radius relationships

In assessing the damage caused either by a chemical or nuclear explosion, it is not only necessary to compare blast damage (both overpressure and dynamic) of each type but also to consider the thermal light and nuclear radiation effects of atomic payloads versus that of conventional explosives. In visualizing a complex damage volume for a nuclear explosive let us define the parameters that we will use as standards.

Full Kill A target within the confines of this damage area at the moment of detonation is considered to be totally destroyed. Partial kill - a volume extending out past the total kill volume; a target in the sector may be severely damaged but not totally destroyed. No kill volume is that segment of the surrounding media where superficial or no damage is done to the target. As illustrated: 5 miles from the center of origin the peak overpressure in psi of a nuclear explosive is equal to that of a chemical explosive at 50 yards. In other words, the blast radius of damage of a nuclear explosive is 176 times as great as a chemical explosive using equal weights of each type. The same relationship will hold true in the partial kill zone. The drag or dynamic pressure of a nuclear bomb is many thousands of times greater than that generated by a chemical explosive. The thermal effects of chemical reaction are almost negligible in comparison with the thermal radiation from an atomic detonation. At about seven-tenths of a millisecond from detonation the fireball of a 1-megaton payload will have reached a diameter of 440 feet. The fireball increases to maximum diameter of about 7200 feet at plus 10 seconds. It is then rising at the rate of approximately 200 mph.

Through the use of a scaling law it is possible to compute the fireball "R" for a nuclear weapon of "W" kilotons equivalent.

$$R\ (12\ \text{feet}) = 230\ W^{2/5}$$

The nuclear radiation potential of a chemical explosive is non-existent, while 15% of the total energy yield of a typical nuclear weapon is distributed in the form of nuclear radiations. All of the nuclear radiation discussed thus far has been the result of fission reactions. However, for the present, comparison between fission and fusion reactions will be disregarded and they will be treated similarly. The initial nuclear radiation is generally defined as that emitted within the first minute after the explosion. At one minute after the detonation, the radioactivity from 1-3/4 ounces of fission products from a 1-kiloton explosion is comparable with that of a hundred thousand tons of radium.

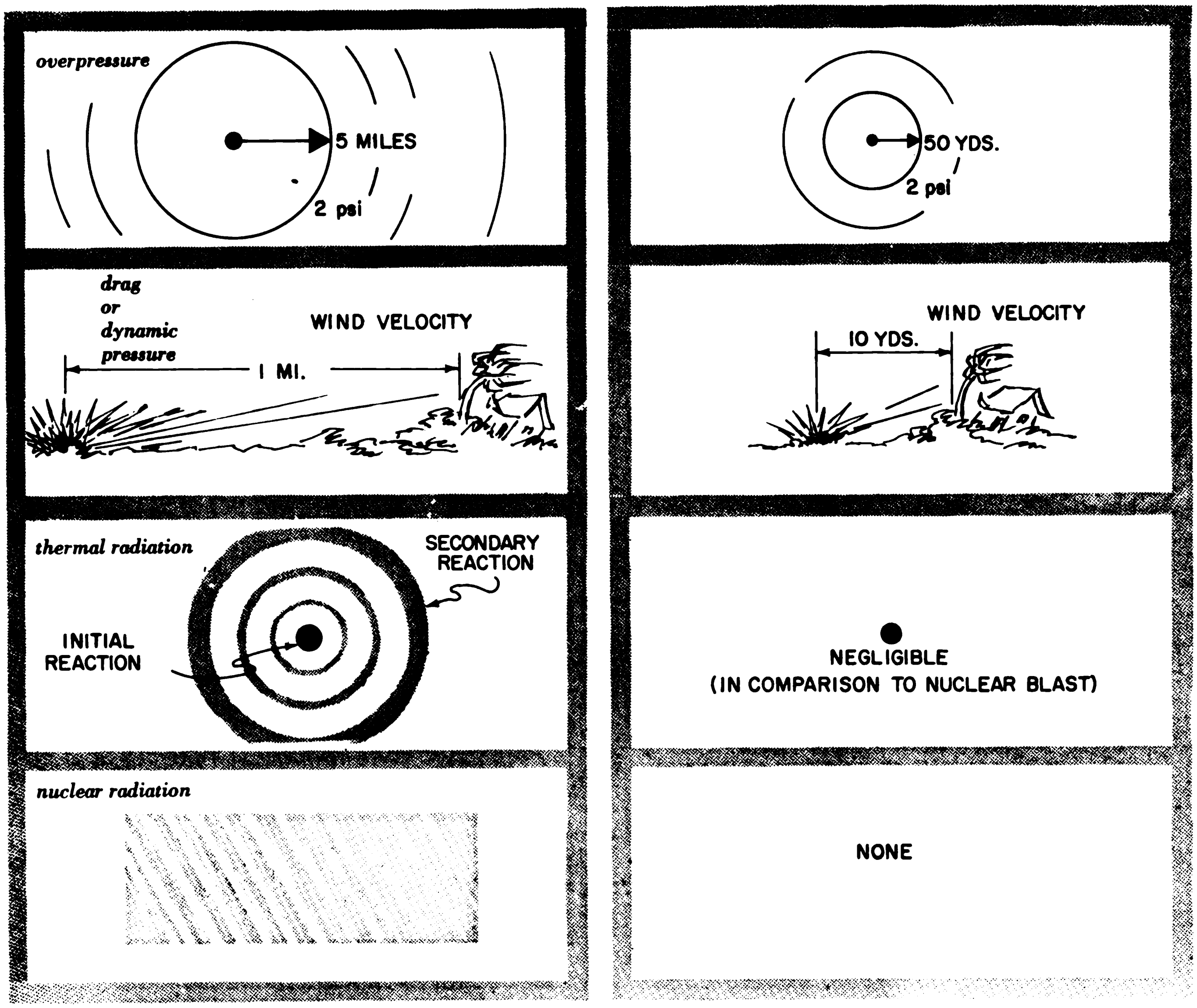
When sufficient cooling has occurred, the fission products become incorporated with particles of earth sucked up by the drag effects of the wind columns, and then these particles fall gradually back to earth. This effect is referred to as fallout. The extent of the damage radius of fallout particles can range between wind extremes and is dependent on the height of the explosion, the nature of the surface beneath point of burst, and environmental conditions. In the case of an air burst occuring high in the earth's atmosphere, the inherent radiation will be widely dispersed by wind conditions. On the other hand, a nuclear detonation occurring near the earth's surface results in strong dosages of contamination within a prescribed area. A percentage of fallout will be carried by minute particles of matter for long distances and may take months or even years to reach the surface.

A measurement of this radiation can be guessed at, the present time.

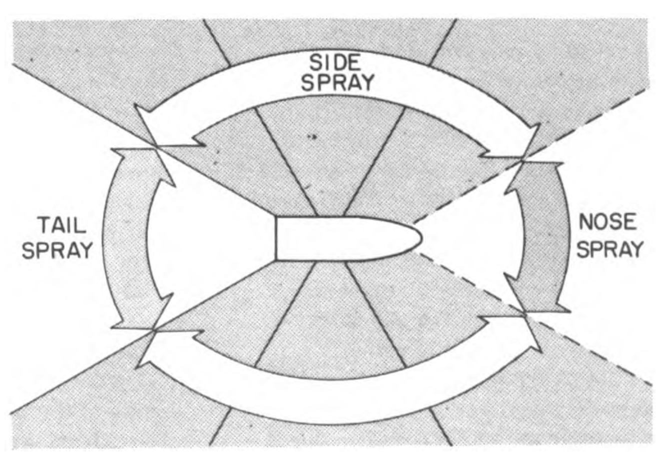
FUNDAMENTALS OF FRAGMENTATION WARHEADS

When a charge of high explosive detonates within a closed container, the container is blown into fragments. These fragments of the container are hurled outward at high velocities and, in effect, become projectiles capable of inflicting damage upon objects in their vicinity. The amount of damage which these fragments can cause is dependent upon size, velocity, and distribution. A container which bursts into tiny particles or into a few very large pieces is of little value. Knowledge of fragmenta-

Comparison of characteristics of Nuclear vs Chemical Explosion
(assuming equal weight of explosives)



tion propagation is therefore a basic requisite in designing many types of warheads. Terminal ballistic studies attempt to determine the laws and conditions governing the speed and distribution of fragments, the sizes and shapes that result from the bursting of different types of containers, and the damage aspects of the bursting charge fragmentation. Fragmentation warheads are used principally as means for destroying air targets, producing personnel casualties, and attacking light military equipment.



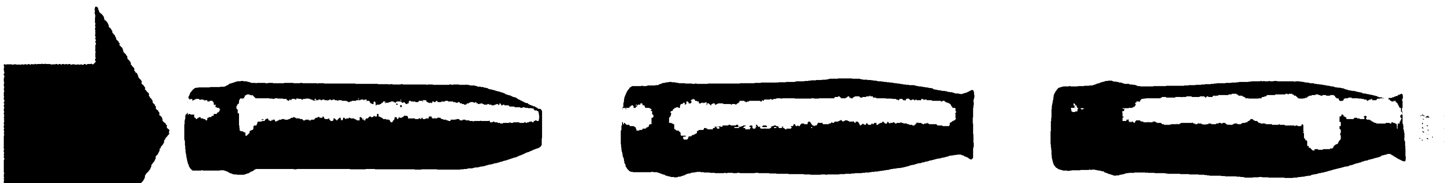
fragmentation pattern of 20 mm projectile

fragmentation process

The detonation of a high explosive in a warhead causes the metal case to expand very rapidly. This expansion is the result of the rapid rise in internal pressure of the products of the detonation. Expansion continues until the case ruptures. The following flash radiographs of a tetryl-loaded 20mm projectile, detonated statically, illustrate the phenomenon of fragmentation.

note

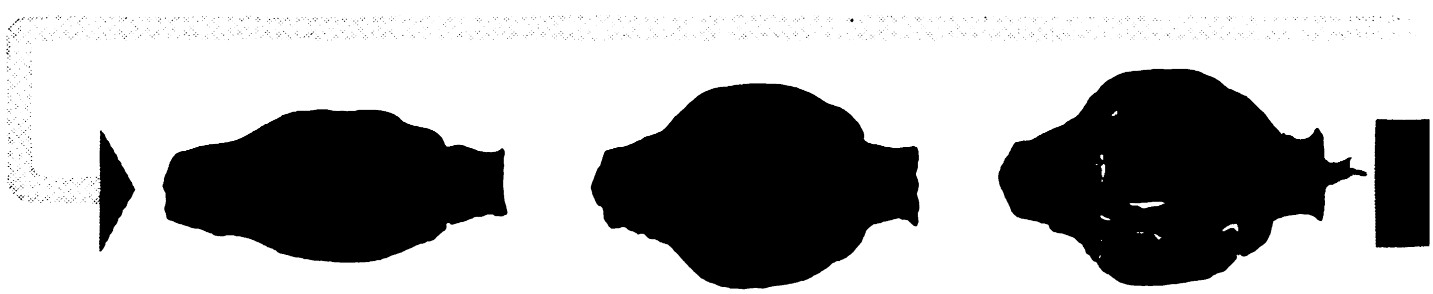
The fragment propagation is non-isotropic. Most of the fragments are propelled radially from the sides of the projectile case.



PROJECTILE BEFORE DETONATION

2 MICROSECONDS after initiation of the bursting charge

5 MICROSECONDS after initiation showing the projectile case in the process of swelling



11 MICROSECONDS after initiation cracks can be seen in the projectile case which has expanded to almost twice its original girth

20 MICROSECONDS after initiation showing continued lateral movement of the projectile case fragments. The expanding gases are escaping through the failure cracks

34 MICROSECONDS after initiation showing continued growth of fragmentation perpendicular to the longitudinal axis of the projectile

About forty percent of the gas energy produced by a warhead detonation is normally absorbed in the fragmentation process, while the balance of available energy is used to create a shock front. The fragments resulting from such a detonation are propelled at high velocity for a very short distance and then pass through the shock wave. The rate at which the velocity of the shock front accompanying the blast decreases is generally much greater than the decrease in velocity of case fragments which occurs due to air friction. Therefore, the advance of the shock front lags behind that of the fragments. The radius of effective fragment damage thus exceeds considerably the radius of effective blast damage. Whereas the effects of an idealized blast warhead are attenuated by a factor roughly equal to $1/R^3$ (R is equal to distance from origin), the attenuation of idealized fragmentation effects will vary as $1/R^2$ and $1/R$, depending upon the specific design of the warhead. The principle advantage of a fragmentation warhead lies in the fact that a particular weapon system using such a warhead can afford a greater miss distance and still remain effective because its attenuation is less. The disadvantage of this type of warhead is that it does not achieve the sudden spectacular results of a blast warhead since the holes produced in a target by fragmentation may not have a crippling effect upon the target, particularly at the maximum range of fragment effect.

Obtaining ballistic data on individual fragments is difficult and since statistical analysis of fragmentation of the container itself provides suffi-

39 MICROSECONDS
after initiation projectile starts to burst

54 MICROSECONDS
after initiation showing the extent to which the fragments fly off in the direction perpendicular to the surface of the casing. The "Side Spray" which exceeds the "Nose" and "Tail Spray" in intensity of fragmentation is the main instrument of damage in most fragmentation warheads

92 MICROSECONDS
after initiation showing the wide variance in size and shape of fragments. All the fragments have by now received their initial velocities

cient practical data, the necessity of obtaining data on individual fragments is eliminated. An ideal fragmentation projectile is one which would break up into uniform fragments with a size and velocity that fulfills the predetermined tactical requirements involved. Although this ideal has not yet been reached, limited control of the size and shape of fragments has been obtained. The problem of determining optimum fragments illustrated the need for knowledge of fragment flight characteristics and the vulnerability of the prospective target in terms of fragment mass and striking velocity.

FRAGMENT VELOCITY.

The initial velocity of a fragment depends primarily on two factors, namely:

1. The charge-to-mass ratio, C/M, where C is the mass of explosive per unit length of projectile and M is the mass of metal per unit length of projectile.
2. The characteristics of the explosive filler, particularly its brisance and power.

The following table illustrates the relationship between the charge-to-mass ratio and the initial velocities, Vo, of the fragments, as determined from a series of tests performed using cylinders with a two-inch internal diameter and uniform wall thicknesses as indicated. The explosive filler used was TNT.

Wall Thickness (inches)	Charge-to-Mass Ratio (C/M)	Initial Velocity Vo (ft/sec)
1/2	0.165	2870
3/8	0.231	3240
5/16	0.286	3800
3/16	0.500	5100
1/8	0.775	6100

The primary reason for the initial velocity of fragments being lower for the containers with the greater wall thicknesses is that a large part of the energy released by explosion is absorbed in rupturing these cylinders. These results could be changed considerably, either through the use of different explosives (TNT or RDX) or different wall materials (cast iron or forged steel).
The best way to achieve high initial velocities of fragments is to have a high charge-to-mass ratio, usually obtained in practice by using a thin wall container.
When a projectile must be designed with a thick wall to withstand setback forces or to produce larger fragments, a more powerful explosive, one having a higher brisance than that used in a thinner wall cylinder, must be used. For example, explosives containing RDX have both high brisance and high power. They are therefore ideal for producing high velocity fragments.
Velocities of fragments from an air burst have higher values than those obtained from detonation of a warhead by impact. This is because of additional velocity imparted to the fragments by the velocity of the moving missile itself at time of detonation. This is one of the reasons that proximity or time fuzing provides more effective fragmentation.

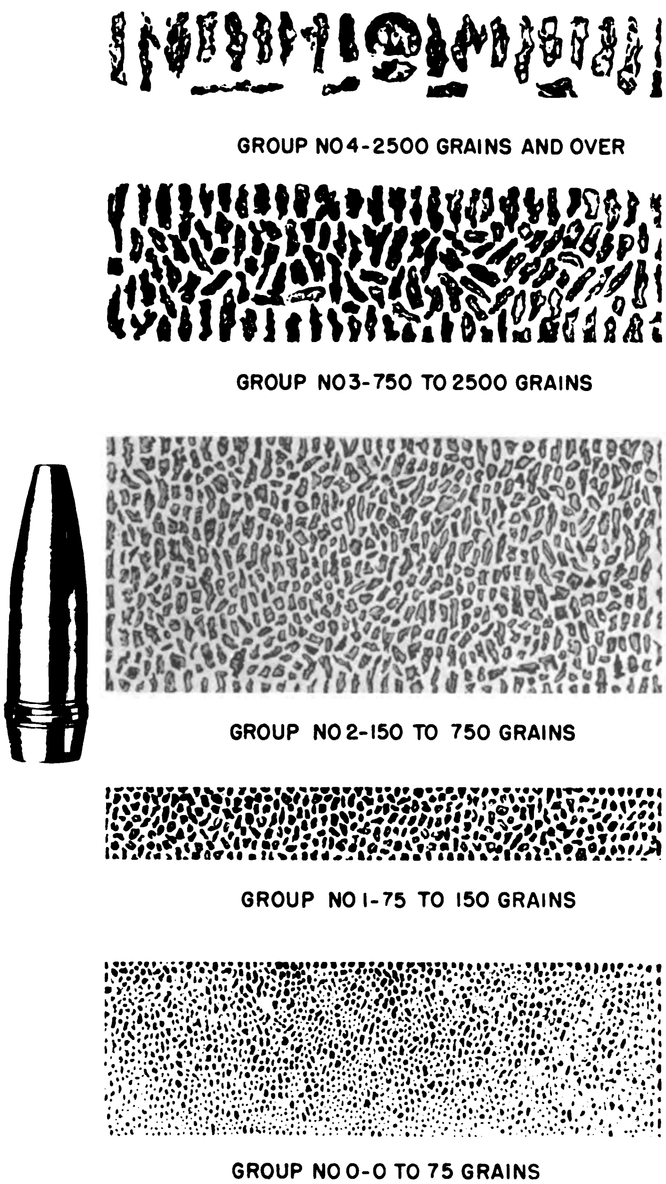
FRAGMENT FLIGHT.

The fragments of a projectile normally travel outward in a direction perpendicular to the surface of its casing. This can be seen by referring to the illustration of the detonation of a 20-mm projectile. The static detonation of a large bomb, suspended nose down with its nose about seven feet from the ground, is also illustrated. The tracks of the fragments are made luminous by their heat. The black area represents the unoxidized solid products of the explosion. The cone of tracks opening upward is called the forty-five degree spray and originates from the section where the cylindrical part of the casing connects to the tail. The fragments almost parallel to the ground constitute the main side spray and originate from the cylindrical side walls of the bomb. The picture does not fully indicate the great density of fragments in the side spray. The slight upward deflection of the side spray is the result of the bomb being detonated, statically, with detonation starting from the bomb nose.
If either the bomb or projectile had been detonated by fuze action while in flight, the side spray would have had a slight forward thrust which is the result of the initial radial fragment velocity and the forward velocity of the bomb or projectile.

static nose-down detonation of a bomb

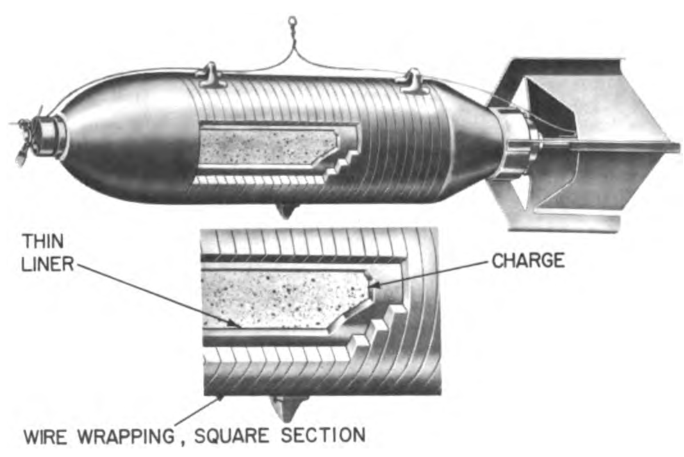
FRAGMENT NUMBER, TYPE AND SIZE.

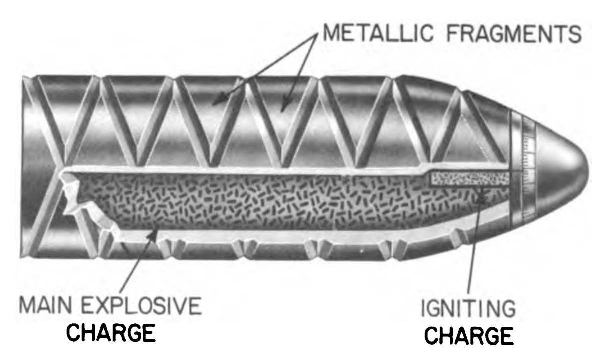
The damage produced by a fragment with a given velocity depends upon the mass of the fragment. It is therefore necessary to know, for each warhead, the approximate distribution of mass for the fragments large enough to cause damage. Mass distribution of warhead fragments is determined experimentally by means of a static detonation in which the fragments, or a portion of them, are caught in sand pits. Usually, the side spray contains the most important part of the fragmentation. It will, therefore, generally have a different mass distribution than the fragment from the rest of the missile. The illustration shows the fragments of a six-inch high capacity gun projectile. Generally, the sharp of the fragments vary considerably. Many of the fragments appear flat, with their smallest dimension corresponding to the thickness of the swollen case stretched by the expansion that follows detonation.



fragments from projectile fragmentations

Some fragmentation bombs have a light casing wrapped with a metal helix of square cross-section. This type of casing is used to control, to some extent, the size and, therefore, the distribution of mass among the fragments. When the warhead bursts, the helix is broken into pieces of comparatively uniform size as compared to the fragments of a warhead without such fragmentation control. In warheads such as General Purpose (GP) bombs, where the size of fragments is not controlled, fragmentation may vary from tiny dust-like particles to relatively large pieces.
While adjustment of the charge-to-mass ratio in projectiles can be used to change the degree of fragmentation of the projectile wall, the size and number of fragments resulting from the shattered wall can also be adjusted by altering the material used for the wall. As an example, grey cast iron is an inherently brittle material which shatters into very small sand-like pieces, which have low killing power and, in addition, have little momentum and are thus short range. This is unfortunate since the casting of projectile casings is desirable from an industrial point of view. Other manufacturing methods for the production of casings are deep drawing and pressure forming. However, metals which deform without splitting in such forming processes are, on the whole, tough and hard to shatter. Hence, some advantage in both production and fragmentation might be realized if a cast iron were available which could be made to shatter into fragments of optimum size.





basic construction of fragmentation warhead

damage effects of fragmentation

In evaluating the effectiveness of fragmentation of specific warheads, the types of damage considered are personnel casualties, and normal perforation of mild steel of 1/8 inch, 1/4 inch, and 1/2 inch thicknesses. A casualty may be considered to be a hit by a fragment with at least 58 ft.-lbs. of energy. Normal perforation of mild steel occurs when a fragment, traveling perpendicular to the face of the plate, passes completely through the plate. Fragments which perforate 1/2 inch mild steel are effective against light armored vehicles, railway rolling stock, and targets of a similar resistant nature.

The fragment damage indicated in the following table gives the number of effective fragments per square foot of the target area at a given distance from the burst of a 240 lb. eight-inch high capacity projectile based on an initial fragment velocity of 2500 feet per second. The average number of effective fragments per square foot was computed on the basis of an average value for different directions from the burst point. The actual density of fragments in the most dangerous direction from the warhead is about six times the average densities given in the table.

240# 8" PROJECTILE
INITIAL FRAGMENT VELOCITY 2500 f/s

Distance in feet	Total Number Effective Fragments			Average Number Effective Frag. per ft^2 for Casualties
	Casualties	1/8" steel	1/4" steel	
20	1920	1450	760	.382
60	1620	1060	585	.036
100	1480	820	445	.012
200	1260	565	248	.005
700	700	180	NONE	.0001

Note that for any given distance, as the damage requirements increase, for example from 1/8-inch to 1/4-inch steel, the total number of effective fragments available decreases. Thus, at 20 feet, 1450 fragments at the same distance can pierce 1/4-inch steel.

As would be expected, as the distance from the burst increases, the number of fragments available to produce any given damage effect decreases.

The fragmentation effects of the above projectile may be compared with a bomb of comparable size by consulting the table which follows.

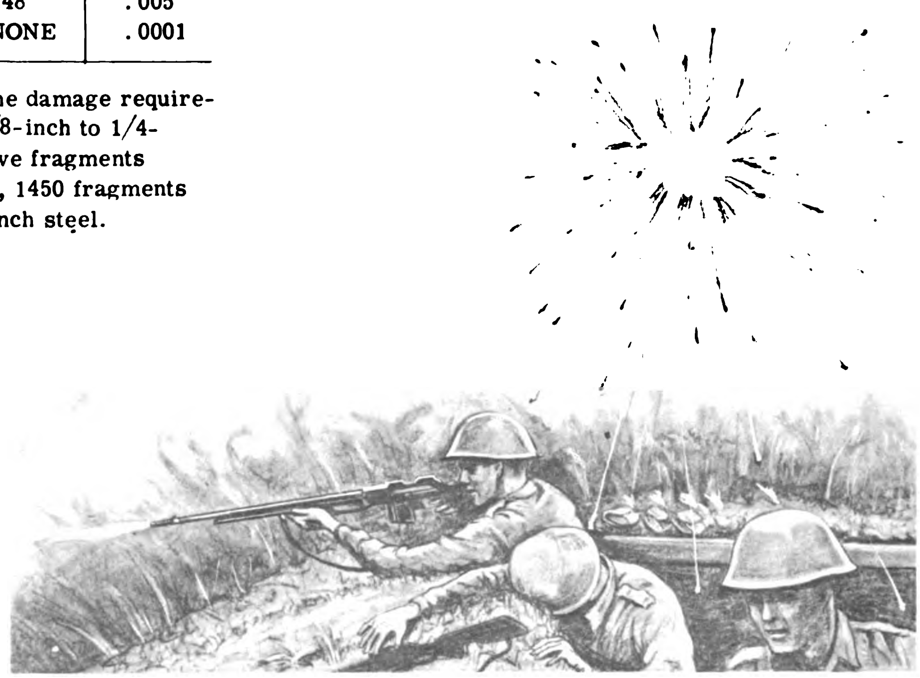
220# FRAGMENTATION BOMB

INITIAL FRAGMENT VELOCITY 4420 f/s

Distance in feet	Total Number Effective Fragments			Average Number Effective Frag. per ft^2 for Casualties
	Casualties	1/8" steel	1/4" steel	
40	8650	6600	3180	.706
60	7400	5600	2520	.268
100	5800	3750	1860	.076
200	3700	1750	820	.012
700	1500	NONE	NONE	.0004
1000	850	NONE	NONE	.0001

The significance of comparing the data of the two tables is that, although the projectile and bomb are of about the same weight, the bomb is far more effective as a fragmentation warhead, which is directly attributable to the requirement that a projectile be designed with heavy walls which can withstand the shock of firing and impact. The delivery of a bomb presents no such requirement and the cases of fragmentation bombs are therefore relatively light. This results in higher initial fragment velocities and a corresponding increase in the radius of effectiveness. However, other factors, such as the tactical situation, desired accuracy of delivery, vulnerability of aircraft, or volume of fire, may often favor the use of projectiles rather than bombs as fragmentation warheads.

The actual fragmentation pattern is greatly affected by the terminal velocity of the warhead and, for surface targets, by the angle of fall and height of burst.

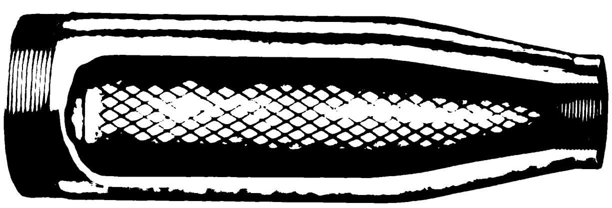


controlled fragmentation

The important characteristics of missiles designed for fragmentation, as discussed in the preceding paragraphs, are as follows:

1. Direction of flight of fragments
2. Velocity of fragments
3. Mass distribution (number, size, and shape) of fragments

A controlled fragmentation warhead is one in which these factors can be predicted to some degree. Control may be achieved in a variety of ways, the most common being: serration or scoring of the warhead body, and the use of notched wire or grooved rings of square cross section.



controlled fragmentation warhead

DIRECTION OF FLIGHT OF FRAGMENTS.

The direction of flight of fragments will depend, among other things, upon the shape of the missile. The best shape is one that directs the flight of fragments in a manner which yields a maximum effective area of fragment spray. The shape of the missiles, however, is not determined by terminal ballistic considerations alone, and therefore a compromise in the ideal shape of missiles is necessary to meet the interior and exterior ballistic requirements.

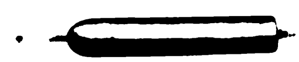
VELOCITY OF FRAGMENTS

The velocity of fragments is another point to be considered. The initial velocity of the fragments depends on the charge-to-mass ratio of the missile, the type of explosive used, and the shape of the fragments. However, the charge-to-mass ratio of a missile is governed, to some extent, by design considerations other than the terminal ballistic requirement, such as the type of explosive used.

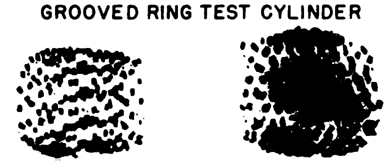
DIRECT MECHANICAL CONTROL OF FRAGMENTATION.

Direct mechanical control of the number, size, and shape of fragments provides the best opportunity for controlling the fragmentation effectiveness of missiles. A more recent technique is the use of grooved rings. The grooves are equally spaced around each ring and are relatively shallow in depth and triangular in cross section. The rings are assembled on a liner and pressed together as tightly as possible. Retainers are then fitted on the liner and made sufficiently tight to hold the rings in place.

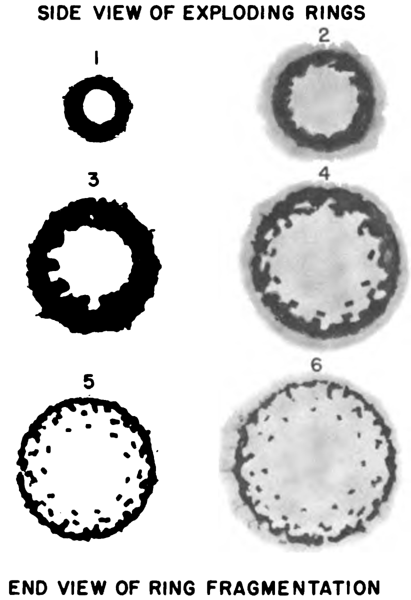
Size control of fragments has been achieved through the use of both notched wire and grooved ring shells by accurately spacing of the notches or grooves. This spacing is based on the assumption, thus far borne out in test firings, that there is an optimum width of fragment for each size and type of projectile. In controlling the size of fragments in this manner, their mass has also been controlled.



GROOVED RING TEST CYLINDER



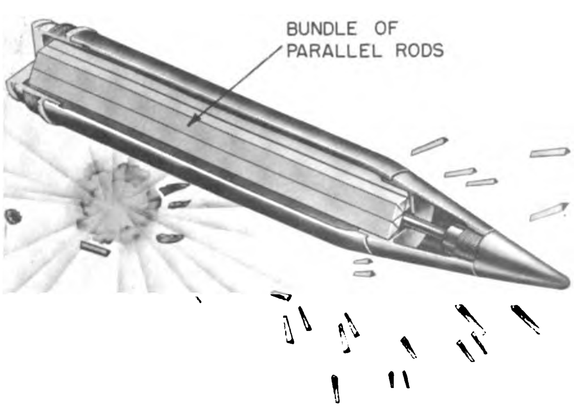
SIDE VIEW OF EXPLODING RINGS

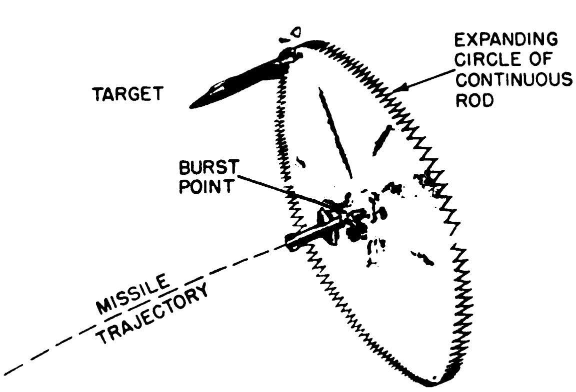


END VIEW OF RING FRAGMENTATION

CONTINUOUS ROD WARHEAD

A significant new development in fragmentation warheads is the continuous rod warhead, which was designed for Navy missiles intended for use against aircraft. In this type of warhead, the warhead energy is used to expand rods radially into a ring of metal which can lengthen and thus increase its diameter rather than produce an expanding shell of small fragments. The continuous rod warhead is non-isotropic. The metal density and thus the effectiveness of the continuous rod warhead decreases inversely as the distance from the point of detonation, whereas the metal density of the expanding zone of normal fragments attenuates with the square of the distance. Early warhead experiments with short, straight, unconnected rods had shown that such rods could chop off propeller blades and engine cylinders, whittle off wings, and, in general, could inflict severe damage to a fighter aircraft. However, rod warheads were ineffective against larger planes because the nature of most bomber aircraft structures permit a number of short cuts in their skin without lethal damage occurring. It was found, however, that long continuous cuts would do considerable damage to a bomber; therefore the continuous rod warhead was developed. Upon detonation the continuous rod warhead expands radially into a ring pattern. The intent is to cause the connected rods, during their expansion, to strike the target and produce damage by a cutting action. Connected rods generally form two semicircles as they expand. This prevents disintegration of the circular pattern when maximum expansion is reached.



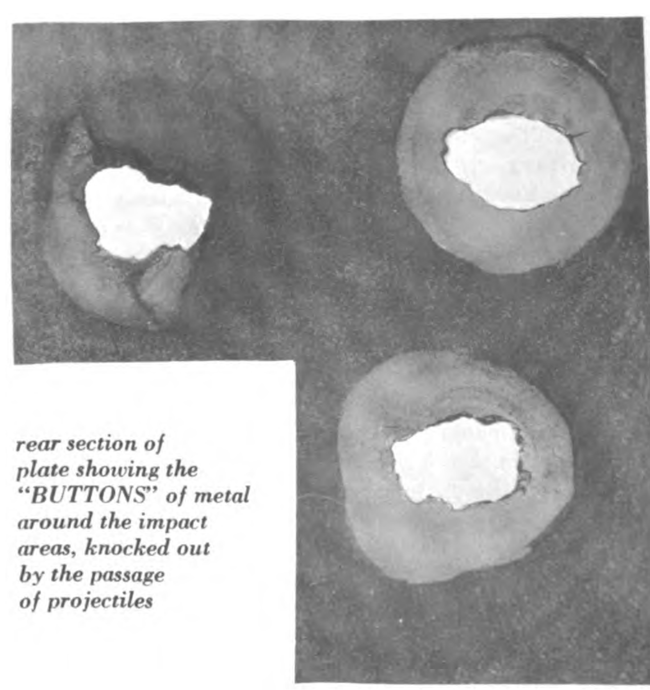


FUNDAMENTALS OF

Shaped charge warheads are designed primarily to pierce armor, but differ from standard armor-piercing warheads in principle of operation. Armor-piercing warheads are designed to force their way into the interior of a target to cause explosive damage as shown by the illustrations. In such cases, the undeformed warhead is the instrument of damage, and the degree of penetration is dependent upon the striking velocity of the missile. For the shaped charge, on the other hand, the thickness of material it can penetrate is essentially independent of its striking velocity and, in fact, the warhead itself remains at the outer face of the target, producing a destructive jet which pierces the target.

projectile penetrating plate

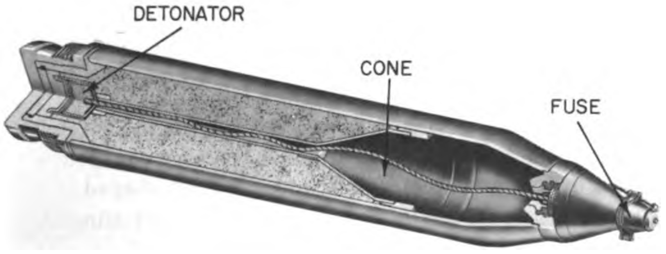
front section of plate showing area knocked out by passage of projectile



rear section of plate showing the "BUTTONS" of metal around the impact areas, knocked out by the passage of projectiles

SHAPED-CHARGE WARHEADS

A shaped charge warhead consists basically of: a thick-walled explosive-filled case open in front; a thin front liner for the explosive of inert material such as metal usually conical or hemispherical in shape, although other shapes may be used (the concave side always facing forward); a fuze and detonating device; and a thin nose cone for proper aerodynamics. The shaped charge is characterized by the concave shape of the front liner.

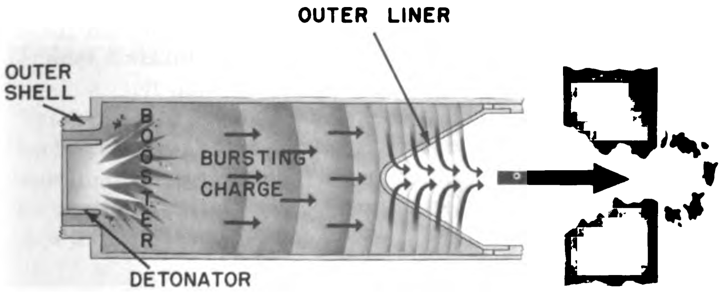


typical shaped charge warhead

functioning

When the shaped charge warhead strikes a target, the point-detonating nose-fuze fires a length of detonating cord which leads to a booster in the rear of the warhead. The booster in turn detonates the main charge and a detonation wave travels forward causing the metal front liner to collapse. Collapse of the liner starts at its apex. When the liner collapses, it ejects a narrow jet of explosive products and metal particles from the face of the liner out the front end of the thick casing at velocities from 10,000 to 38,000 feet per second. This process is illustrated by the series of ultra high speed radiographs of an experimental shaped charge liner with a 45° steel cone liner showing the mechanism of cone collapse.

The charge was 50/50 pentolite, having a base diameter of 3/4 inch and the detonation wave moving from right to left. The time noted in microseconds for each radiograph denotes the time after the detonation wave has passed the apex of the cone. The jet breaks up into fine particles early in the process of its formation, but retains its jetlike characteristics. There is a gradient in the velocities of the particles along the jet. The particles in front move faster than those in the rear, thus causing the jet to lengthen and thereby reduce its average density with time. The jet is followed by the major portion of the now completely collapsed cone. The latter is generally referred to as the slug.



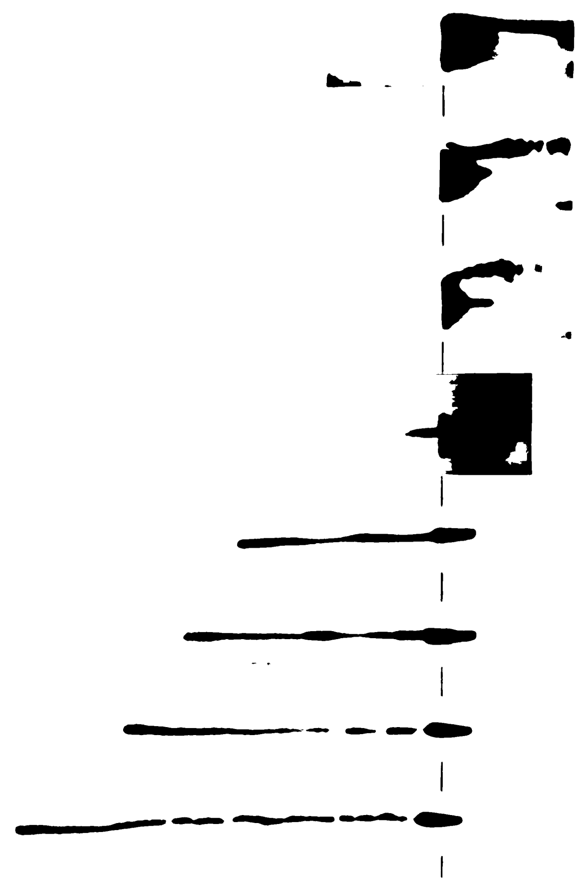
shaped charge explosive train

JET FORMATION.

The following considerations will facilitate an understanding of jet formation from a shaped charge:

1. The outer shell, which is massive and strong, neither moves nor shatters.
2. As the detonation wave arrives at the apex of the cone, the cone material and explosive gases leave normal to the cone surface.
3. As the detonation wave envelops the cone, the cone material and explosive gases continue to leave normal to the cone surface.

The departure of cone material and explosive gas in a normal direction is indicated schematically by small arrows in the illustration. The symmetrical inbound radial components of these forces cancel (thus minimizing side spray of the jet) but they greatly increase jet pressure. The axial components of the forces are all additive and thus greatly increase the kinetic energy of the jet.



ultra high speed radiograph of shaped charge detonation (jet moves from right to left)

JET PENETRATION.

When a jet strikes a target of armor plate or mild steel, pressures of about 250,000 lbs. per sq. in. are produced at the point of contact. This pressure produces stresses far above the yield strength of steel and the target material flows out of the path of the jet as would a fluid. In fact both the target and jet may be considered as fluids. There is so much radial momentum associated with the flow of target material that the diameter of the hole produced is considerably larger than that of the jet. The difference in diameter between the jet and the hole it produces depends upon the characteristics of the armor plate. However, the depth of penetration into a thick slab of mild steel will be only slightly greater than that into homogenous armor.

penetration of steel by a shaped charge

As the jet particles strike the target, they are carried outward radially along with the target material. Thus, the jet is used up as it strikes the target and becomes shorter and shorter until finally the last jet particle strikes the target and the primary penetration process stops. The actual penetration continues for a short time after cessation of jet action because the kinetic energy, imparted to the target material by the jet, must be dissipated. The additional penetration caused by this afterflow is called secondary penetration.
Although exceptions may be found to the following rule, it has been found generally that the depth of main penetration P' is related to the length of the jet L, the density of the target material p, and the average density of the jet pj, by the following expression:

$$P^1 \approx L\sqrt{\frac{pj}{p}}$$

The strength of the target material has no appreciable effect on the depth of primary penetration because the target is fluid in nature during penetration. The jet density, pj, is dependent to a large degree on the density of the particles of the cone liner which are dispersed throughout the primary jet.
The standoff distance is defined as the distance from the base of the cone to the surface of the target when detonation occurs. This distance is extremely important in obtaining maximum penetration of the jet. An increase in standoff distance permits an increase in the length of the jet L, but at the same time decreases the average density, pj. From the expression above, however, it would appear that an increase in standoff distance increases the depth of primary penetration, since P' varies directly as L but only as the square root of pj. This is true up to a certain distance but beyond that distance irregularities developed in the jet cause the jet to spread somewhat, causing, in turn, a decrease in the depth of primary penetration.

performance

The performance of shaped charge missiles is independent of the striking velocity as such and, therefore, of range. This would appear to make these charges ideal for armor piercing warheads. When shaped charge warheads are caused to rotate by the rifling in the guns from which they are fired, their performance drops from 70% to 30%. Non-rotated, fin-stabilized shaped charges can be made to perform almost as well when detonated by impact as when they are detonated statically, provided their striking velocity is not so great that the operation time of the fuze will allow the cone liner to become deformed before complete detonation. Thus, shaped charges are better suited for use in rocket warheads than in gun projectiles and are particularly useful in anti-tank weapons.
Various theories have been advanced as to the action that takes place within a tank when it is perforated by the jet of a shaped charge missile. Tests quite definitely show that for shaped charge missiles with explosive charges weighing 15 pounds or less there is no appreciable blast effect, pressure rise, or temperature rise inside the tank. The jet does not spread out in a cone shaped spray as it exits from the armor plate. If the jet perforates the armor plate with enough energy to spare, it continues along its original path however, and ammunition, fuel, etc., in the path of the jet will often be set afire and crew members in its path will be killed. It is entirely possible, however, for one man to be killed and the man immediately adjacent to him, but out of the path of the jet, to escape serious injury. Damage is actually produced by the tiny, hot, high velocity fragments of liner metal making up the jet. Depending upon the quality of the armor plate and upon the amount of explosive in the charge, varying amounts of metal may be spewed off the rear face of the armor at sufficiently high velocities to injure crew members. Fire very often results because the cramped space and very large amounts of explosive and inflammable fuels make it highly improbable for a jet to perforate a tank without striking such inflammable material.
The damage caused by the jet itself, along with the damage caused by the spewing of metal of the rear face of the tank, makes it highly unlikely that escape from the tank is possible once it has been penetrated by the jet of the shaped charge.

SPECIAL PURPOSE WARHEADS

Many targets may be attacked more effectively by other means than by blast or fragmentation which are, of course, the most common types of warheads. There are several types of warheads of a more specialized nature that are worthy of examination. These warhead types include: thermal, biological and chemical, radiation, illuminating, and psychological.

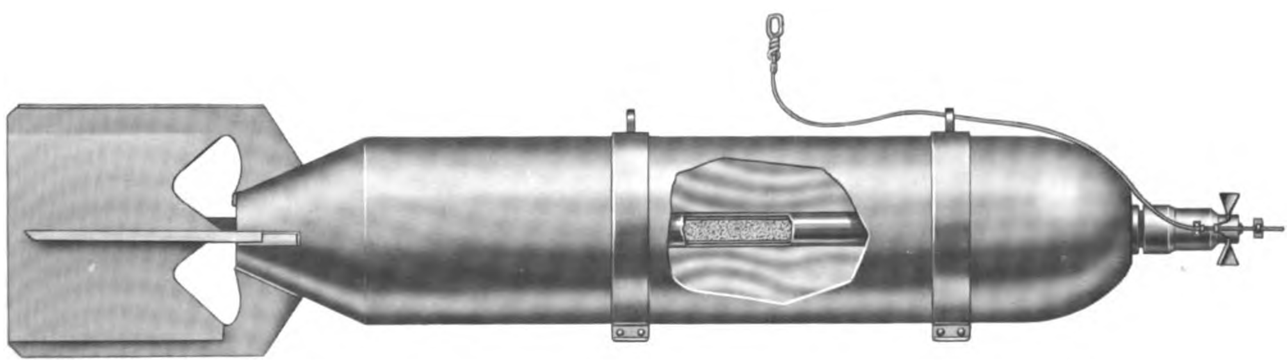
THERMAL BIOLOGICAL AND CHEMICAL RADIATION ILLUMINATING PSYCHOLOGICAL

thermal

The purpose of thermal warheads is to start fires. Thermal warheads may employ chemical energy to kindle fires with subsequent uncontrollable conflagrations, or nuclear energy to produce direct thermal destruction as well as subsequent fires.

Thermal warheads of the chemical type may be referred to as "incendiary" or "fire" warheads.

Many area targets are more effectively attacked by fire than by blast or fragmentation. Thermal warheads, principally in the form of aircraft bombs, have been developed for use against combustible land targets where large and numerous fires will cause serious damage.



an incendiary bomb

biological and chemical

A biological warhead utilizes bacteria or other biological agents for accomplishing its purposes of causing sickness or death, and is of extreme strategic importance since it is capable of destroying life without damaging buildings or materials. The poisoning of water supplies is probably the single most efficient way of destroying enemy personnel. The war potential of the enemy, such as guns, missile launching sites, etc., are thus left intact and at the disposal of the attacker. The biological agent may be chosen so that it causes only temporary disability rather than death to enemy personnel, thereby making it relatively simple to capture an enemy installation. A small explosive charge placed in a biological warhead is useful in ejecting the initial dispersion of biological agents.

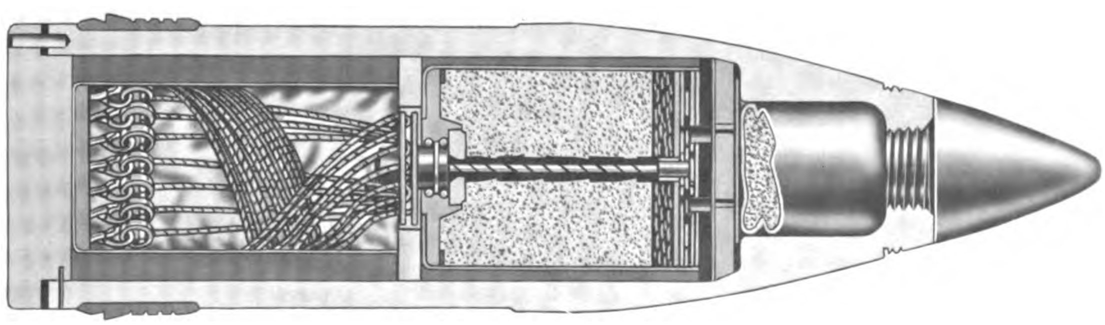
A chemical warhead is designed to expel poisonous substances and thus produce personnel casualties. Although the use of poisonous gases in warfare appears to be obsolete, the possibility of their use remains as a threat.

radiation

Radiological material may be employed in warheads in the same manner as chemical and biological agents. The payload may consist of matter prepared especially for the purpose, radioactive by-products of another process, or an atomic warhead designed to produce an abnormally large amount of radioactive material.

illuminating

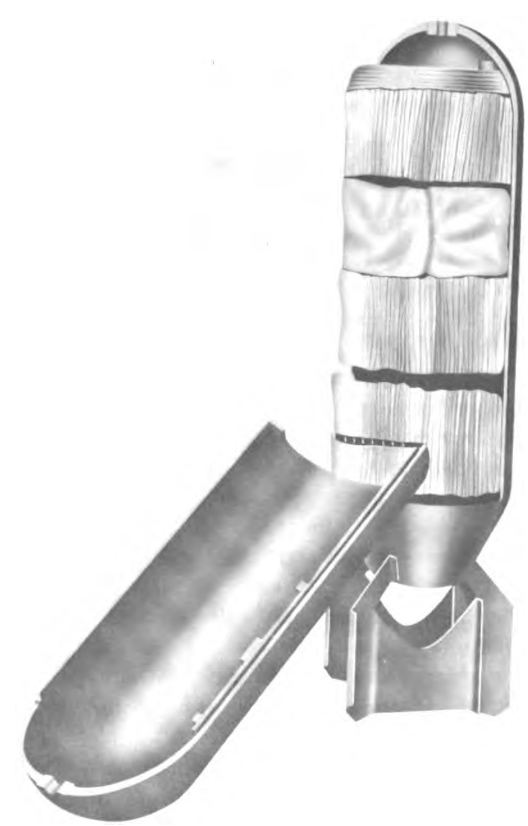
Illuminating warheads usually contain a flare or flare candle which is expelled by a small charge and is parachuted to ground. During its descent the flare is kindled. The illuminating warhead is thus of great usefulness during night attacks in pointing out enemy fortifications. Of particular use is the illuminated projectile which is used with great effectiveness in shore bombardments. Illuminating warheads are also used in aircraft bombs and rockets to assist in the attack of the ground targets and submarines.



typical illuminating projectile

psychological

This type of warhead does not carry a lethal or destructive agent, but carries material designed to create a psychological effect on the enemy rather than actual damage. Payloads such as propaganda leaflets, mysterious objects which appear to be dangerous, and inert or dummy warheads are examples. In this class of warheads can be included decoys such as window, which causes false radar echoes, and noisemakers, designed to foil the sonar operators of antisubmarine ships.



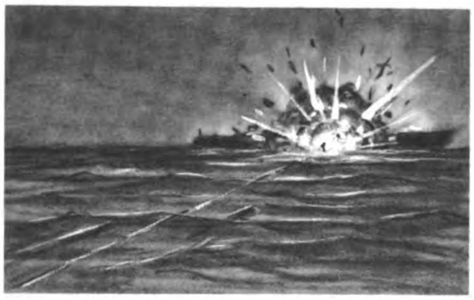
leaflet bomb

SECTION 2

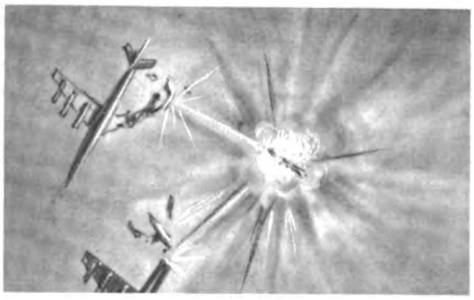
FUZES

TIME

IMPACT



PROXIMITY



AMBIENT

introduction

A fuze is that portion of the warhead which causes detonation of the payload. A fuze may perform its function while the warhead is traveling toward a target or awaiting the approach of a target. In either case, a stimulus is required to initiate operation of the fuze. A fuze is but a small part of the complete weapon system, but unless it functions exactly as intended, the entire effect of the weapon may be lost.

The following characteristics are of primary concern in fuze design:

a. Simplicity of design and construction.
b. Streamlining for good ballistics.
c. Reasonable economy of manufacture.
d. Compactness for ease of storage and handling.
e. Ease of loading.
f. Safety in handling and use.
g. Freedom from deterioration in storage.
h. Strength to withstand the forces of firing or launching.
i. Certainty of action.

Some of these characteristics conflict.

For instance, the addition of safety features may greatly complicate the problem of design and increase the cost of manufacture. The safety features will be considered as part of the safety and arming function of the warhead.

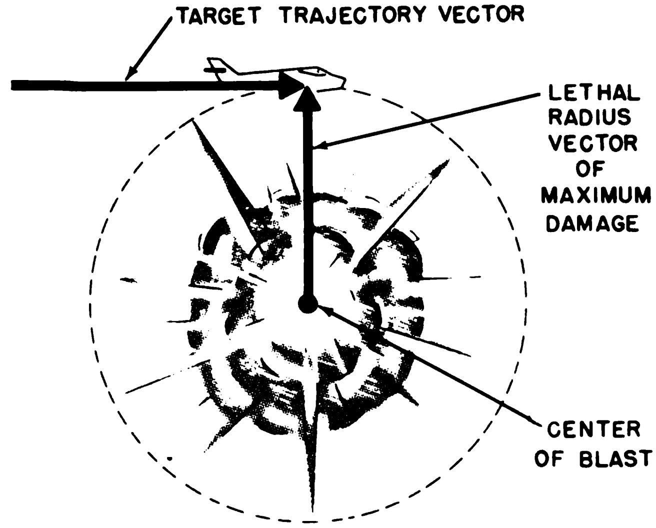
Each listed characteristic becomes more or less critical, depending upon the type of missile being considered. Thus, compactness is essential to the fuze of a projectile, yet becomes less important for large missiles where space is not so restricted. Furthermore, simplicity is often sacrificed for reliability when the missile is to carry a large and expensive warhead.

Compact fuzes used in projectiles, small rockets and mortars must perform the same functions as larger fuzes used in guided missiles. Essentially, they must initiate detonation at the optimum time, while assuring that detonation does not occur prematurely (an S & A function). Since the fuze must be safe when launched, the fuze designer must take advantage of forces or effects available during and after launch to prepare the fuze for firing and to activate it.

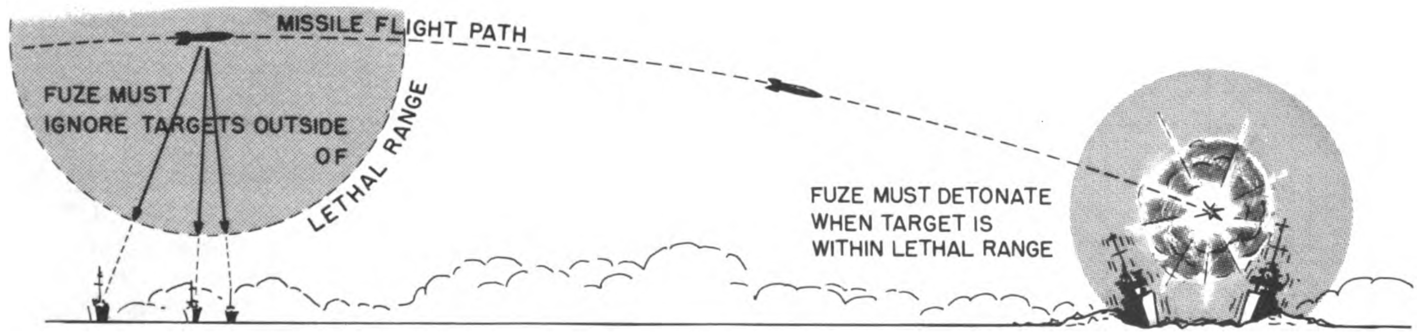
In genral, the modern fuze operates in conjunction with an explosive train. The elements of the explosive train are usually part of the S & A, but may be part of the fuze itself. The fuze acts to detonate the first charge in the train, which is small and quite sensitive. By its action, each charge in the train initiates the next charge until the payload is detonated. The explosive train is discussed more fully under Safety and Arming, later in this chapter.

BASIC THEOREMS

The function of a weapon (such as a missile) is to deliver a warhead to a position relative to a target where detonation of the warhead will cause maximum destruction. The time at which the weapon reaches this point is known as the optimum time, the component of the missile warhead which initiates detonation is called the fuze.
The condition which must be met to ensure that the target is damaged requires that the destructive parameter of the warhead be active in the same volume of space occupied by the target and at the same time. To cause maximum damage to the target, activation of the warhead destructive parameter should occur when the target trajectory intersects the radius vector of maximum damage effect.
The radius vector of maximum damage effect, and hence the optimum time of burst, is a function of the missile trajectory and thus will vary with the type of target being attacked and the attack geometry involved.
If the fuze is effectively designed, it will choose the optimum burst time for a target whether or not the weapon has followed an optimum trajectory to that target. It is therefore possible for the fuze to initiate detonation of the warhead at a point which, although optimal for the trajectory in question, is still too far away to be effective. This would constitute undesired employment of the warhead if a target is available at a lesser distance. Thus, the fuze must be capable of refusing targets which are not within the effective damage range of the warhead. If weapon and target velocities are constant, and the relative weapon velocity vector (vector sum of the weapon and the negative target velocities) passes through the target, the weapon will hit, and is said to be on a collision course with the target. If a fuze detonates the warhead at some point along the weapon trajectory, the emissions from an isotropic warhead will be sent in several directions, and all will have the same speed relative to the weapon. It is obvious that only a portion of these emissions can travel in a direction to be on a collision course with the target. If the weapon is



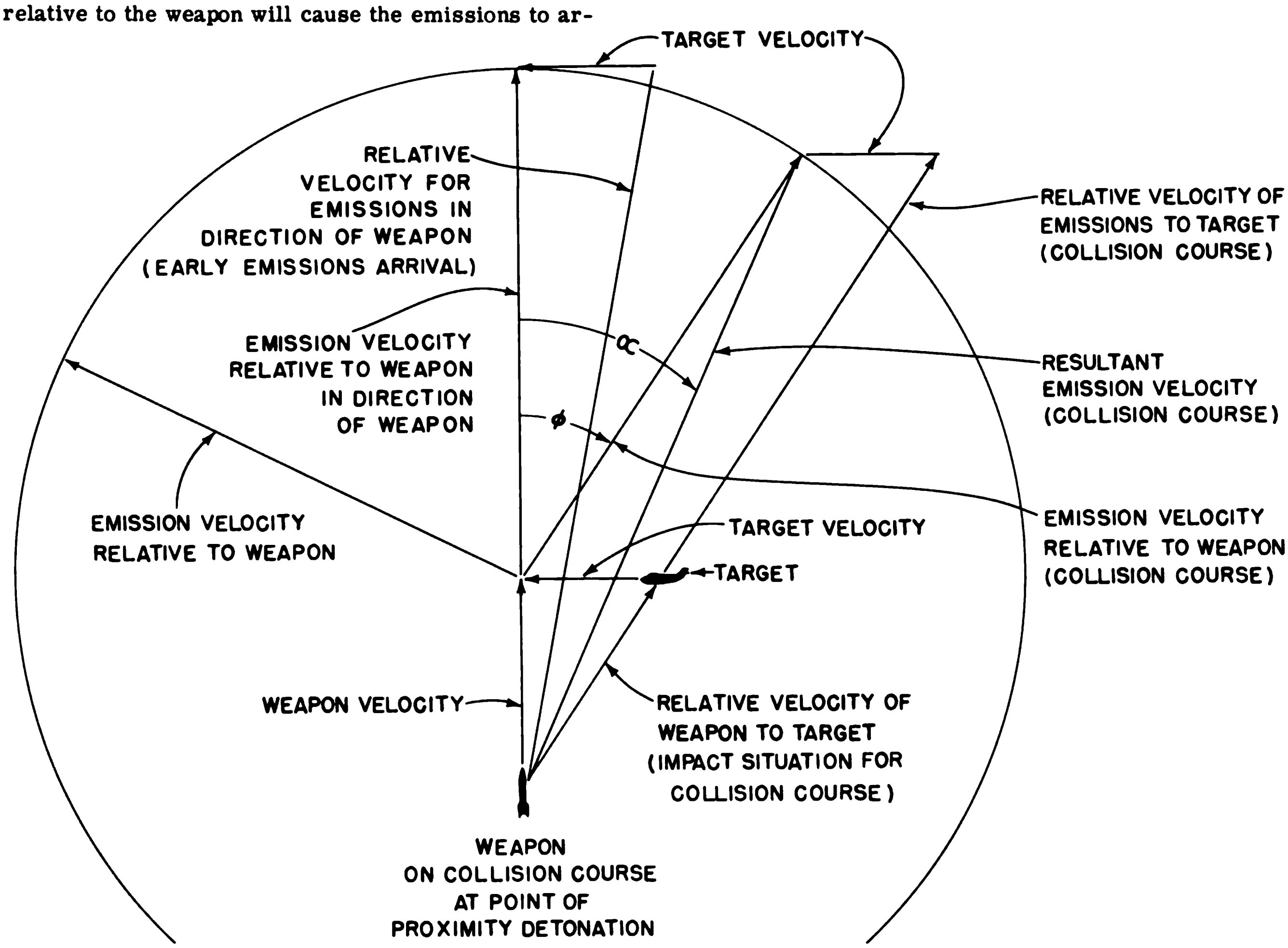
target trajectory intersection within lethal range

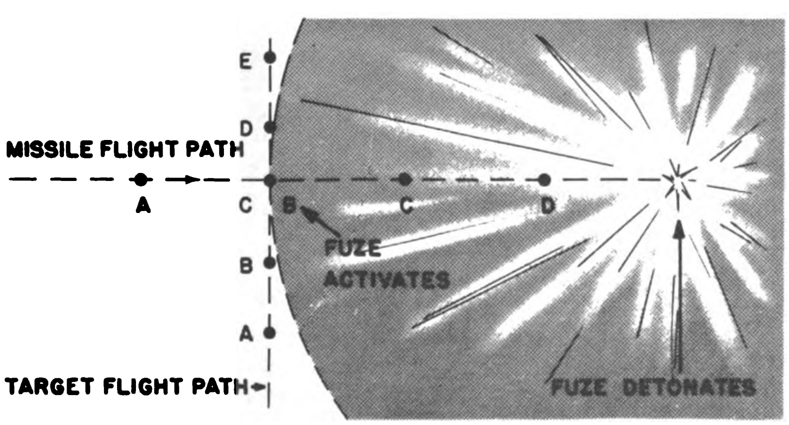


target recognition

Target recognition for guidance purposes generally requires that the target be recognized from a distance and that it be kept in sight until it is encountered. Since a target viewed from a distance appears as a point. It may be treated as such; this type of recognition therefore lends itself to relatively simply mathematical analysis. On the other hand, target recognition for fuzing purposes is many times more complicated because of the need for recognizing the target with sufficient detail to permit detonation of the payload at a point quite close to one of its vulnerable areas. Consequently, target recognition must be at close range. At close range, the near-zone phenomena (i.e., the effects of the size and shape of the target upon natural phenomena, such as electromagnetic radiation, which are used to sense target proximity) must also be considered.
Detailed target recognition is still further complicated by the high-velocity characteristics of present day aircraft and missiles which have greatly reduced the period of time during which the fuze is within sensing distance of the target. As seen in the illustration, a missile with a damage volume radius of 150 feet will not damage the target if the payload is detonated 0.1 second after fuze initiation. The target and missile are traveling at 1000 ft/sec. and 2000 ft/sec., respectively.

traveling on a collision course, then those emissions that travel in the same direction as the weapon, relative to ground, will miss the target unless the encounter is a pure head-on or tail-on type. Since the collision course computation was based on the speed of the weapon, it follows that the added velocity of the emissions relative to the weapon will cause the emissions to arrive at, and pass, the theoretical point of collision prior to arrival of the target. The vectors in the direction of ϕ and α are the relative velocity vector of the emissions to the weapon and the resultant velocity vector, respectively. The emissions having these velocity vectors will be on a collision course with the target.





fuze agent

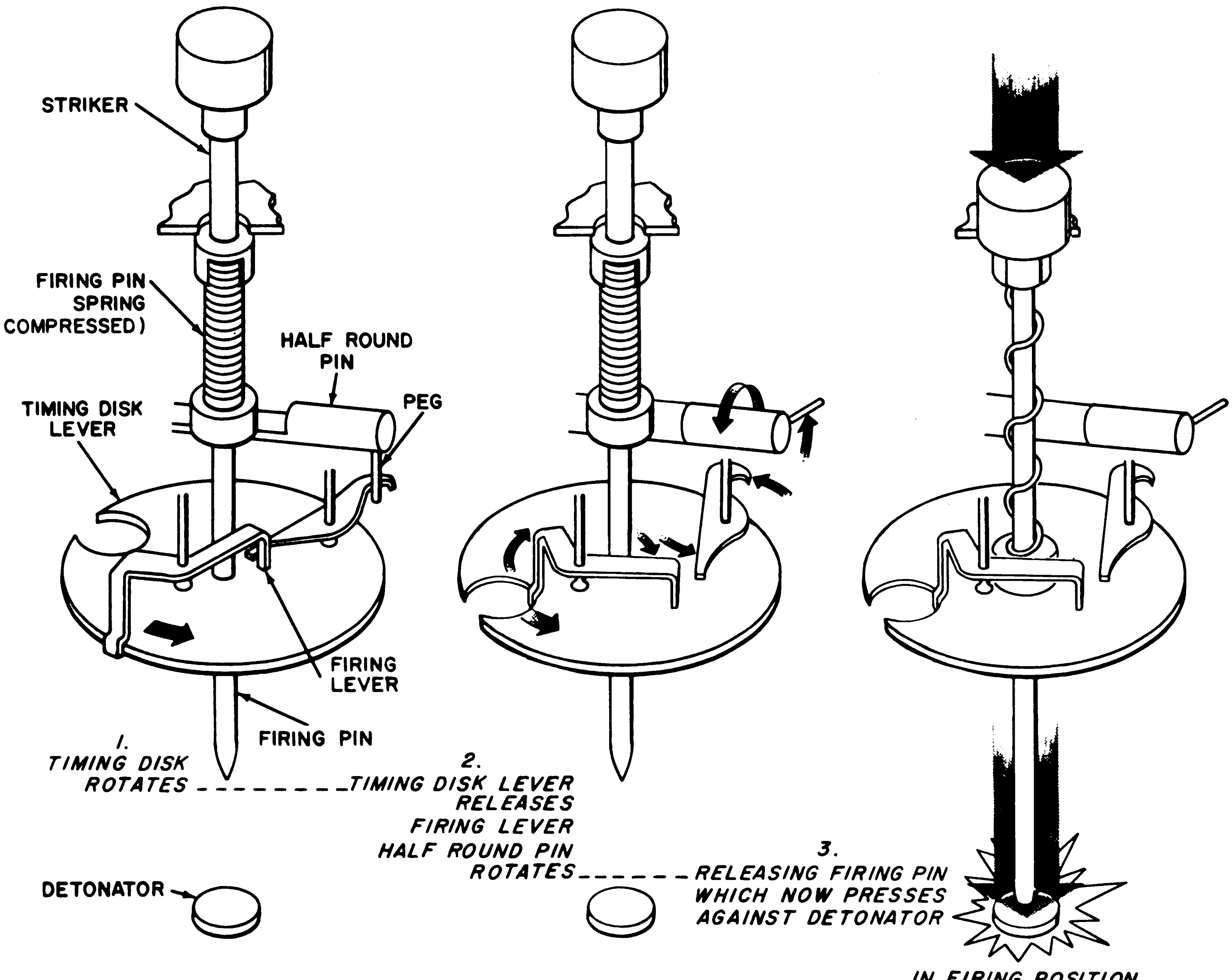
It can be seen from the above discussion that target characteristics such as size, shape, and velocity impose strict limitations upon the fuze agent selected for a particular attack. A fuze agent is any mechanism or phenomenon which may be used as a means to accomplish fuzing of a warhead. Some of the fuzing agents used are: time, impact; proximity agents such as electromagnetics, radioactivity, acoustics, magnetostatics and electrostatics; and ambient agents, such as cosmic ray distribution, and barometric or hydrostatic pressure distribution.

TIME FUZES

Time fuzes detonate a payload at some predetermined time, using a timing device of some sort as the means of detonation. This device, or "clock" mechanism, is actuated either by signals from the fire control and guidance systems, which represent the time of flight necessary for the weapon to reach a future predicted position of the target, or is preset if the target is stationary, in which case the time of flight can be determined beforehand. The clock mechanism can thus detonate the payload at the moment necessary to cause target damage. For a situation where the time of flight is known beforehand, a mechanical or powder-train type of "clock" mechanism is generally used; these fuzes are referred to as "mechanical" time or "powder-train" time fuzes.

mechanical time fuzes

The mechanical time-clock mechanism consists of a gear train and escapement mechanism driven by weights which are rotated centrifugally. The mechanism begins to operate as soon as the weapon is launched and, after a preset time interval, actuates a firing pin. The firing pin then causes detonation of the payload.



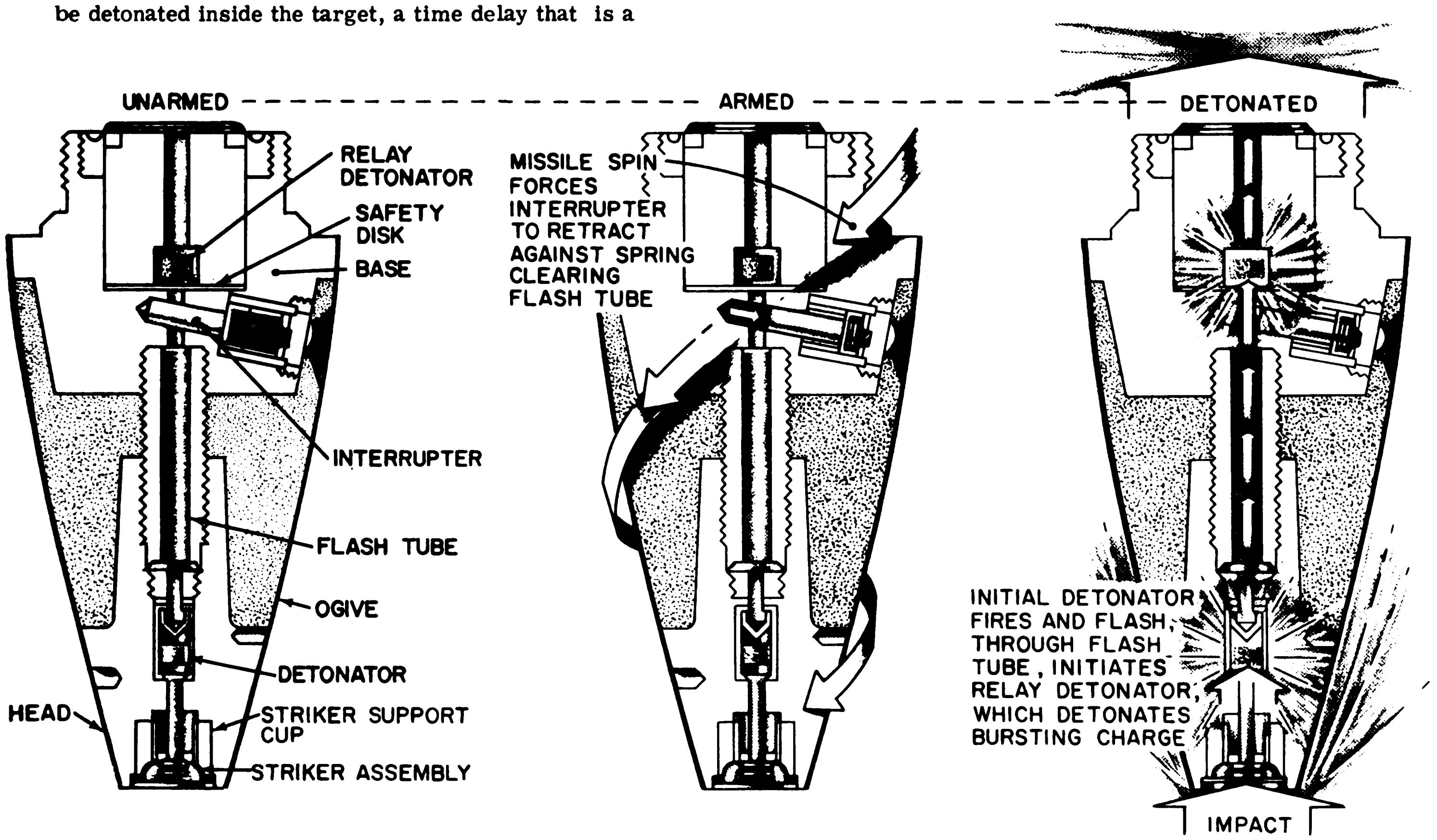
powder-train time fuze

The powder-train time fuze contains a timing device consisting of a compressed train of black powder. The powder train is graduated, and, since the burning characteristic of the powder is known, the burning action of the train can be set for a predetermined time. At the predetermined time, the powder train initiates the explosive element of the fuze which in turn detonates the payload.

IMPACT FUZES

Impact fuzes detonate the payload upon contact with the target. Detonation can be made to occur just outside or inside the target, depending upon the design of the fuze. Difficulty arises in the design of impact-type fuzes for targets which are not simple plane surfaces perpendicular to the line of flight and not extremely solid, such as an airplane. The "skin" of an airplane generally is so thin that it does not decelerate the missile enough, as it penetrates the skin, to actuate the impact element. This is true, especially in the case of high-speed missiles for which the impact element must be designed to withstand such high speeds. In addition, the warhead may strike at an acute angle, rather than perpendicularly to a target, because of the irregular shape of the target. In such case, the warhead may glance off the target without the impact element being actuated, because of the velocities present. When it is intended that the payload be detonated inside the target, a time delay that is a function of the striking velocity relative to the target surface is required. If the time delay were preset at some fixed value, it would, of course, be in error most of the time. However, if the delay problem is considered to be one of distance rather than time and the fuze is placed to the rear of the warhead, the required result may be obtained, since the warhead will penetrate the target before the fuze can function. If this location of the fuze does not prevent its operation, the intended delay is obtained.

If the target is a hard plane surface, such as a submarine, the above-mentioned problems are of no consequence, and neither is target motion, since surface or subsurface velocities are quite slow when compared with airborne missile velocities. The major problem associated with this type of target is that of functional reliability in the presence of large impact forces.



fuze actuation

An impact fuze is actuated as a result of physical contact with a target. The means of actuation may be through the acceleration of a mass, the joining of two points, or the removal or rupture of a barrier. The acceleration of masses may be used to initiate percussion detonators, break the light beam of a photo electric device, generate voltage by moving magnets, release chemicals, etc. The joining of two points may complete a circuit, and the opening of a barrier may permit the flow of some material and thereby cause actuation. Fuzes initiated at impact with the target generally afford the simplest solutions for fuzing requirements although, as previously stated, the design of such fuzes is by no means simple. All functioning components activate upon initial contact of missile and target. When properly designed, an impact fuze can be used to produce the detonation of a bursting charge at any point from the surface of the target to a measured distance within the target. The electrical or mechanical action of impact fuzes is usually actuated by moving a firing pin, completing an electrical circuit, or having a piezoelectric transducer initiate an electronic detonator.

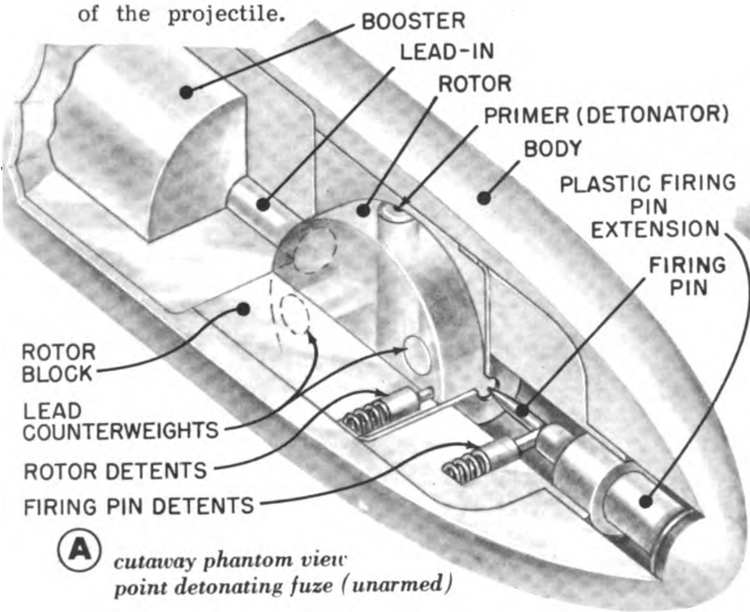
inertia firing

Inertia firing of fuzes is an important aspect of fuze initiation. The force of inertia tends to move all fuze components in a rearward lateral movement at the first moment of acceleration in the bore of a gun, or at the instant of launching a guided missile. The term used to describe this action is known as "setback".

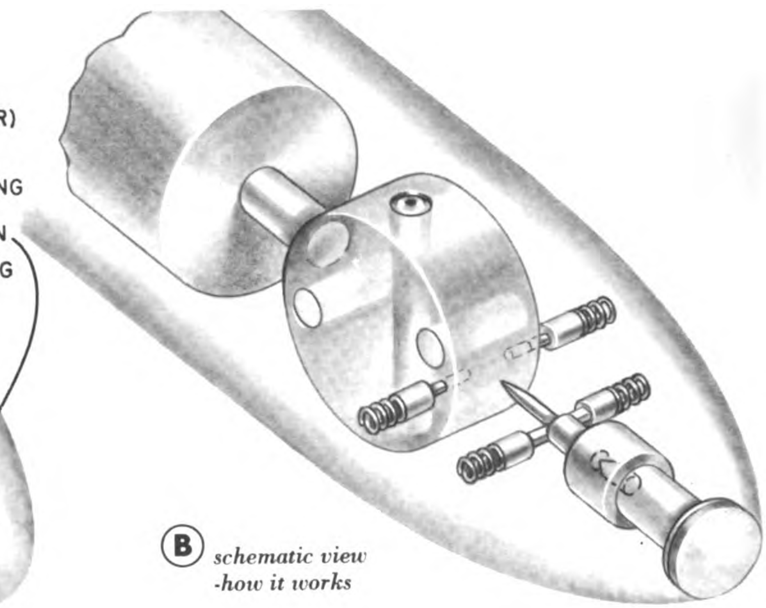
Angular setback refers to the force of inertia which tends to resist the initial rotational acceleration of a projectile in the gun. In flight, a projectile's spin brings into play centrifugal forces which tend to move all fuze parts radially away from the axis of the projectile.

Fuzes may be classified according to a location, such as nose or base, or functioning type, such as "point detonating" or "base detonating". A point-detonating fuze, located in the nose of the warhead, is designed to function upon impact with the target. These fuzes have the advantage of being faster acting after impact than base-detonating fuzes.

Illustration A shows a point-detonating fuze for use in projectiles. The fuze is designed to detonate a projectile with high-order detonation almost instantaneously

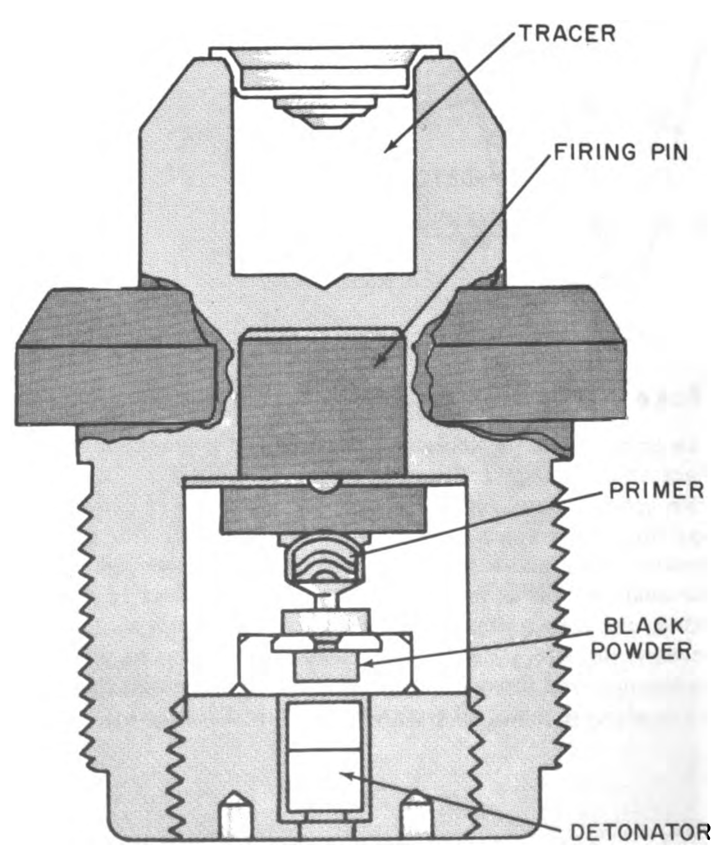


Ⓐ *cutaway phantom view point detonating fuze (unarmed)*

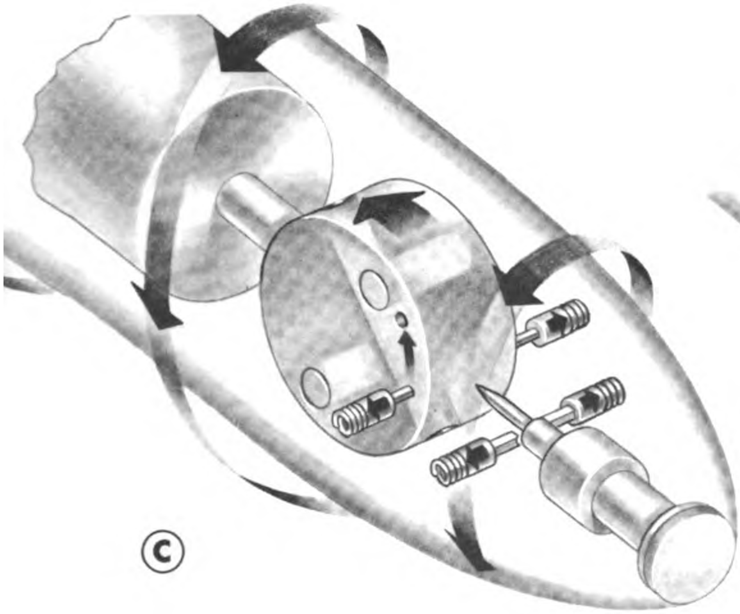


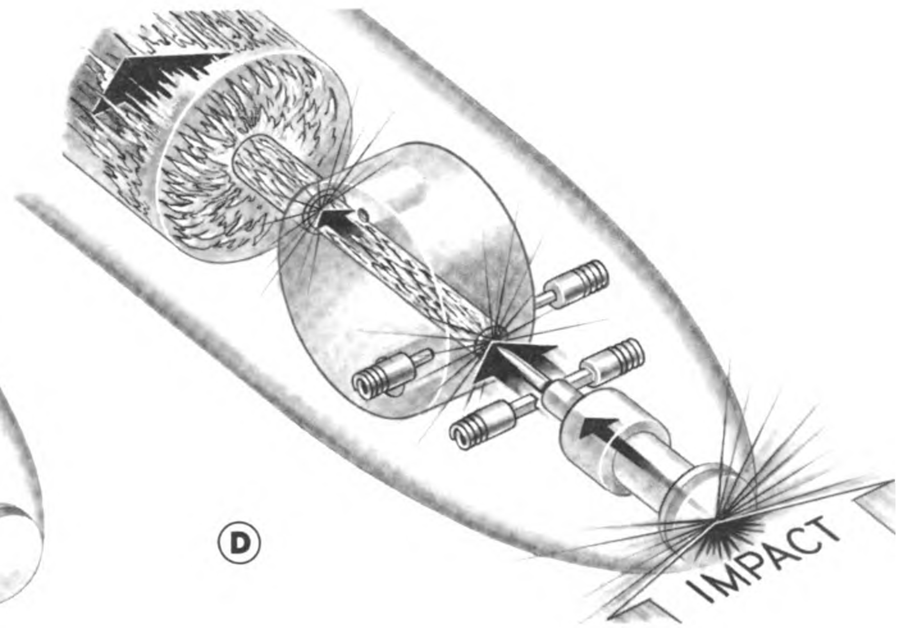
Ⓑ *schematic view -how it works*

Base-detonating fuzes, located at the base of the warhead, are of the delay-action type. Base-detonating fuzes are primarily designed for armor-piercing use. These fuzes are frequently used in dual-purpose projectiles in combination with nose fuzes. All base-detonating fuzes initiate detonation some measurable time after impact. One of the basic methods of delaying the initiating action of the fuze is by the use of delay pellets which burn for a predetermined time. After this slow burning time the detonator explodes, actuating the explosive train and culminating in the detonation of the main explosive charge.

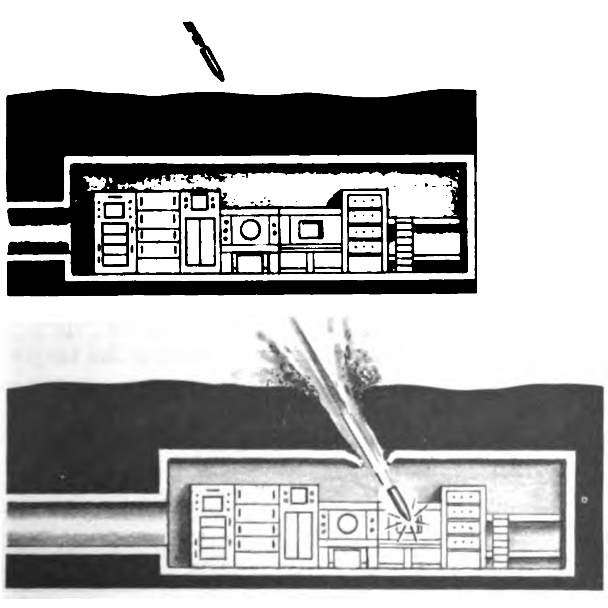


at the moment of impact. The action is as follows: the firing pin, as shown in illustration B, cannot move because of the locking action of the firing pin detents. Also, the detonator is not in line with the firing pin, so that in any eventuality the firing pin could strike the rotor body only, and not the detonator. When the projectile is fired, the following arming functions occur: The projectile, as shown in illustration C, spins as a result of the rifling in the gun, and the centrifugal force generated causes the firing-pin detents to move outwards, freeing the firing pin. The rotor detents also move outwards, freeing the rotor. The counterweights in the rotor body tend to move outwards, causing the rotor body to rotate until the detonator is in line with the firing pin. The fuze is now armed. Upon impact, as shown in illustration D, the firing pin is rammed backwards, striking and exploding the detonator. The detonator explodes the booster, which in turn detonates the explosive charge of the projectile. This instantaneous functioning is characteristic of point-detonating fuzes.

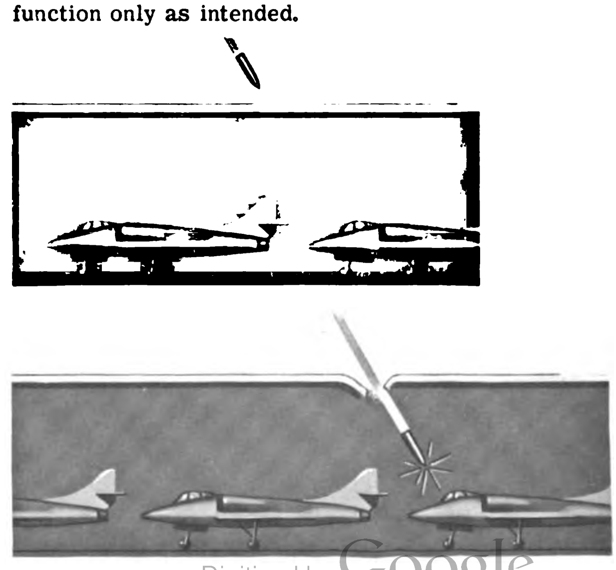




Deceleration of a missile in flight, caused by gravitational forces exerted upon the missile itself, and by changes in air resistance in the various media through which the missile travels, tends to move the fuze components in a forward direction. This inertia force is called "creep effect". The final moment of deceleration, which takes place at impact, must be at the point when explosive-train action is initiated. The fuze must be capable of response to the specified forces and incapable of response to all others.

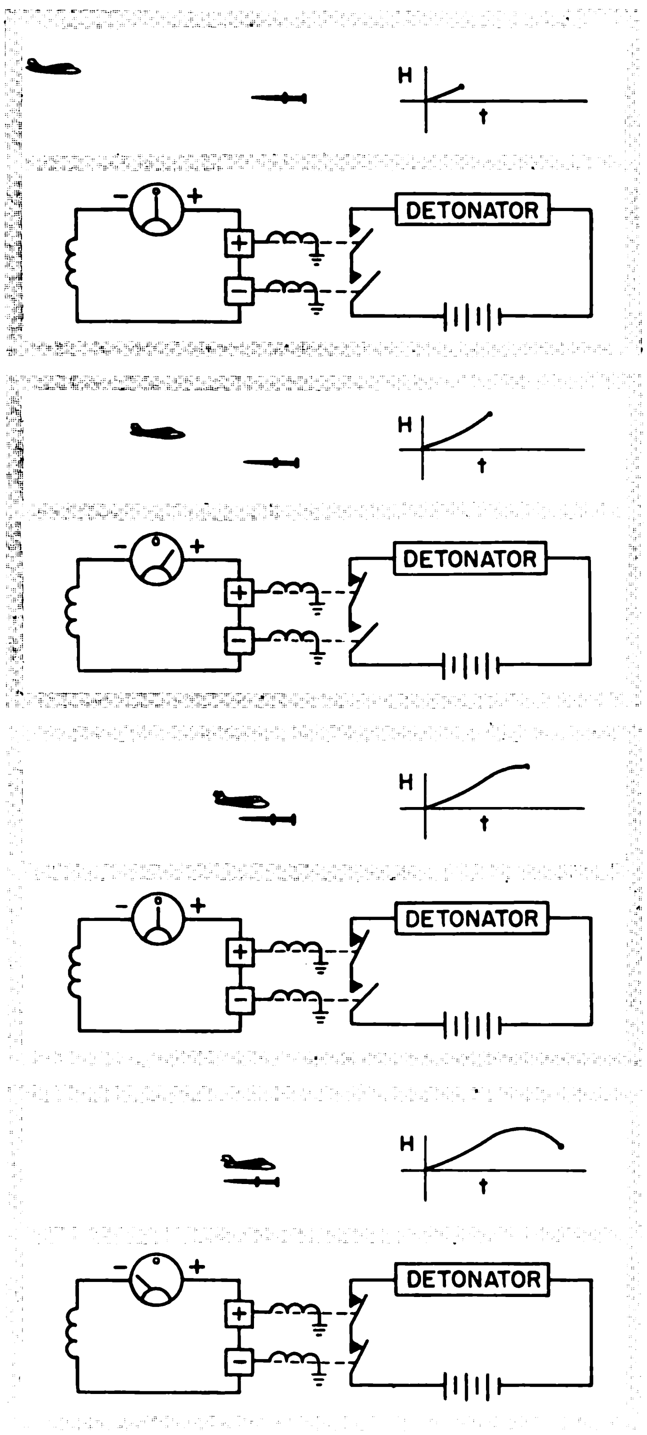


Delay fuzes are manufactured with varied built-in delay time. These fuzes are designed to penetrate material targets before initiating bursting action, and are usually employed in armor-piercing projectiles to attain complete penetration of the armor before the shell bursts. When a missile is fired for ground impact, a larger crater is formed. The functioning delay depends upon the design, and may vary from a few microseconds to 0.25 second. The location of the fuze unit is selected to protect the fuze during penetration of the target and provide a definite high probability that the fuze will function only as intended.



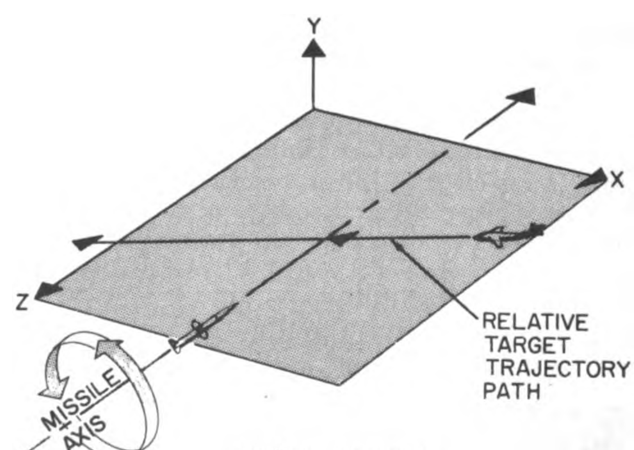
PROXIMITY FUZES

The function of a fuze is to initiate the detonation of a warhead at a time and place for maximum damage to be inflicted upon an enemy target. Proximity fuzes accomplish their purpose through "influence sensing" with no contact between missile and target. These fuzes are actuated by some characteristic feature of the target rather than physical contact with it. Initiation can be caused by a reflected radio signal, an induced magnetic field, an interrupted light wave, a pressure measurement, or an acoustical impulse.



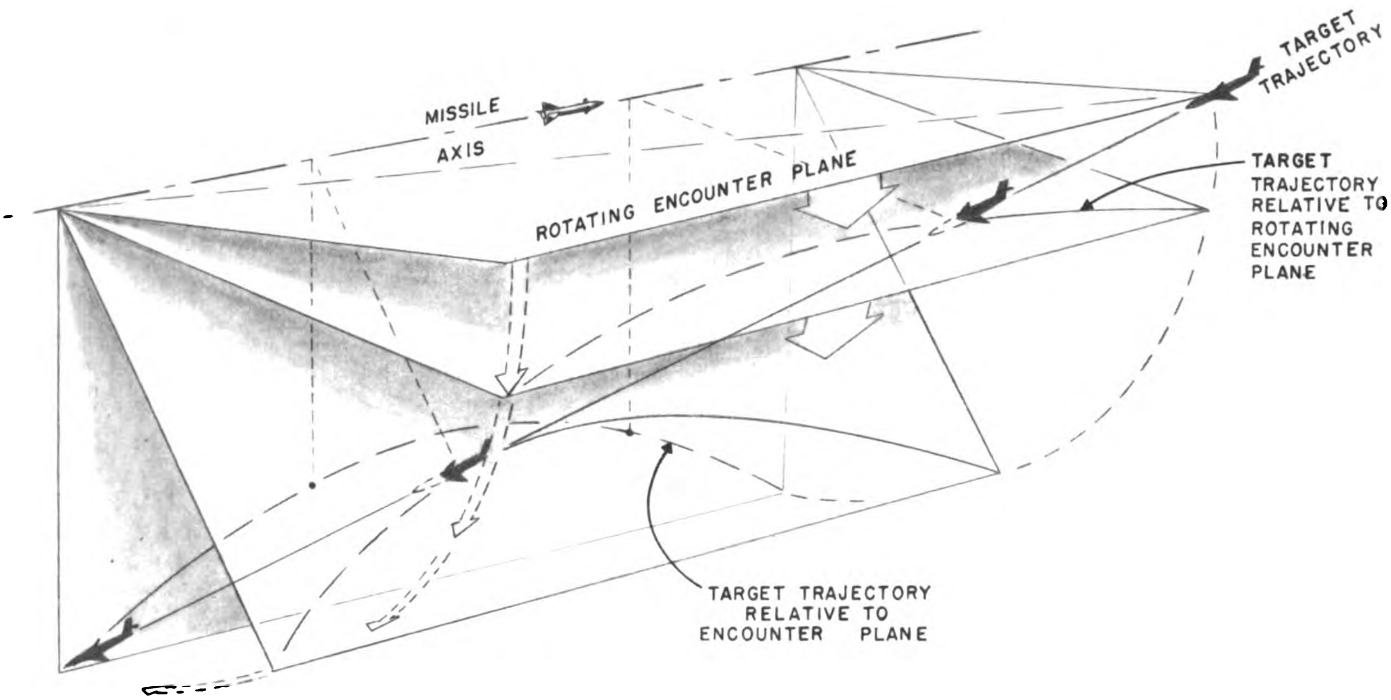
Since the warhead damage volume is preset, the achievement of maximum damage depends on the time of fuze initiation. "Proximity sensing" results in detonation of the bursting charge in the vicinity of the target or targets, a direct hit not being necessary for the net effect to be an enlarged damage volume. It is the function of the guidance system of the missile to deliver the warhead to a point where its kill capabilities are exploited to the fullest. Because of the speed and maneuverability of air targets, the missile carries its own sensing equipment for detection and fuze initiation. Since targets may exist simultaneously in large numbers, the sensing equipment must be able to isolate a selected target. The problem of fire control of proximity fuzes can be divided into three integrated areas: 1) Sensing the correct target within a particular frame of reference, 2) deciding the optimum moment for fuze initiation, and 3) selecting the fuzing agent to be employed. An understanding of the relationships existing in space between missile damage volume parameters and target trajectories is necessary in selecting the optimum moment of fuze initiation.

The coordinate system used to determine proximity for fuze action against air targets is based on the missile and is represented by a plane passing through the missile axis and the target. This plane will be referred to as the "encounter plane". In the analysis it is assumed that the missile and target are traveling with constant velocity and that their trajectories are straight lines. If the missile axis and the relative target trajectory (target path) intersect within the encounter plane, the encounter is coplanar.



coplanar encounter

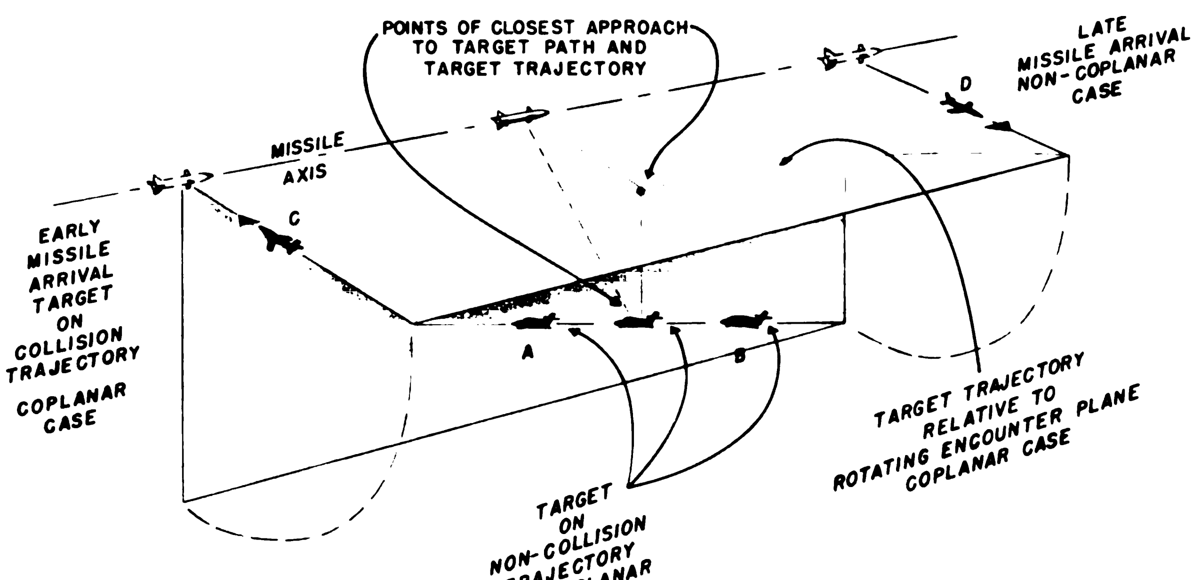
In the coplanar case the encounter plane remains stationary until the target passes the missile axis and then rotates 180° instantaneously. After the target has passed the missile axis, the encounter plane is again stationary. Thus the encounter in the coplanar case may be represented by two straight lines, one representing the target approaching the missile axis, the other representing the target moving away from the missile axis. These lines, of course, intersect at the missile axis.



If the missile axis and the relative target trajectories do not intersect, the encounter is referred to as non-coplanar. When the encounter is non-coplanar, which is the more common of the two types of encounters, the encounter plane (containing the missile axis and the target) rotates as the target moves either toward or away from the missile.

In the non-coplanar encounter, the encounter plane continuously rotates about the missile axis as the target moves and the target path usually appears as an hyperbola whose asymptotes are the straight-line target paths of the coplanar case. Collision trajectory occurs when the target path and the missile axis intersect at the missile.

It is more common, however, for the point of closest approach of the target path to be either forward of the missile, referred to as "early missile arrival", or aft of the missile, referred to as "late missile arrival"; thus, the target is attacked either before it reaches the missile axis or after it crosses the missile axis.



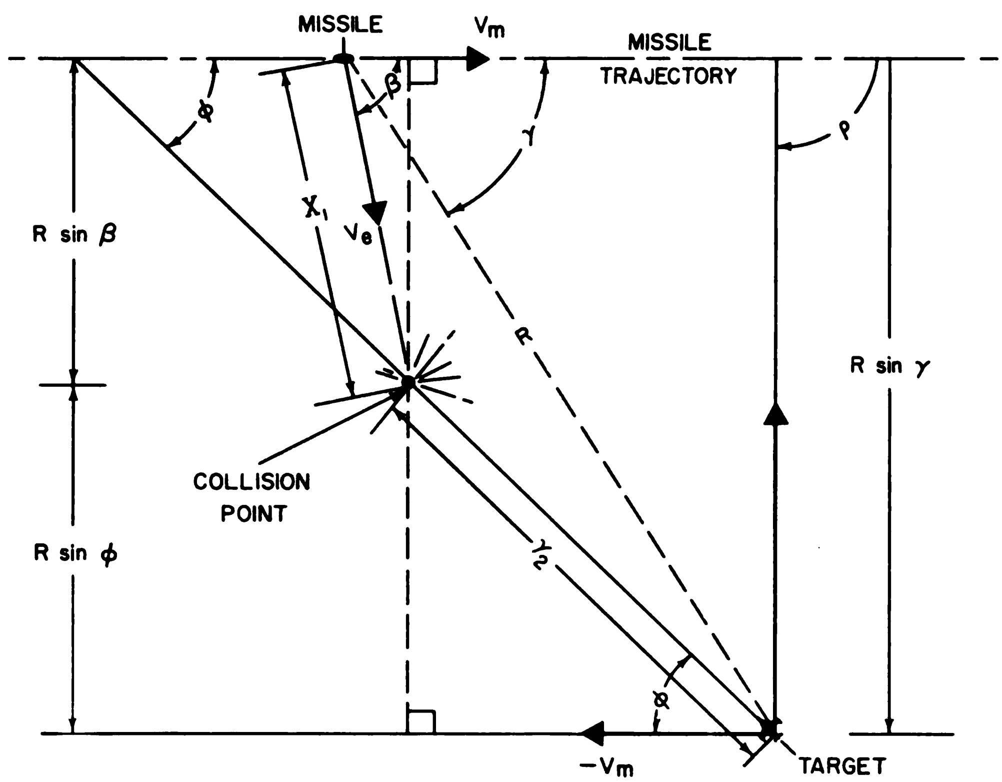
Each of these preceding cases, i.e., early and late missile arrivals, will now be examined in detail, and the following parameters, all of which are referred to the encounter plane, will be used:

V_m = Missile velocity vector
V_t = Target velocity vector
V_r = Target relative velocity vector
V_e = Warhead emission velocity vector

ρ = V_t elevation angle
ϕ = V_r elevation angle
β = V_e elevation angle
R = Line of sight from missile to target
γ = R elevation angle

EARLY MISSILE ARRIVAL

A vector diagram represents an early arrival encounter. It is assumed that at time zero the target and missile are in the positions shown. Time t′, the time which must elapse from the instant of sighting to detonation to ensure that the warhead emissions and the target will collide, must be calculated.



$$R \sin\gamma = X_1 \sin\beta + X_2 \sin\phi \qquad (1)$$

$$R \cos\gamma = X_1 \cos\beta + X_2 \cos\phi \qquad (2)$$

Therefore

$$\tan\gamma = \frac{X_1 \sin\beta + X_2 \sin\phi}{X_1 \cos\beta + X_2 \cos\phi} \qquad (3)$$

If t is the time required for the target to arrive at the collision point,

then $X_2 = V_r t$ (3a)
and $X_1 = V_e (t-t')$

Substituting these values of x in equation (3), we obtain the following relationship:

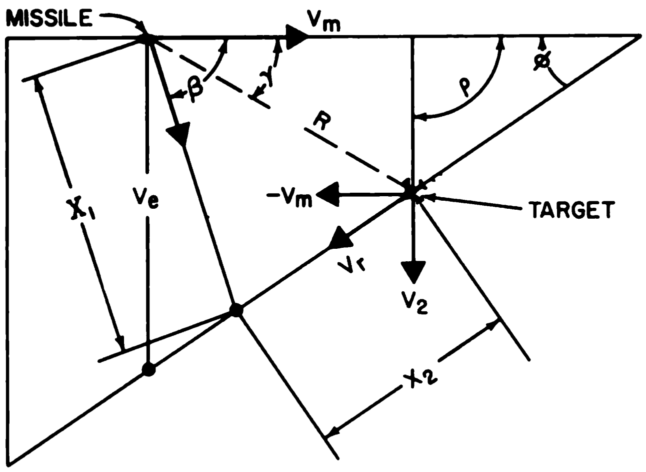
$$t' = \frac{(t)\ \left[V_e \sin\beta + V_r \sin\phi - \tan\gamma\,(V_e \cos\beta + V_r \cos\phi)\right]}{V_e\,(\sin\beta - \cos\beta\,\tan\gamma)} \qquad (4)$$

This expression gives t′ as a function of t which is generally unknown. We must therefore obtain an expression which is independent of t. If we assume the condition of zero time delay (t′ = 0), or the situation in which the warhead is detonated immediately upon sighting the target, then equation (4) becomes

$$\tan\gamma = \frac{V_e \sin\beta + V_r \sin\phi}{V_e \cos\beta + V_r \sin\phi} \qquad (5)$$

Thus, angle γ, between the missile velocity vector and the line of sight to the target, is defined.

Equation (5) indicates that if the warhead is initiated when the target relative velocity vector, V_r, has the elevation angle defined above, the warhead emission and the target are on a collision course regardless of the miss distance between the target and the missile.



LATE MISSILE ARRIVAL

In the case of the late missile arrival illustrated above,

$$\mathrm{Tan}\gamma = \frac{X_1 \mathrm{Sin}\beta - X_2 \mathrm{Sin}\phi}{X_1 \mathrm{Cos}\beta + X_2 \mathrm{Cos}\phi} \tag{6}$$

and

$$t' = \frac{(t)\left[V_e \mathrm{Sin}\beta - V_r \mathrm{Sin}\phi - \mathrm{Tan}\gamma (V_e \mathrm{Cos}\beta + V_r \mathrm{Cos}\phi)\right]}{V_e(\mathrm{Sin}\beta - \mathrm{Cos}\beta\ \mathrm{Tan}\gamma)} \tag{7}$$

Again, assuming delay time $t' = 0$, we obtain:

$$\mathrm{Tan}\gamma = \frac{V_e \mathrm{Sin}\beta - V_r \mathrm{Sin}\phi}{V_e \mathrm{Cos}\beta + V_r \mathrm{Cos}\phi} \tag{8}$$

By comparing equations (5) and (8), it can be seen that the early and late missile arrivals differ only with respect to the sign of the second term in the numerator.

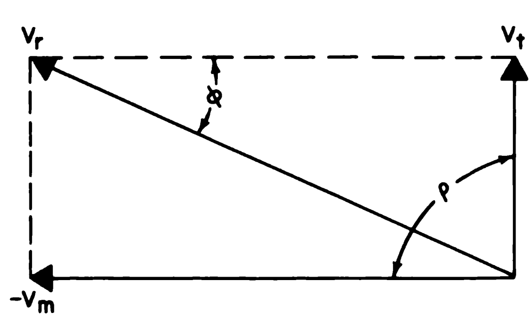
If parameters ϕ and V_r can now be related to the missile and target velocities so that ϕ can take on both positive and negative values through its entire range, then a single expression may be obtained for early and late missile arrival in which ϕ is positive for the early arrival situation and negative for the case of late arrival.

This can be accomplished through the use of another coordinate system represented by a plane identical to the encounter plane but fixed with respect to Earth, as illustrated. It can be seen that

$$V_r = -V_m + V_t \tag{9}$$

$$V_r \mathrm{Cos}\phi = V_m + V_t \mathrm{Cos}\rho \tag{10}$$

$$V_r \mathrm{Sin}\phi = V_t \mathrm{Sin}\rho \tag{11}$$



Thus, we have in the case

where $0° \leqq \rho \leqq 180°$ (11a)

Early missile arrival:

$$\mathrm{Tan}\gamma = \frac{V_e \mathrm{Sin}\beta + V_t \mathrm{Sin}\rho}{V_e \mathrm{Cos}\beta + V_m + V_t \mathrm{Cos}\rho} \tag{12}$$

Late missile arrival:

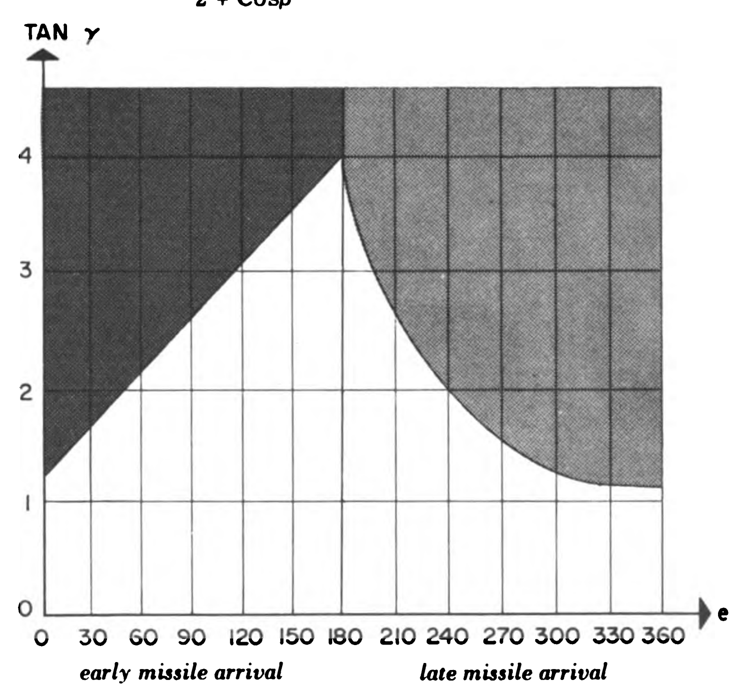
$$\mathrm{Tan}\gamma = \frac{V_e \mathrm{Sin}\beta - V_t \mathrm{Sin}\rho}{V_e \mathrm{Cos}\beta - V_m + V_t \mathrm{Cos}\rho} \tag{13}$$

Equation (11) may be used for either early or late missile arrival provided the following limits are used:
Early arrival: $0° \leqq \rho \leqq 180°$
Late arrival: $180° \leqq \rho \leqq 360°$
The quantities, V_e, V_m, and β, are generally fixed for a particular missile design. The unknown parameters are then target velocity V_t and its elevation angle, ρ. If we assume the following values:

$$\beta = \frac{\pi}{2},\ V_e = 4\ V_t,\ V_m = 2V_t \tag{13a}$$

and substitute them into equation (12), the equation reduces to

$$\mathrm{Tan}\gamma = \frac{4 + \mathrm{Sin}\rho}{2 + \mathrm{Cos}\rho} \tag{14}$$



A plot of this equation with ρ varying between 0° and 360° is shown. It can be seen that γ varies only about 26° for all directions of attack.
Location of the target with respect to some reference point may be established through the use of a rectangular coordinate, a cylindrical coordinate system, or a spherical coordinate system. The location of the target may also be expressed in terms of time intervals or polar measurements with respect to a reference event. Refer to the discussion of Reference Frames and Coordinate Systems, Appendix A, for a complete explanation of these methods of location.

modes of operation

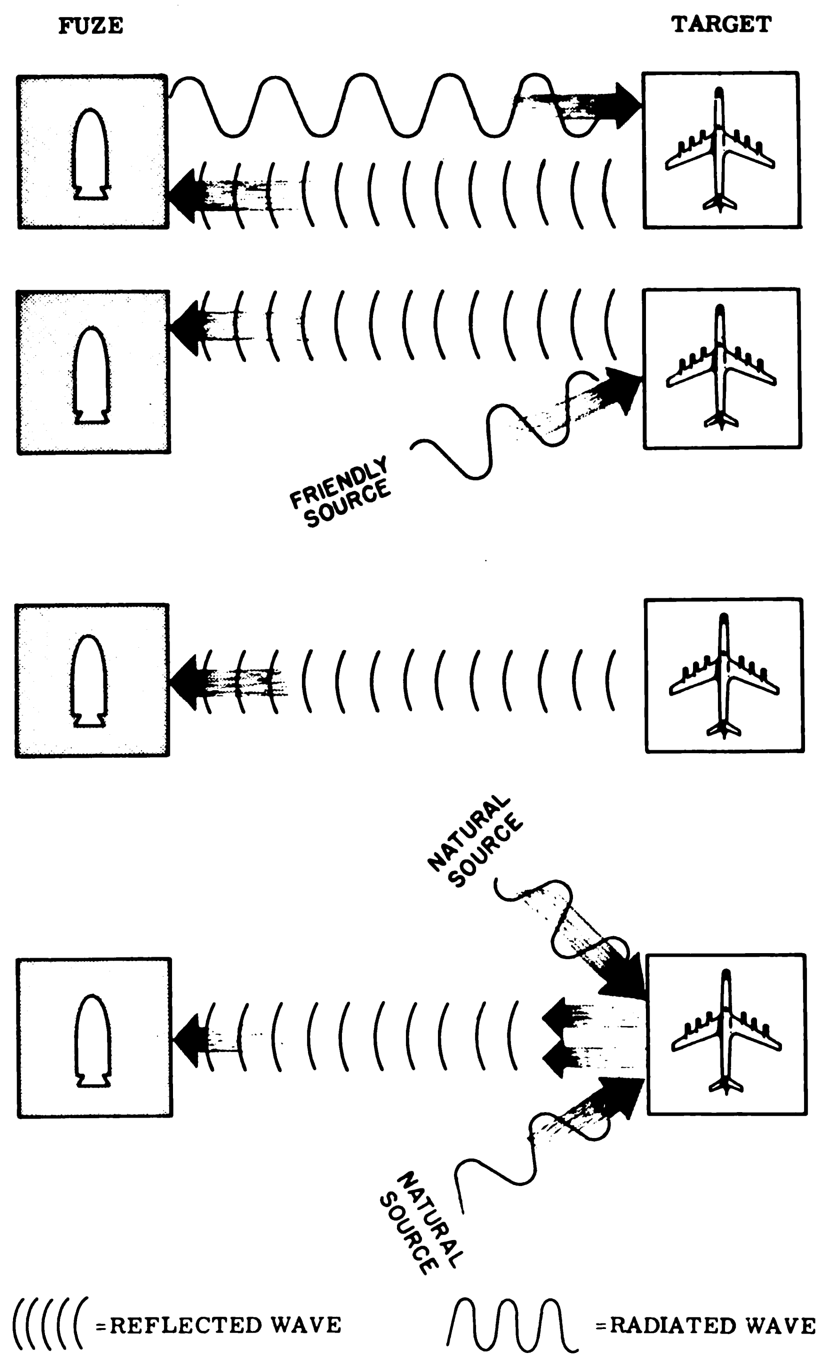
A fuze agent may be any mechanism or natural phenomenon used to initiate fuzing. For example, there are four modes of operation for the proximity fuze which employs some type of radiated energy as its fuzing agent:
This method of classification applies even when radiated energy is not used for fuzing. For instance, in a fuze where the agent is magnetostatic, the active mode would correspond to the fuze having its own magnetic field which the presence of the target would modify; it may measure the inherent magnetic field of a magnetized target, which corresponds to the passive mode, or it may detect the change in Earth's magnetic field caused by the target, which corresponds to the semi-passive mode. The active, semi-active, and passive modes are more commonly used than the semi-passive.

the active fuze
radiates energy, a portion of which is reflected back by the target detected by the fuze.

the semi-active fuze
receives energy reflected from the target which is transmitted from a friendly source other than the fuze.

the passive fuze
receives energy radiated by the target itself.

the semi-passive fuze
receives energy reflected from the target, which is radiated from an outside natural source.



fuze agents for proximity fuze initiation

Agents used for proximity fuzes include electromagnetic, magnetostatic, acoustic, and electrostatic fuzing. The sensors employed in such proximity fuzes are described in more detail in Chapter 2 of Volume 3. The electromagnetic spectrum has great applicability to fuzing in all modes. Certain basic laws govern the behavior of energy transmitted and received through all sections of the electromagnetic spectrum. Radiation patterns, attenuation and propagation constants, principles of generation, reflection and reflector surfaces, absorption, and other characteristics determine the applicability.

environmental factors

Conceivably all portions of the electromagnetic spectrum may be utilized as fuzing agents. Practically, however, consideration of propagation, attenuation, and other parameters affected by the radiation determine the applicability. The portions of the spectrum having the greatest utility are the radio, radar (microwave), infrared and ultraviolet. Propagation of radio and radar waves is similar to light propagation in the infrared to ultraviolet regions. Microwaves travel slower near Earth's surface than at high altitudes because the air is denser and the water vapor content is higher at lower altitudes. Thus, the waves tend to bend and follow the curvature of Earth at this time, and make fuze initiation difficult and complex. Another factor of importance in fuze initiation by electromagnetic means is the continual turbulence of the troposphere. This turbulence produces areas of low and high densities which affect wave propagation. Similarly there will be variations in water vapor content and temperature. These variations in the atmosphere produce in effect a scattering of the incident wave. There is an evident increase in transmission attenuation with increased rainfall, by scattering loss rather than absorption loss. At radar frequencies, the raindrops are large enough as compared with the wavelengths to scatter or reflect incident waves quite effectively, and, with countless raindrops in the path, a considerable portion of the incident energy can be scattered and lost. Some of the scattered waves can be picked up by the receiver and may show up as "grass" on the radar presentation. Weather radars make use of this phenomenon to chart the progress of storms. An electromagnetic fuze, particularly one operating in the radar region, may be constructed to operate much like a miniature radar set. It must transmit, receive, and identify electromagnetic pulses. However, the received signal in the fuze initiates an explosive train when the signal meets specific built-in requirements. The basic proximity fuze of the electromagnetic type has the following elements:

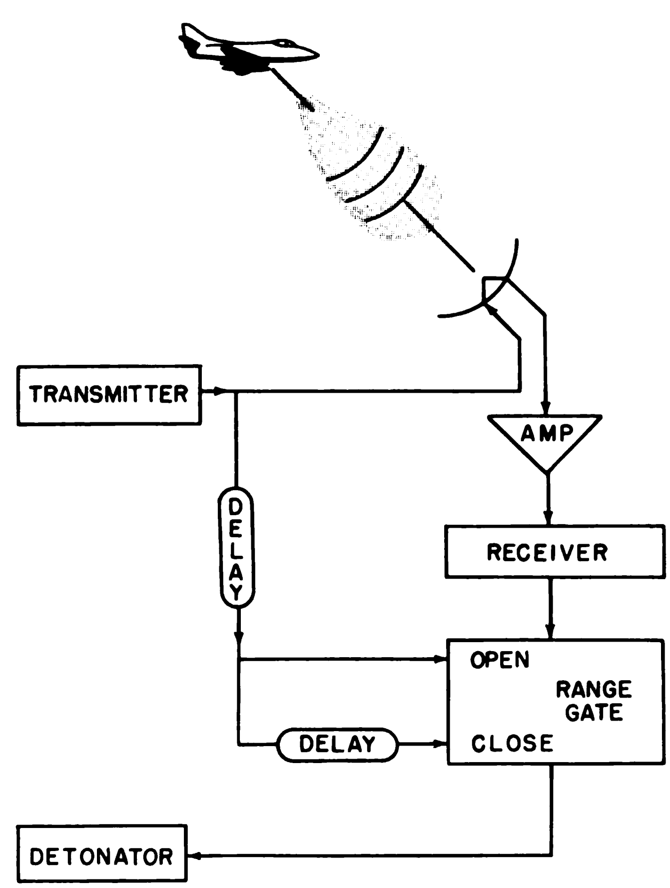
A TRANSMITTER-RECEIVER composed of either subminiature vacuum tubes or transistors, which is capable of delivering the required power for transmission and sensitive enough to sense the weak signal return.

AMPLIFYING CIRCUITRY to magnify the return signal so that it will activate the firing circuit and initiate the detonator. The receiver and amplifier circuits are designated to select the desired signal.

ELECTRICAL SAFETY DEVICE to prevent premature firing if error occurs. There are numerous devices of this sort, most of which operate on the premise that a delay time is imperative to place the warhead vehicle out of the range of friendly forces. A fuller description of these devices appears later in this chapter under Safety and Arming.

POWER SUPPLY to generate and provide electrical power for the fuze.

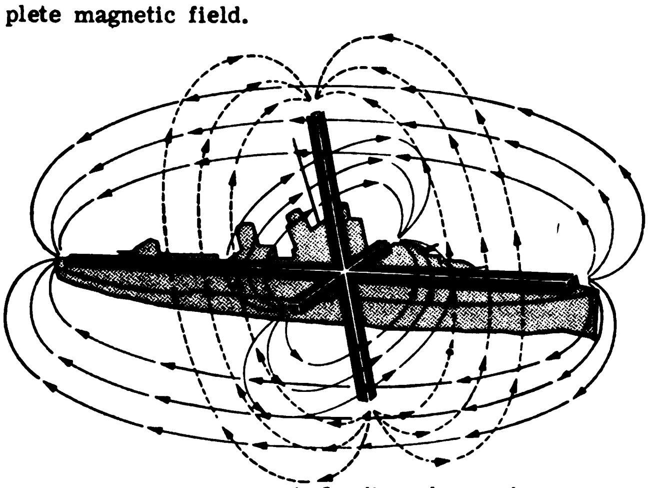
One means of signal selection makes use of the radar principle in which the elapsed time between a transmitted and received pulse is a function of range between target and missile. A "range gate" circuit set for a given distance will pass the signal to initiate the payload when the elapsed time reduces to a predetermined value. Another means makes use of the Doppler principle in which frequency of the received signal varies as a function of the relative velocity between missile and target. This permits the classification of targets according to their radial velocities, which is useful in the selection of a primary target in a heterogeneous target cluster.



magnetostatic fuzing

Magnetic sensors measure changes in Earth's magnetic field or the presence of a source of magnetic flux. The magnetic field that surrounds Earth can be visualized as being similar in character to the arrangement of iron filings which occurs when placed in the vicinity of a magnet. At any given point on its surface, Earth has a magnetic field, the value of which remains practically steady.

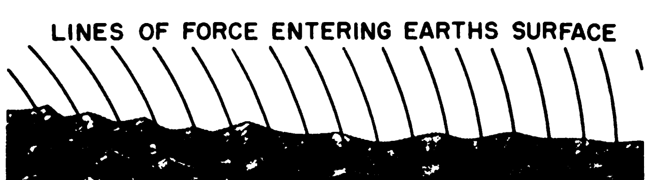
Every steel ship has a permanent magnetic field of force peculiar to itself. During ship construction, the metal hull lies on the ways in a fixed relationship to Earth's flux lines, which induces a permanent magnetic field in the steel of the hull. A ship's magnetic field has three main components: vertical, longitudinal, and athwartship, the sum total of which comprises the complete magnetic field.



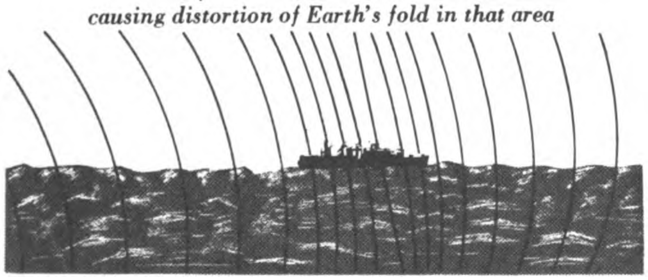
permanent magnetic flux lines about a ship

INDUCED FIELD

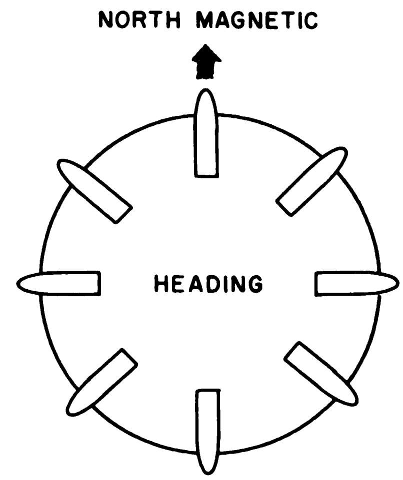
The steel in a ship also has the effect of causing Earth's lines of force (flux) to move out of their normal positions and be concentrated at the ship. The variation in Earth's magnetism caused by the ship is called the "induced field", which varies with the heading of the ship. The manner in which the longitudinal and athwartship components of a ship's induced field vary with changes in heading is shown.



ship has greater permeability, therefore lines of force tend to concentrate in her causing distortion of Earth's fold in that area

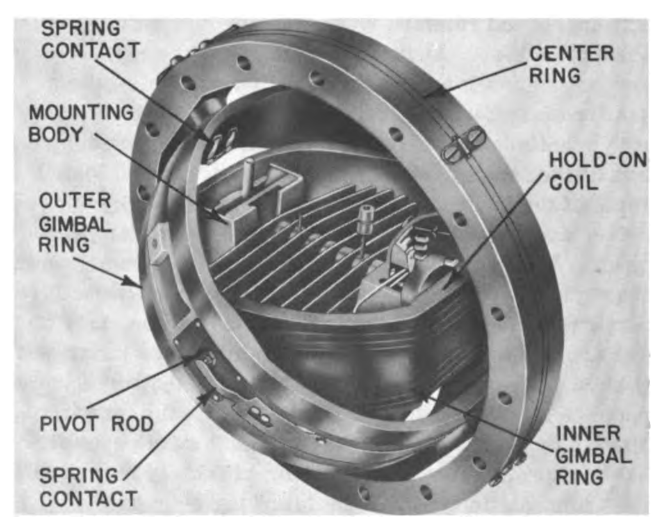


effect of ship's magnetic permeability on Earth's magnetic field



charges of induced longitudinal and athwartship magnetic components

A ship heading directly toward the north or south magnetic poles has no athwartship component in its induced magnetic field. A ship heading east or west (in the magnetic sense) has no longitudinal component in its induced magnetic field. For all other headings, a ship has a greater or lesser amount of induced athwartship and longitudinal magnetism in addition to its permanent magnetism. A ship's total magnetic field or "magnetic signature" at any point on Earth's surface is a combination of its permanent and induced magnetic fields. A ship's magnetic field may be reduced substantially by using degaussing coils, often in conjunction with the process of "deperming" (neutralizing the permanent magnetism of a ship); but for practical purposes it is not possible to eliminate such fields entirely. Degaussing coils are coils through which a current is passed in order to set up an electromagnetic field to counteract the permanent field of the ship or the effect of the ship on the Earth's magnetic field.



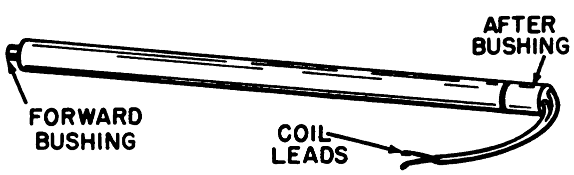
mechanisms of magnetic dip needle

The principles of magnetostatic fuze indicating devices are of two general types: 1) magnetic dip-needle, and 2) induction. The dip-needle mechanism of a magnetic mine is a combination of two or more magnetic needles acting as a single unit.

The dip-needle mechanism illustrated has a total of eight separate magnets which constitute the needle unit. When the firing mechanism is properly adjusted for a given latitude and longitude, the needle will stay in its nonfiring position as long as Earth's magnetic field remains normal. If a moving ship's vertical magnetic component disturbs Earth's magnetic field, the needle closes the open switch in the mine's firing circuit, initiating the explosive train and exploding the mine. Polarity of mine and ship are important. Magnetic dip-needle mines must have the same polarity as a ship in the planting area. In the northern magnetic hemisphere the N (+) pole points toward the north magnetic pole, and in the southern magnetic hemisphere the S (-) pole of the ship-induced magnetic field points toward the south magnetic pole. The firing mechanism of the mine must have the same polarity. A dip-needle proximity fuze is actuated by a change in Earth's normal magnetic field.

The speed of this change has no effect on initiating the fuze operation. As soon as the change becomes great enough, the needle initiates the detonating action. Even a stationary ship within the mine's effective range could actuate a dip-needle-type fuze when the last safety device ceased to operate.

INDUCTION-TYPE MECHANISMS employ the basic principle that a magnetic field induces current in a conductor as the field changes with respect to the conductor.



a magnetic search cell

Movement of a ship near a conductor or search coil produces a changing magnetic field, inducing a voltage in the extremely sensitive search coil. The small voltage in the search coil is amplified and energizes the firing current which in turn explodes the detonator. Some induction mechanisms use a sensitive relay for this purpose; others employ a highly complicated electronic or electromechanical system.

The needle-type mechanism operates upon the magnitude of the change in the magnetic field, whereas mechanisms of the induction type require both magnitude of change and rate of change. This feature gives the induction mechanism a practical advantage by allowing a wider diversity of selective firing. Induction firing mechanisms can be used in mines with ferromagnetic cases, while needle mechanisms require mine cases of nonmagnetic material. Induction mines are usually more sensitive than dip-needle mines, and much more difficult to sweep.

Magnetostatic fuzing is also used for subsurface targets. The magnetic field disturbance fuze for subsurface targets is also actuated by a change in the surrounding magnetic field; any change in the magnitude of the magnetic field activates the fuze. A magnetic-type fuze provides the possibility of damaging a target without a direct hit. This is important, as the damage potential is greater when the explosion of the warhead takes place several feet below the hull, rather than at the side, near the surface of the water. The most advanced methods of fuze initiation operated by a ship's magnetic field employ an electromagnetic detecting system. Such a system operates on what can be called the "generator principle". Essentially an electric generator consists of a coil of wire rotated in a magnetic field to produce a voltage. Similarly a small voltage is developed across a coil of wire (the search coil) when it comes in contact with a moving or changing magnetic field. However, a complex problem can occur, due to the fact that the movement of the interceptor itself (torpedo) through the water creates its own change in the field gradient and initiates the fuze at an earlier time than intended. This has led to the development of the gradiometer, a device attached to the torpedo, which has two search coils approximately one foot apart and connected in opposing series. As the now magnetically balanced torpedo moves in Earth's magnetic field, equal and opposite voltages are induced in the coils, and no net voltage results.

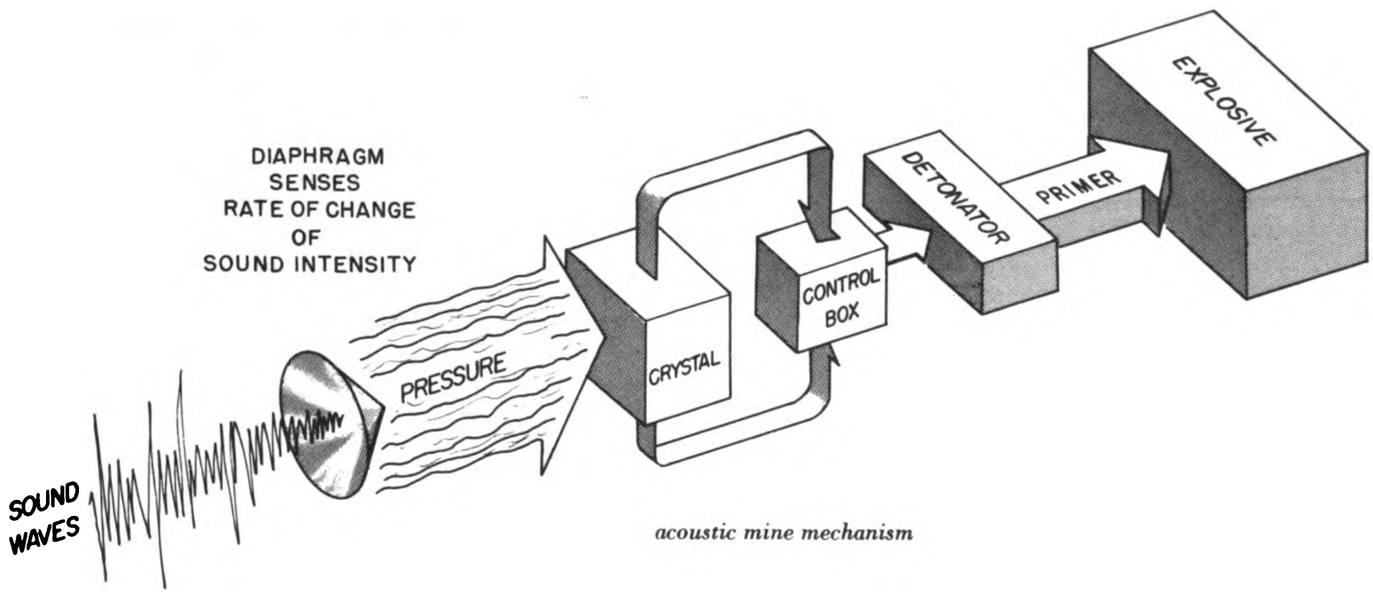
In the vicinity of a steel vessel, the situation is different. One of the two coils is slightly closer to the vessel than the other, and a slightly different voltage will therefore be induced in it. This difference is small but, when properly amplified, it causes the detonator to explode the main charge of the warhead.

MAGNETOSTATIC FUZING IN AIR SYSTEMS is being developed for use in missiles. Earth's magnetic field variations, magnetic disturbances, and limitations upon operation too close to the magnetic poles add to the problem of developing a dependable and accurate magnetic-type fuze agent. The most common sensor used in magnetic measurement is the magnetometer. This device measures various characteristics of Earth's magnetic field, which are useful in both sensing and fuze initiation such as: 1) lines of equal magnetic duration (isogonic lines), 2) lines of equal magnetic inclination or dip (isoclinic lines), and 3) lines of equal magnetic intensity. A line of equal magnetic intensity exists uniquely through a set of points and can be measured and charted. Any variation in the intensity can act as a fuzing agent. Equipment designed to measure the intensity of Earth's magnetic field (flux value) at a point or in a reference frame are still in the developmental stages. Variations in Earth's magnetic field would be sensed as flux density changes which in turn would position and guide a destructive agent to the target. This type of equipment would be used in conjunction with guidance systems in missiles. A principal advantage of such a system is that it is capable of sensing objects under water as well as in the air. Also, jamming of this equipment is a negligible factor in its usage.

acoustic fuzing

Acoustic disturbances, such as propeller and machinery noises or hull vibrations, invariably accompany the passage of a ship through the water. The intensity or strength of the sound wave generated depends upon several factors, such as ship size, shape, and type; number of propellers; type of machinery, etc. Therefore, a ship's acoustic signal is variable, and acoustic fuzes must be designed to prevent an intense signal from actuating the fuze at distances well beyond the effective explosive radius of the warhead.

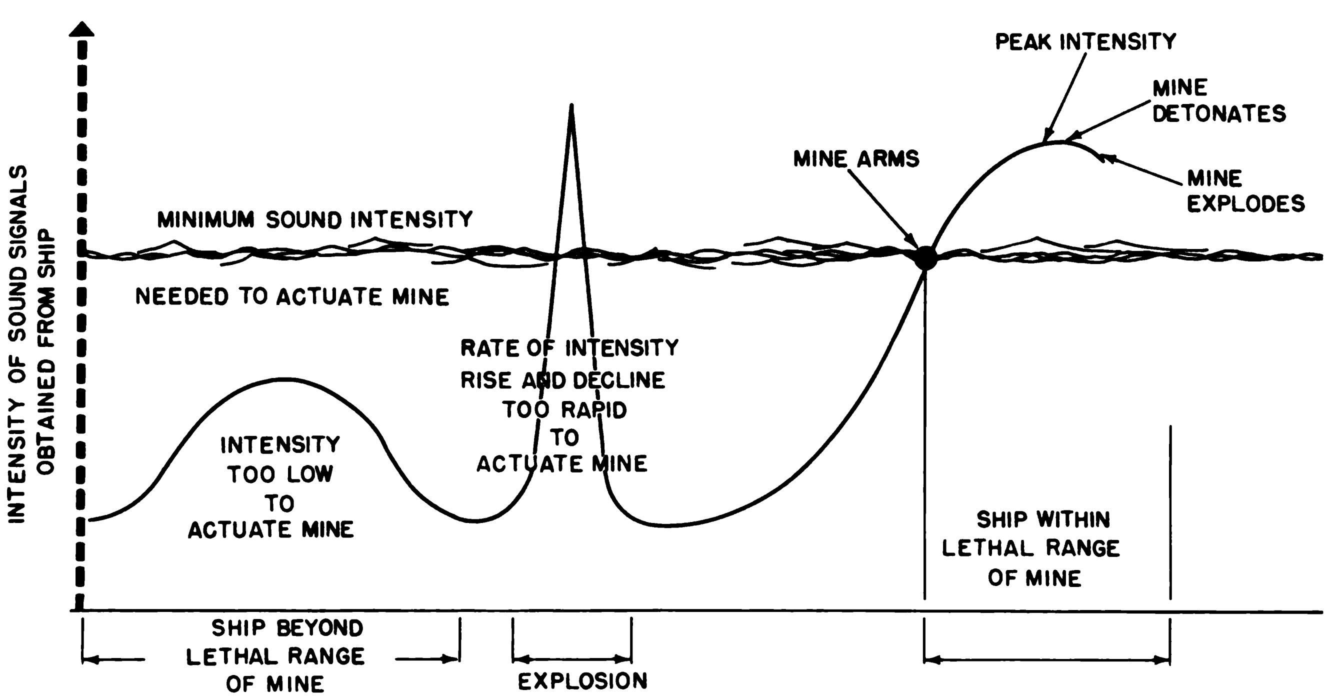
In an acoustic fuze, the diaphragm of the fuze's receiving element, called "hydrophase" or "transducer", vibrates when struck by a sound wave. Pressure exerted upon a crystal or other material generates a small electric current or voltage. To accomplish the change from mechanical energy (vibration) to electrical energy is the function of the transducer. The electrical signal activates the firing circuits, either directly or through an amplifier, to detonate the charge.



acoustic mine mechanism

Acoustic devices must be designed so as to be unresponsive to sounds representing non-target objects, such as a passing school of fish. Another design requirement is to prevent these fuzes from being easily actuated by countermining noises, such as nearby explosions.

Acoustic proximity fuzes are usually initiated by actuating the mechanism at a predetermined level of sound intensity. If the received sound builds up too quickly, or too slowly, the discriminating features incorporated in the fuze circuitry will not allow the fuze to operate.



Acoustic fuze mechanisms are used in torpedoes as well as mines. There are two operating modes of acoustic torpedoes: the active and passive types. The passive type has a homing device which guides the torpedo in the direction of the strongest target noise. The active type employs a sonar set which emits a series of sonic "pings" which are reflected back to a receiver in the torpedo. The principle is similar to that of a radar set. As the torpedo approaches the target, less time is required for a signal to travel to the target and return. At a predetermined critical distance, initiation of the firing circuits begins.

The speed of surface ships or submarines is relatively slow, in comparison with the speed of sound in water, whereas the speed of sound in air is Mach 1, which is lower than most missile and target velocities; therefore, the value of acoustic effects is limited primarily to underwater applications.

An acoustic firing mechanism generally utilizes an underwater microphone, called a hydrophone, as a detector to sense the presence of a target. A typical hydrophone functions much the same as the human ear. A diaphragm (corresponding to an eardrum) vibrates from the impact of underwater sound waves. These vibrations are transmitted through an oil medium to a crystal, which converts mechanical energy (sound vibrations) to the electrical energy required to initiate the firing mechanism. The selectivity of the firing mechanism is so critical that pulses of the required characteristics only are sent to the detonator. Selectivity is necessary because of the varied sounds which are received by the mine. To distinguish between these many sounds, acoustic firing mechanisms must possess a very selective type of hearing. For example, a 2000-ton tramp steamer may have one large propeller turning over rather slowly, or a 2000-ton destroyer may have two propellers turning over much faster. An acoustic firing mechanism can distinguish between the steamer, and the destroyer, and fire on the selected target. When the firing mechanism detects a sound which has the required characteristics (including intensity and rate of change of intensity), the mechanism initiates the firing circuitry and fires the detonator.

The U.S. Navy has no distinctive acoustic mine, but employs acoustic firing mechanisms as accessories designed to be interchangeable with other firing devices in several different mine cases.

electrostatic fuzing

Electrostatic fuzing has application over short distances. Fuzing of this type may utilize the active, semiactive, passive, or semipassive modes of operation.

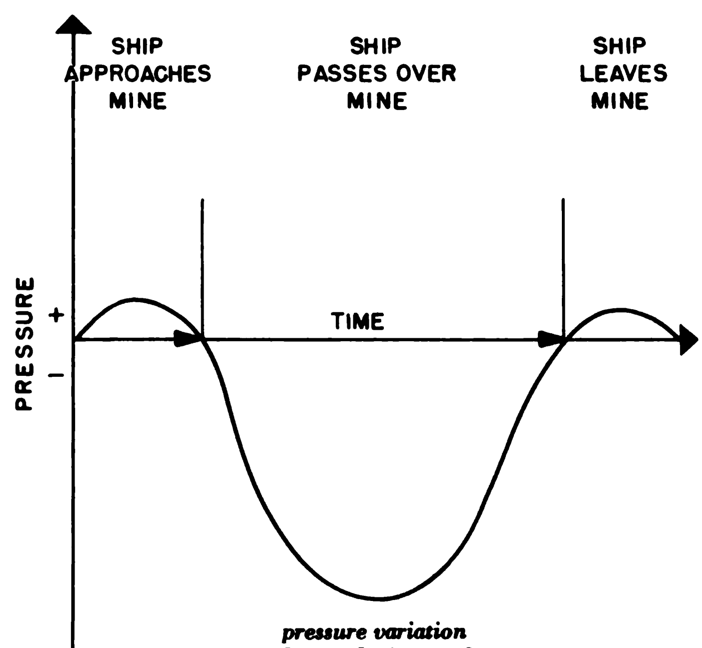
In the active mode, an electrostatic charge is placed on the weapon; any rearrangement of the charge due to the proximity of the target can then be sensed by the fuze. In the passive mode, the fuze is equipped with the capability of sensing an electrostatic charge, and any target having such a charge then becomes vulnerable. In this regard, it should be noted that air targets become electrostatically charged as they travel through the atmosphere.

In the semipassive mode, the distrubance of Earth's electric field by the target is sensed by the fuze.

One of the serious problems encountered in the use of electrostatic fuzes is caused by water vapor or rain dissipating the charge on either the weapon or the target.

hydrostatic fuzing

Ocean swells and surface waves produce pressure variations of considerable magnitude. Moving ships displace water at a finite rate. This continuous water flow is measurable, at considerable distances from the ship, as pressure variations which normally exist in the water. Various pressure measuring mechanisms can be used in fuzes to detect such variations. The pressure differential becomes more pronounced when the ship is moving through confined waters, but is still appreciable in the open sea even at a considerable depth. This pressure variation, called the "pressure signature" of a ship, is a function of its speed and displacement and the water depth. Therefore, to avoid premature firing because of wave action, pressure-firing mechanisms are designed so as to be unaffected by rapid pressure fluctuations. Pressure firing mechanisms are seldom used alone, but are generally combined with other influence firing devices.



pressure variation due to ship's travel

miscellaneous effects

There are other methods of obtaining proximity action. Possible proximity fuze agents include interpretation of the infrared radiations of a target, gravitational devices that indicate true vertical and gravitation effects, and detection of degree and direction of trajectory changes by employment of inertia devices. However, factors concerning the effectiveness of these methods are still under development. At this time, the electromagnetic spectrum offers the most convenient agent for missile fuzing.

AMBIENT FUZES

An ambient fuze is a type of proximity fuze which must be activated by some characteristic of the target's environment rather than the target itself. Assume that the guidance system of a missile or fire control system of a gun is adequate to bring the warhead damage volume to a position where it encloses at least part of the target; it then becomes necessary to initiate the fuzing action at the optimum moment. The primary consideration for correct fuze initiation is that the surrounding environment should be stable relative to the target. In the air, the environmental characteristic that lends itself to measurement and comparison is air pressure. The measured atmospheric pressure can determine the altitude or height of an object above sea level, and measurement is usually made by a barometer. Unfortunately, when used against a moving air target, an ambient type fuze system could not react swiftly enough to follow the almost instantaneous changes of altitude that a missile might undergo. However, an air pressure measuring device or altimeter can be useful in selecting the optimum fuze initiating moment against a stationary ground target. Since the target is on the ground, and its defense characteristics are known, the warhead can be detonated at a fixed height above the ground to make the warhead most effective. Other factors to be considered, and for which compensations must be made, are humidity, density, and temperature. Weather reports of a surrounding target area are essential for maximum damage potential.

The fire control problem for ambient fuzes operating against sub-surface targets involves a study of hydrostatics and underwater sensing devices that activate detonating circuits in moving or stationary warhead vehicles. Water pressure varies with depth, and is easily measurable. After a target is sensed and a warhead damage volume is in a position of maximum effectiveness, the fuze is initiated by a hydrostatic device.

ambient fuze agents

SURFACE AND AIR TARGETS

The mass of air surrounding Earth and attracted to it by gravitational forces exerts pressure upon Earth's surface as expressed by the equation:

$$p = hdg$$

where h = height of air layer
d = density of air layer
g = gravitational attraction of Earth

This atmospheric pressure can be measured by any one of several methods. The most common is an ordinary mercury column (barometer). The height (h) of the mercury column serves as an indicator of atmospheric pressure. At sea level and a temperature of 0° C, the height of the mercury column is approximately 30 inches, which represents a pressure of 14.7 pounds per square inch. The 30-inch column is used as a reference standard. At higher altitudes, the atmospheric pressure is less than at sea level, hence the height of the column of mercury settles lower. These variations in the height of the mercury column represent changes in atmospheric pressure, which may be calibrated in terms of altitude with reference to sea level. The change in atmospheric pressure with altitude is approximately exponential, changing more rapidly at lower altitudes because the compressibility of the air is greater at lower altitudes.

Atmospheric pressures create forces which are used to initiate fuze systems. The force produced by atmospheric pressure is calculated by the use of the equation:

$$F = pA$$

where F = total pressure on a surface
p = atmospheric pressure (lb./sq. in.) under specific conditions of altitude and density
A = area of the surface (sq. in.)

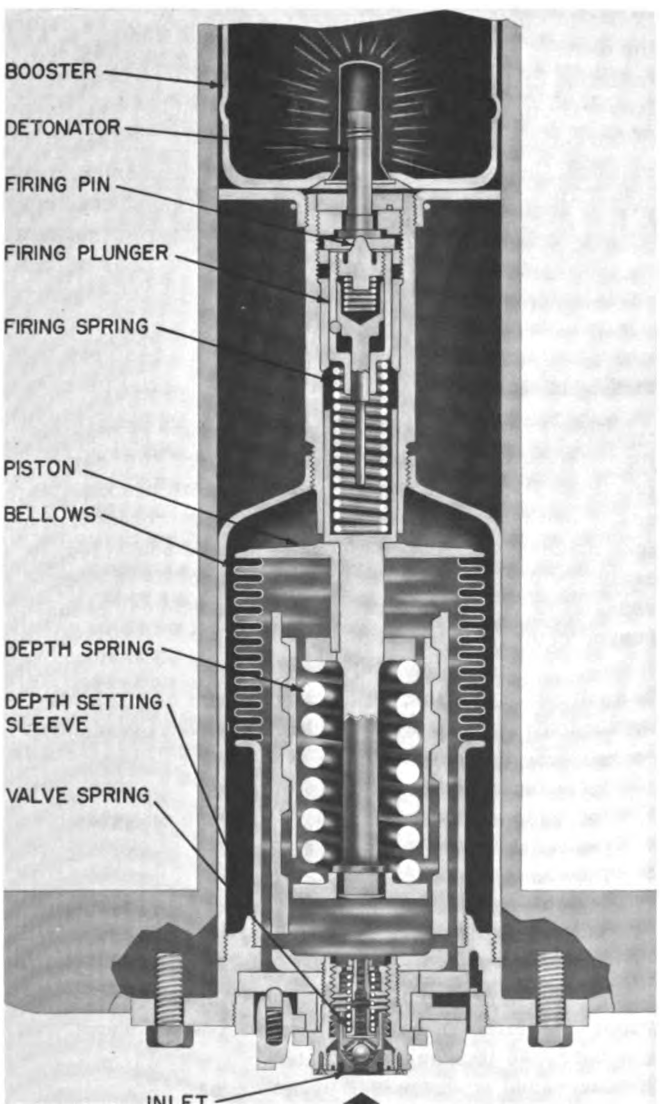
Altimeters are used to measure altitude. Two main types of altimeters are: pressure altimeters and absolute altimeters, or radio altimeters. A pressure altimeter is a mechanical aneroid barometer registering atmospheric pressure, calculated in feet instead of inches of mercury. The pressure altimeter, although calibrated in feet, actually measures atmospheric pressure at flight level, and interprets this value in terms of feet above a certain pressure level. Since the pressure at any level does not remain constant, the readings are only approximate, and its use as a fuzing agent is restricted.

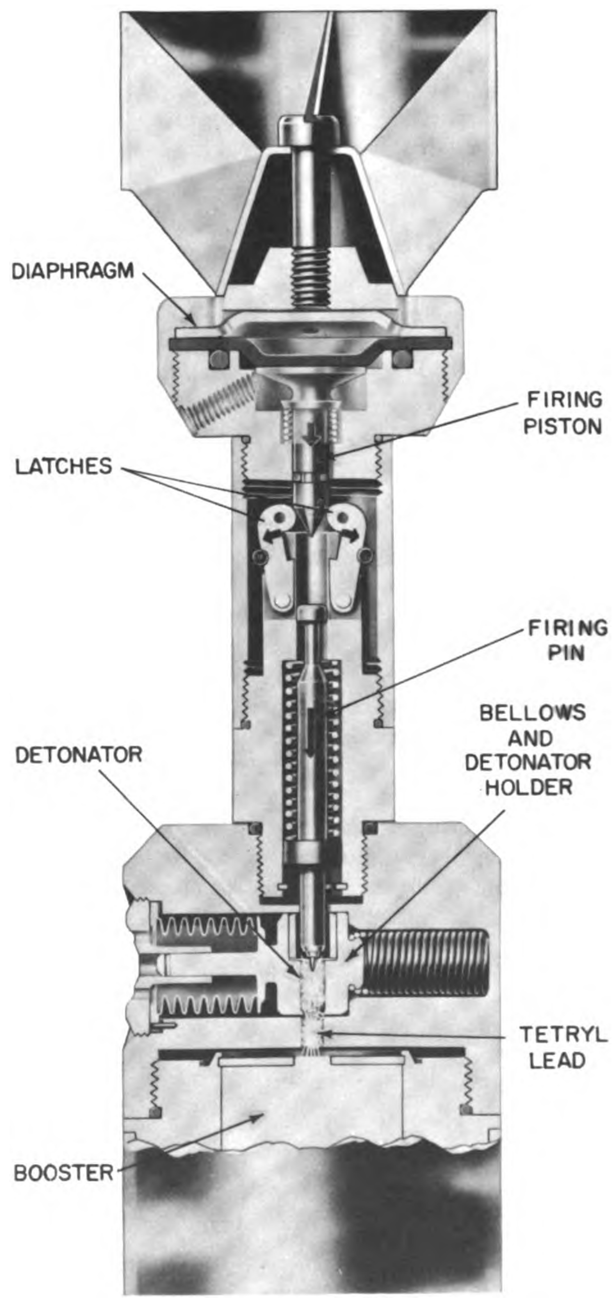
Frequency-modulated radio altimeters can be used automatically to maintain an aircraft or a missile at a preset altitude. It is also possible to use the radio altimeter as an ambient fuze-detonating device. In FM radio altimeters, a transmitter radiates a frequency-

modulated wave towards Earth. The reflected frequency is mixed with the carrier frequency, and the difference frequency generated measures directly the height of the warhead. The primary method of fuzing against surface targets, other than by impact or time, is by measuring the air pressure or altitude above a target area. Advantages of this method of fuze initiation are: the determination of altitude is rather simple, and the fuze possesses a high degree of immunity to countermeasures. One disadvantage is : the characteristic density, temperature, etc., of the environment surrounding the target area must be known. (Since a target may be thousands of miles away with the fuze preset to activate at a certain atmospheric pressure, it is difficult to make reliable predictions as to the accuracy of calculation.)

SUB-SURFACE TARGETS

The basic fuzing agent for sub-surface employment is the hydrostatic pressure indicator.
Pressure fuzes contain a firing mechanism that is actuated by a change in water pressure. Depth charges are thin-walled containers filled with a heavy charge of explosive and designed to explode at a predetermined depth by the action of a pressure-type fuze. The depth at which the explosion will occur is controlled by setting a hydrostatic fuzing mechanism to initiate the fuze at the intended depth. It is extremely difficult to obtain a direct hit with a depth charge because the exact position of the submarine is not known. Depth charges damage targets by underwater blast effect.





DESIGN CONSIDERATIONS

philosophy

There are few devices that must satisfy as many stringent requirements as a military fuze. Fuzes are designed to meet an almost limitless number of tactical situations and to function under all conditions with a high degree of probability. They are used with various types of projectiles, torpedoes, mines, rockets, and missiles. Each type has its own set of tactical requirements, environmental conditions, and safety and arming parameters, which govern the final arming design of the fuze. To be reliable, the fuze must activate the firing circuitry at the first optimum moment of subjection to the proper stimulus. It must also be safe in handling, free from deterioration in storage, simple in design and operation, and easy to manufacture and load. As complexity increases, the design problems become too numerous for one person. Therefore, it is necessary to divide the overall project into separate teams of specialists, each skilled in a particular phase of the project. A project engineer coordinates their work and presents pertinent information to all groups.

fuze specifications

Fuze specifications are originated by various agencies and research groups which specify what is required, what target characteristics may be encountered, environmental conditions that must be taken into account, launching sites to be used, carrier size, and other details pertinent to the fuze design problem.

design team

The specifications are studied and measured against existing devices. The length of time available may make it mandatory to modify an existing fuze rather than develop a new fuze. If the study proves that a new fuze design is called for, several teams are set up by the team head, which include research, design groups, production engineers, safety and arming engineers, test engineers, drafting section, and a liaison group which keeps each group informed of the progress of all other groups. Finally, a report and administration group sends reports to the field representative, furnishes information, etc. The final product may be for a guided missile, or a new influence-sensing mine. There may be a single approach, or a series of competitive designs. The final fuze must meet the specifications set forth.

reliability

The reliability of fuzes depends to a large extent on type of design (simple or complex), type of stimulus (impact, time, proximity, ambient), type of environmental conditions under which it must operate (air surface underwater), and type of countermeasures that are utilized against it. The operation of a fuze relative to a target is a function of many variables, including guidance and launching. In practice testing, keeping variables at optimum conditions, it is possible to measure the fuze activation mechanism and firing characteristics, and develop data that will include all deviations from the average. Fuzes are considered to perform in one of four ways: early, optimum, late, or dud. An early fuze is one in which the initiation of the fuze took place before the desired time. An optimum fuze is one in which the initiation occurs within a prescribed time interval (several microseconds). A late fuze is one in which the initiation occurs after the prescribed time interval. In a dud, the fuze fails to initiate. This dispersion characteristic of fuze performance is a consequence of being unable to build devices that are uniform in every manner.

redundancy

Additional reliability is achieved by the use of multiple fuze mechanisms, all designed to perform the same function. Redundancy is the term used to denote this costly method of increasing the reliability constant of a fuze mechanism. These mechanisms are connected in series or parallel to increase the probability of occurrence of fuze initiation at the optimum interval as discussed below.

In a parallel situation, either mechanism may initiate the explosive train. The effect of parallel redundancy on fuze function would show:

1. Increase in the incidence of earlies
2. Increase or decrease in the incidence of optimums
3. Decrease in the incidence of lates
4. Decrease in the incidence of duds

The result of a possible decrease in optimum fuzing is signified by an increase in the incidence of earlies. In the series situation, the fuze mechanisms are connected so that they must function simultaneously to cause initiation. The effect of series redundancy would show:

1. Decrease incidence of earlies
2. Increased or decreased of optimums
3. Increased incidence of lates
4. Increased incidence of duds

The decision on how to utilize fuzes in redundancy fashion depends on the probability distribution curves or earlies, optimums, lates, and duds of the component fuzes. The problem of redundancy is also applicable to safety and arming devices. A discussion of the probabilities involved is given in the section on Safety and Arming.

redundancy theory

A fuze can be considered to operate in four ways. It may initiate the payload at the optimum time (t), it may initiate the payload early (e), late (l), or it may be a dud (d). Mathematically this may be stated as:

$$t + e + l + d = 1$$

If two fuzes were used, such that if either could initiate the payload, the combination of optimum, earlies, lates, and duds would become:

$$(t + e + l + d)\ (t + e + l + d) =$$

$$t^2 + e^2 + l^2 + d^2 + 2td + 2te + 2tl + 2de + 2dl + 2el$$

Therefore t^2 is the probability of the two fuzes operating simultaneously, 2te is the probability of simultaneous occurrence of an optimum and an early, which is, of course, impossible. It can be seen that through the use of more than one fuze there are many more probabilities of target damage. Redundancy is thus the use of more than one fuze to increase reliability of fuze operation and therefore the probability of target damage.

We will now use capital letters to denote probability under redundant conditions. Thus the probability of optimum under redundant conditions is T, of earlies is E, etc. In addition, the fuzes may be used in a parallel arrangement (the case shown above) for which operation of either fuze will cause initiation, or in series, where it is required that both fuzes operate simultaneously to cause initiation. These operations will be noted by the subscripts p and s. Thus, optimums resulting from a parallel arrangement will be indicated as T_p and optimums resulting from a series arrangement will be denoted as T_s.

If we eliminate those terms in equation (2) which can not occur simultaneously in the optimum case, such as 2te, and all other terms which do not include t, then the probability of an optimum with two fuzes in parrallel is:

$$T_p = t^2 + 2dt + 2tl$$

By going through the same procedure for earlies, lates, and duds we may obtain the following relationship:

$$E_p = e^2 + 2te + 2de + 2el \tag{4}$$

$$L_p = l^2 + 2dl \tag{5}$$

$$D_p = d^2 \tag{6}$$

Equations (3), (4), and (5) may now be manipulated and rewritten as:

$$T_p = t\ (1 - e + d + l) \tag{7}$$

$$E_p = e\ (2 - e) \tag{8}$$

$$L_p = l\ (1 + 2d) \tag{9}$$

By examining equations (6) through (9) we can draw the following conclusions about two fuzes in a parallel arrangement. The incidence of optimums may be increased, decreased or remain unchanged; the incidence of earlies always increases; the incidence of lates generally decreases, and the incidence of duds always decreases.

We will now examine the probabilities for two fuzes operating in series. Remember that in a series arrangement both fuzes must operate simultaneously.

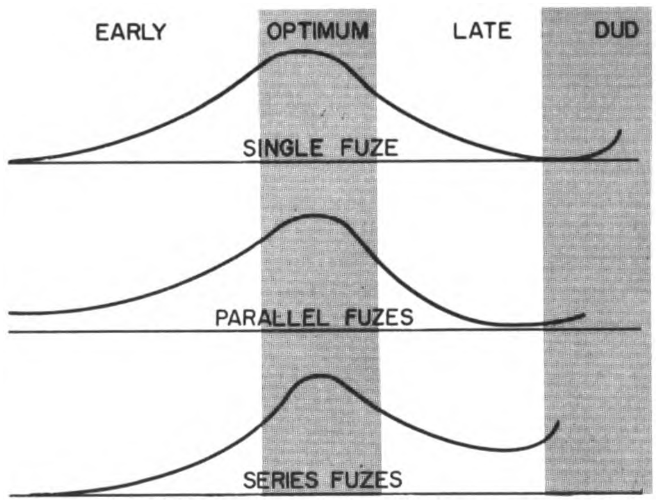
$$T_s = t\ (1 + e - d - l) \tag{10}$$

$$E_s = e^2 \tag{11}$$

$$L_s = l\ (2 - t - 2d) \tag{12}$$

$$D_s = d\ (2 - d) \tag{13}$$

It can be seen from these equations that optimums may again increase, decrease or remain unchanged; the incidence of earlies always decreases and the incidence of duds and lates always increases. Choice of a series or parallel arrangement depends on experience for a particular type of fuze as to its probability of optimums, earlies, lates, and duds. Similar derivations can help determine the increases in reliability when more than two fuzes are used in a series or parallel arrangement, or in a combination of both.



effects of redundancy on fuze accuracy

We have discussed redundancy and have shown that it provides more reliable fuzing. We will now examine its effect upon fuze accuracy, that is, how the number of optimum detonations is affected. For this examination we will make use of probability density functions which examine the probability of a situation occurring at a particular time or place during a series of trials.

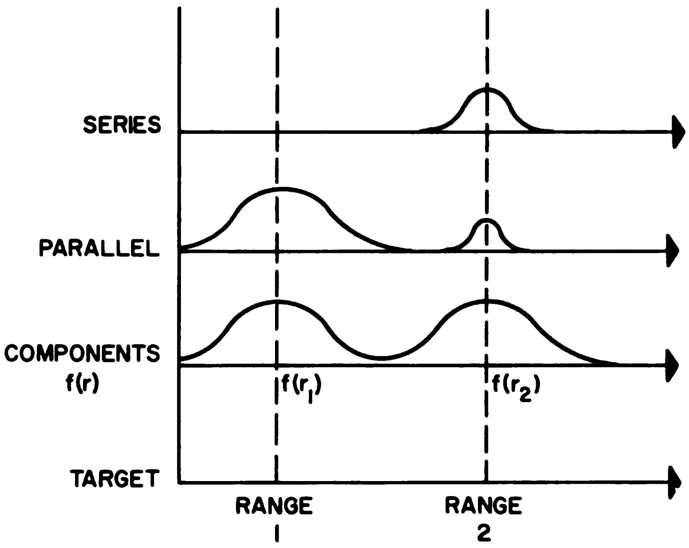
In the case of a fuze, the probability of its functioning is one minus the probability of a dud occurring, or 1-d. If we now denote the probability density as a function of range f(r), then the probability of fuzing at any range is:

$$F = \int_0^\infty f(r)dr \tag{1}$$

$$F = 1 - d \tag{2}$$

Since the following analysis concerns itself with the probability density function of a redundant system instead of an individual fuze system, the concept of early, optimum and late detonation will no longer be considered.

We will now check the effect of redundancy on the probability density function for the parallel and series cases.



parallel and series redundancy

The illustration shows two fuzes which are completely identical with the exception that they are preset for different ranges. The fuze with the greater preset range has a density function f (r_1), and the fuze with the lesser range, f (r_2).

The corresponding function probabilities, F_1 and F_2, are 0.8, and the dud probabilities, d_1 and d_2, are 0.2. The joint parallel function occurs whenever at least one fuze functions and the range at which functioning occurs is the range at which the first fuze functions.

We will now consider the probability of joint parallel function which is the sum of the probabilities that:

(1) both fuzes function
(2) first fuze functions; the other is a dud
(3) first fuze is a dud; the second functions

Since the probability of two events occurring is the product of the probability of each event occurring separately:

$$(F_1 + d_1)\ (F_2 + d_2)$$
$$F_1F_2 + F_1d_2 + F_2d_1 + d_1d_2$$

In a parallel fuze system, total probability (F_{t_p}) of the system initiating is:

$$F_1F_2 + F_1d_2 + F_2d_1 + d_1d_2$$

Since system initiation is the only criterion, d_1d_2 is eliminated. Therefore, conditions have been satisfied:

F_1F_2 (both fuzes function)
F_1d_1 (first fuze functions; the other is a dud)
F_2d_1 (first fuze is a dud; the second functions)
$F_{t_p} = F_1\ (F_2 + d_2)\ F_2d_1$

since $(F_2 + d_2)$ equals unity
$F_{t_p} = F_1 + F_2d_1$

This equation may be expressed in terms of a density function (f(r)):

$$F_{t_p} = \int_0^{\infty} f(r_1)dr + d_1\int_0^{\infty} f(r_2)\ dr$$

For the values of F_1, F_2 and d_1 given, the total probability is:

$$F_{t_p} = 0.8 + 0.2(0.8) = 0.96$$

For the series case, we must consider the situation after both fuzes function. If either or both fuzes are duds, the system fails to operate. Thus, in a series fuze system, total probability (F_{t_s}) of the system initiating is:

$$F_1F_2$$

which satisfies the condition stated. This may also be expressed in terms of the density function $[f(r)]$:

$$F_{t_s} = F_1\int_0^{\infty} f(r_2)dr = F_2\int_0^{\infty} f(r_1)dr$$

and for the values given:

$$F_{t_s} = (0.8)\ (0.8) = 0.64$$

There are many more complicated situations for which the effects of redundancy on the probability density function could be given: for example, the situation where the preset ranges of the fuzes are the same. Such situations, however, will not be discussed further in this text, since it is felt that the foregoing examples are sufficient to indicate the extent of the problem.

target and flight environment

Tactical requirements vary for specific fuzes, but every fuze will undergo a series of environmental stresses from manufacture to usage. The environmental conditions will influence choice of materials, packaging and sealing methods, operating temperatures, storage temperatures, and waterproofing techniques. The design problems differ for the various environments in which the fuze is to perform. A fuze operating against an air target must be able to withstand the accelerations and decelerations to which it will be exposed, while a fuze employed in a sub-surface device must operate against various pressures and water actions. The basic mission of a fuze is to function reliably when subjected to the proper target condition. Another important design consideration is safety in manufacture, loading, transportation, storage, and assembly to the carrier. In some cases the forces against which the fuze must be protected may be greater than the environmental and flight forces to which it is subjected.

Safety enters every facet of fuze development. Many fuzes are required to have at least two independent safety features to prevent initiation before firing. This philosophy is based on the remote probability that two features will fail simultaneously.

SECTION 3

introduction to SAFETY AND ARMING

PURPOSE

A fuze is said to have intelligence when it has the capability of controlling the detonation of the payload, until the time and under the circumstances desired. Fuze action may be subdivided into two phases: 1)functioning and 2) safety and arming (S & A). The functioning process has as its ultimate aim the energizing of the bursting explosive charge of the payload at the optimum moment of maximum damage potential. The safety and arming mechanism has a dual objective. Safety wise, it must prevent initiation of the payload until it receives a unique signal or series of signals that correspond or can be measured against built-in comparator devices. Once the correct sequence of events occurs, then the arming mechanism has the responsibility of removing or cancelling the safety features designed into the fuze circuitry, allowing the transfer of energy from the fuze initiating circuits to the payload via an explosive train. The fuze must be so designed that the payload will not be initiated or detonated prior to the desired time, regardless of forces or conditions to which it is exposed.

SAFETY

The safety system used to prevent premature detonation of the payload must be capable of meeting and controlling an extremely wide range of internal and external forces that are capable of initiating the fuzing action prematurely. The warhead must be able to withstand the rigors of transportation (vibration, acceleration, impact), the storage and operating temperatures to which it will be subjected and the environmental forces exerted on it during launch and in flight, and still be capable of selecting the proper stimuli for arming detonation. Technological advances in the design of missiles, new launching techniques, lower operational depths of submarines, and an increase in the complexity of countermeasures employed against such systems, have all led to more advanced techniques of science and engineering to insure against the probability of undesired detonation.

ARMING

Arming is the process of removing the safety features built into the warhead, and readying the warhead for initiation at the optimum moment. The shift in status from an unarmed to an armed mode can be accomplished by proper utilization of the natural phenomena which occur during the flight or trajectory of a projectile or missile. Sensing circuits measure and evaluate certain characteristics of the environment, and with built-in compensating networks to account for variations in stimuli, the arming sequence occurs at the optimum moment, and in a zone of maximum damage volume, increasing the destructive efficiency of the weapons in which they are used.

S AND A

will be used as the abbreviation of

SAFETY AND ARMING

TRANSFER FUNCTION OF S AND A

The primary purpose of the safety and arming (S & A) mechanism is to insure the transfer of energy from the fuze to the payload at the optimum moment and to prevent the energy transfer from occurring prior to the optimum moment. The transfer must occur after completion of the armed mode. Arming may be defined as the act of completing or aligning the signal or explosive path from fuze to payload.

For maximum destructiveness, the payload contains a powerful but fairly insensitive charge of explosive that requires an energy level of some magnitude to trigger or initiate it. As the output energy level of the sensing device is normally low, several stages of energy amplification are necessary to raise the level to a value that will insure actuation of the payload. An explosive train or energy amplifying network is designed to amplify the energy output of a sensing device to a desired energy level, either by electrical or chemical means. In its truest sense then, the transfer action of an S & A mechanism encompasses a detonation or energy amplifying path from fuze to payload.

DETONATION PATH

An identified response signal from a sensing device initiates fuze action. Fuze initiation methods essentially are either mechanical or electrical in character, and often are used in combination. Mechanical initiation takes place by stab, percussion or pinch action, and is primarily utilized in contact or impact type fuzes. In mechanical fuze types, contact sensing is converted directly into mechanical movement of a firing pin which, in turn, initiates a relatively small sensitive primer. The transducer action of converting mechanical into chemical (explosive) energy is the primary function of this stage. A mechanically operated fuze is comparatively simple to design and manufacture, and it has a high degree of reliability. Unfortunately it is inherently slow in operation, and cannot be utilized when required operational timings are measured in microseconds.

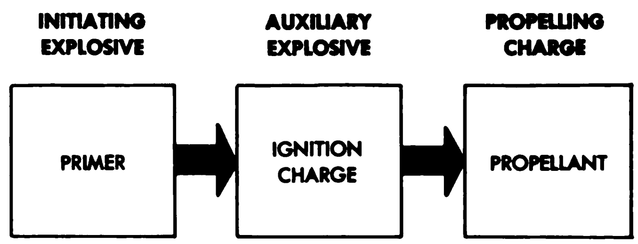
Electrical fuze initiation is used when the required duration of action from sensing and initiation to final detonation is a very short period of time. An example is the field of guided missiles and interceptors, where flight paths and damage volumes intersect for micro-increments of time and where instantaneous response characteristics are an absolute necessity. Other advantages of electrical fuzes are that they may be operated by remote control equipment, and can be easily sensitized by electronic timers or delay instruments. No matter which type of fuze initiation is utilized, the output energy of the sensitive device is amplified or boosted to the necessary level, by having smaller sensitive stages trigger larger less sensitive stages of greater potential until the required level of energy is reached.

explosive train types

There are two types of explosive trains which differ both in the type of energy they transmit and amplify, and on the terminology applied to their components.

PROPELLANT EXPLOSIVE TRAIN

In the propellant explosive train the initiating explosive is incorporated in a primer that provides for percussion or electrical firing. The primer does not detonate, but produces energy output in the form of a flame that initiates the explosive reaction. The energy level output of the primer (lead azide) is not of sufficient intensity to insure the combustion of the insensitive propellant. An auxiliary explosive (called an ignition charge or igniter) is initiated by the primer, which in turn ignites the propellant. The propellant burns to produce the hot, high-pressure gases which propel the missile. The basic propellant explosive train, with examples of typical explosive train, with examples of typical explosives used in gun ammunition, is shown in the illustration.



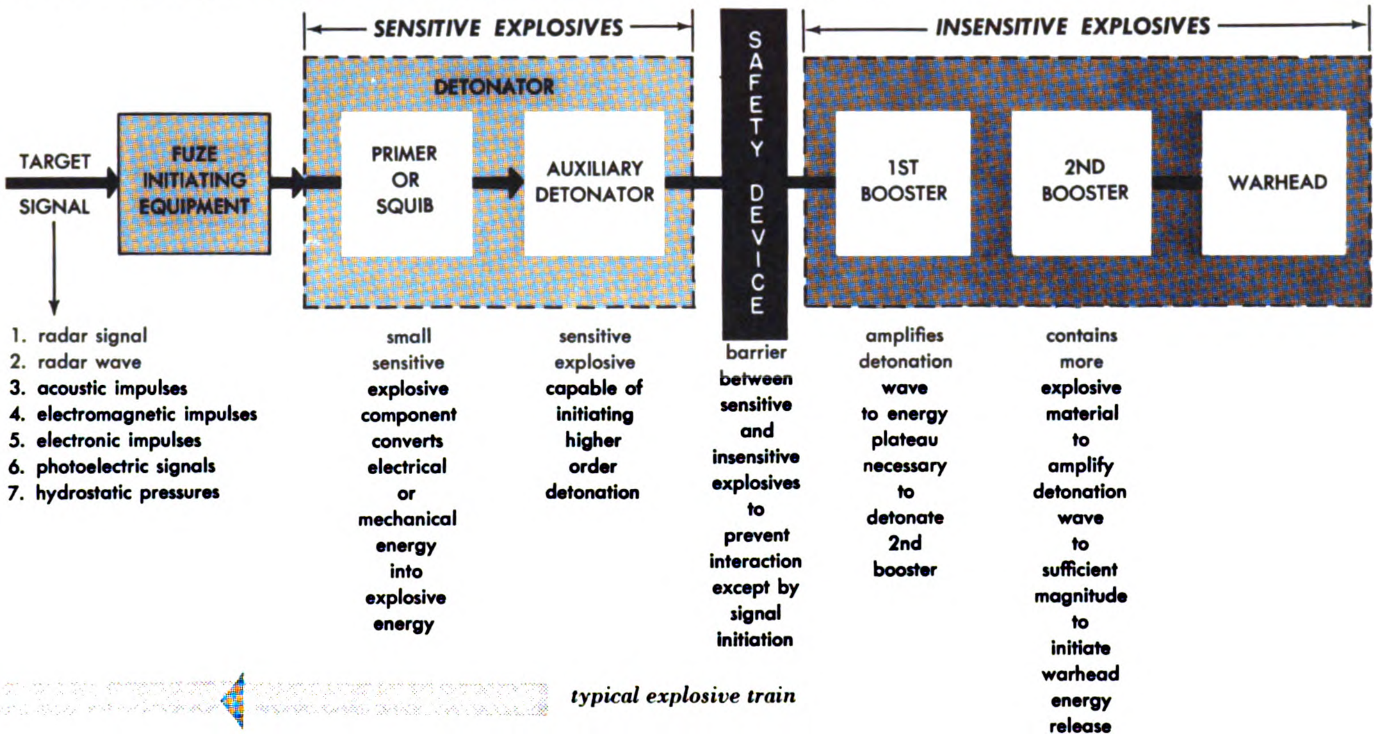
propellant explosive train

HIGH EXPLOSIVE TRAIN

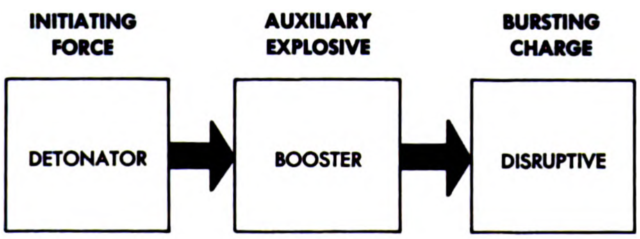
The same explosive train technique used with low explosives is utilized with high explosives. The high explosive train is usually composed of a primer-detonator, booster, and bursting charge. Many explosive types require several booster stages. In the high explosive train, the sequence of operation depends upon the transmission and amplification of shock rather than of heat (flame). The initiating explosive which produces the shock necessary to start the reaction is called a detonator; the auxiliary explosive is known as a booster.

explosive train

A typical detonating train consists of a series of stages which function to increase the fuze signal energy level to a point sufficient to detonate the explosive payload. The output signal of the sensing equipment is routed to a primer or squib, the first element in the explosive train. This device is capable of converting electrical or mechanical energy into explosive energy. Its explosive energy is minimal, but of a sufficient order to trigger an auxiliary detonator. This combination of primer and auxiliary detonator in the same casing is known as a detonator. Following the initial sensitive stages of the train are the insensitive explosive booster stages which are initially triggered by the output power of the detonator. In turn, each following booster stage is initiated by the previous stage until detonation of the payload occurs. In general, the successive impulses are transmitted by small quantities of sensitive explosives to larger quantities of less sensitive explosives. The explosive train will be in an operative condition only when the safety and arming barriers between the sensitive and insensitive elements of the train are removed. This physical barrier prevents spontaneous functioning of the train in the unarmed state.



typical explosive train



high explosive train

To fulfill the functional requirements of various explosive trains, it is evident that explosives of different types and characteristics are needed. The S & A problem varies with the type of train, sensitivity of the explosive, and conditions of temperature, shock, vibration, etc. that may be encountered. To fulfill the required high standards of safety, an arming barrier of extreme sensitivity, yet capable of complete safety, must be employed.

arming barrier

A fuze may exist either in an armed state or an unarmed state. Numerous safety devices are used to maintain the nonfunctioning characteristics of the fuze until the optimum moment for initiation. No matter how well designed the system may be, circumstances may exist that will spontaneously initiate the fuze. It is vital to safeguard personnel and material against the effects of accidental detonation. The basic principle that must be observed, and provision made for is that under no condition must a chance initiation be allowed to force a segment of the explosive train past a measured energy plateau (usually the sensitive fuzing device). Input circuitry to the fuze is usually completed at the last possible moment to prevent this undesired action from occurring. Physical barriers are placed between the sensitive and insensitive elements of the train, and are not removed until arming status is required, and finally the structure of the fuze casing is often made capable of containing an unwanted burst of energy from the detonator. The arming barrier is usually metal, and acts as a fuze might, i.e., normally adjacent elements are placed in a disconnected status.

SAFETY

definition of terms

Certain terms associated with S & A need definition to clarify the discussion.

A MISFIRE is a failure of the missile's propellant or booster to ignite at the launching sight.

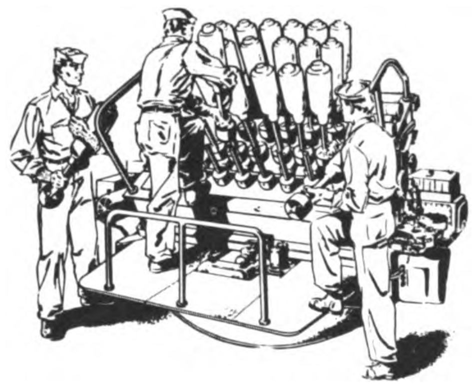
A PREMATURE is an initiation of the warhead prior to the desired time the S & A is designed to arm. It is a safety failure, and is an indication that the S & A malfunctioned.

AN EARLY is the initiation of the payload after the warhead is armed, but before the fuze should properly operate. It is usually caused by faulty fuze operation, or enemy countermeasures, rather than any Safety and Arming malfunction.

The terms
PREMATURE and EARLY
ARE OFTEN CONFUSED.

The first is a safety failure, the second is a probability variable. A premature is a system malfunction, and a "dud" is a warhead that failed to operate.

A safety system must encompass the life of a fuze from manufacture to detonation. Various chemical, mechanical, and electrical means are used to insure safety during storage, handling, launching, and firing. The missile during its trajectory from launching path to termination, undergoes various environmental pressures and forces, and must therefore be designed bore-safe or launch-safe. Under the influence of forces arising after launch, safety devices must engage or disengage so that signal paths or explosive trains can function. When an S & A employs electronic devices, safety can be obtained by interrupting the circuits, by short circuiting certain elements, by timing the operation of battery power, or by utilizing timing circuits (discharging capacitors through resistance) to obtain time delays. As much effort has been expended on warhead safety as on development of warheads and vehicles of greater speed, range, and lethality. Safety and reliability are essential characteristics of all fuze systems. New advances in nuclear and special type warheads, and the complicated guidance and launching circuitry associated with them, make current missiles extremely expensive and difficult to manufacture. To waste one because of a fuze malfunction would be extremely uneconomical. Also, a premature explosion might cause death or injury to friendly troops or civilians, in addition to a reduction of military potential.



handling safety

A safety program can be divided into two categories. Handling safety refers to the features that protect the fuze device against spontaneous initiation, from manufacture to installation in a missile being made ready for flight. Functional safety refers to the features designed into a fuze and S & A system that cause it to function in a reliable mode during launch, flight, and termination.

Handling safety tests are devised to demonstrate that the S & A will remain safe and operable after exposure to various test parameters. The permanence, ruggedness and reliability of the fuze are determined by exposing the device to a wide variety of the same environmental conditions and physical forces that it would be subjected to during tactical use. Drop tests, jolt tests, and jumble tests check the ruggedness of manufactured components, sensitiveness of explosive components, and the probability of spontaneous firing when the fuze' is subjected to severe strain and impact forces. In some cases, drop tests from 40 to 100 feet simulate free-fall conditions, and indicate results of aircraft drops of ammunition for delivery or accidental release on landing. The device must remain safe, either for use or for disposal after being dropped from these heights. Environmental tests include measurement of waterproofness, effects of humidity, temperature, and altitude on the fuze device. Any manufactured device tends to deteriorate at some rate dependent on the environmental forces to which it is subjected, and on the protective systems designed to safeguard and prolong its efficiency. Fuzes are adversely affected by corrosive atmospheres (salt water) and extremes of heat or cold. Protective coatings are used to decrease the effect of harmful chemical combinations, and proper loading and storage techniques guard against deterioration of the explosive. Procedures are available to test various forces, including the tensile strength of the fuze container to withstand the combustion of the sensitive primer without damage to the surrounding devices or injury to personnel. In brief, the safety factors are intended to insure that the chance of a fuze arming at any time except when intended, is practically nil.

basic safety methodology

Any mechanism which depends on energy stored prior to launching should not be used as a safety device. To do so would invite an initiation of the explosive train anytime from installation or launch to a premature disastrous detonation. For maximum safety, the S & A must be designed so that all of the energy required for arming is derived from environmental parameters during normal flight behavior. Thus, arming, even if early, will occur only along the warhead's normal flight path, and not on the launcher or in a service or handling area.

Another basic safety design feature is to have the S & A react to parameters which do not require the device to be physically connected to any part of the weapon outside of the actual warhead. For example, if missile skin temperature were used as a primary arming agent, the S & A would require attachment to some temperature measuring device. A device of this type could be activated, unintentionally, by man-made heating sources (blow torch, etc.) and initiate the fuze action spontaneously.

Also, the probability of malfunction increases due to the necessity of adjustments and connections. It is desirable that the S & A be a complete, independent package sealed off as completely as possible. For this reason, acceleration is a parameter well suited to warhead safety requirements. No outside measuring instruments are needed to operate in conjunction with the device, and by means of acceleration indicators (accelerometers) sealed within the S & A, the warhead may be armed at the optimum moment.

primary and secondary safety mechanisms

PRIMARY SAFETY

This mechanism is exclusively a safety functioning device. Its sole purpose is to safeguard the missile in which it is incorporated. As previously stated, it is improper to incorporate as a primary safety device any mechanism which depends on energy stored prior to missile flight, or on environmental parameters existing outside the missile.

SECONDARY SAFETY

It should be realized that the S & A can be designed to actuate upon reception of information from various indicating devices, including guidance system resolvers to remote control apparatus. The S & A can be designed to measure a derivative of acceleration, a given time interval, or the cessation of propulsion, and to arm at a predetermined interval of time if various other conditions are satisfied. Even though alignment of the explosive train occurs, the warhead's guidance system may delay complete arming until specific impulses in the guidance sequence have occurred. A safety system in a complex missile design will move from a "ready arm" status to an "active arm" status only upon the occurrence of a unique series of events acting as the final arming stimuli. Its purpose is not primarily safety, but to help overcome countermeasures, and to keep the warhead "safe" in the event of a miss. This additional arming function is called "secondary arming". The necessity of a secondary arming system became necessary because of the advances made in countermeasure techniques which simulate various environmental conditions useful for sensing purposes.

countermeasures

Sensors detect, analyze, measure, and test the forces that are applied to the S & A, and the characteristics of the environment that surround it. The information and data sensed are recorded and measured against the rigid specifications programmed into the S & A, and the S & A will attain active arming status only when all the desired specifications are matched in a unique sequence. Countermeasure decoys include the launching of small unmanned craft from conventional bombers which are fitted with signal simulating devices to force premature or late initiation of S & A and fuzing systems. To confuse or delay the S & A, frequencies indicating false Doppler shifts may be generated and transmitted to camouflage the target's speed or position. Temperature measurements can be altered by using insulating shields, or by deflecting engine exhaust gases. Flares of various intensity may be set off along a target's projected trajectory. The most common type used is the pyrotechnic flare. Similar type devices may be used to act against all types of sensors, and the art of countermeasure and counter-countermeasure grows increasingly complex.

low probability of undesired arming

The design goals for safety and arming mechanisms must be set extremely high (a safety factor of 1,000,000 to 1, and a reliability of 99.5 percent) to provide maximum safety to personnel and a high degree of reliability of warhead initiation. In order to achieve the high reliability essential for safety, performance tests are devised that will determine the acceptability of an S & A system. In view of the complexity of design and the high cost per unit, it is not economical to test large numbers of the devices (many of which would be destroyed in the process). While time-tested methods are available to test explosive train action, this procedure is not possible in the case of arming mechanisms. S & A devices are normally actuated by environmental conditions and sequence probabilities that cannot be easily or economically reproduced in laboratories. Without the benefit of a complete "test out", the designer must include "redundant" circuitry into his design to provide assurance of attaining the desired probability constant.

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redundancy

Redundancy, as discussed under Fuzes, is the use of multiple mechanisms all designed to perform the same function. These mechanisms are connected in series or parallel to increase the probability of occurrence of some function of the warhead. In the case of S & A mechanisms, that function is arming of the warhead.
Simple probability theory tells us that the probability of several statistically independent events happening simultaneously is the product of the probabilities that the individual events occur. If three completely independent arming mechanisms exist, and the chance of a premature arming of any one of them is less than 1 in 100, then the probability of simultaneous premature arming is less than 1 in 1,000,000. If the three devices were connected in series, the incidence of prematures would be minute. It should be realized, however, that to achieve such reliability, the price exacted is high. To achieve the required safety, each of the arming mechanisms must be placed in the weapon series, and thus arming can occur only if all of the mechanisms actuate. Unfortunately, each device also has the probability that it will fail to arm when required. The probability that arming will occur when required is the probability that all the mechanisms will arm when required. If three mechanisms are used, and each has a .94 probability of arming when required, then the probability of the system arming when required is ($.94^3$) or approximately .830. To achieve an overall probability of arming of .99 in a three-component S & A system requires that each mechanism have a probability of operating of about .997, a degree of reliability difficult to produce. One solution is to produce a more elaborate system. For example, two three-component systems can be connected in parallel so that arming is achieved if either system arms. The probability that a malfunction may occur would be approximately doubled, but the reliability would become .991. The theorem that the probability of two events occurring simultaneously is the product of the probabilities of either event occurring is valid only if a single stimulus cannot simultaneously produce both events. To solve this problem, use is made of components that differ in operating principles, in design, and location within the warhead. Thus, by using devices differing in nature, it is hoped that the devices become truly independent and provide the required safety and reliability.

ARMING

INITIATION OF ARMING

The arming process is a transition between the safe mode and the functioning mode. Arming involves the removal of barriers on the transfer path, and an alignment of explosive train elements to attain a ready position. Arming mechanisms usually operate upon reception of energy from a source contained in the fuze, or from a potential created by an external environment such as acceleration, spin, or pressure. The moment of initiation of the arming process is controlled so that the fuze cannot function until the occurrence of a unique sequence of events. The optimum time for arming is some moment just before the target comes within the effective damage volume of the warhead, and after it is beyond dangerous range of friendly forces and material. The S & A cannot think, it merely carries out a procedure for which it was designed. By measuring various characteristics of its environment, and by making use of "arming agents", the S & A activates at the desired moment. An arming agent may be defined as a force that an S & A can sense and measure. When the arming agent acts upon the warhead to a designed degree, or achieves a desired magnitude, the S & A responds by arming.

use of flight characteristics in arming

The function of each electronic or mechanical component of an S & A system involves some physical principle and/or property which has been found applicable to the specific requirement. Many of the components are simple in form, and perform a single fundamental operation; other components may operate simultaneously or in conjunction with other S & A elements to perform highly complex operations. The basic structure of an S & A system requires a discussion of the trajectories and the environmental forces exerted on missiles and projectiles. To understand the relationship between an arming agent and arming methodology employed by a weapons systems, the following terms require definition:

INERTIA is the property of matter by which it tends to remain at rest, or, if in motion, tends to follow the same straight line or direction, unless acted upon by some external force.

VELOCITY is a representation of speed and a specific direction of motion. The velocity of an object may be constant or varying, and the variation may be in speed or in direction, or in both.

ACCELERATION is the time rate of change of velocity. It represents motion in which the velocity changes from point to point. Acceleration is not the time rate of change of speed; it is the time rate of change of velocity treated as a vector quantity. High values of acceleration, both axial and lateral, are achieved during missile flight, but similar levels may also be reached during handling or launching. The single and double time integrals of measured acceleration, which represent the velocity and the distance traveled, are significant. Over an extended period of time, they can serve as unique indications of correct missile flight. There is no handling environment which will duplicate such velocities. The distance traveled by the warhead, assuming it is traveling in the proper direction, is a basic parameter in determining the time for arming, since it establishes the warhead in space relative to its target. Velocity is useful as an indication of successful missile flight. Distance alone is not sufficient as an arming agent, because distance may be traveled at low velocities by an S & A outside the missile. Therefore, the combination of high velocity and distance is used for determining whether, and when, to arm.

A useful phenomenon associated with acceleration is known as setback, which is the relative rearward movement of component parts in a missile during acceleration. The force necessary to accelerate the device is balanced by a reaction force of equal magnitude called setback force. It may be calculated by determining the acceleration (a) of the missile, and multiplying it by the mass (m) of the part affected ($F = ma$).

DECELERATION

A missile encounters air resistance during flight, causing the missile to decelerate. Two important phenomena associated with deceleration are creep and impact. Creep is the continuing force of inertia that tends to move parts of a S & A system forward during deceleration. Since the deceleration which causes creep is caused by air resistance, creep is a function of warhead velocity and air density. In ballistic missiles, this force reaches very large magnitudes during the re-entry phase. Impact occurs at termination of flight, or when a warhead strikes a target. The extreme deceleration which takes place upon impact results in a force of extreme magnitude moving in a forward direction and initiating the arming and firing mechanisms.

AMBIENT PRESSURE is a parameter often used for initiating arming action in sea mines and depth charges. Ambient pressure when employed in a hydrostatic device can be derived as follows: $P=du$, where u is the density of the water, and d is the depth. Arming circuits are activated at a predetermined level or pressure gradient. Atmospheric pressure variations that occur during missile flight may be used as an indication of trajectory position; for example, a ballistic missile may leave the atmosphere during part of its trajectory. Thus, the sequence of normal pressure going to very low pressure and back to normal pressure can be used for the determination of arming initiation. Also barometric pressures can be employed to arm a fuze at a specific altitude.

AMBIENT TEMPERATURES

The skin temperature of a warhead, which may reach high levels on some missiles, has possibilities as an arming agent. Of course, in itself it may not indicate that the warhead is in flight. For example, the warhead could conceivably be in close proximity to an unwanted source of heat (fire). In conjunction with other parameters, however, temperature is a good indication of warhead flight, giving some measure of velocity and distance.

CENTRIFUGAL FORCE is the most commonly used means of arming a fuze in spin-stabilized missiles and projectiles. Centrifugal force created by a rotation of the warhead in flight tends to move all moveable parts radially away from the longitudinal axis.

AERODYNAMIC AND HYDRODYNAMIC FORCE

The travel of missiles, rockets, and anti-submarine devices (torpedoes) through an environment creates forces of sufficient magnitude to rotate mechanical parts such as vanes or propellers. Torque is the product of such a force and the intensity of the force created depends upon the velocity of the air or water flow past the vanes or propeller blades. The power developed is a function of area and the radius of the blades, as well as the density and velocity of the air or water stream. The rotation is designed to arm the warhead after a certain number of revolutions. A hydrodynamic arming fuze is illustrated in the section on ambient fuzes.

GAS PRESSURE

In rocket propulsion systems gas under pressure is a by-product of the burning of the rocket motor propellant. The pressure of these gases is fairly constant during burning, and is of the magnitude of several thousand psi. Entrance of the gas into the S & A can be controlled to act as a time delay device to activate the arming of the fuze.

RAM AIR PRESSURE is a measurable quantity not unique but an important indicator in combination with other parameters.

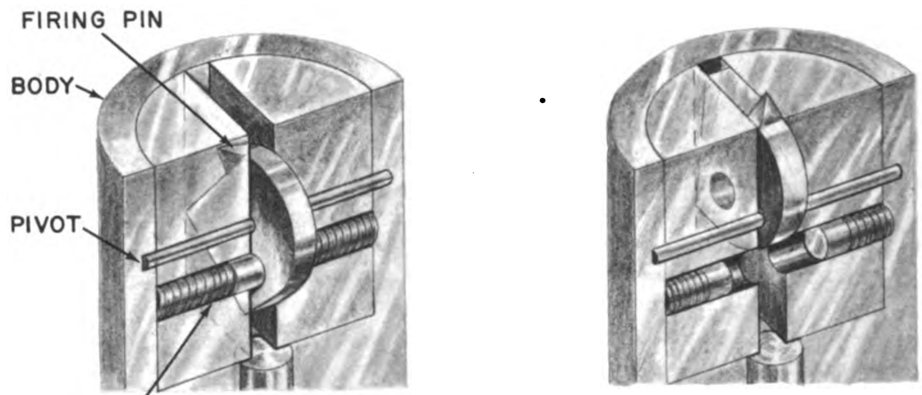
STATIC PRESSURE and COSMIC RAY DISTRIBUTIONS are at the present time considered secondary in importance but in the future might be useful as arming agents. The sequence of such measurements can be used as arming agents, since pressure or cosmic ray distributions vary sharply at three points in the missile trajectory: on the way up, at the peak height attained, and on the way down.

Regardless of the environment measured, or the force utilized, there must be a characteristic uniqueness present to employ the parameter as an arming agent.

MOTIONS DURING FLIGHT AND THEIR USE

centrifugal forces

A commonly used method of arming a fuze,as mentioned, is by centrifugal force. This force is caused by the rotational effects of spin, inherent in projectiles and other bore launched devices. Centrifugal forces caused by missile rotation are effective in moving safety devices out of the explosive train path, in a plane perpendicular to the spin axis of the missile. Spring loaded safety mechanisms such as centrifugal pins, interrupters, and plungers are devised to resist movement until the missile has attained a certain velocity of rotation, at which time the rotational or centrifugal forces generated are strong enough to affect its requirements. The centrifugal plunger is a typical example of an arming mechanism used in base detonating fuzes. The firing pin "g" is mounted on its pivot "j" in a slot in the plunger body "f". In the unarmed position, safety pin "h" keeps the firing pin from rotating. When the missile attains a certain velocity, the resultant centrifugal force causes the safety spring to compress, thus removing the safety pin. With the removal of the safety pin and the application of the centrifugal force, the firing pin rotates on its pivot and sets in the armed position. The force of setback during launch is strong enough to overcome the centrifugal force and contains the firing pin in the safety position until the missile clears the launcher. After cessation of the setback force, centrifugal forces rotate the firing pin about its pivot and place it in the armed position.

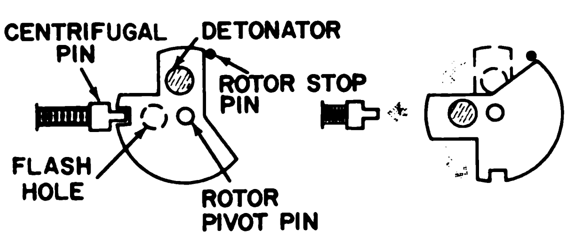


simple centrifugal plunger

rotors

Some arming mechanism components are pivoted so that they may rotate through a specified angle. The axes of the rotating members may be parallel to, or perpendicular to the missile axis. The rotor illustrated is an eccentrically weighed rotating body mounted so that its rotating axis is parallel to the axis of the missile. Until the missile leaves the launching device, the centrifugal safety pins lock the rotor in a misaligned state with the remainder of the explosive train.
After the missile leaves the launch device, the centrifugal force acting against the safety spring moves the centrifugal safety pins out of the rotor, which then rotates about the rotor pin until it comes to rest against the stop pin. In this position the detonator is lined up with the flash hole leading to the booster.

IN UNARMED POSITION **IN ARMED POSITION**



centrifugal interrupters and slide assemblies

The centrifugal interrupter is similar in operation to a centrifugal pin, but larger. It is used to block off the flash path of an explosive train and is moved out of the explosive train by a force exerted on a safety spring. A slider assembly is a device similar to, but larger than, an interrupter. It contains the primer or primer detonator and it is held in a safe position until the centrifugal force acting against it slides it into an armed position.

acceleration

An arming system requires that the mechanisms, as far as possible, react to the parameters to be utilized as arming agents without requiring a physical interconnection to the measurement transducers. A parameter well suited to this criterion is acceleration. Acceleration represents motion of a missile in which the velocity changes from point to point in the trajectory. The units of acceleration depend upon the unit of velocity as well as upon the unit of time during which the velocity changes. The first integral of acceleration in a three dimensional coordinate system, is a measure of incremental velocity changes; the second integral is a measure of incremental distance changes. The two terms, incremental velocity and incremental distance, are important in that they pin-point a missile's position in reference to a launching site both as a function of velocity and of distance traveled. If the missile is traversing a fairly straight trajectory path, simple measurements can determine if the missile has moved out of range of friendly personnel.
The measurement of incremental quantities requires a velocity and distance measuring system that is similar to a basic inertial guidance system. Functions can be computed by measuring the actual acceleration of the missile. The inertial device used to carry out the measurement is an accelerometer. The basic principle of operation of an accelerometer consists of the measurement of the inertial reaction of a mass to an acceleration. There are two principal types of accelero-

meters. In the first type, the inertial reaction force of the mass causes a displacement of the device which is measurable by any one of several methods. The second type operates on a fundamentally different system. In this system a minute deflection of the mass is detected, and a force is instantly applied to prevent any further motion. Acceleration is indicated by the magnitude of the force applied to produce a balanced status. The accelerometers detect missile velocity changes without the need of an exterior reference. This was accomplished by integrating acceleration signals to obtain velocity components. If the velocity signal is then integrated, the result is missile distance, and is measurable from the point of origin of the launcher. Usually measurements and integration of the axial component of acceleration are sufficient. By basing computations on this axial component of acceleration, it is feasible to determine the axial velocity attained relative to the launch site, and to compute the distance traveled along the trajectory. When a predetermined sequence of events occurs, the arming mechanism will be initiated.

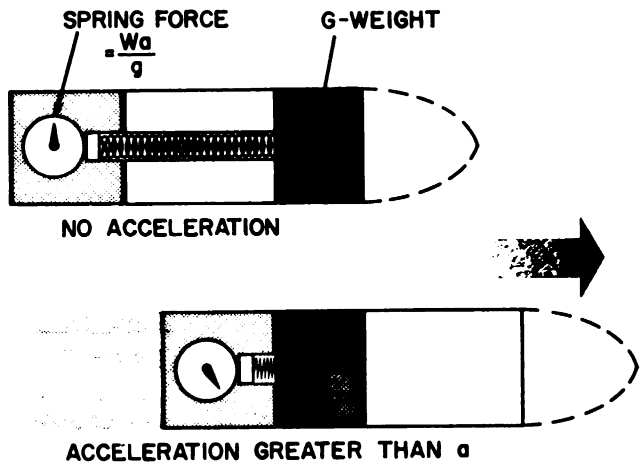
"G" weight

A type of device which may be used to sense the axial acceleration of the warhead is a weight, called a G-weight, restrained to move only along the warhead axis and assumed to be on frictionless bearings. The motion of the G-weight may be restrained by a spring or by some mechanism which it is designed to drive. If the motion of the G-weight relative to the warhead is minute, then it accelerates in a similar manner to the missile and the measure of its movement can be related to that of the missile. Such a G-weight is shown in the illustration. This weight is held in position by the spring when the acceleration of the missile is zero. When the warhead accelerates to some value greater than a, the weight moves aft against the force of the spring. Such a weight could be designed so that its motion starts a time device (clock) which measures the time interval t when the warhead acceleration exceeds the value of acceleration a. Since this incremental acceleration acts during t seconds, an incremental distance of at least $at^2/2$ will have been traveled. For any kind of motion, the distance S an object is displaced from its starting point is obtained as the product of average velocity V_{av} and time t, $S = V_{av}\, t$. When the motion is uniformly accelerated (a is constant) the relationship between the average velocity V_{av} and the initial velocity V_0 and final velocity V_f is expressed by the equation $V_{av} = (V_0+V_f)/2$. With an initial velocity, the distance S is given by the equation $S = V_0 t + \frac{at^2}{2}$.

For the G-weight to be accelerated it must be acted upon by a force. The force is derived from the spring or constraint and is a function of the acceleration of the missile. If the mass of the G-weight is m, the restraining or spring force is then $F(t)=ma(t)$, where a(t) is the acceleration of the missile. Since the force exerted on the G-weight is equal to the force exerted by the weight of the spring, the G-weight exerts a force on the spring proportional to the acceleration of the missile, and the resultant movement can initiate the arming action.

effects of gravity

There are numerous forces acting on all parts of a missile, of which gravity is one. Gravity is a force always forcing a body toward the earth. This force is constantly applied to an S & A system and the effects of gravity must be considered as a parameter when acceleration characteristics are used as arming agents. An unfortunate property of inertial measuring systems is that they cannot differentiate between the force applied due to acceleration and that applied by gravity. Because some accelerometer outputs contain a component due to gravity as well as missile acceleration, a method of separating the two is needed. The position of the missile in its trajectory determines the strength of the gravitational component. If the component of the signal due to its position on trajectory is nullified, a pure acceleration component remains. The force of gravity may be computed on an instant to instant basis, and the results fed to a computer network where calculation may be made and feedback circuits utilized to neutralize the effects of gravitational forces. The remaining force will be the result of acceleration only.



a typical G-weight

deceleration

A missile encounters air resistance during flight and decelerates. Creep is the tendency of component parts of a missile to move forward as the missile decelerates. This is similar to setback but its force is of a smaller magnitude and acts in the opposite direction. The inertial force is calculated by multiplying the weight of the part by the deceleration of the missile. The greatest applicability for an S & A system utilizing deceleration as a parameter is to be found in missile re-entry into the atmosphere.

DESIGN CONSIDERATIONS

basic safety and arming methodologies

A properly designed S & A system must meet several specific requirements that are common to all S & A systems:

1. No device should utilize as an arming agent, energy stored prior to launch.

2. For maximum safety, the S & A should be so designed that the energy required for arming initiation should be derived from environmental parameters that exist during normal flight.

3. The S & A device should not be initiated by parameters that exist outside the actual warhead.

4. The missile must be protected against malfunctioning by a primary safety mechanism.

5. The process of removing the safety features built into a warhead must occur only under the impulse of a unique sequence of events.

6. A secondary safety system is needed to combat or neutralize the effects of countermeasures.

7. Redundancy in design when necessary to assure a high degree of probability of function occurrence of the S & A.

8. Devices to prevent or make the warhead incapable of spontaneous detonation.

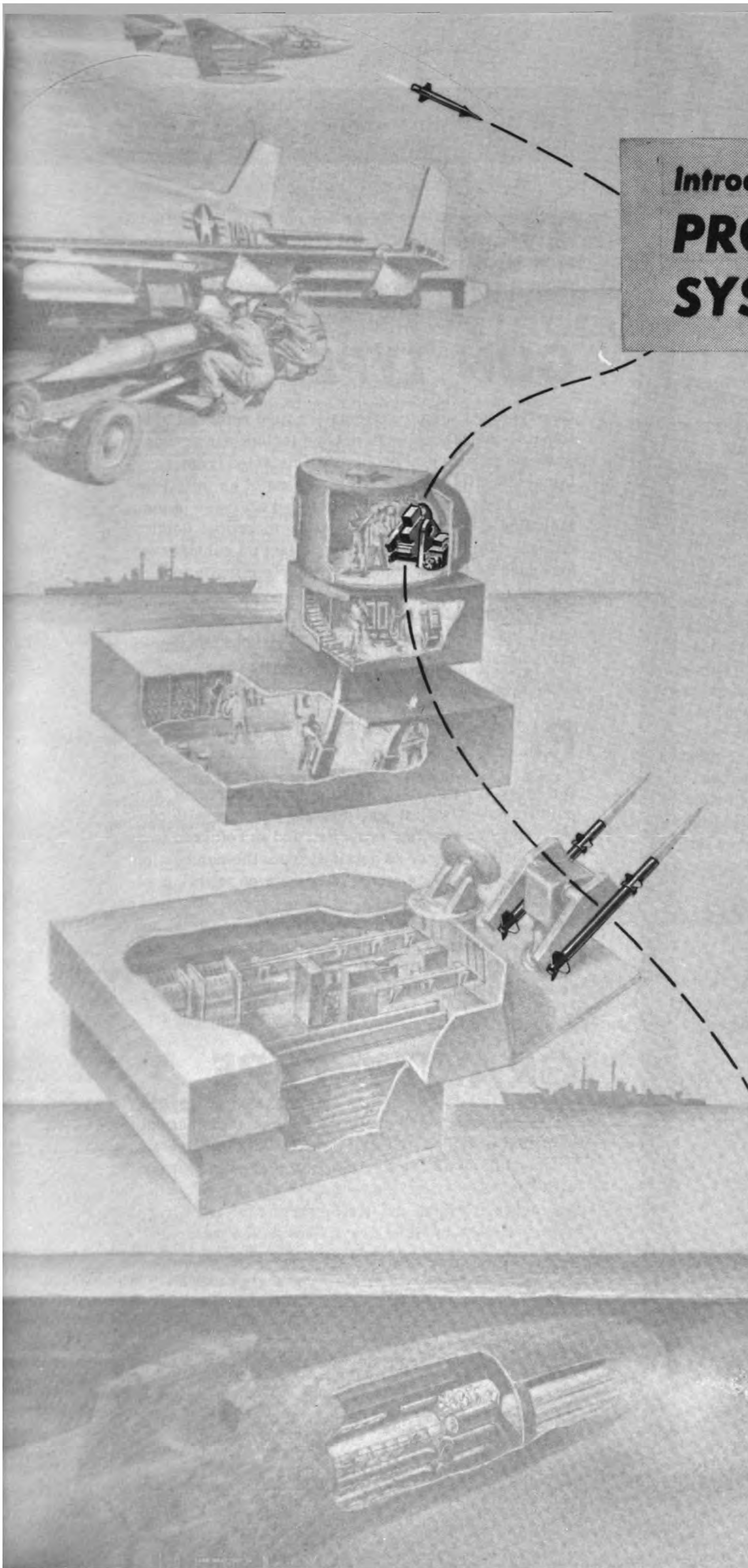
The choice of optimum devices to perform the above listed functions of an S & A system depends primarily on the specifications or requirements set forth by the interested governmental agency or military developmental group. All necessary parameters must be defined, both tactical and operational, to permit the development of a new device or the alteration of an existing one to conform to the specification set forth by the appropriate body.

tactical and operational considerations of designs

S & A systems are designed to meet fundamental, tactical situations. They are employed in various weapon systems, from sea mines, to rockets and guided missiles. Within a particular series of fuzes, specified parameters are utilized to initiate arming action. Therefore, before undertaking the development of an S & A mechanism, a designer must not only be thoroughly familiar with the tactical requirements of the S & A but of the fuze as well. The tactical purpose of the device and the operational considerations that may influence a choice of parameters are usually evaluated and analyzed in respect to existing mechanisms and their applicability to correctly function in the system component. New devices if needed require a complete research and developmental program, and may evolve a system of greater complexity than is presently available. Therefore before undertaking the development of an S & A system, a designer must be thoroughly familiar with the tactical requirements of the S & A, and the conditions prevalent in the system concerned. All S & A devices,regardless of tactical use, must satisfy basic operational requirements, and above all, safety considerations.

The S & A mechanism to be designed will have been specified for 1) arming agent, 2) operational probability expected, 3) degree of handling or functional safety required, 4) explosive train components to be controlled and armed, and 5) environmental conditions to be encountered. Upon completion of the design effort, all S & A devices are subjected to an extensive laboratory and flight test evaluation.

The laboratory evaluation is divided into two categories - 1) performance reliability tests, 2) safety probability tests. An S & A device that satisfactorily passes or meets both requirements is considered to have met the design goals. Numerous operational checks are made of the performance of the device, after laboratory approval is given, to corroborate the results obtained in the basic tests. Only after a specified number of "operational runs" are concluded without malfunctions is the device accepted and approved.



CHAPTER 3

Introduction to PROPULSION SYSTEMS

The underlying principle of propulsive movement has been stated by Newton in his Third Law of Motion: To every action there is an equal and opposite reaction. Every forward acceleration or change in motion is a result of a reactive force acting in the opposing direction. A person walks forward by pushing backwards against the ground. In a propeller-type airplane, the air through which it is moving is driven backward to move the airplane forward. In a jet-propelled plane or a rocket, a mass of gas is emitted rearward at high speed, and the forward motion of the plane is a reaction to the motion of the gas. Matter in the form of a fluid, a gas, or a solid may be discharged as a propellent force, expending its energy in a direction opposite to the desired path of motion, resulting in a predetermined acceleration of the propelled body along a desired trajectory.

SOURCES OF STORED ENERGY

The power required to propel a warhead to its target is obtained through the controlled release of stored energy. The energy source may be the end product of a chemical reaction, a compression of gases or liquids, or the activation of a solid that is potentially explosive.

An explosive propellant may be a chemical compound, or a mixture of compounds such as TNT and ammonium nitrate comprising amatol, or a mixture of one or more compounds and one or more elements such as potassium nitrate, sulfur and carbon comprising black powder.

Propellants may be found in either solid, liquid, or gaseous states. TNT and nitroglycerin are examples of solid and liquid explosives, respectively.

The chemical composition of military explosives can be divided into 3 catagories:

1. Organic compounds that include:
   a. Nitroglycerin and nitrocellulose
   b. TNT and picric acid
   c. Mercury fulminate and lead styphnate
2. Inoganic compounds
   a. Lead oxide and ammonium nitrate
3. Mixtures of oxidizable materials (fuels) and oxidizing agents.
   a. Black powder
   b. Pyrotechnic compositions

To prevent serious disruption of the launch device (gun, rocket tube, missile launch pad), the explosive process must be subjected to close control and the inherent brisance of the explosive must not exceed the characteristic resistance of the launcher.

TYPES OF

Weapon propulsion systems are normally classified as gun type, reaction type, and gravity type. The object to be propelled in each case will be referred to as a projectile, a missile, or a bomb, respectively.

GUN TYPE

Gun-type propulsion systems are also referred to as impulse-propulsion systems and include all weapons systems in which a projectile is ejected from a container (usually a long tube) by means of an initial impulse. Under this classification fall all types of guns, including K-guns, which are used to propel depth charges from ships. Systems reliant on compressed air, such as torpedo tubes or Polaris missile tubes, are also considered to be gun-type systems, at least for the "ship-clearing" portion of their journey, after which they depend on internal propulsion systems for movement.

REACTION TYPE

Any weapon which carries an internal source of propulsive power with it as it travels to a target is considered to be reaction propelled and is referred to as a "missile". Power is obtained from the combustion of fuel by a reaction motor of some type, either a reciprocating heat engine, a jet, or a rocket. The means for providing thrust include propellers, and jets, and rocket exhausts, respectively. The jet engines are classified as pulsejets, turbojets, or ramjets; and rockets as liquid or solid-fuel types.

GRAVITY TYPE

Those weapons which obtain no further assistance other than that provided by inertia and gravity after they are released from aircraft or ships to make the journey to their objectives are classified as gravity propelled. Air-dropped bombs and depth charges (after hitting the water) are examples of gravity-propelled weapons.

GUN-TYPE SYSTEMS

INTRODUCTION

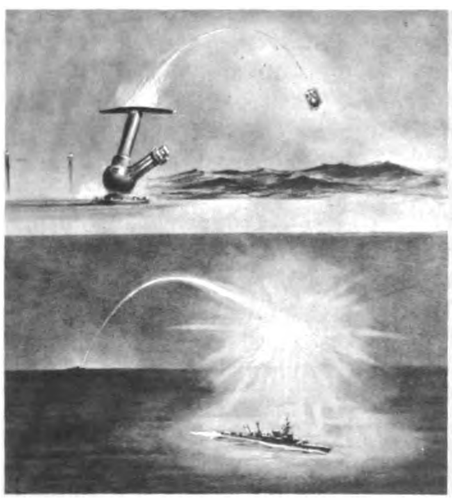
The expulsion of gun-type projectiles at the high velocities demanded by modern warfare requires tremendous forces. The energy source furnishing these forces must be capable of being readily manufactured, easily transported and stored, and must be usable in a safe yet effective manner. Chemical explosives, gun powders in particular, have proven most suited to this task. It has been proposed that energy from a source other than explosives be used in standard gun-projectile systems. Such energy sources would include compressed air, electromagnetic force, and centrifugal force. To date, only compressed air has been found useful. The study of the action within a gun at the time of fire is referred to as "interior ballistics". It comprises a study of a chemical energy source, a working substance (high-pressure gas), and the equipment to release and direct the working substance. To provide the high speed of response needed in a gun, the propellant must transfer its energy of reaction to the projectile by means of the expanding gaseous products of explosion, which are considered the working substances.

There are several points of view from which a propellant is studied. From its process of decomposition, the temperature at which it must be kept and the manner in which it must be handled are determined. The energy released by a propellant can be precalculated from a study of its thermodynamic characteristics and the quantity being used. The dynamics of the gases evolved during propellant firing must also be studied, since the kinetic energy of the expanding gases is a significant portion of the total energy of the process. Calculating the motion of a missile in a gun is not simply a matter of applying Newton's laws to the motion of the projectile considered as a point mass, but, rather, is an involved study of the rate at which the working substance is evolved from the propellant, its dynamics, and its effects on projectile motion. The gun tube or barrel is stressed as the projectile is forced through it, and its interior is subjected to sliding friction. At times, the barrel can become so heated by the passage of high-temperature gases and the great pressures generated by the propellant that chemical interaction occurs between the metal and the gases.

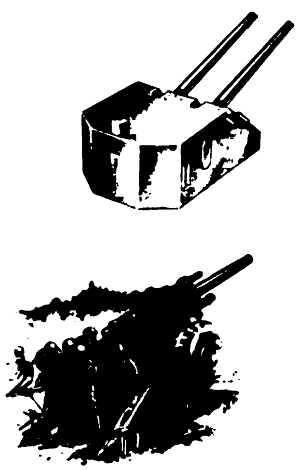
SOURCES OF IMPULSE ENERGY

In the impulse propulsion system, the ejection of the projectile from the gun barrel is accomplished through the action of expanding gases. Three sources are commonly employed.

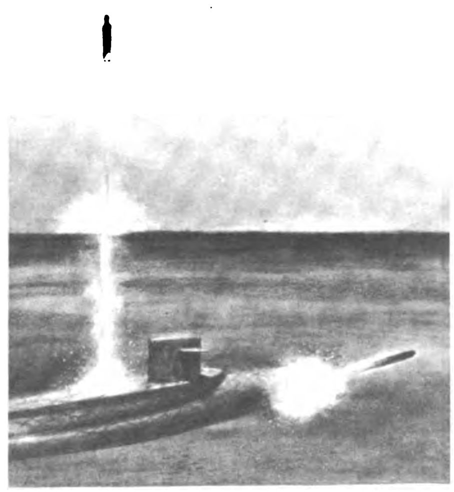
black powder



gunpowder



compressed air



gunpowder was probably invented by the Chinese around 200 A. D. and used for bombs, fireworks and rockets

one of the earliest of hand weapons was the Straight Gun between 1250-1350 A. D.

black powder

The oldest known explosive is black powder. It was first used as the propellant in guns in the beginning of the 13th Century and was the only propellant for firearms until the latter half of the 19th Century, when nitrocellulose powders were developed. Originally known as gunpowder, black powder has undergone little change in composition from its conception to present times. Basically it is a mixture of 75 percent sodium nitrate (saltpeter), 15 percent charcoal, and 10 percent sulphur. Black powder is an undersirable propellant for a number of reasons:

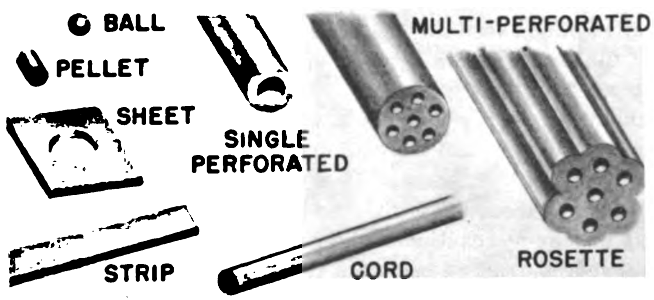
1. It burns incompletely, leaving large amounts of residue in the bore.
2. It creates high temperature when burning, thus causing rapid erosion of the gun bore.
3. It creates great billows of black smoke.
4. It detonates rather than burns. Its reaction speed is thus too fast, even with very large granulations.

In addition, black powder must be stored in airtight containers because it deteriorates when exposed to moisture. As it deteriorates, it becomes more unstable. Black powder is not subject to spontaneous combustion at normal storage temperature, or even at moderately high temperatures, but it is highly flammable and extremely responsive to friction, shock, and sparks. When ignited, its reaction is extremely rapid and violent; however, the larger the granulations, the less rapid the reaction. Black powder dust is highly dangerous and its accumulation during handling should be prevented. Black powder is the most dangerous of all explosives handled on warships, and is used mainly as a primer or igniter for propelling charges in standard-impulse propulsion systems. It is also used for various above-surface, low-pressure, booster-type applications, including K-guns, torpedo tubes, and catapults.

gunpowder

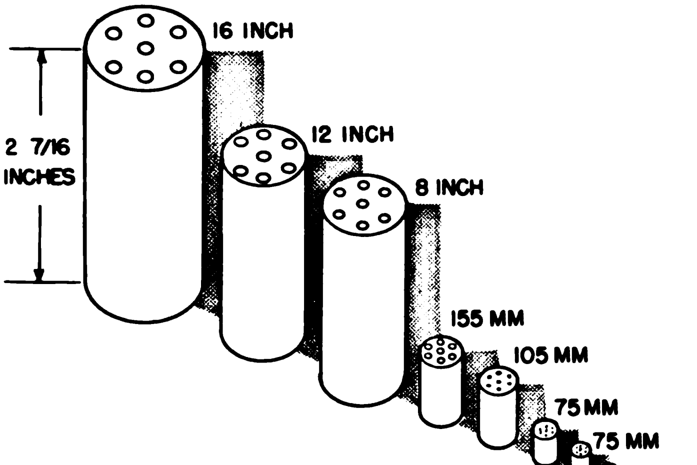
Gunpowders, or smokeless powders, are a fairly recent development in firearms. Up until their introduction near the close of the last century, black powder was the sole propellant used in guns and was referred to as gunpowder. The basis for the manufacture of smokeless powders is nitrocellulose, which is produced by the action of nitric acid on cotton. The first attempts to use nitrocellulose as a propelling agent failed because of its highly explosive nature. However, it was observed that by colloiding nitrocellulose with ether and alcohol, a low, or burning explosive was formed, the rate of combustion of which could be controlled. Furthermore, the substance was found to be efficient, safe to use, easy to handle, and fairly stable in various storage environments.

Smokeless powders, which have come to be known as "gunpowders," are neither completely smokeless nor are they true powders. Since granulation and shape were found helpful in controlling burning rate, they are produced in the forms of sheets, cord, strips, and cylinders.



Smokeless powders may be considered to be of two classes: single-base powders and multibase powders. In the single-base powder, nitrocellulose is the only explosive present. Other ingredients are used to obtain suitable form, desired burning characteristic, and stability. The standard single-base smokeless powder used by the Navy is a uniform colloid of ether-alcohol and purified nitrocellulose to which, for purposes of chemical stability, is added a small quantity of diphenylamine.

The multibase powders may be divided into double-base and triple-base powders, both of which contain nitroglycerin to facilitiate the dissolving of the nitrocellulose and enhance its explosive qualities. Although double-base powder produces less gas than single-base powder, it is thermodynamically more efficient, since it burns at higher temperatures and thus produces more heat. However, this heat increases gun-bore erosion. To achieve the effects of single-base powder and the manufacturing advantages of double-base powder without incurring the disadvantages, a third explosive, nitroguanidine, is added in making triple-base powder. Nitroguanidine is a "cool-burning" explosive which keeps the maximum temperatures of the triple-base powder comparable to those of single-base powder, yet produces greater propulsive force.



comparative sizes of some powder grains

CHARACTERISTICS OF SMOKELESS POWDER

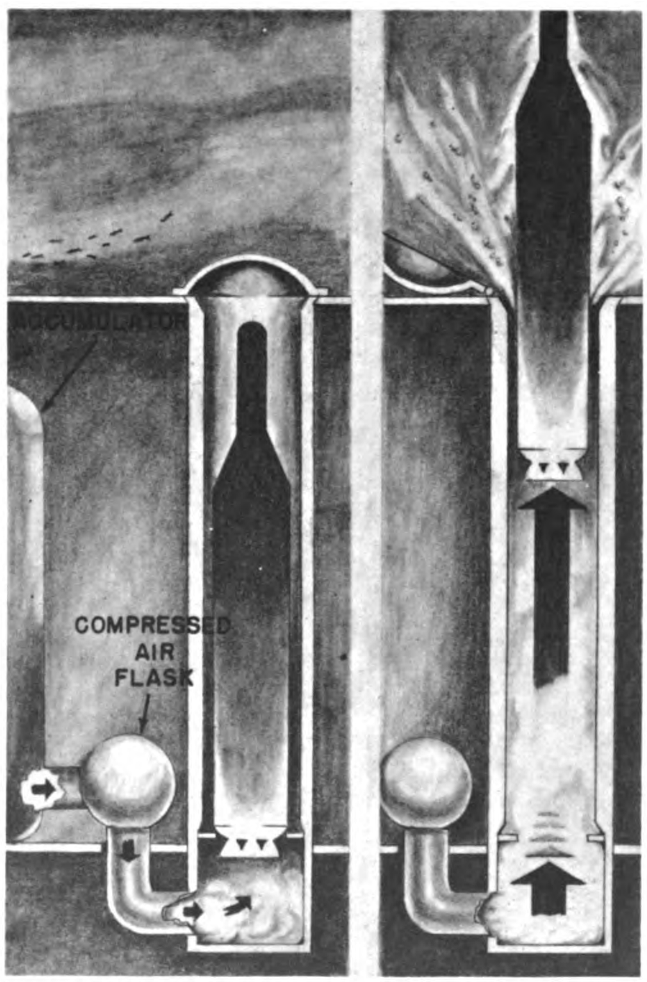
Grains of smokeless powder have a hard, smooth finish. As powder ages, it changes in appearance from translucent amber to dark brown, to black, and finally to opaque. No loss of stability is indicated by these changes.

Chemical decomposition is very gradual in smokeless powders but in time may result in spontaneous combustion, unless the appropriate preventive measures are taken. Smokeless powders, not unlike other explosives, are in a state of unstable chemical equilibrium and consequently are easily influenced by any impurities within them. If any part of the powder decomposes, the products of decomposition are acid in nature and will accelerate further decomposition. The stability of smokeless powders has been greatly increased by the addition of diphenylamine to counteract this acidity. A powder made chemically dangerous by partial decomposition is not unsafe for use in a gun, since the part of decomposition which normally is caused by the sudden evolving of hot gases within the gun has already taken place. The powder has lost a corresponding number of heat units.
Although stability is relatively unaffected at temperatures below 60°F, the decomposition rate rises sharply at temperatures above 70°F, becoming high at 90°F, and dangerous at temperatures over 100°F. Maintaining a uniformly low temperature in the powder storage areas is therefore essential.
Since decomposition is increased by the presence of moisture, smokeless powders must be stored in airtight containers. A leaky container will not only admit moisture but also may allow volatiles to escape through evaporation, especially if the air is subjected to temperature changes. The loss of volatiles may increase powder burning speed to such a degree that the pressures within the gun will become too great. Such powder is considered ballistically unsafe.

Triple-base powder is by far the most stable of the smokeless powders because of its relatively low nitrocellulose content (only 19 percent), extremely small content of volatile components, and low hygroscopicity. The gases evolved from a triple-base powder have much less erosive effect on a gun barrel than those produced by a double-base powder. This is because the large percentage of nitroguanidine (55 percent) produces lower gas temperatures. Triple-base powder also burns more completely, thus leaving less residue in the gun. Although it usually requires more ingredients than the other gun powders, it costs less to produce.

compressed air

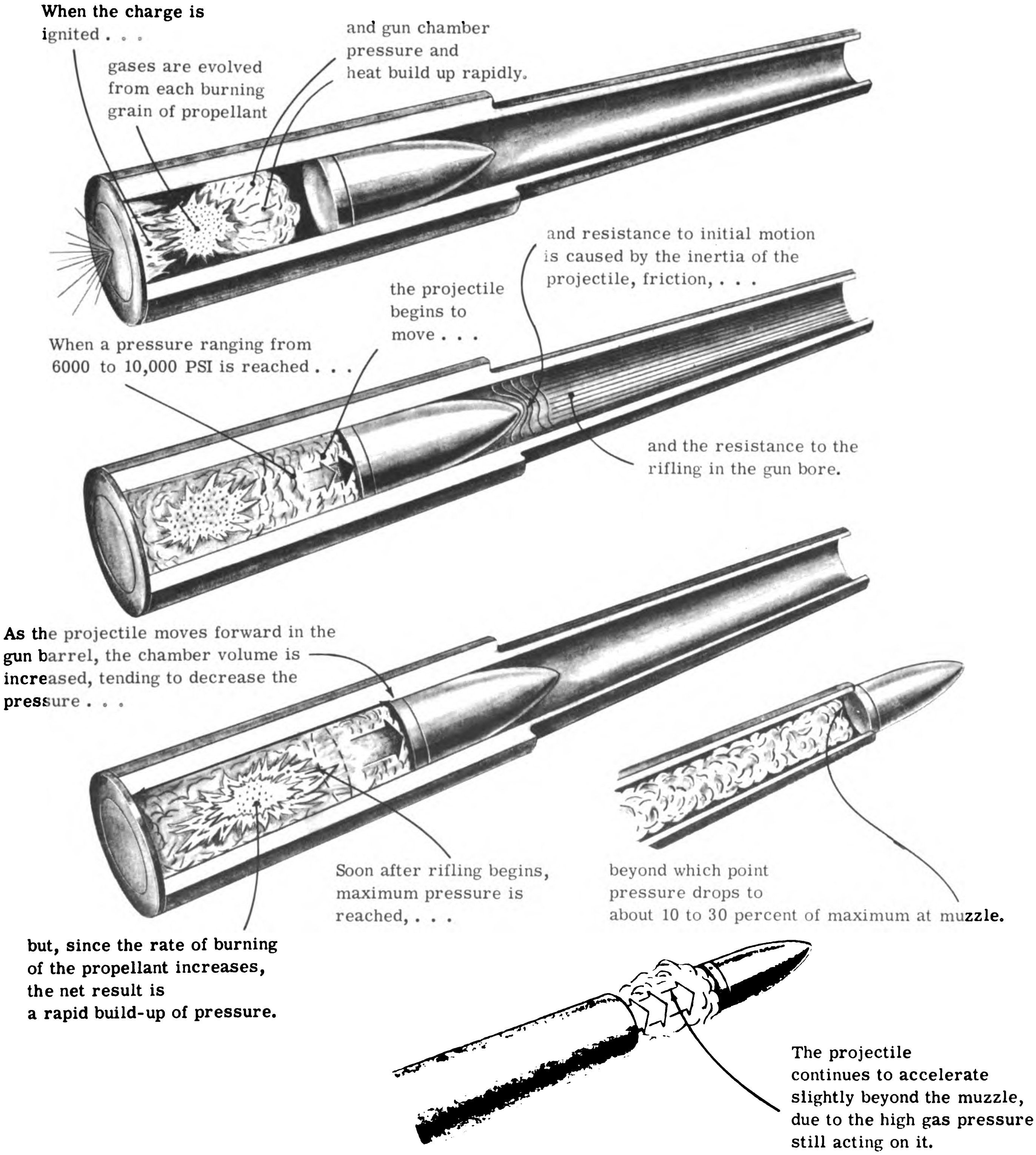
Compressed air is used for the ejection of torpedoes and missiles such as the Polaris. Particularly useful in submarines, compressed air has the advantage of being readily available and easily controllable without the obvious disadvantages of explosives (sensitivity to shock, and flame, storage problems, etc.). In addition, the high pressures obtainable from gunpowder are not needed in these applications, since the expanding gas serves primarily as a booster, with a sustaining propulsion system taking over once the missile is clear of the submarine.



IMPULSE-PROPULSION PRINCIPLES

action inside a gun

A gun is essentially a heat engine. Its action resembles the power stroke of a gasoline engine, with the expanding hot gases driving a projectile instead of a piston.



pressure-travel curves

The pressure-travel curve represents pressure acting on the base of a projectile at any point in its passage through the gun tube. The areas under the curve represent the work done per unit of the cross sectional area of the projectile during its travel through the gun. Therefore, if the areas under the two curves, A and B, are equal, then the work performed per unit area in each case is the same. Since

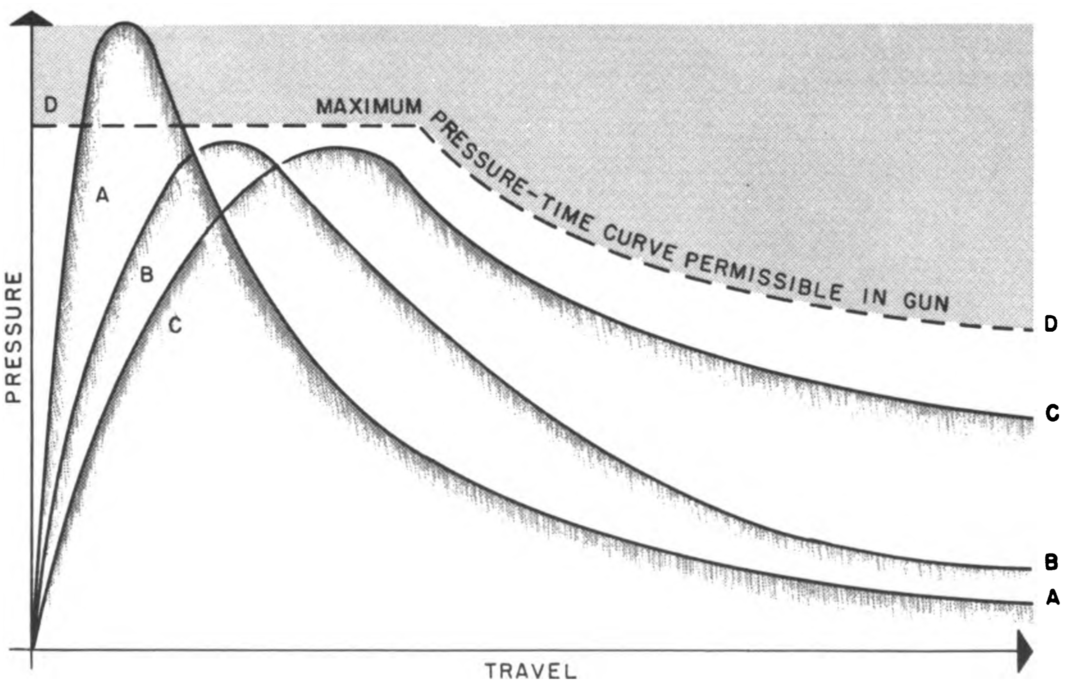
$$\text{Work} = KE = 1/2\ MV^2$$

the muzzle velocity produced in each case will be the same. Curve D represents the maximum permissible pressure based on gun tube strength. Any charge producing a pressure curve exceeding this limit, such as the one represented by curve A, would not be permissible. All guns are designed with a specific pressure-travel curve in mind, so that the desired muzzle velocity can be achieved without causing damage to the gun.

To increase muzzle velocity, the area under the pressure-travel curve would have to be increased without exceeding the pressure limits of the gun. For example, the area under curve C is greater than that under curve B; therefore, the muzzle velocity in C is greater than that in B. It would seem that the ideal charge would be one that produced a pressure-travel curve which coincided with the maximum permissible pressure curve of the gun. However, this is not true, since a charge capable of producing such an effect would have many disadvantages. Erosion would be increased, causing a decrease in weapon durability. The powder chamber would have to be enlarged, which would add to weapon weight and thus decrease weapon mobility. In addition, because of the high muzzle pressure that would exist, excessive muzzle flash and irregular muzzle velocities would be created. The velocity prescribed for a particular gun is always somewhat below the maximum it is possible to achieve. The charge used will produce the desired muzzle velocity round after round without exceeding the permissible pressure at any point in the gun.

The actual pressure-travel curve of a gun system is dependent on many factors:

(a) Interior ballistic variables
(b) Ignition characteristics
(c) Powder grain characteristics
(d) Variation in gun-projectile system
(e) Variation in service conditions



use of Pressure-Time curve for examining suitability of powders for a given gun

distribution of energy

In a medium-caliber gun, the total energy released, assuming complete combustion of the propellant, is distributed as follows:

ENERGY ABSORBED	PERCENT OF TOTAL
Translation of projectile	32.00
Rotation of projectile	00.14
Friction	2.17
Total work done on projectile . . . . . . (muzzle energy) (Area under pressure-travel curve)	34.31
Translation of recoiling parts.	00.12
Translation of propelling gases	3.14
Heat loss to gun and projectile	20.17
Sensible and latent heat losses in . . . . propelling gases	42.26
Total energy	100.00

efficiencies of gun and charge

The overall performance of a given gun-charge-projectile system can be expressed in terms of its piezometric efficiency and its ballistic efficiency.

Piezometric efficiency is found by dividing the mean pressure by the peak pressure, where the mean pressure is that pressure which, if uniformly exerted on the projectile over the entire length of the barrel, will produce the muzzle velocity observed. The higher the piezometric efficiency the flatter the pressure-travel curve. A high piezometric efficiency permits a shorter and lighter barrel to be used but requires a larger chamber. Because of high muzzle pressures, however, non-uniform muzzle velocities and greater muzzle flash may result.

Ballistic efficiency is defined as the ratio of the total work done on the projectile to the total work potential of the charge, or

$$\text{Ballistic Efficiency} = \frac{\text{Muzzle energy}}{\text{Potential energy}} \times 100$$

A high ballistic efficiency is obtain by burning the charge as early as possible in the projectile's passage through the bore, in order that there be as little residual pressure as possible. A high explosive would be ideal for high ballistic efficiency but would be impractical in gun design. Therefore, by considering both ballistic efficiency and piezometric efficiency, the most useful compromise may be chosen.

interior ballistic variables

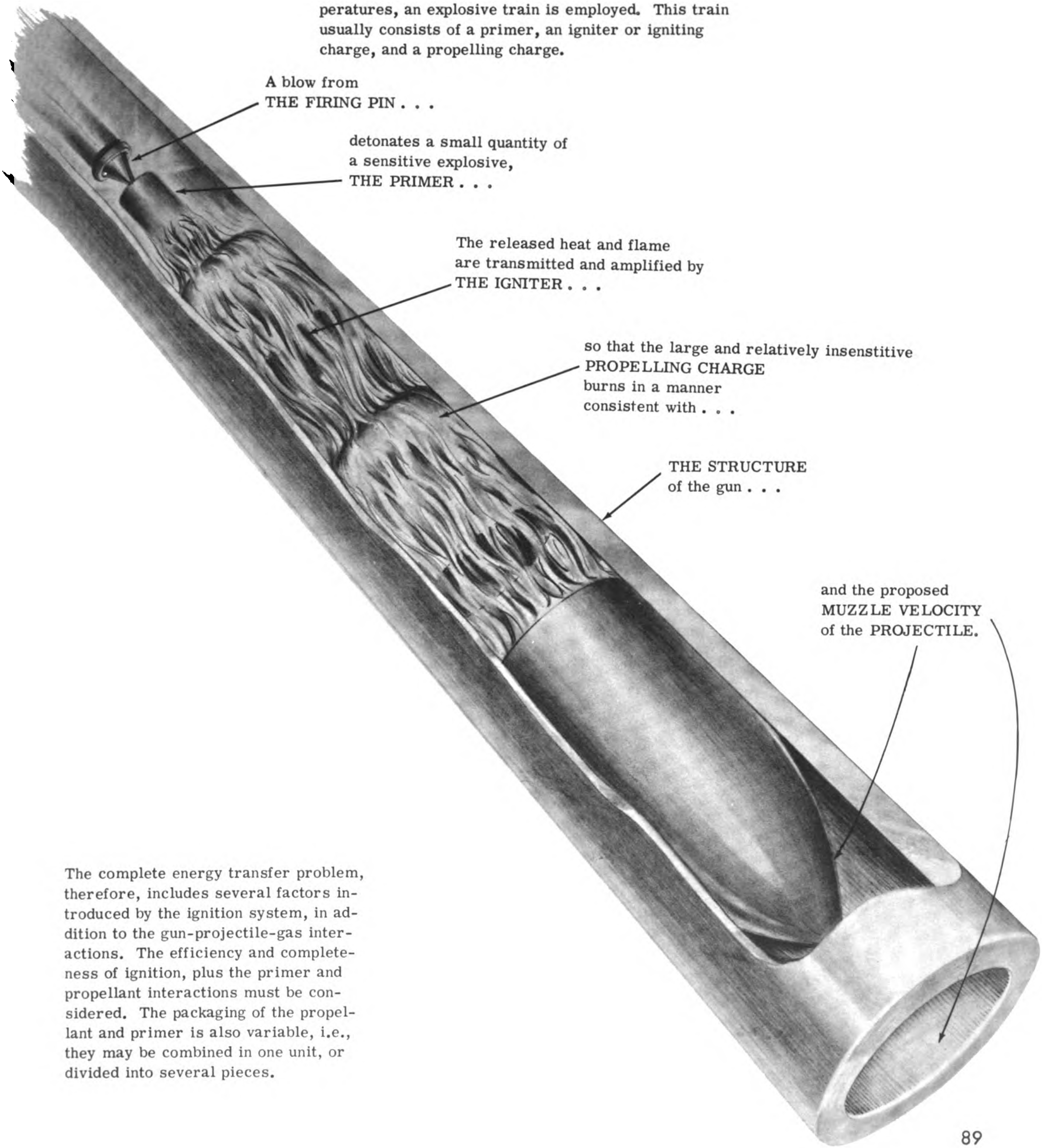
Interior ballistic performance is controlled by many variables including variations in:

(a) Ignition characteristics
(b) Grain geometry (surface factors)
(c) Chemical composition of the powder
(d) Rate of burning
(e) Charge weight (density of loading)
(f) Environmental factors

To evaluate the influence of all variables on a particular problem requires theoretical analysis, use of established empirical relationships, and detailed experimentation.

ignition principles

Propellant powders must be ignited by high temperature rather than shock, since the latter may result in detonations. In order to achieve the necessary high temperatures, an explosive train is employed. This train usually consists of a primer, an igniter or igniting charge, and a propelling charge.



The complete energy transfer problem, therefore, includes several factors introduced by the ignition system, in addition to the gun-projectile-gas interactions. The efficiency and completeness of ignition, plus the primer and propellant interactions must be considered. The packaging of the propellant and primer is also variable, i.e., they may be combined in one unit, or divided into several pieces.

In the ammunition for most large case guns, the primer and igniter are fabricated as a single unit, which is referred to as the "primer". Where the propellant is small enough to be ignited by the primer, an igniter is not required. This is the case in most small-arms cartridges, the components being a percussion primer and a propelling charge. The primer is exploded by the action of the firing pin and ignites the charge.

Heat is transferred to the main charge by two means: radiation and conduction. When the ignition charge is surrounded by the propelling charge, the overall radiations between the charges is a function of the surface areas, the emissivity, and the fourth power of the temperatures of the radiating bodies; that is:

$$\frac{dq}{dt} = cEA\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]$$

where

$\frac{dq}{dt}$ = rate of radiant heat flow (q = heat)
c = radiation coefficient, 0.172
E = emissivity or specific radiating ability
A = area of radiating surface
T_1 = absolute temperature of the hot body (primer)
T_2 = absolute temperature of the cool body (charge)

The emissivity, E, of most solids is up to ten times as high as that of gases. Since luminous flames contain large quantities of solid particles in suspension, they radiate much more intensely than non-luminous flames. Because its products of explosion contain large amounts of solids, such as potassium carbonate and sulphate which radiate intense heat, black powder is ideal for use as a priming compound.

The flow of heat from a hot gas to a solid by conduction is dependent on the temperature, the mass, and the velocity of the gas, that is:

$$\frac{dq_c}{dt} = KC_pAT_1 \frac{0.25G^{0.8}}{D^{0.2}} (T_1 - T_2)$$

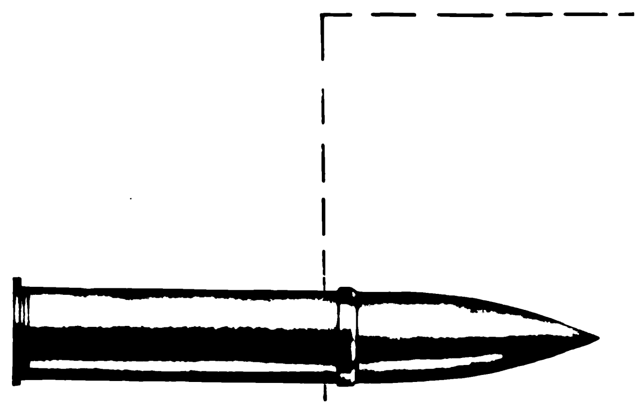
where

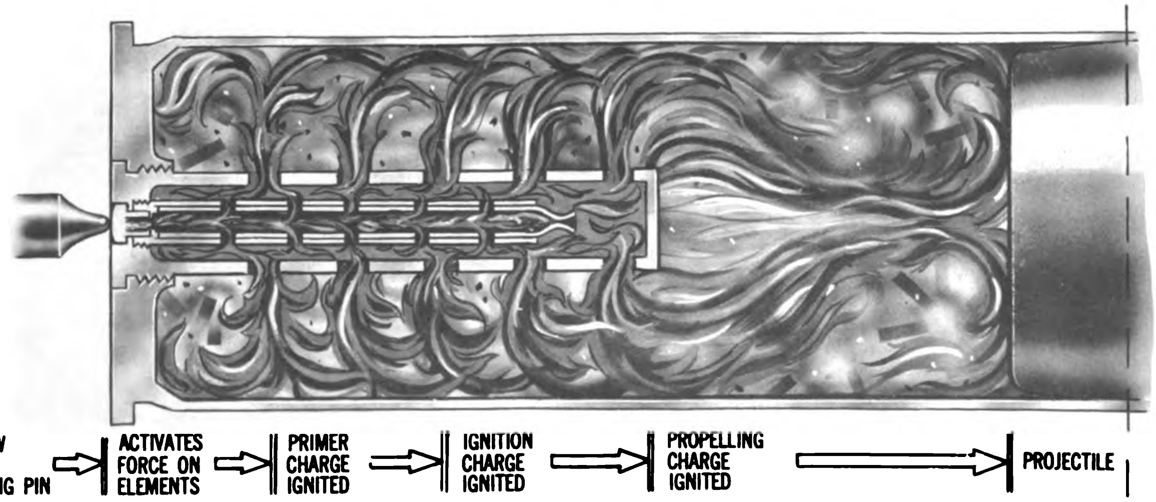
K = conductivity coefficient
C_p = specific heat at constant pressure of the hot gas
A = area of the solid to which heat is flowing
T_1 = absolute temperature of the hot gas
T_2 = absolute temperature of the cold surface
G = mass velocity of gas in contact with solid
D = average diameter of the solid

When luminous solid particles are present in the gas, the radiation effects exceed those obtained by conduction alone.

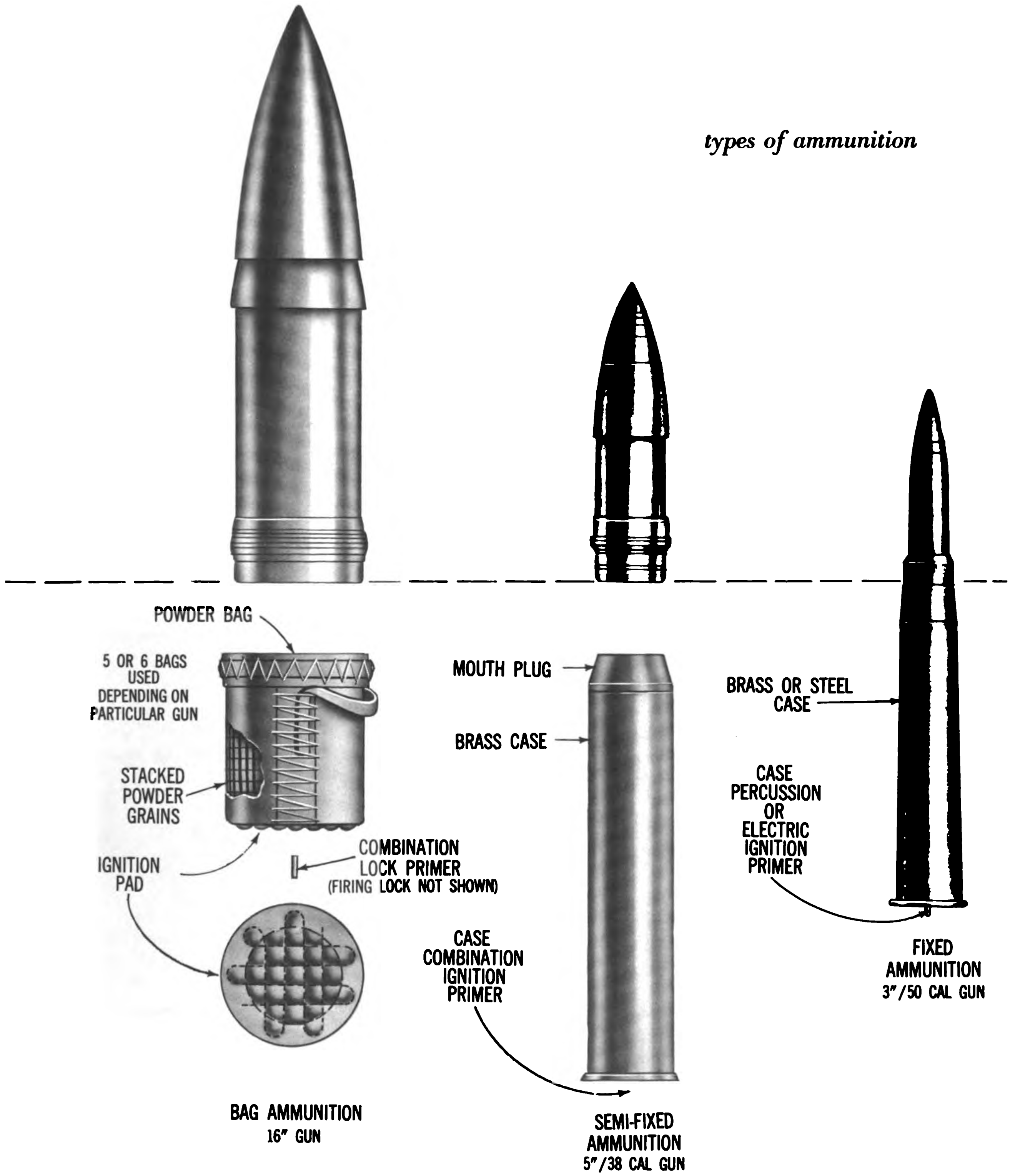
An ideal primer would ignite each gain of powder in the propelling charge at the same instant. Theoretically, the most satisfactory primer would be an explosive gas which would permeate the entire propelling charge, liberating solid particles. To date, black powder has proven the most practical solution.

Originally, the primer-igniter combination was comparatively short, being about one-quarter of the total length of the case. Because this design was found to be both unsafe and inefficient, primers now used are almost as long as the case itself. However, no additional black powder is used. It is simply spread out over the longer length.





types of ammunition



In weapons where the propellant is separate from the projectile, such as systems employing powder bags, ignition has been greatly improved by placing a core of black powder through each powder bag, or by attaching an igniter to several parts of the charge.

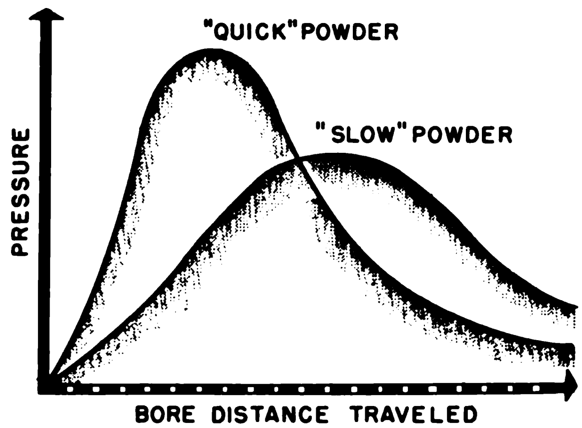
CHARACTERISTICS OF PROPELLANTS

The important properties of propellants are determined by four characteristics:

GRAIN COMPOSITION
GRAIN CONFIGURATION
GRAIN SIZE and
DENSITY OF LOADING

grain composition

A definite residual moisture content and volatility product is specified for each powder composition and granulation. The powder may also contain small percentages of stabilizer (dephenylamine), graphite, and inert materials. Any changes in composition will affect "quickness". (Powders are referred to as "quick" or "slow" for a particular gun depending on their rate of burning.) The pressure-travel curve for a charge of "quick" powder reaches a high peak value very early in the projectile's travel through the bore.



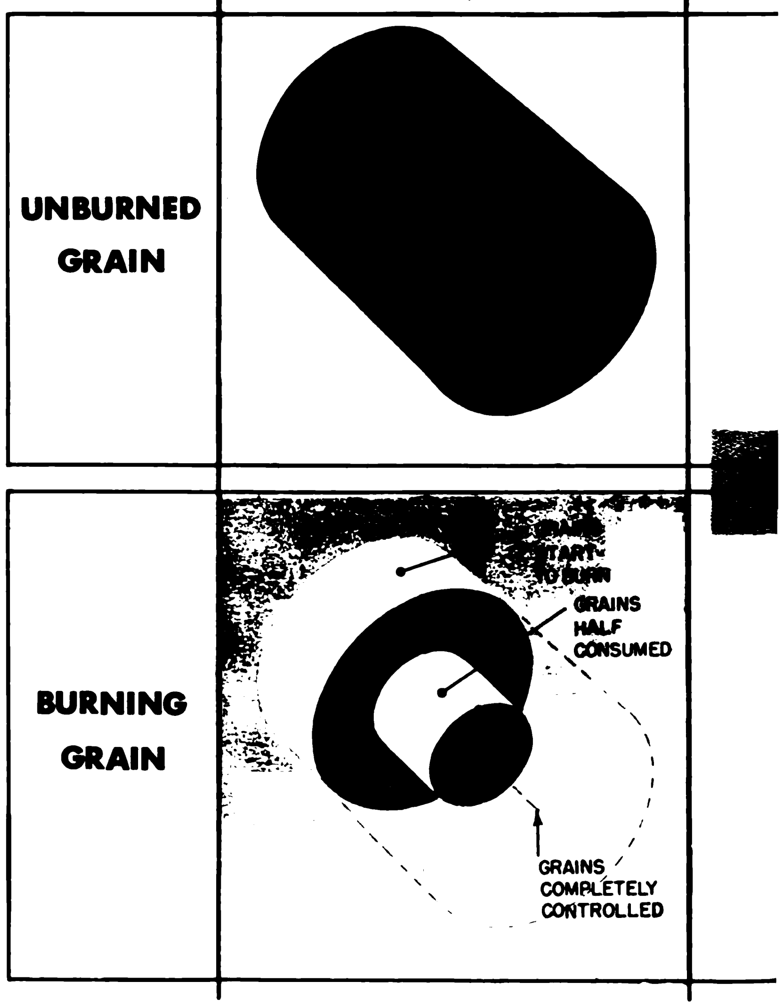
characteristic gun pressure curves with weight of charge as a constant

grain configuration

Gun powders are manufactured in grains of various shapes and sizes. Each grain of powder burns in layers parallel to the ignited surfaces. The greater the exposed surface area of a powder grain, the faster it will burn.

If the area of the burning surface of the powder grains continually decreases during combustion, they are termed

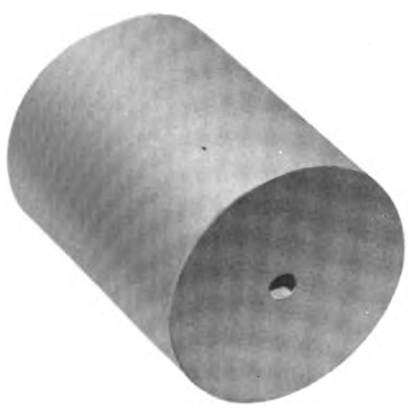
DEGRESSIVE
(solid)

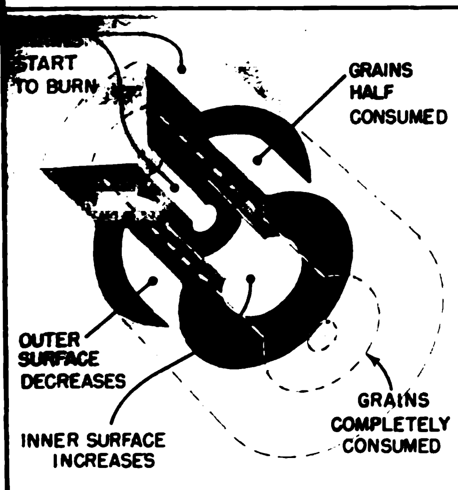


The powder grain which maintains an approximately constant burning surface is called

NEUTRAL

(single perforated)

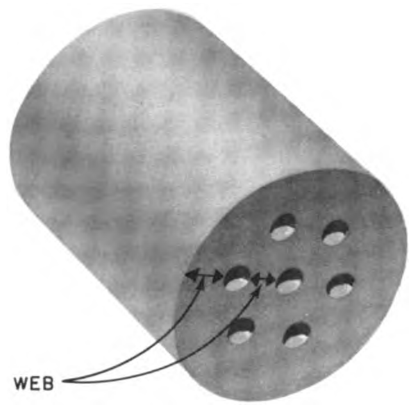


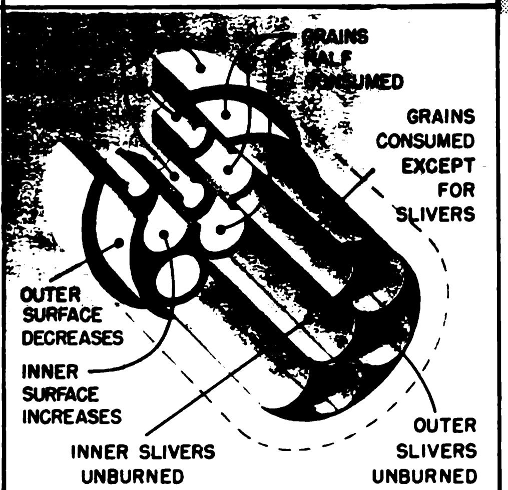


If the burning surface increases during combustion, the grain is considered

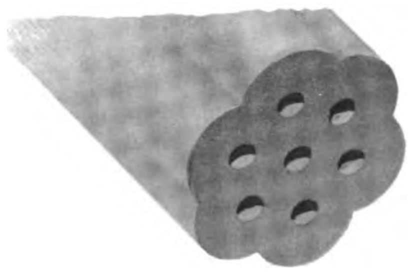
PROGRESSIVE

(multi-perforated)





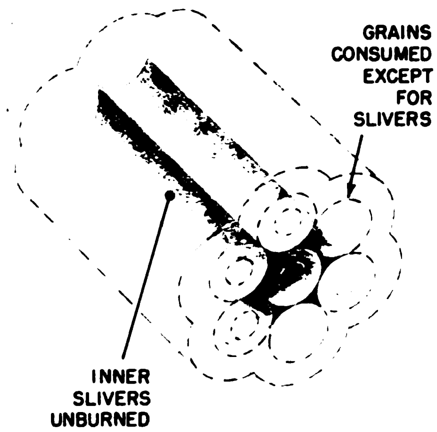
Most forms of grain will be completely consumed when the thinnest web thickness has been burned through. In multi-perforated cylinders, however, some slivers remain after the web has been burned through. Usually they are consumed before the projectile leaves the gun, but in a short weapon may remain unburned and be ejected along with the projectile.



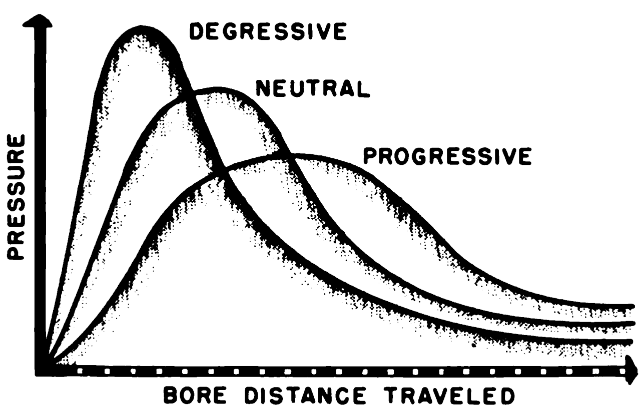
To elminate the outer slivers,

ROSETTE

construction is sometimes employed.



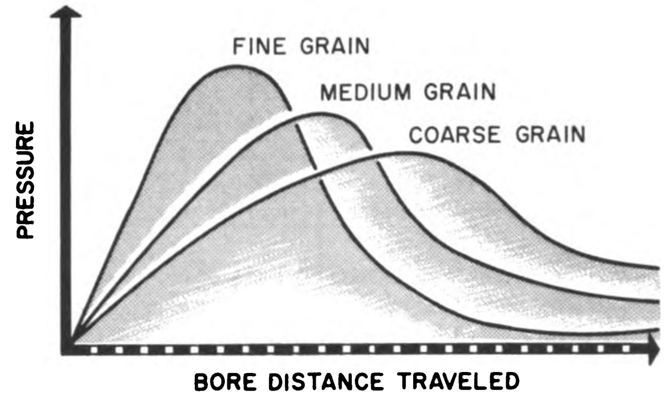
In achieving any desired muzzle velocity, a progressive powder reaches maximum pressure much later than a degressive powder and loses pressure much more gradually. Since the work done in both cases is the same, the degressive powder produces a much larger maximum pressure. The maximum pressure is dictated by the design of the gun. Thus, for a particular pressure limit, a progressive powder would produce higher muzzle velocity than a degressive powder.



pressure-travel curve

grain size

The rate of burning, or quickness, of a gunpowder varies inversely with the size of the grain. Thus, if two powders are made up of grains which are identical in composition but different in size, the powder with the smaller grains will be consumed first. If two powder charges are of equal weight, the one made up of the smaller grains will be the quicker.

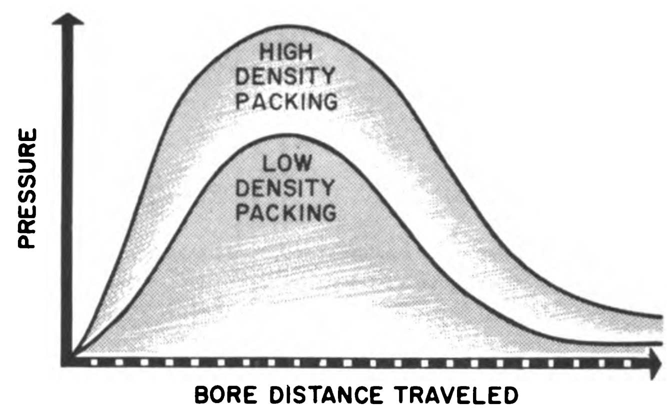


pressure-travel curve, illustrating the inverse relationship of grain size to quickness

density of packing

Density of packing is the ratio of the weight of the charge of powder to that of the volume of water which, at standard temperature, would fill the powder chamber. It is a measure of the amount of space in which the gases of combustion may expand before the projectile begins to move. High density of packing leaves little space for initial expansion; consequently, pressure builds up rapidly. Therefore, maximum pressure behind the projectile is recorded early in the projectile's travel through the gun. With low density of packing, there is more space for gas expansion before the projectile begins to move; consequently, maximum pressure occurs later and is less than that achieved with high density of loading. Other factors remaining equal, increased density of packing increases maximum pressure, muzzle velocity, and muzzle loss.

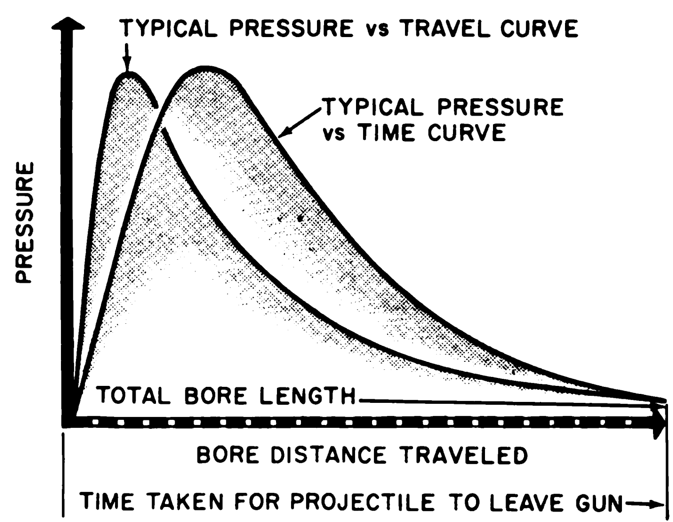
In choosing a propelling charge for a specific gun system, a compromise in the foregoing powder-grain characteristics (composition, size, and density of packing) must be made. For instance, small arms such as hand and shoulder weapons employ quick, degressive, small-grained powders in order to minimize muzzle blast even though high peak pressures result. On the other hand, large guns with high-inertial projectiles and long barrels have design problems resulting from low peak pressures. They employ slow, progressive, large-grained powders.



characteristic pressure-travel curve illustrating the direct relationship of density packing to reaction pressure at a given bore distance traveled by the projectile

pressure-time relationship

The pressure-time curve shown in the illustration has the same peak value but is not as steep as the pressure-travel curve. Pressure builds up with time, but the projectile does not begin its travel down the bore until the pressure has reached an appreciable value (usually in the order of 10,000 psi).



velocity-travel relationship

The LeDuc equation for velocity as a function of travel is based on the translation of a hyperbolic curve, the general equation of which is:

$$v = \frac{au}{b+u}$$

where v = velocity of the projectile (ft/sec)
a = empirical constant (ft/sec)
u = distance traveled by the projectile in the bore (ft)
b = empirical constant (ft)

Muzzle velocity is given by:

$$v = \frac{au}{b+u}$$

where v = muzzle velocity (ft/sec)
u = length of the bore (ft)

If u is allowed to approach infinity, then the muzzle velocity is equal to the limit of the general equation, as:

$$v = \lim_{u \to \infty} \frac{au}{b+u}$$

By employing L'Hospital's rule:

$$v = a$$

Therefore, a is the theoretical value of the projectile's muzzle velocity in a gun of infinite length.
The kinetic energy of the projectile when v = a is:

$$K = \frac{1}{2} \left(\frac{w}{g}\right) a^2$$

By equating the potential energy expression for the propelling gases to the kinetic energy of the projectile at v = a, an expression for a can be determined. From the results of calculation and experimentation:

$$a = K\left[\frac{w_c}{w_p}\right]^{1/2} \Delta^{1/2}$$

where
w_c = weight of the charge (lbs)
w_p = weight of the projectile (lbs)
Δ = density of packing
K = a constant determined from the potential of the propellant

The relationship between the constant (b) and the distance traveled (u) is found by differentiating the general equation:

$$v = \frac{au}{b+u}$$

$$\frac{dv}{dt} = \frac{ab}{(b+u)^2} \quad \frac{du}{dt}$$

since $\frac{du}{dt} = v$

$$\frac{dv}{dt} = \frac{ab}{(b+u)^2} \quad \frac{au}{(b+u)} = \frac{a^2bu}{(b+u)^3}$$

$$\frac{d^2v}{dt^2} = \frac{(b+u)\, a^2b - 3a^2bu\, (b+u)^2}{(b+u)^6} \quad \frac{du}{dt}$$

$$= \frac{(b+u)a^2b - 3a^2\, bu}{(b+u)^4} \quad \frac{du}{dt}$$

$$= \frac{a^2b\, (b+u) - 3u)}{(b+u)^4} \quad \frac{du}{dt}$$

when $\frac{a^2v}{dt^2}$ 0, $\frac{dw}{dt}$ is a maximum

and b-2u = 0

$$u = \frac{u}{2}$$

Therefore, maximum acceleration $(\frac{dv}{dt})$ occurs when the bore travel (u) is equal to one-half of the constant (b).
The propellant force acting on the projectile is given by:

$$F = \frac{wp}{g} \quad \frac{dv}{dt}$$

Therefore, the pressure is:

$$P = \frac{\frac{wp}{g} \; \frac{dv}{dt}}{A}$$

where A = area of the base of the projectile

From this equation, it is seen that the point of maximum acceleration will also be the point of maximum pressure. Therefore, the constant (b) is equal to twice the distance traveled (u) at the point of maximum pressure.

The empirical formula for b, used in all calculations by the LeDuc method, is:

$$b = B\left(1 - \frac{\Delta}{s}\right)\left(\frac{S}{W}\right)^{2/3}$$

where B = powder constant
S = specific gravity of the powder
s = volume of the powder chamber

For each powder manufactured, a powder constant representing the relative quickness of the powder is determined. The value of this constant varies inversely with the velocity of burning and therefore is largest for "slow" powders. Since b is directly proportional to the powder constant, a "slow" powder will cause the point of maximum pressure to occur farther down the bore than a "fast" powder.

GUN AND PROJECTILE VARIABLES

Variations in the structure of the gun and projectile for a charge of fixed size and nature affect pressure and muzzle velocity.

gun chamber volume

The density of packing for a fixed charge varies inversely with the volume of the powder chamber. If the projectile is not seated uniformly, or if a different projectile or different gun is employed, the chamber volume varies. Decreasing chamber volume (increasing density of packing) increases muzzle velocity and maximum pressure. The length of travel to the maximum pressure point is correspondingly decreased.

gun tube length

Increasing the caliber or length of the gun tube increases the muzzle velocity. This is clearly indicated in the pressure-travel curves. Since energy from the expanding gases remains after the projectile leaves the weapon, lengthening the gun tube would enable more of this energy to be coupled to the projectile, thus increasing its muzzle velocity. However, lengthening the gun tube increases the weight and thus decreases weapon mobility. Therefore, beyond a certain point, the added velocity is not worth the addition in weight.

projectile weight

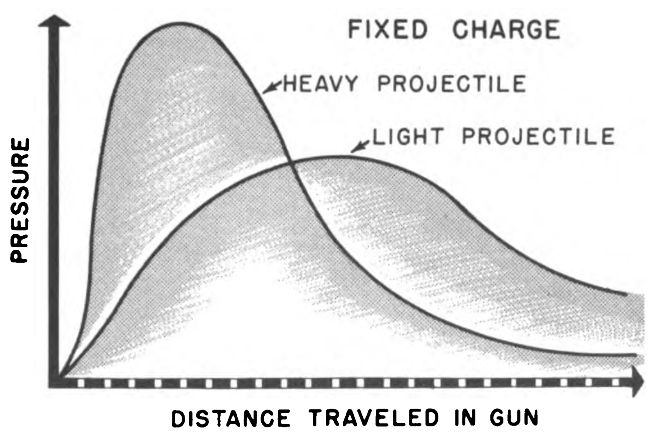
Increasing projectile weight has the effect of increasing powder "quickness": that is, the heavier the projectile, the greater the force required to move it; thus, maximum pressure is greater and is reached earlier in the projectile's travel. The increased weight will lower muzzle velocity. Since the work done in each case will be approximately the same,

$$\text{and Work} = KE = \frac{1}{2} mv^2 = \frac{1}{2} \frac{w}{g} v^2$$

$$\text{then } v = \frac{2\,(KE)\,g}{w}$$

That is, muzzle velocity will vary inversely with the square root of projectile weight. A more accurate expression of this relationship would be:

$$v = Kw^{-h} \quad \text{where } .35 \geq h \geq .5$$



SERVICE ENVIRONMENT

Environmental conditions of ambient temperatures, gun temperatures, deterioration of ammunition in storage, etc., may adversely affect ballistic performance. Various methods are used to take such conditions into account.

temperature of the powder

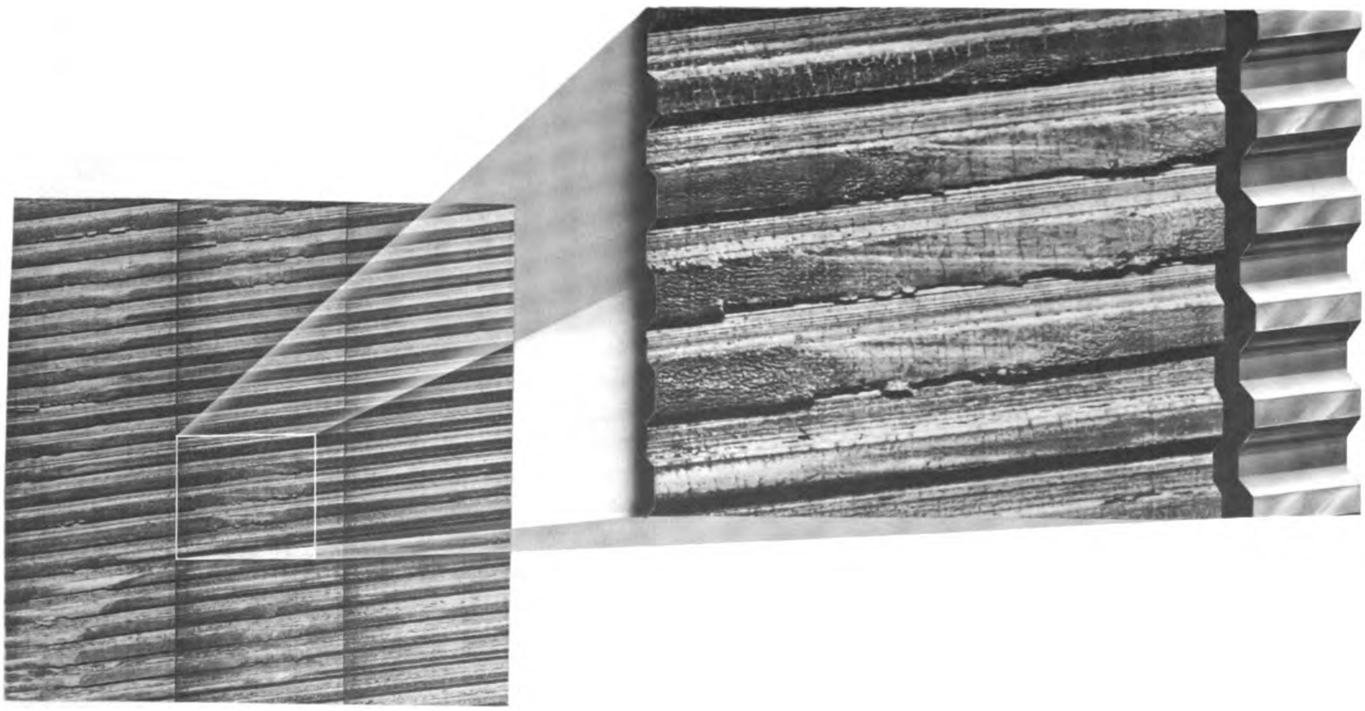
Range tables are based on a powder temperature of 90°F at the time of firing. Increasing this temperature increases the burning rate and maximum pressure of the propellant, resulting in greater muzzle velocity. In a 5 in. gun, for example, the muzzle velocity will be increased approximately 2 feet per second for each degree of increase in temperature; a decrease in temperature will cause a corresponding decrease in velocity.

temperature of the gun

Rapid fire produces high gun temperature which, if allowed to act on a round for any length of time, can have an appreciable effect. As the temperature of the powder is raised, the muzzle velocity is also increased. If the gun is hot enough and the round is allowed to remain unfired in the breech long enough, spontaneous firing occurs. This is commonly known as "cook off".

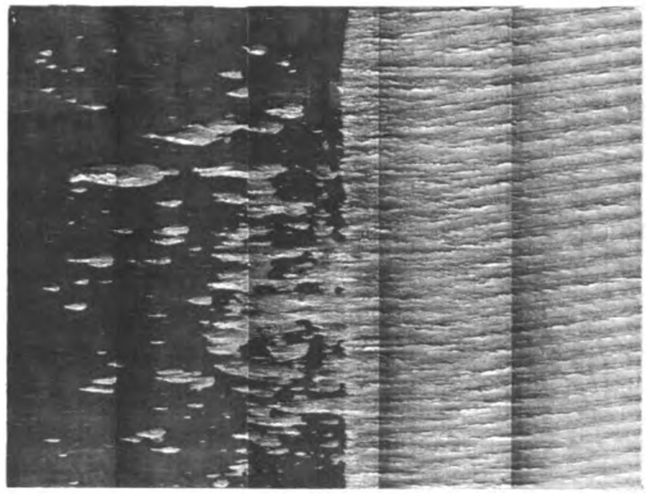
gun bore erosion

The accuracy of a gun is dependent on the condition of its bore, which is determined by the degree of wear present. There are two types of wear: erosion, which is the result of firing; and corrosion, which is the result of neglect. Erosion is greatest at the origin of rifling where the temperature of the propelling gases is highest. The process of erosion wears down the lands, which are the ridges between the grooves in the bore, so that the bore becomes irregular and enlarged. Erosion results in loss of pressure and muzzle velocity, ultimately enlarging the bore to the extent that spin is no longer imparted to the projectile. Consequently, the projectile tumbles and its flight is not accurately predictable.



CAUSES OF EROSION

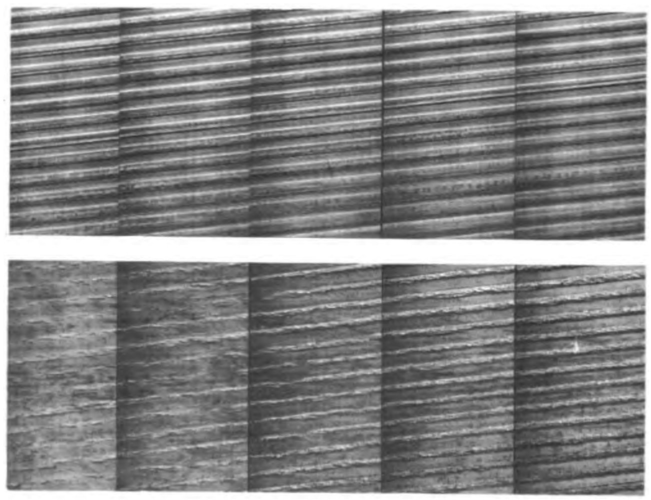
The phenomenon of erosion is not entirely understood, but the probable principal causes are as follows:
1. The surface layer of bore metal is melted and then disarranged or moved by the hot high-velocity gases.
2. The bore is mechanically abraded by the projectile, the gas, and the unburned solid particles of the charge.
3. The lands of the bore are easily worn away by the chemical reaction of the escaping gases.
4. The bore is scored as a result of leakage of gas escaping past the rotating bands of the rifling. This form of erosion can be intensified by mechanical defects in the manufacture of the original rifling and leads to a reduction in pressure and muzzle velocity.



gas erosion

CONTROL OF EROSION

All erosion factors are related to the temperatures of the expanding gases and the length of time in which these gases are present in the bore. Therefore, large guns with slower powders and larger barrels undergo more erosion per round fired than small guns. In small guns, the problem becomes one of cooling, since their rapid rate of fire does not permit adequate cooling time between rounds. Various means have been employed to control erosion. Hard plating of gun bores has greatly reduced erosion. In smaller guns, water jackets have been employed around the barrels. Large guns use coolant between their barrels and liners. In addition, cooler-burning propellants have been developed to facilitate erosion control.



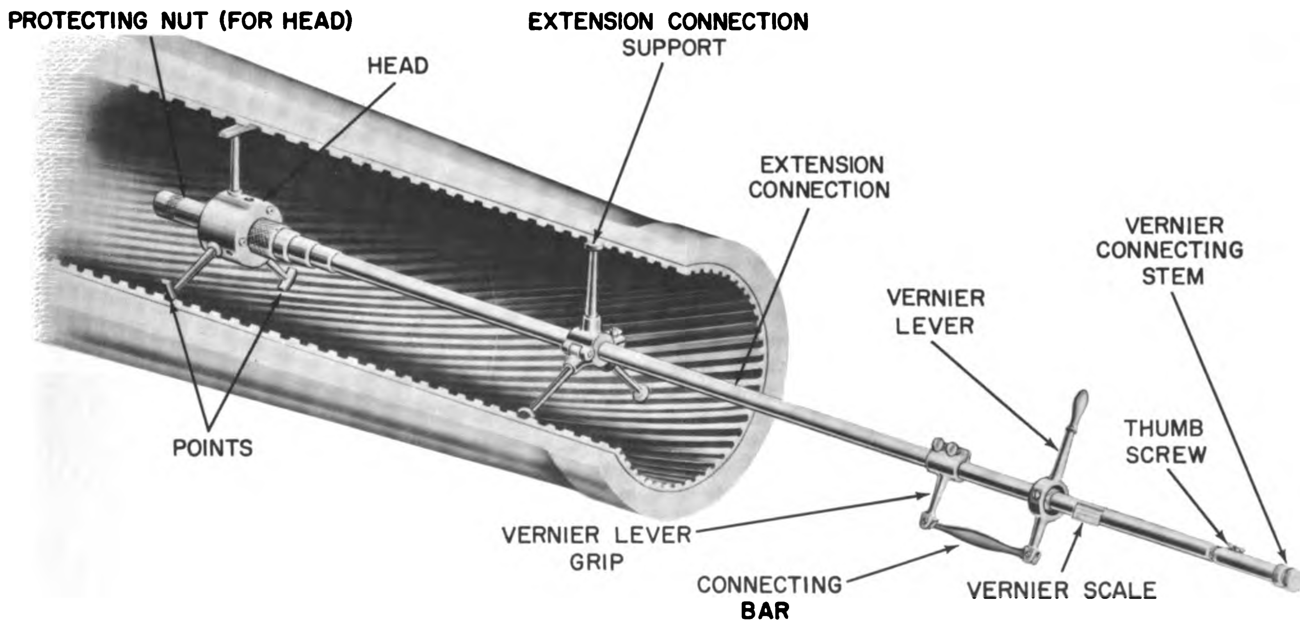
gas erosion and heavy scoring

EROSION MEASUREMENT

Erosion can be controlled, but not eliminated, and eventually will render the gun ineffective or unsafe. The symptoms of erosion are a progressive loss in accuracy and muzzle velocity, and erratic fuze behavior. For each class of gun, there are curves which show the relationship between the enlargement of the bore and the muzzle velocity of the gun. Thus, by checking the bore diameter, the velocity loss can be predicted and compensated for. In addition, barrel liners can be replaced before the performance of the gun becomes noticeably erratic. Therefore, particularly with large guns, it is important that this point be anticipated and that tube liner replacements be made before the effectiveness of the gun is seriously reduced.

THE EROSION GAGE, OR WEAR GAGE, is a truncated cone which can be inserted directly into the breech; the further in it fits, the greater the erosion. The wear gage is used in small caliber guns where measurement of bore enlargement is made only at the origin of rifling. A graduated scale on the gage permits reference to standard tables or curves of muzzle velocity versus gage reading for each type of gun.

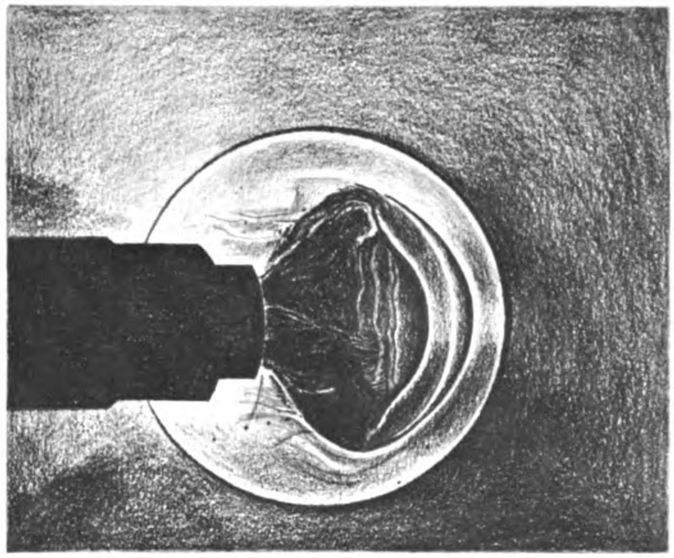
THE STAR GAGE is a form of inside calipers which permits bore diameter to be measured at several points along the length of the barrel. There are three radial measuring points 120 degrees apart at the head of a long shaft with means for extending the points radially until they contact the inner diameter of the bore, which is indicated on a vernier scale. Star gages are generally not available on combat ships but can be found on tenders and shore-based repair installations.



THE EQUALIVALENT SERVICE ROUND (ESR)
Each round fired from a given gun causes some erosion, and this erosion is progressive, increasing with successive firings. Curves relating the number of rounds fired to muzzle velocity loss have been determined experimentally for each gun. The ESR is a standard unit of measurement of the amount of erosion which takes place with each firing of the gun, averaged out over the effective lifetime of the bore. An ESR is available for each type of projectile and each charge to be used with any given gun. Thus, by logging all firings, a fairly accurate estimate of velocity loss can be determined between star gagings. Periodic gagings is necessary to "calibrate" the ESR data for a particular gun.

INITIAL CHARACTERISTICS OF

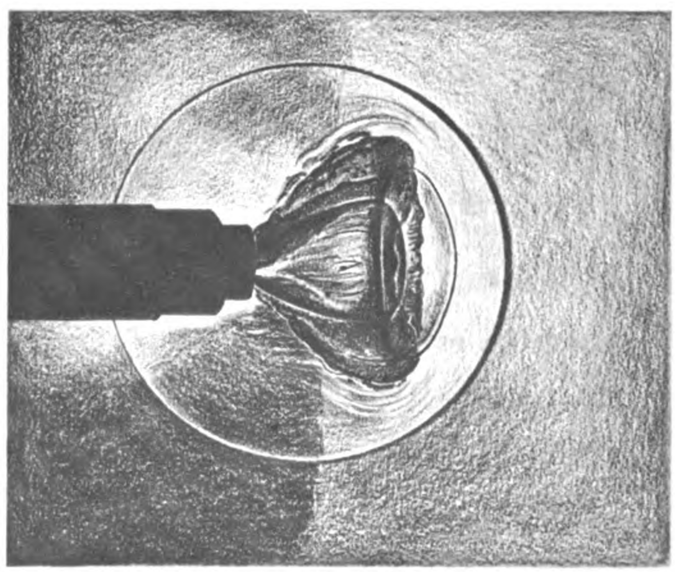
In launching a projectile from a gun, a number of phenomena occur in the vicinity of the muzzle, i.e., between the areas of interior and exterior ballistics. In the past, most of the problems in this area were dealt with by interior ballistics. Lately, however, they have come to be known as transition ballistics, and consist essentially of the initial effects of air, vertical jump, and lateral jump.



initial shock wave

initial air effects

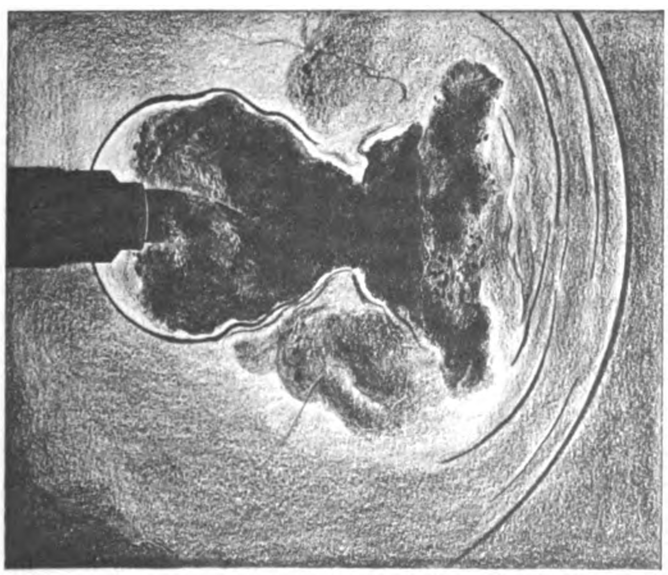
The projectile moving forward in the barrel pushes the air mass in front of it, causing the latter to emerge from the muzzle first. The internal air mass, now traveling at a high velocity, strikes the outside air, which is at rest, and creates a shock wave which develops spherically and disturbs the outside air. This condition is immediately followed by a rush of small amounts of powder gas which have forced their way in front of the projectile, and hence emerge from the muzzle ahead of it. As the base of the projectile clears the muzzle, the main mass of the propellant gas begins to pour out into the already turbulent outside air. At this instant, the velocity of the gas is equal to that of the projectile, but, because of the tremendous pressure present, the velocity of the gas increases suddenly, causing it to overtake the projectile and pass it rapidly. During this phase, the gas develops a maximum velocity of more than twice that of the projectile and consequently imparts to the latter an additional thrust, thereby causing it to reach its maximum velocity, not at the muzzle, but at some short distance beyond the end of the muzzle.



emerging powder gas

GUN LAUNCHED PROJECTILES

The gas loses velocity very rapidly, because of its small mass and the resistance to motion which it meets. In small arms, for example, the bullet overtakes the gas at approximately 35 cm from the muzzle. Shortly thereafter, the projectile overtakes and pierces the report wave, which produces the noise of the exploding propellant. At this instant, the projectile is accompanied by the normal head wave which is defined as the projectile shock wave. It should be noted that a shock wave cannot form on the projectile unless the relative velocity of the projectile and the surrounding gaseous medium equals or exceeds the speed of sound. During the time the projectile is passing through the powder gas envelope, this condition does not exist, and hence no head wave is formed. However, at the instant the projectile pierces the report wave, the required conditions exist and a head wave is formed. Obviously, in guns with a high cyclic rate of fire, some exterior effect must be present, due to gas stagnation, since the turbulence of the gaseous medium in the vicinity of the muzzle creates a condition of instability. The stagnation, or pressure limit, created in front of the muzzle is the result of high-velocity propelling gases emerging and compressing the initially still air, thus creating a marked retardation effect. An envelope of gas is thus formed with maximum pressure existing at the intersection of the stagnation line and a prolongation of the axis of the bore.



emerging projectile

The cycle of events, described by a related series of Schlieren photographs, continues as long as gas under high pressure continues to emerge from the muzzle. As the pressure subsides, the stagnation line moves toward the muzzle of the gun. The effects of the air disturbances described here are directly associated with the cause and control of muzzle flash.

vertical jump

Vertical jump is the angle of error between the line of elevation of the gun and the line of departure of the projectile to the target. When a gun is standing idle, the axis of its barrel tends to bend downward. This is known as gun droop; the longer the barrel, the greater the droop. When the gun is fired, the projectile whips the barrel upward. A similar action takes place when water is forced through a coiled hose, with the result that the gun barrel is bent slightly upward at the instant the projectile leaves the muzzle. This action is known as muzzle whip. A projectile twisting clockwise in a gun bore will impart a counterclockwise torsion to the gun tube. The vertical component of the torsion will contribute to vertical jump. Because of vertical jump, the projectile leaves the gun at an angle greater than the angle of elevation of the gun. A number of factors influence vertical jump. One factor which can give rise to vertical jump is the lack of stability and rigidity of the gun at the time of firing. This effect is most noticeable with hand and shoulder weapons. Shipboard gun emplacements are usually rigid enough to make this effect negligible. When the projectile moves towards the muzzle, the center of gravity of the system also moves in the direction of the muzzle, causing the muzzle of the gun to move toward the ground. Thus muzzle whip is somewhat reduced.

lateral jump

The angle of error between the line of sight of the gun to the target and the line of departure of the projectile to the target is known as lateral jump. It is usually negligible and, even when considered, has a magnitude much smaller than vertical jump. Its causes are similar even to the point of a lateral muzzle whip. This is due to the action of the projectile against a lateral bend in the gun tube. The unwanted curve in the gun barrel is usually the product of poor machining.

SPECIAL CASES

The gun propulsion principles discussed previously apply to the standard concept of gun-projectile design. With the advent of modern, high-speed aircraft that can travel at speeds in excess of mach 2, defense became a matter of fire control and weapon response time. The new developments in attack probabilities have in turn demanded the design of special guns that can successfully counter the threat of the attack weapon. Special gun designs are constantly being developed that employ variations in or higher level improvements of previously used gun-manufacturing techniques. Two classes of special guns are the hypervelocity gun and the recoilless gun.

THE HYPERVELOCITY GUN and THE RECOILLESS GUN

hypervelocity gun

PRINCIPLES

Since interception of high-speed targets presents a difficult fire-control problem, in which a predicted future position of the target is being fired at, the less time the projectile spends in the air, the less time the targe has for defensive maneuvering. Thus, demand for higher muzzle velocities of gun projectiles has developed and this demand continues to increase. The basic principle of muzzle velocity increase is to increase the area under the projectile pressure-travel curve. For an optimum gun, this requires approaching a pressure-travel curve that is as nearly flat as possible.

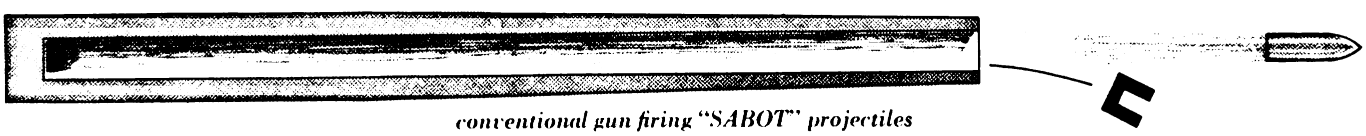
means of increasing muzzle velocity

In a hypervelocity-type gun, higher muzzle velocity is obtained by using lightweight or subcaliber projectiles which effectively reduce the force requirements of the propellant. Higher muzzle velocity can also be obtained by increasing the pressure or propellant force applied to the projectile, using machanical devices which serve as gas seals to prevent leakage of propellent energy. Of the various means employed to achieve higher muzzle velocities, the following have proven the most practical.

LIGHTWEIGHT PROJECTILES. By firing lightweight projectiles from conventional guns, the muzzle velocity for standard projectiles can be increased from 2700 feet to about 4000 feet per second. This type of projectile, being greatly affected by air resistance, is useful only at short ranges.

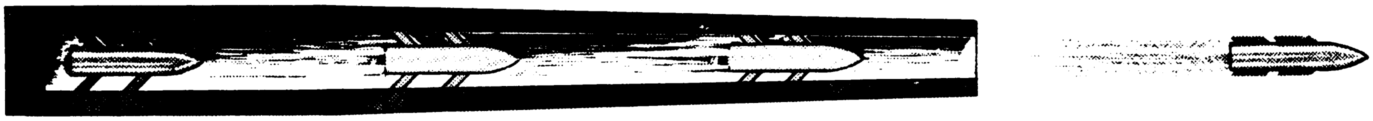
conventional gun firing LIGHTWEIGHT full-diameter projectiles

SUBCALIBER PROJECTILES. By using a projectile of smaller caliber than the gun, the projectile weight and the resistance encountered during flight are reduced. In order to seat the projectile firmly in the gun, it is fitted with a lightweight bushing, called a sabot, which falls away from the projectile after it leaves the muzzle. The main disadvantage of this type of system is the danger which the released sabot presents to friendly forces in the vicinity.



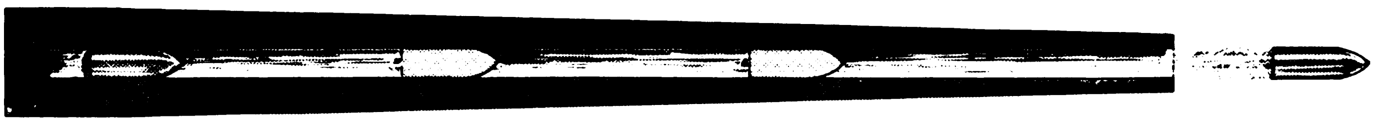
conventional gun firing "SABOT" projectiles

To obtain the advantages of a sabot-fitted projectile without endangering friendly forces, guns with tapered bores have been used. The projectile employed is fitted with flanges, or skirts, which act as a gas seal.



tapered bore gun firing "SKIRTED" projectiles

HIGH-STRENGTH, EXTRA-LONG GUNS. Conventional projectiles, fired from high-strength or extra-long guns can attain a velocity limited only by the amount of pressure the gun can withstand and by the amount of pressure that can be developed with the available type of powder. Such guns would be heavier and require considerably more space aboard ship than guns which impart conventional velocities to standard projectiles.



HIGH STRENGTH long gun firing conventional projectiles

MOVING PROPELLANT. Another method of achieving higher muzzle velocities, currently under development, is the employment of a moving propellant. A mechanism enables the propelling charge to travel down the tube with the projectile. One concept is that of the rocket-assisted projectile, which was first used by the Germans. The rocket action was initiated during flight. Although higher velocities and greater ranges were achieved, destructive potential was decreased, since part of the payload had to be eliminated to make room for rocket fuel. In addition to increasing range and velocity, rocket-assisted projectiles permit a lighter gun barrel and mount to be used. With the incorporation of the latest rocket propulsion techniques, rocket-assisted projectiles show great promise for shipboard applications.

conventional gun firing ROCKET-ASSISTED projectiles

Advantages

INCREASED RANGE. For a given projectile, an increase in muzzle velocity will increase its effective range.
INCREASED ARMOR PENETRATION. With improvements in armor, the target-piercing ability of projectiles must improve. For a given armor-piercing projectile, an increase in velocity will increase its armor-piercing ability.
DISADVANTAGES
GREATER SIZE AND WEIGHT. The muzzle velocity that can be achieved with conventional projectiles is entirely dependent on the pressure the propellent can produce and the pressure the gun can withstand. A high-velocity gun employs higher pressures and is, of necessity, larger and heavier than a conventional gun. The installation of one high-velocity gun on a ship would require the removal of several guns of normal velocity.

Disadvantages

SLOWER RATE OF FIRE. Loading is complicated by the additional propellant required, and it becomes more difficult to maintain a high rate of fire.
REDUCED LIFE EXPECTANCY. A high-velocity system employing conventional guns cannot safely increase maximum pressure. Therefore, it relies on building up the average pressure over the length of the bore, which also means an increase in muzzle pressure. Thus, it inherits all the disadvantages of high muzzle pressure, including increased unburned powder, muzzle loss, muzzle flash, irregular velocities, and increased erosion. Because of increased erosion, high-velocity systems wear out more quickly than those designed to fire projectiles of conventional velocities.

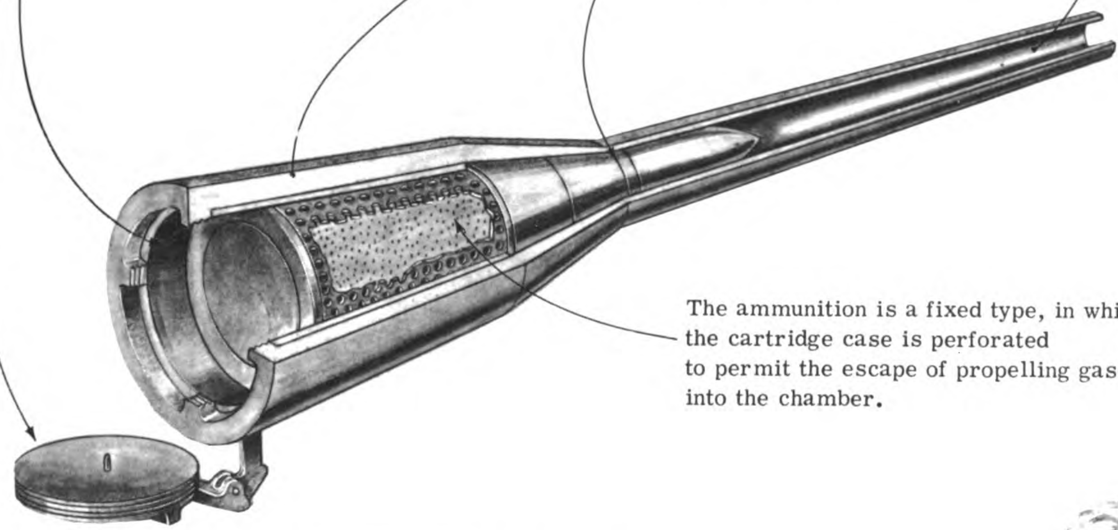
recoilless gun

PRINCIPLES OF OPERATION

A recoilless gun delivers no recoil to its mount or, if shoulder fired, to the person firing it. It is a light-weight, accurate weapon of large caliber and high mobility. The principle behind the recoilless gun is that part of the expanding gas pressure is used to counteract recoil.

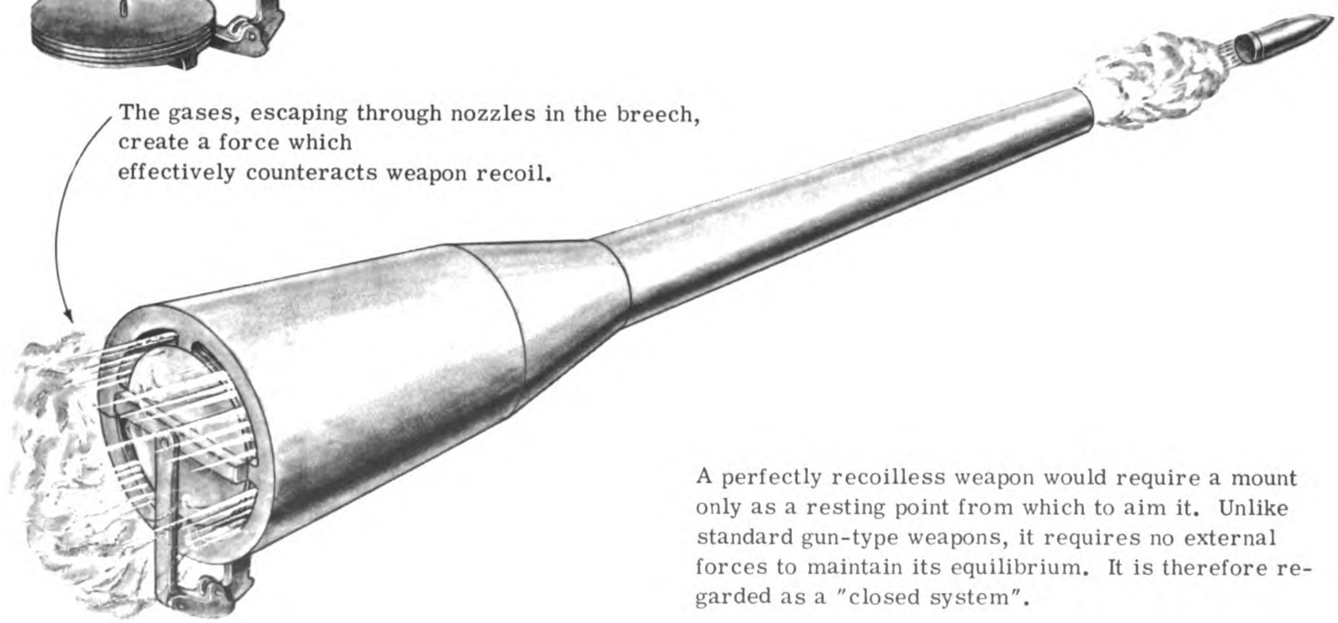
The rifle has an extra large chamber which is partially closed by a breechblock containing nozzles for the rearward release of gas. . . . . .

The recoilless weapon has a rifled bore and fires a conventional projectile with a standard fuze.



The ammunition is a fixed type, in which the cartridge case is perforated to permit the escape of propelling gases into the chamber.

The gases, escaping through nozzles in the breech, create a force which effectively counteracts weapon recoil.



A perfectly recoilless weapon would require a mount only as a resting point from which to aim it. Unlike standard gun-type weapons, it requires no external forces to maintain its equilibrium. It is therefore regarded as a "closed system".

That is to say, the total change in momentum imparted to all the elements of the system in a given period of time must equal zero.

Considering the four movable parts of the system to be the projectile, the gases forward, the gases rearward, and the gun itself, and representing the mass of each as m_1, m_2, m_3, and m_4, respectively and the velocities of each as v_1, v_2, v_3, and v_4, then:

$$m_1 v_1 + m_2 v_2 + m_3 v_3 + m_4 v_4 = 0.$$

If the rifle is recoilless, then $v_4 = 0$, and the equation reduces to:

$$m_1 v_1 + m_2 v_2 + m_3 v_3 = 0$$

$$\text{or } m_3 v_3 = -m_1 v_1 - m_2 v_2.$$

That is, the combined momentum of the projectile and the forward-moving gases is equal, but opposite in direction, to the momentum of the gases discharged through the breech. If this is true for all instants during the weapon's firing period, which is about a hundredth of a second, then the momentum of the weapon must equal zero at all times during firing and the weapon can be considered perfectly recoilless.

In practice, however, this is generally not the case. A recoilless weapon is usually recoilless in the mean; i.e., although the total momentum applied to it over the firing period is zero, the sum of the momentums at any instant is not necessarily zero. The weapon undergoes large unbalanced forces during some parts of the pressure interval, and oppositely directed forces during other parts of the pressure interval. Thus, its recoillessness is an average rather than an absolute value.

A recoilless rifle just out of the factory may recoil slightly because the nozzles, being somewhat undersize to allow for erosion, restrict the rearward flow of gases, m_3. As $m_3 v_3$ is reduced, the total momentum must still remain zero; therfore, the weapon momentum, $m_4 v_4$, is no longer zero, but a rearward, or positive, recoil. As the nozzles erode, i.e., become larger, the weapon reaches a point where it is actually recoilless ($m_4 v_4$ equals zero). Further enlargement of the nozzles, however, increases the rearward flow of the gases, m_3, with the result that the momentum of the gun, or so-called recoil, must now be forward, or negative, in order to balance the momentum equation.

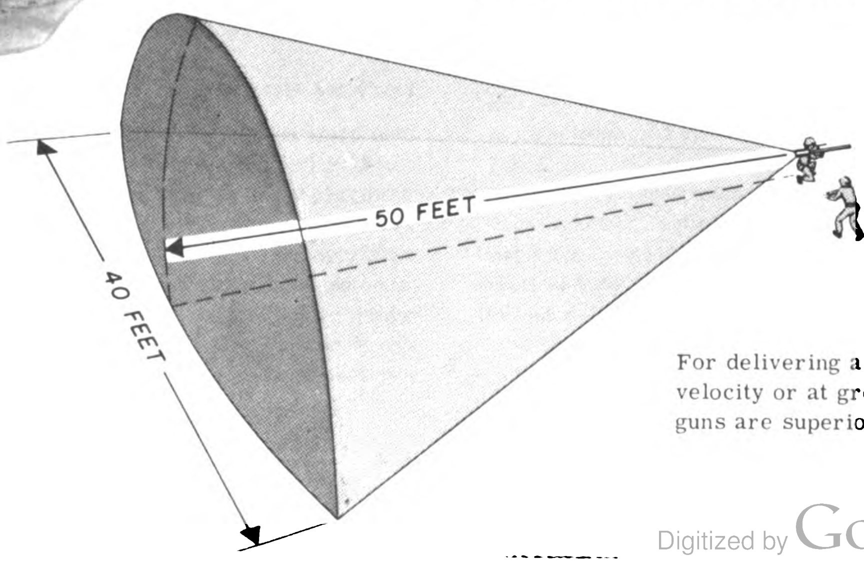
ADVANTAGES

The main advantage of recoilless guns over conventional guns of comparable caliber is their much lighter weight, which gives them much higher mobility. The price paid for this advantage is a slight reduction in velocity and range, and a large (between 2.5 and 3 times as much) increase in propellant.

Recoilless weapons can be tactically employed where conventional guns cannot. For instance, they are used when terrain is inaccessible to guns or when recoil forces must be taken into account, as in small sea and air craft.

DISADVANTAGES

The main drawback of recoilless weapons is the tremendous blast to the rear of the gun caused by the escaping gas. The danger zone for a comparatively low caliber (57 mm) rifle is a conical section 50 feet long and 40 feet wide diverging rearward from the breech.



For delivering a given warhead at high velocity or at great range, conventional guns are superior to recoilless ones.

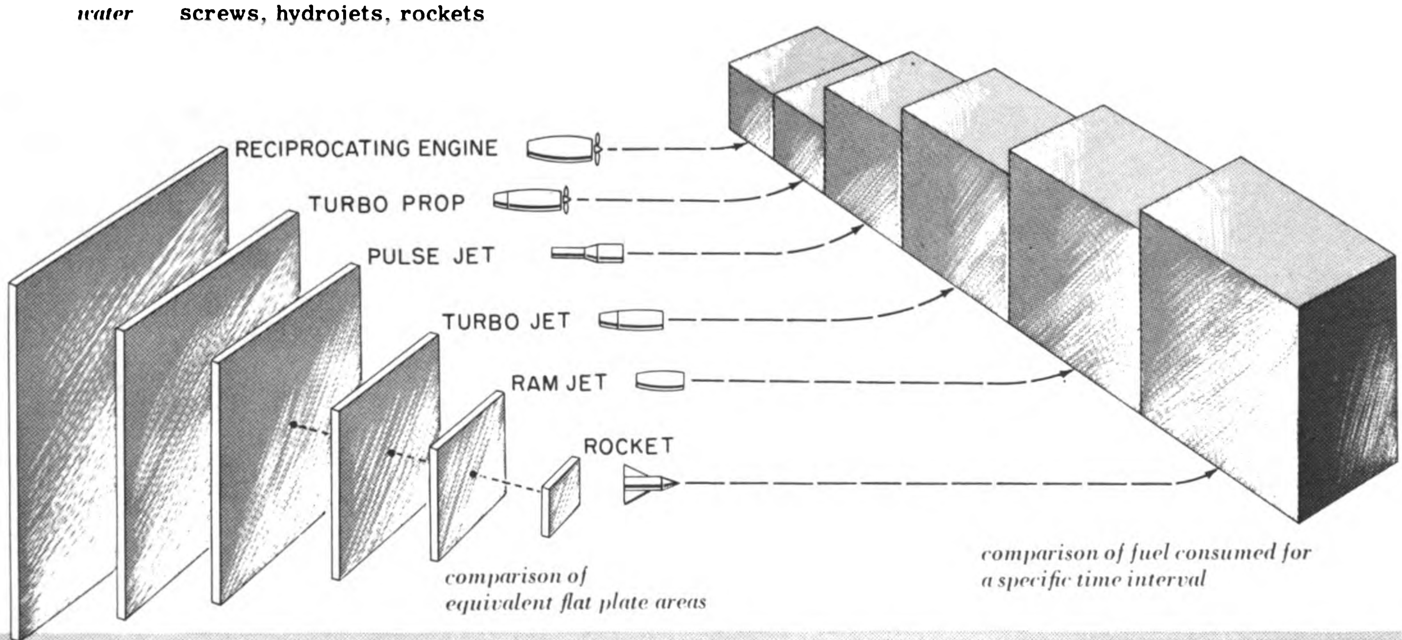
REACTION-TYPE PROPULSION

INTRODUCTION

In contrast to popular belief, reaction motors do not obtain thrust by "pushing" against the medium in which they are operating. Thrust is developed by increasing the momentum of the working fluid and creating a pressure differential therein. Newton's Third Law of Motion states that for every action there is an equal and opposite reaction. This principle forms the basis for the motion of all self-propelled objects. Pressure differentials are employed to move propeller-driven aircraft as well as rockets and jet-propelled missiles. By definition, a reaction-propelled vehicle is one containing within its structure its own source of propulsion, i.e., a reaction-type motor. The mediums in which reaction motors operate and the types that have been used in weapon systems are:

air	propeller engines, turbojets, ramjets, pulsejets, rockets
vacuum	rockets
water	screws, hydrojets, rockets

The performance required of modern propulsion systems, insofar as range, velocity, and control are concerned, is considerably in excess of that which can be accomplished by heretofore conventional methods. Until the advent of guided missiles that travel at supersonic speeds to lessen the probability of interception, the reciprocating engine-propeller combination was considered satisfactory for the propulsion of aircraft. The generation of shock waves as the speed of sound is approached limits the development of airplane thrust, and thus the speed of propeller-driven craft. Although future developments may overcome the limitations of propeller-driven vehicles, it is necessary at the present time to use jet propulsion for missiles traveling at high subsonic and supersonic speeds. A minor disadvantage of jet propulsion is that tremendous quantities of fuel are consumed per second of flight time. The efficiency ratings of jet engine types, however, are so superior to those of propeller-driven types that the disadvantage as a measure of degree is minimal. Also, the only means of self propulsion that will operate in a vacuum is jet propulsion with rockets.



comparison of equivalent flat plate areas

comparison of fuel consumed for a specific time interval

CLASSIFICATION OF

thermal jet engines (air breathing)

The missile accepts or breathes in a predetermined quantity of air at its input end and compresses it. Liquid fuel is then injected into the compressed air and the mixture is fired in a combustion chamber. As the resulting hot gases are expelled through a nozzle normally located at the rear of the vehicle, heat energy is transformed to kinetic energy and thrust or propulsive motion is thereby created. Missiles using air-breathing propulsion systems are incapable of operating in a vacuum.

rocket engines

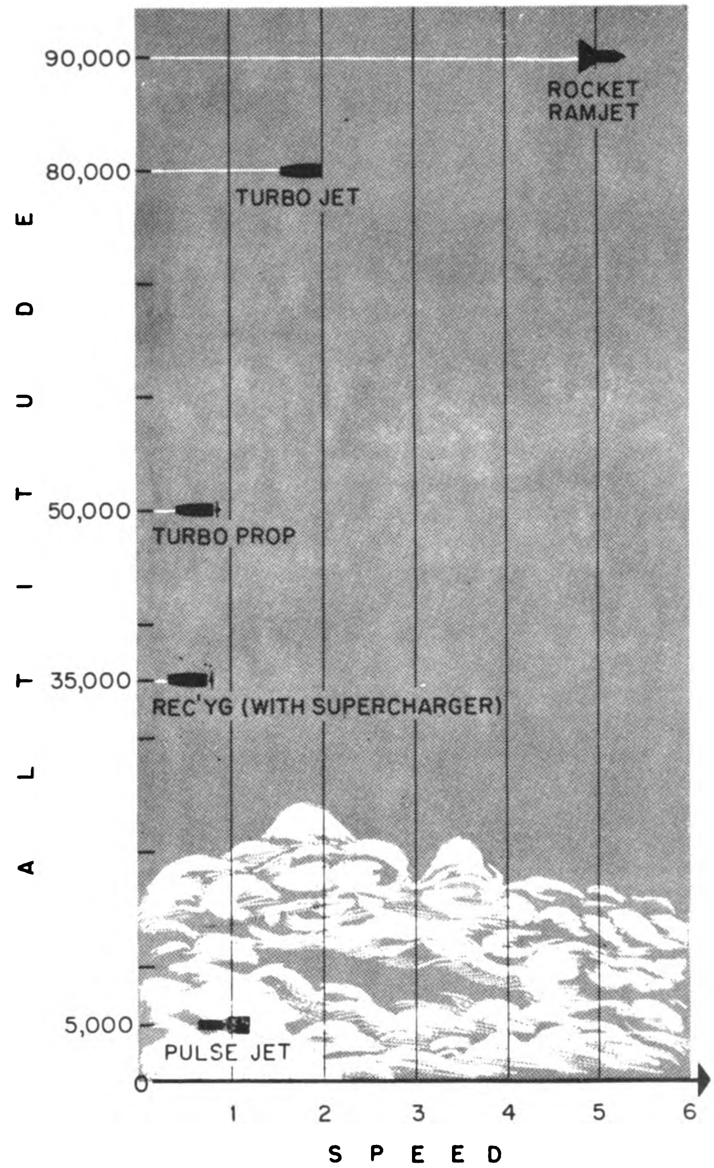
The basic difference between a rocket jet engine and an air-breathing jet engine is that a rocket carries its own oxidizer, while an air-breathing engine relies on oxygen obtained from the atmosphere. A child's balloon filled with compressed air becomes a simple type of rocket when the air is allowed to escape through the air-inlet, which now acts as a nozzle. In addition, the thrust developed by an air-breathing engine is dependent on the difference between the jet velocity and the velocity

SYSTEMS

JET PROPULSION

Jet propulsion is a means of locomotion obtained from the momentum of matter ejected from within the propelled body in the form of a fluid jet. The fluids used for producing the jet include water, steam, heated air, and gases produced by chemical reaction. The fluid may be drawn into the body from the very medium in which the body is moving, such as air in a jet engine or water in a hydrojet engine, or it may be self contained, as in the case of rocket engines, where the working fluid is composed of the gaseous products of rocket fuel combustion. A jet-propulsion engine consists essentially of an air inlet section (diffuser) for air-breathing jets, a propellant supply system, a combustion chamber, and an exhaust nozzle. The purpose of the propellant system and the combustion chamber is to produce large volumes of high-temperature, high-pressure gases. The exhaust nozzle then converts the heat energy into kinetic energy as efficiently as possible. In jet engines and in liquid-fuel rockets, the fuel is pumped into the combustion chamber to be burned. In a solid-propellant rocket, the combustion chamber already contains the fuel to be burned.

Popular terminology makes a distinction between jets and rockets: a jet takes in air from the atmosphere; a rocket needs no air supply, as it carries its own supply of oxygen. Both types of engine operate by expelling a stream of gas at high speed from a nozzle at the after end of the vehicle. For our purposes, a rocket can be considered a type of jet engine. Jet-propulsion systems used in missiles may be divided into two classes: thermal jet engines and rockets.



comparison of altitude and speed

The chart illustrates maximum altitudes and speeds of air-breathing engines. Since a rocket requires no air intake, it is not restricted to atmospheric flight and therefore is omitted from the chart.

JET-PROPULSION ENGINES

of the air entering the engine, while in a rocket the thrust depends on the velocity of the jet only. Besides being independent of flight speed, the thrust is also completely independent of the surrounding environment. Consequently, there is no altitude ceiling. Furthermore, the thrust developed by a rocket per unit-of-engine frontal area and per unit-of-engine weight is the greatest of any known type of engine.

Rockets are distinguished by the means used to produce exhaust material. The most common type of rocket engine obtains its high-pressure gases by burning a propellant. This propellant consists of both fuel and oxidizer and may be solid, liquid, or both. A rocket using both solid and liquid propellants is known as a hybrid, and is employed to gain some of the advantages of both types. Actual rockets depending on the release of stored compressed gas have limited application. They are used mainly as small vernier rockets and as spin-producing rockets in ballistic missile re-entry bodies.

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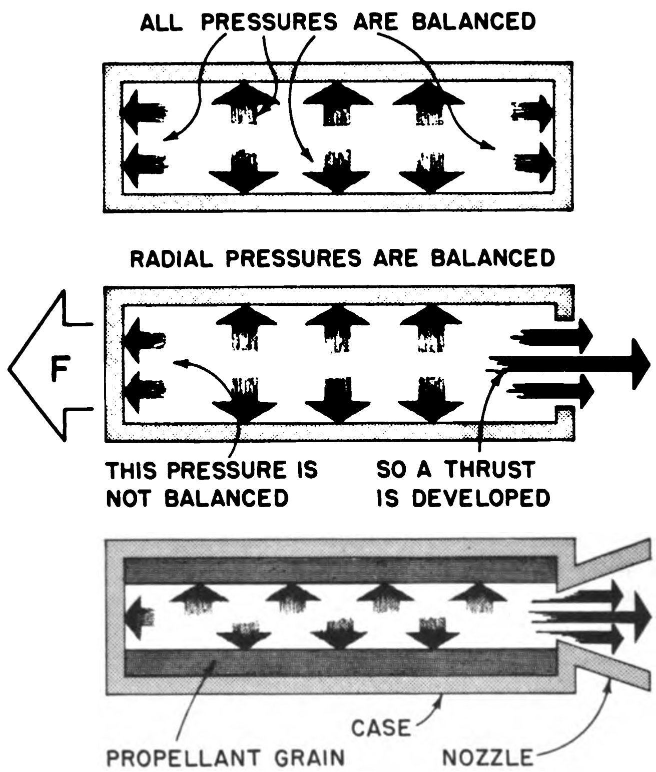
PROPELLANTS

The fuels and oxidizers used to power a jet engine are called propellants. The chemical reaction between fuel and oxidizer in the combustion chamber of a jet engine produces high-pressure, high-temperature gases that, when channeled through an exhaust nozzle, are converted into kinetic energy used to propel the missile.

propellant performance

The transformation of heat energy into kinetic energy within a combustion chamber creates a force to act in a direction opposite to the flow of the exhaust gases from the nozzle. This propulsive force, termed thrust, is a function primarily of the velocity at which the gases leave the exhaust nozzle and the weight of the gases, and is independent of the velocity of flight. In order to develop a high thrust constant, grains or charges of propellant are employed with large burning surfaces so that a high rate of mass flow is developed.
The duration of burning of a propellant charge is determined by the web of the grain and the burning rate. Since the combustion chamber has fixed dimensions and capacity for propellant, the thrust may be either great but of short duration or low but of long duration. The thrust developed by a reaction motor is the resultant of static pressure forces acting on the interior and exterior surfaces of the combustion chamber. The static pressures acting upon the interior surfaces depend upon the rate at which propellants are burned, the thermochemical characteristics of the gases produced by their combustion, and the area of the throat of the dispersing nozzle. As the internal forces are several times greater than the external forces, and as the forces acting normal to the longitudinal axis of the combustion chamber do not contribute to the thrust, the thrust developed is primarily the resultant axial component of the pressure forces.

If the thrust-time curve obtained from firing a rocket is integrated over the burning duration, the result is called total impulse and is measured in pound-seconds.



development of thrust in rocket motor

The performance characteristic of a specific propellant, called the specific impulse of the propellant (solid-fuel propellants), or specific thrust of the propellant (liquid-fuel propellants), is expressed in consumption rates of the propellant and can be defined as $I_{sp} = F/W \times (g/g_o)$ where g is local acceleration due to gravity and g_o is the standard gravitational acceleration. The mixture ratio of oxidizer and fuel are instrumental in determination of the propellant consumption which is a measure of propellant thrust. The weight of propellants consumed in developing a specific impulse is

propellant types

With regard to their physical state, propellants may be classified as 1) solid, or 2) liquid. Gases are rarely used as missile propellants. The density of a solid or a liquid is higher than that of a gas, even when the gas is compressed; thus, a larger quantity of solid or liquid propellant can be carried in a given space. Moreover, a higher level of energy transference occurs when a solid or liquid substance is transformed to a gaseous state than when a gaseous substance is merely accelerated to a higher velocity.

solid propellants

Solid propellants consist of a fuel (usually a hydrocarbon) and an oxidizer, so combined as to produce a solid of desired chemical potential. A complete entity is called a stick or grain, and a combination of grains constitutes a charge. Normally, the solid propellant grain is one of two types, either double base or composite. The double-base propellant consists of nitrocellulose and nitroglycerin plus additives in small quantity. Fuel and oxidizer are not separated. The molecules are unstable and break up and rearrange themselves upon ignition, liberating large quantities of heat. These propellants lend themselves well to smaller rockets. In the composite type, now predominant, separate fuel and oxidizer chemicals are used, but are mixed together in the solid grain.

called specific propellant consumption. The most important characteristic of a propellant system is the exhaust velocity developed by the propulsion system. The average rate of propellant consumption can be determined with a high degree of accuracy, but is practically impossible to measure the jet pressure exactly, so that it becomes necessary, when discussing propellant performance characteristics, to introduce a ficticious exhaust velocity (effective exhaust velocity) that is a function of thrust and mass rate of propellant flow.

total impulse

Total impulse comparisons between propellants are made by determining their total impulse coefficients. Total impulse is a product of the thrust in pounds times firing duration in seconds.

$$I_t = Ft$$

where I_t = total impulse (lb-sec.)
F = thrust (lbs)
t = time (sec.)

specific impulse

The impulse per unit weight of a propellant is termed the specific impulse of a propellant and is expressed in pounds-seconds per pound of propellant. Solid propellants are rated on the basis of specific impulse. Specific impulse is the amount of impulse produced by one pound of the propellant, that is:

$$I_{sp} = \frac{I_t}{W}$$

where I_{sp} = specific impulse ($\frac{\text{lb-sec.}}{\text{lb}}$ or sec.)
I_t = total impulse (lb-sec.)
W = Weight of solid propellant (lbs)

specific thrust

Liquid propellants are compared on the basis of specific thrust. Specific thrust is equivalent to the specific impulse for solid propellants, but is derived in a slightly different way. Specific thrust is defined as the thrust in pounds produced, assuming that the propellant is consumed at the rate of one pound per second.

Specific thrust can be expressed:

$$T_{sp} = \frac{F}{\frac{dW_f}{dt}}$$

where T_{sp} = specific thrust ($\frac{\text{lb}}{\text{lb/sec}}$ or sec.)
F = thrust (lbs)
W_f = propellant weight

specific propellant consumption

This is another term of importance in liquid propellant systems. It is the reciprocal of specific thrust, and is defined as the propellant flow in pounds per second necessary to produce one pound of thrust; that is:

$$SPC = \frac{\frac{dW_f}{dt}}{F}$$

where SPC = specific propellant consumption

mixture ratio

This term designates the relative quantities of oxidizer and fuel used in the propellant combination. It is numerically equal to the weight of the oxidizer flow divided by the weight of the fuel flow.

exhaust velocity

Exhaust velocity is determined theoretically on the basis of the energy content of the propellant combination. The actual velocity of the exhaust gases is less than the theoretical value, since jet engines cannot completely convert the energy content of the propellant into exhaust velocity.

effective exhaust velocity

The effective exhaust velocity is used and determined on the basis of thrust and propellant flow, that is:

$$V_e = \frac{F}{m_f}$$

Then $$V_e = F \div \frac{1}{g}\frac{dW_f}{dt} = \frac{Fg}{dW_f/dt}$$

$$V_e = I_{sp}g$$

The oxidizer comprises as much as four-fifths of the whole propellant mix. Common oxidizers include ammonium nitrate, potassium chlorate, and ammonium chlorate. The fuels used are hydrocarbons, such as asphaltic-type compounds or plastics. Since the oxidizer has little structural strength, the fuel must also supply the necessary form and rigidity to the grain. The specific impulse, I_{sp}, of solid-fuel rockets is between 100-300 seconds and there is not much hope of any significantly large increase.

Some examples of solid rocket propellant I_{sp} are:

Oxidizer-Fuel Combinations	I_{sp}
Potassium percholorate + thiokol or asphalt	170-210
Ammonium perchlorate + polyurethane	210-250
Ammonium nitrate + rubber	170-210
Ammonium nitrate + nitropolymer	210-250
Aluminum metal components + oxidant	200-275
Magnesium metal components + oxidant	200-250
Perfluoro-type propellants	250 up

CLASSIFICATION OF SOLID PROPELLANT CHARGES

The two most common types of charges are restricted burning charges and unrestricted burning charges. Restricted burning charges allow burning to take place only on desired surfaces of the charge, lengthening the burning duration, and permitting a predetermination of combustion pressure for a given charge. Unrestricted burning charges permit burning on all surfaces simultaneously, so that a large amount of energy is made available for thrust, but for a short duration.

BURNING RATES

The burning rate of a solid propellant is the velocity in inches per second at which the grain is consumed. Burning rates of propellants depend upon the chemical composition of the propellant, the combustion chamber temperature and pressure gradient, gas velocity adjacent to the burning surface, and the size and shape of the grain or the geometry of the charge. Three types of burning previously discussed are neutral, progressive, or digressive burning. Propulsive specifications determine the advisability of a particular type. Propulsive thrust depends upon the velocity of the jet for a determined interval of time, or the mass rate of flow for a change in the velocity of the gas. Therefore, by varying the geometry and arrangement of the charge, the thrust coefficients can be greatly influenced. Propellant compositionsburn at characteristic linear rates which are affected by the initial temperature and the pressure under which burning takes place. An increase in pressure or in initial temperature causes an increase in the characteristic burning rate of the propellant. The range of burning rates of solid propellants under a pressure of 2000 psi is about 1 to 2 inches per second.

Charge design is based on obtaining the optimum combustion pressure as a function of burning duration. Most Navy propellants utilize either a cruciform grain or a cylindrical grain with an axial hole and radial perforation. The latter is characterized by three ridges spaced 120 degrees apart and running longitudinally along the grain. The cruciform grain in cross section is a symmetrical cross. If all of the exterior surface of this grain were permitted to burn, there would be a gradual decrease of area, and the burning rate would be regressive. Since a uniform burning rate is desired, a number of slower burning plastic strips or inhibitors are bonded to certain parts of the area exposed on the outer curved ends of the arms. These control or slow the initial burning rate, and gas production rate is approximately uniform over burning time.

Hollow cylindrical grains have inherently uniform burning rates and normally require no inhibitors.

Further configurations include end burning grain, internal burning star grain, and rod and tube grain.

End burning grain configuration takes advantage of the principle that burning of a solid propellant surface occurs in parallel layers. Under these conditions, the burning surface recedes parallel to itself with no change in burning area. Thrust is therefore proportional to the cross sectional area, and burning duration is a function of grain length.

Internal burning star grain is a further development of tubular grain principles in which the inside burning area increases while the outside burning area decreases, maintaining a nearly constant burning area. The internal star configuration has great flexibility, since the number of star points, diameters, and angles can be varied to perform a predetermined function. Also the grain can be designed for either neutral, progressive, or regressive burning. This grain configuration has wide application in large, solid propellant rocket motors.

The rod and tube grain configuration utilizes the exterior surface of the rod and the internal surface of the tube as the burning surfaces. As burning proceeds, the burning area of the rod decreases and that of the tube increases, and by having different propellant formulations for the rod and tube, the pressure time curve can be controlled within practical limits to give any desired variations of the combustion pressure as a function of time.

TEMPERATURE LIMITATIONS

An important aspect of the performance characteristics of a propellant is its ability to resist temperature variations. The initial temperature almost always affects grain performance. A grain will generate more thrust with an increase in temperature. A propellant designed to produce 1000 pounds of thrust at 70° F may produce only 500 pounds of thrust at 45° F. The percentage change of thrust per degree Fahrenheit temperature change is referred to as the temperature sensitivity of a propellant, and determines the optimum propellant for use under a given set of operating conditions. Temperature also affects the physical composition of a solid propellant. Extremes of temperature can adversely affect the performance of a propellant. At low temperatures, grains become brittle and often crack, increasing the burning area and therefore the combustion chamber pressure. If this pressure increase exceeds the limitations of the chamber, an explosion may result. Conversely, a propellant exposed to high temperatures may lose its shape and thus its potency, and an undesired performance may result. Allowable temperatures for the majority of solid propellants range from about 25° F to 120° F.

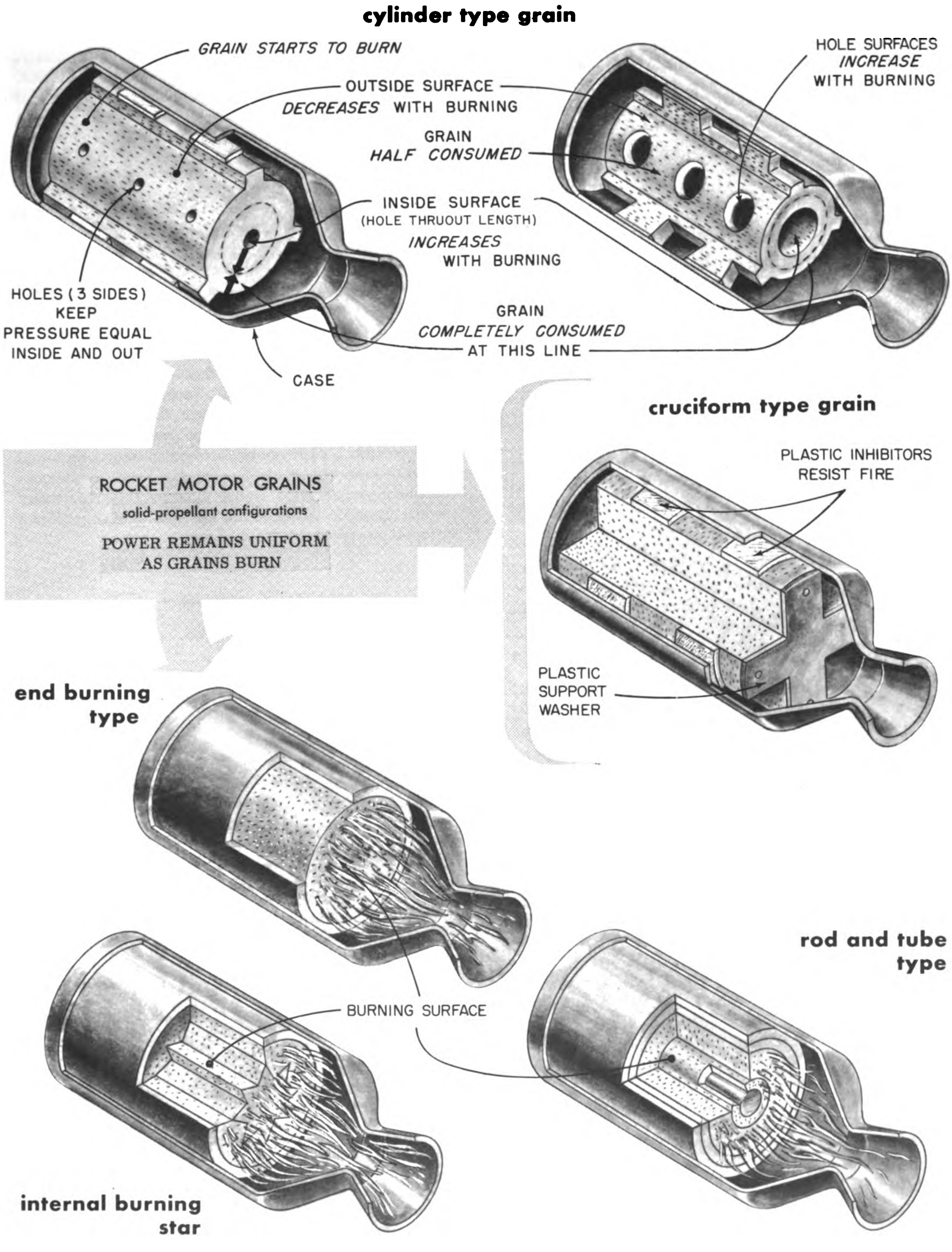
PRESSURE LIMITATIONS

Propellants sustain combustion at relatively high chamber pressures. Below a certain chamber pressure, the combustion rate becomes unstable, and faulty performance results. For a given propellant with a specified burning area, chamber pressure is a function of exhaust nozzle area. If the throat area is too great, large amounts of energy are dissipated, and an unwanted loss of thrust results. Proper chamber pressure must be maintained to develop the thrust necessary for missile propulsion.

STABILITY FACTORS

The tendency to absorb moisture and the tendency to decompose are weaknesses of solid propellants. Hygroscopic tendencies are undesirable, since change in moisture content causes change in gaseous energy output. Stability is extremely important in solid propellants, since almost all of them contain nitrocellulose, which is unstable in the presence of moisture. To insure maximum stability, a stabilizer or additive is included to counteract the effects of moisture upon the propellant.

If a propellant is stored for a long period of time at ordinary temperatures, decomposition takes place at a rate dependent upon the temperature. Acceleration of the rate of decomposition is prevented, however, by using a stabilizer which acts as a neutralizer. When deterioration has reached the point where the stabilizer content is a few tenths of 1 percent, this neutralizing action is no longer significant, and the rate of decomposition is accelerated.



cylinder type grain
GRAIN STARTS TO BURN
OUTSIDE SURFACE DECREASES WITH BURNING
GRAIN HALF CONSUMED
INSIDE SURFACE (HOLE THRUOUT LENGTH) INCREASES WITH BURNING
HOLE SURFACES INCREASE WITH BURNING
HOLES (3 SIDES) KEEP PRESSURE EQUAL INSIDE AND OUT
GRAIN COMPLETELY CONSUMED AT THIS LINE
CASE
ROCKET MOTOR GRAINS
solid-propellant configurations
POWER REMAINS UNIFORM AS GRAINS BURN
cruciform type grain
PLASTIC INHIBITORS RESIST FIRE
PLASTIC SUPPORT WASHER
end burning type
BURNING SURFACE
rod and tube type
internal burning star

liquid propellants

Liquid propellants are normally stored in tanks outside the combustion chamber and are injected into the combustion chamber. The injector vaporizes and mixes the fuel and oxidizers in the proper proportions for efficient burning. When oxygen is used as an oxidizer, the best liquid fuels are those rich in carbon and hydrogen.

VELOCITY OF CHEMICAL REACTION

The velocity of a chemical reaction is a function of the temperature, pressure, and concentration conditions under which it is carried out. The velocity of a chemical reaction sometimes is increased by the presence of a catalytic agent. Inert additives, which take no part in the chemical reaction, are sometimes combined with the liquid fuels, and although such additives add no energy to the system, they contribute to the attainment of higher thrust by increasing the rate of mass flow through the system.

CLASSIFICATION OF LIQUID PROPELLANTS

Liquid propellants are usually divided into two types: monopropellants and bipropellants. In a monopropellant, the fuel and oxidizer are mixed, while in a bipropellant they are kept separate until firing.
A liquid monopropellant may be a chemical compound such as nitromethane or a mixture of chemicals. An engine employing a liquid monopropellant is a simpler design than one using a bipropellant, since only a single liquid is involved. Burning is initiated either by a pyrotechnic igniter, an electrically heated glow plug, a spark plug, or a small supply of auxiliary fluid with which the propellant reacts readily. Liquid monopropellants that have been investigated include ethylene oxide, n-propyl nitrate, isopropyl nitrate, nitromethane, diethyleneglycol dinitrate, acetylenic compounds, mixtures of methyl nitrate and methyl alcohol, hydrazine, and mixtures of nitric acid with benzene and water.

Certain liquid chemicals can be made to form hot gas for thrust production by direct decomposition in a rocket chamber. Hydrogen peroxide, for example, when passed through a platinum catalyst mesh, decomposes into hot steam and oxygen which are then ejected by means of an exhaust nozzle to produce thrust. A satisfactory liquid monopropellant is stable under all storage conditions, but decomposes completely when injected into the combustion chamber of a rocket engine. In general, these requirements are conflicting, and greatly restrict the choice of possible liquid monopropellants. As a rule, the larger the thrust output per unit weight of a liquid monopropellant, the greater its sensitivity to shock; that is, the more explosive is its nature. It is doubtful that a liquid monopropellant can be found that will give better performance than the best liquid bipropellants. Consequently, monopropellants will probably not be relied upon for the main thrust in ballistic missiles, but will be used only in small control rockets, auxiliary supplies, and other special applications.
Most liquid-fuel rockets employ bipropellants, since they are less likely to react to shock and heat than monopropellants. A liquid bipropellant helps develop thrust by reacting a liquid oxidizer with a liquid fuel to produce tremendous quantities of high-temperature, high-pressure gas. When separated, the fuel and oxidizer are ordinarily incapable of chemical reaction. Thus, liquid bipropellants are less hazardous than liquid monopropellants. Bipropellants which ignite spontaneously when they come in contact with each other are said to be hypergolic, while those which require an addition of energy to initiate the chemical reaction are said to be diergolic.

A large number of known liquids are suitable for use as fuels, but only a few can serve as practical oxidizers in a propellant. The principal liquid oxidizers are the nitric acids, white fuming nitric acid (WFNA), red fuming nitric acid (RFNA), stabilized (red) fuming nitric acid (SFNA), liquid oxygen (LOX), high strength (80 to 100 percent H_2O_2) hydrogen peroxide (HTP), mixed oxides of nitrogen (MON), and liquid fluorine (LF). The principal liquid fuels are the hydrocarbon fuels, such as jet engine fuels JP-4 and JP-5, aniline and mixtures of aniline with furfuryl alcohol, alcohol-water mixtures, hydrazine, unsymmetrical dimethylhydrazine (UDMH), diethylenetriamine (DETA), anhydrous ammonia, liquid hydrogen, and mixtures made of other fuels with either hydrazine or UDMH. In general, the liquid propellants commonly used yield higher specific impulses than available solids. The following table lists some liquid propellants and their I_{sp}.

Oxidizer-Fuel Combinations	I_{sp}
Acid + JP-4	200-230
Nitrogen tetroxide + ammonia	230-260
LOX + JP-4	250-270
LOX + alcohol	250-270
LOX and fluorine + JP-4	270-300
LOX and ozone + JP-4	270-300
LOX + hydrazine	270-300
Fluorine and hydrogen	300-385
Fluorine and ammonia	300-385
Ozone and hydrogen	300-385

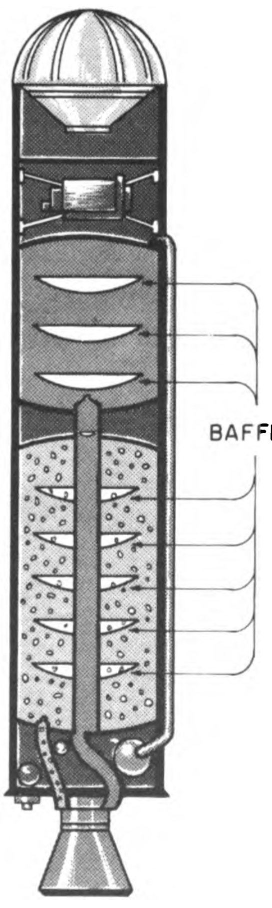
EFFECTS OF STORAGE.

Double-base propellants slowly decompose during prolonged storage. Their decomposition is autocatalytic. Diphenylamine is usually added to such propellants to neutralize the catalytic effect of the initial decomposition products. One such propellant, ballistite, when stored at 140°F for a period in excess of two weeks or at 120°F for prolonged periods, becomes unstable. Composite propellants do not decompose chemically in prolonged storage, but will do so in an atmosphere of high relative humidity. By absorbing moisture, the charge becomes soft and mechanically weak. These propellants are stored and shipped in moisture-tight containers and are not exposed to moisture before use. Military necessity requires that missiles be fired at a few moments notice. This means that they must be loaded and stored in a ready-to-fire condition for extended periods of time. Most solid-fuel rockets can meet these conditions, but, at present, the majority of

liquid-fuel rockets cannot. As a result, much effort has gone into the development of storable liquid propellant combinations. One of the most promising is nitrogen tetroxide + hydrazine.
Nitrogen tetroxide (N_20) can be stored without refrigeration and is not difficult to handle. In addition, it is non-corrosive to steel. To maintain high performance, hydrazine (N H) and unsymmetrical dimethyl hydrazine (UDMH) are the most commonly used fuels with nitrogen tetroxide. Another good storable liquid fuel is perchloryl fluoride (PF), which is also non-corrosive.

SLOSHING EFFECT

A difficult design problem connected with liquid propellant systems is that of propellant sloshing. Ballistic movement of the missile causes propellant motion, or sloshing of the liquid within the tanks. As a result, the center of gravity is effectively shifted and a severe control problem is created. Because sloshing forces vary as the cube of the tank diameter, the magnitude of these forces can reach limits that can cause unstable operation of the system. Sloshing must therefore be carefully controlled. Long, narrow propellant tanks are employed to reduce the effects of sloshing. Baffles can also be employed within the tanks to minimize sloshing.



PRESSURE LIMIT

Propellants may be safely used only at chamber pressures below some critical point. If the critical chamber pressure is exceeded, the propelling charge burns in a violent and unpredictable manner. For double-based propellants, this pressure limit is greater than 12,000 psi. Some composite propellants have pressure limits of 3000 psi and below, which is a disadvantage in certain applications.

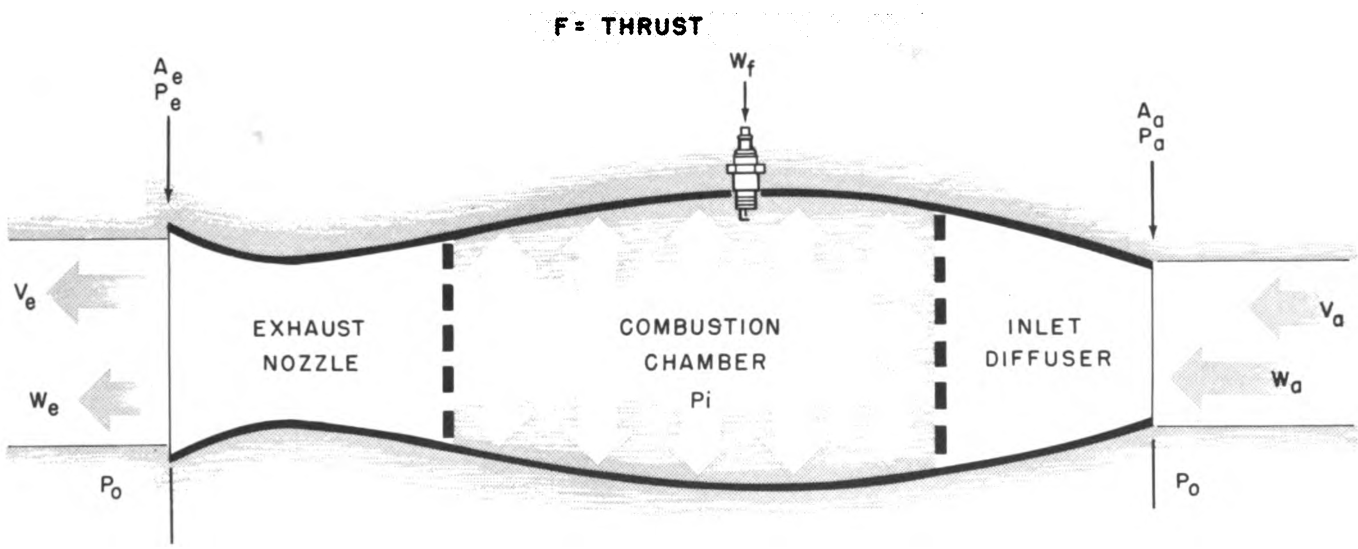
IDEAL CHARACTERISTICS OF PROPELLANTS

Solid Propellants	Liquid Propellants
1. high specific impulse	1. high specific thrust
2. manufactured from easily obtained substance	2. easily manufactured
3. safe and easy to handle	3. high heat of combustion per unit weight of mixture, to develop high chamber temperature
4. easily stored	
5. stable to shock and temperature changes	4. low molecular weight of the reaction products
6. ignites and burns uniformly	5. low freezing point
7. maintains constant burning surface	6. high specific gravity
8. nonhygroscopic	7. low toxicity and corrosiveness
9. smokeless	8. low vapor pressure
10. flashless	9. stability in storage

Although the ideal characteristics of both solid and liquid propellants are listed above, it is highly unlikely that chemical research can develop any single propellant that incorporates all of the ideal advantages without exhibiting unwanted deviations from the optimum.

COMPONENTS OF PROPULSION SYSTEMS

A jet-propelled engine is basically a device for converting a portion of the thermochemical energy developed in its combustion chamber into the kinetic energy associated with a high-speed gaseous exhaust jet. The basic elements of the engine are: 1) the combustion chamber, wherein the transformation of energy from potential to heat form occurs; 2) the exhaust nozzle, wherein thermochemical energy is converted into the kinetic energy necessary to produce an exhaust jet of propulsive potential; and 3) the diffuser (for air-breathing jets only) or intake duct, wherein the high-speed air intake is converted into a low-speed, high-pressure gas for entry into the combustion chamber as an oxidizing element. In the following discussion, air is the fluid medium, but the general principles apply to any medium.



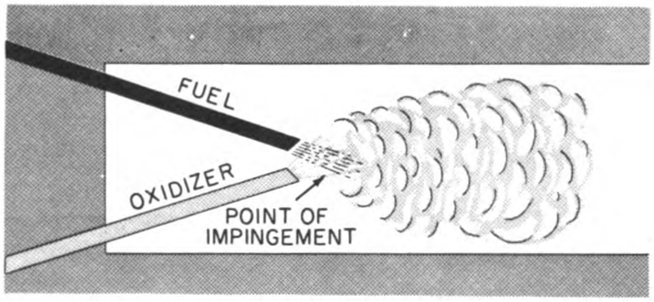
combustion chamber

The combustion chamber is the enclosure within which high-temperature, high-pressure gases are produced, and potential energy is converted to kinetic energy. The chamber is usually a cylinder, although its geometry is determined by the requirement that the optimum gas velocity and pressure be produced at the nozzle exit. Liquid propellants flow from storage tanks through a control valve and injector device into the combustion chamber. The rate of flow of propellant fuel and the mixture ration (oxidizer weight, flow rate, and fuel weight,flow rate) are functions of the respective areas of the oxidizer and fuel orifices in the injector. For a given geometry of the chamber, the design of the injector exerts a significant influence upon the efficiency and stability of the combustion process.

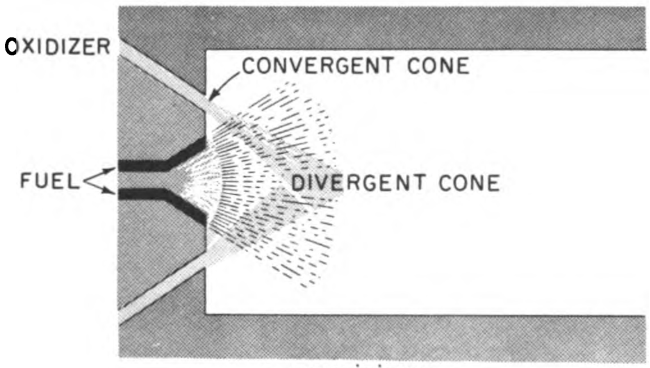
injectors

The function of the injector in a jet engine can be compared to that of a carburetor in a reciprocating engine. It performs the dual function of mixing the fuel and oxidizer in the proper proportions for efficient combustion and vaporizing them. The combustion temperature depends primarily upon the mixture ratio and is a determinant of combustion pressure. Normally, it is desired that the motor operate at a constant value of thrust, and, as thrust is a function of combustion pressure, any variation in the mixture ratio can have an adverse effect on engine efficiency.

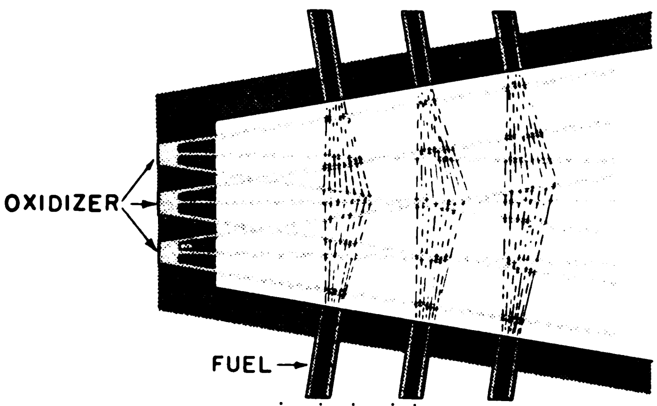
Three basic types of injectors are the multiple-hole impingement type, the spray injector type, and the nonimpinging injector type. In the multiple-hole impingement type, the oxidizer and fuel are injected through an arrangement of separate holes in such a manner that the jet-like streams intersect each other at a predetermined point, where the fuel and oxidizer mix and break up into vapor-like droplets. The spray injector has the oxidizer and fuel orifices arranged in a circular pattern, so that conical or cylindrical spray patterns are produced that intersect within the chamber. The nonimpinging injector is one in which the oxidizer and fuel do not impinge at any specific point, but are mixed by the turbulence within the chamber.



multiple hole impingement injector



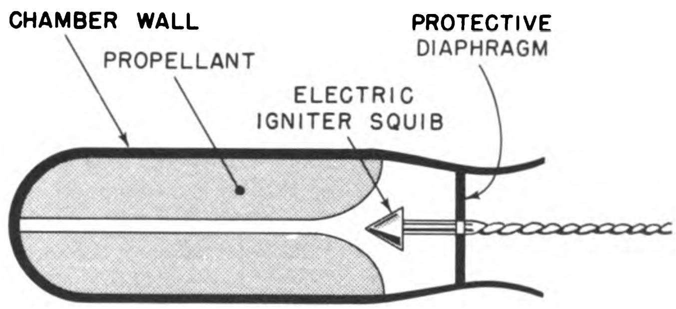
spray injector



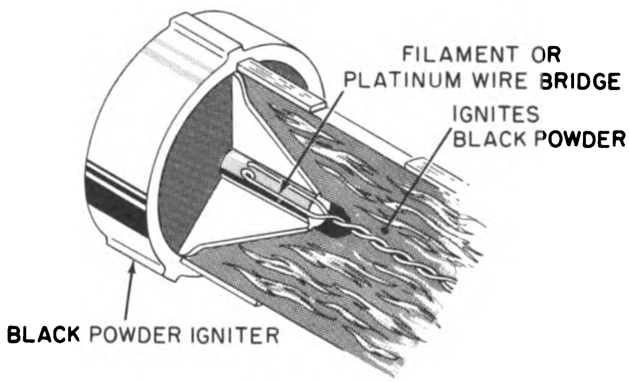
non-impinging injector

ignition systems

A propellant is said to be diergolic when no spontaneous reaction occurs upon chemical combination of the liquid fuel and the oxidizer. A propellant in which such a spontaneous chemical reaction takes place is said to be hypergolic. Some form of igniter is required to initiate the chemical reaction and to cause combustion in a diergolic propellant. Often hypergolic propellants are utilized as igniters for diergolic liquid propellants. The igniter device must be located within the combustion chamber at a point where the injected mixture of fuel and oxidizer can be ignited readily. If either fuel or oxidizer accumulates excessively in the chamber before ignition begins, an uncontrolled explosion may result. Often ignition is originated by a device of the spark plug type similiar to those used in reciprocating engines.



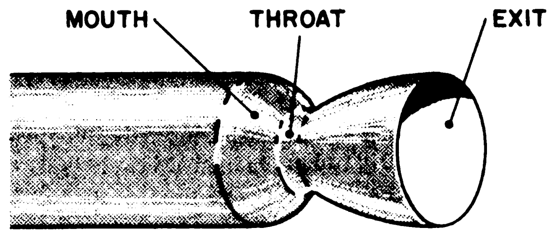
electrical ignition devices



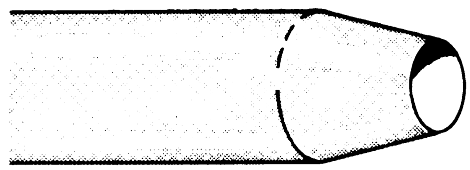
Often, powder-charge ignition systems are employed to initiate solid-fuel propellants. The device consists of a powder squib that can be ignited electrically from a safe distance; it burns for a short time with a flame hot enough to ignite the main propellant charge. Another basic system of ignition is that which introduces a catalyst into the mixture. A catalytic ignition system uses either a solid or a liquid catalytic agent to bring about the chemical decomposition of the propellant.

exhaust nozzles

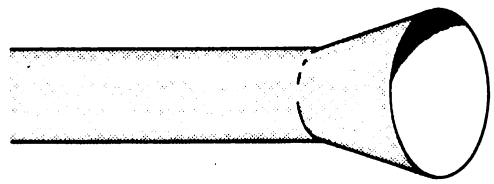
An exhaust nozzle is a mechanically designed orifice through which the gases generated in the combustion chamber flow to the outside. The function of the nozzle is to increase the exit velocity of the hot gases flowing out of the engine so that maximum thrust is extracted from the fuel. Under conditions of steady flow, the weight of gas that passes any cross section in unit time is constant (Bernoulli's theorem). Thus, in subsonic flow, the velocity of the gases will increase at any point where the cross sectional area of the nozzle is constricted, provided the weight ratio of flow remains constant. Where the cross sectional area of the nozzle becomes wider, the gas velocity will decrease. Since thrust from a rocket motor is proportional to the momentum of the exhaust gases ejected per second and momentum is equal to mass times velocity, the efficiency of thrust can be increased at no extra cost in fuel consumption if the exhaust velocity is maximized by proper nozzle design. For rocket engines operating at subsonic speeds, the best nozzle design is the convergent nozzle.



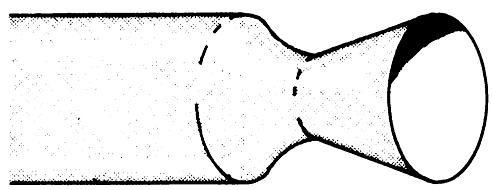
nozzle components



convergent nozzle



divergent nozzle



convergent-divergent nozzle (DeLaval)

convergent nozzles

The speed of flow of gases entering a simple convergent nozzle will increase if the input velocity was less than the local speed of sound. The greater the convergence, the greater the increase in speed, up to the local speed of sound. (When we refer to the local speed of sound, we mean the speed of sound that corresponds to the temperature and pressure at that specific point.) In a simple convergent nozzle, no further increase in speed will take place beyond mach 1 no matter how high the pressure in the chamber is raised. Conversely, if gases at supersonic speeds enter a convergent nozzle they will slow down.

divergent nozzles

When gases enter a divergent nozzle at subsonic speeds, velocity decreases. The decrease in speed also is in accordance with Bernoulli's theorem. Since the cross sectional area increases and the weight ratio of flow remains constant, the velocity of the gas must decrease with a proportionate increase in pressure. When gases moving at supersonic speed enter a divergent nozzle, their speed is further increased. Gases in supersonic flow are in a state of compression, and when they enter a divergent nozzle they expand. A portion of the potential energy contained in the compressed gas is converted into kinetic energy, and increases the velocity of flow.

convergent-divergent nozzles

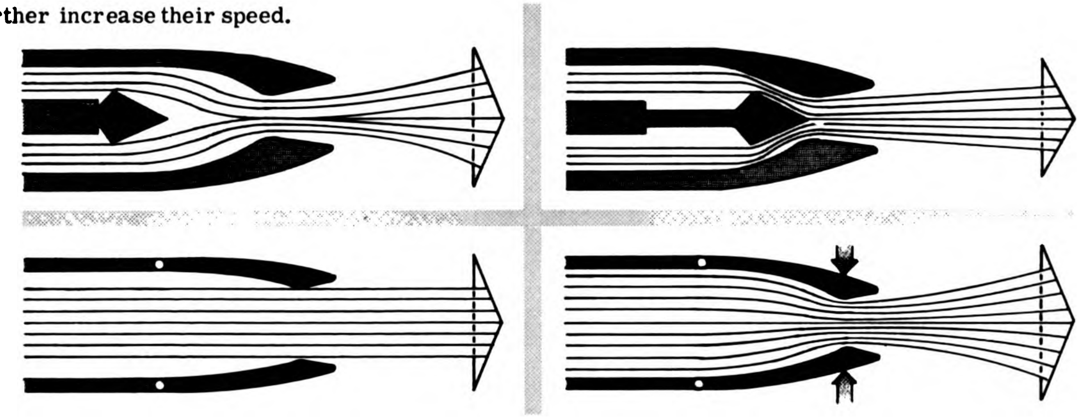
A nozzle designed to extract the maximum benefits of both the convergent and divergent types, the convergent-divergent nozzle, was designed by a Swedish engineer, De Laval. In this type, the exhaust nozzle first converges to bring the subsonic flow up to the local speed of sound and then, at an optimum point, the nozzle diverges, allowing the gases to expand and produce supersonic flow. The convergent-divergent nozzle, if properly designed, can be used to control the expansion of gases after they pass through the throat, and thus obtain higher velocity and increased thrust. The throat area is determined by the weight rate of flow. The area at the exit of the divergent cone is determined by the desired ratio of expansion of the gases between throat and exit.

nozzle efficiency

Efficient nozzle design is necessary to extract the maximum amount of thrust from rocket fuel. The nozzle must be designed for a specific set of propellant and combustion characteristics in order to attain highest practical exhaust velocity with a minimum of turbulence. Rocket nozzles increase the velocity of the combustion gases by first passing them through a convergent nozzle until they reach the speed of sound. This occurs at the narrowest part or throat of the nozzle. Then they are sent through a divergent nozzle to further increase their speed.

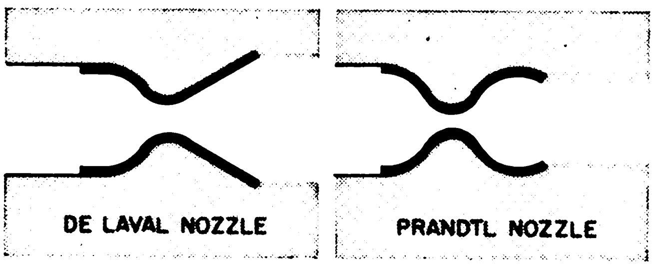
Other nozzles of increasing importance are the adjustable area type in which the nozzle area is varied to suit varying combustion environmental conditions.

Although most nozzle convergent-divergent sections are straight cones, as shown in the De Lavel nozzle, the Prandtl nozzle is more efficient but much more difficult to engineer and produce. It increases the velocity of flow at a higher rate than a normal convergent-divergent type.

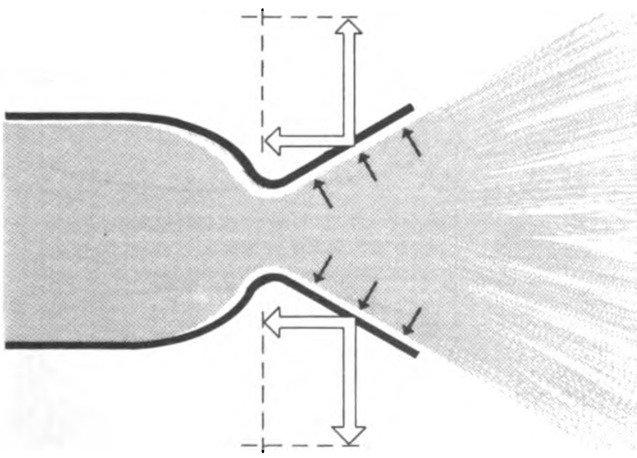


WIDE OPENING = LOW JET VELOCITY

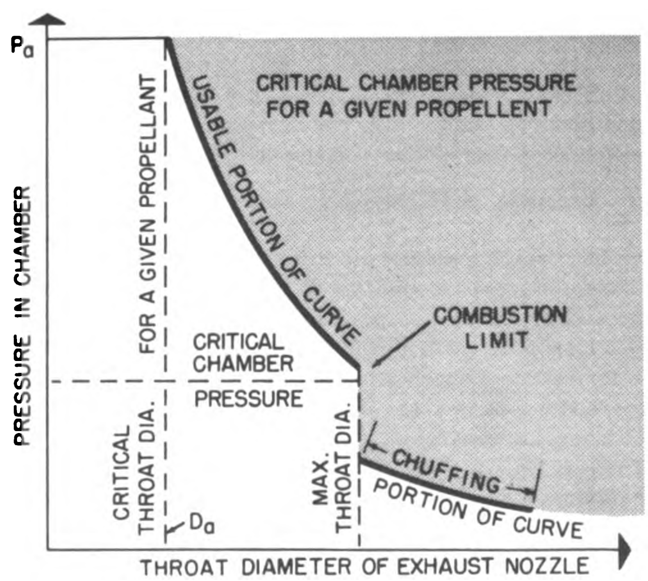
NARROW OPENING = HIGH JET VELOCITY



The shape of the nozzle determines the characteristic of gas flow, which must be smooth. For a smooth, controlled trajectory, thrust must be produced along the horizontal or long axis of the rocket. By tapering the rear of the combustion chamber so that it narrows smoothly toward the nozzle aperture, a smooth, non-turbulent flow of escaping gas is created. This tapered section forms the front half of the nozzle which leads outward from the nozzle aperture, increasing the pressure of the escaping gas, and increasing the forward thrust potential by about 33 percent.

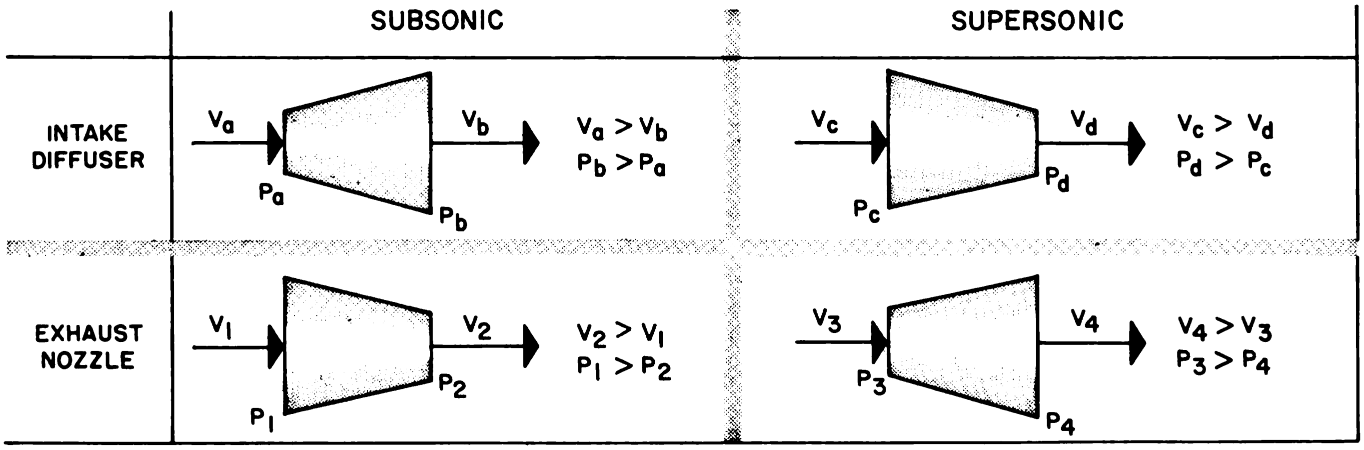


Chamber pressure is controlled by the mass rate of flow onto the motor, and the mass rate of flow of gas out of the exhaust nozzle. It was observed in a series of tests that when the exhaust nozzle throat diameter of a rocket assembly was increased beyond a certain value, the chamber pressures recorded thereafter became erratic and unpredictable. These values fell far below those expected from an extension of the usable portion of the pressure curve plotted for smaller exhaust nozzle throat diameters. The highest throat diameter in the normal section of the curve and the corresponding chamber pressure are referred to as the combustion limit of the propellant. For exhaust nozzle throat diameters below the combustion limit, the curve is smooth, but for values that exceed the combustion limit, the chuffing portion of the curve is erratic and unpredictable.



diffusers

The objective of the air intake and diffusion system is to decelerate the velocity of the air from its free stream speed to the desired speed at the entrance of the combustion chamber with a minimum of pressure loss. When the free stream air is of supersonic velocity, the problem of diffusion is complicated by the formation of shock waves at the inlet. Diffuser types are normally considered to the either 1) subsonic or 2) supersonic. Often, when a vehicle is traveling at supersonic speed, the diffusion process is achieved in two steps: a supersonic diffusion to the speed of sound, followed by a subsonic diffusion.



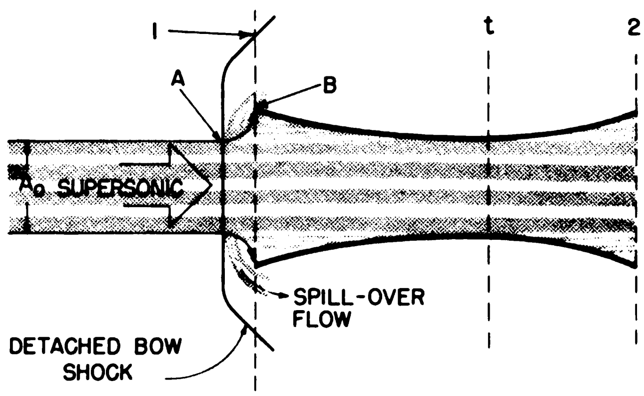
subsonic diffusers

Subsonic diffusers are normally classified as internal compression diffusers or external compression diffusers. An internal compression diffuser is a duct of varying cross section (a divergent duct for reducing the velocity of the air flow) located at the forward end of the engine between the air intake orifice and the injector. Between these two points the diameter increases; as a result, the velocity of air entering the diffuser decreases and the pressure increases. If the pressure change is to be kept small, diffuser length must be increased. Because increased diffuser length could lead to pressure losses resulting from skin friction, a compromise value is usually used.

External compression diffusers obtain air velocity decrease at a point external to the air inlet. It is desirable to obtain compression externally to permit simplified diffuser design, but, unfortunately, although the principles are known (use of airfoils), no completely successful device has been developed.

supersonic diffusers

The function of a supersonic diffuser is to effect the diffusion of a supersonic flow of air to a subsonic stream with a minimum loss of pressure. One of the primary phenomenom to contend with in this transformation is the formation of shock waves at the inlet of the diffuser. Supersonic diffusers are classified as 1) normal shock, 2) converging-diverging, and 3) conical or "spike" diffusers. In a normal shock diffuser, a diverging duct is employed to reduce the process to two steps: the normal shock effects at the input section decrease the velocity to approximately the speed of sound; thereafter the air is diffused to subsonic speeds in the diverging duct. The converging-diverging diffuser principle is similar to that of the De Laval nozzle, except that the process is reversed. Again the transformation is accomplished in two steps. While the air stream is traveling through the converging portion of the duct, its velocity is decreased to the speed of sound (Bernoulli's theorem); and then its velocity is again decreased to the desired velocity through the divergent portion of the diffuser. Conical or "spike" diffusers utilize the principle of a protruding conical nose or spikes placed inside the diffuser assembly to produce a reduction in mach numbers of the free stream. When a supersonic flow of air approaches the cone, a conical shock wave emenates from the cone and the supersonic flow through this shock field is slowed to subsonic velocity. The fluid then enters the subsonic diffuser element for completion of the diffusion process.



THRUST

The transformation of heat energy to kinetic energy of flow causes a force to act in the opposite direction to the flow of the exhaust gases from the motor. This force, termed thrust, is a function primarily of the velocity of the gas leaving the nozzle and the weight of the gases and is independent of the velocity of flight of the missile. Thrust is an applied force measurable in pounds and is not a measure of work or horsepower. A reaction motor which is motionless develops no horsepower. At a velocity of 375 miles per hour, one pound of thrust will develop one horsepower, that is:

$$THP = \frac{\text{Thrust (lbs) x Velocity (ft/sec.)}}{\frac{\frac{550 \text{ ft-lb}}{\text{sec.}}}{\text{HP}}}$$

$$= \frac{\text{Thrust (lbs) x Velocity (mph)}}{\frac{\frac{375 \text{ mi-lb}}{\text{hrs}}}{\text{HP}}}$$

Thus a rocket that develops a thrust of 100,000 pounds at a velocity of 3750 miles per hour is developing 1,000,000 horsepower (10 x 100,000). The general equation for total thrust (F) on a reaction motor is composed of momentum thrust (F_m) and pressure thrust (F_p). Momentum is the product of the mass of a body and its velocity. The momentum thrust is a function of the rate of change of momentum of the working fluid through the engine or propeller. The pressure thrust is a function of the difference between the exhaust pressure, p_e, and the atmospheric pressure, p_o.

$$F = F_m + F_p$$

$$F_m = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt}$$

$$= m_e \frac{dv_e}{dt} - m_a \frac{dv_a}{dt} + v_e \frac{dm_e}{dt} - v_a \frac{dm_a}{dt}$$

Since v_e and v_a are approximately constant,

$$F_m = V_e \frac{dm_e}{dt} - V_a \frac{dm_a}{dt}$$

In air-breathing engines, the ratio of air to fuel in the mixture is large enough so that the mass of the exhaust is always approximately equal to the mass of the air.

Therefore $\frac{dm_e}{dt} = \frac{dm_a}{dt}$

and $F_m = \frac{dm_e}{dt}(V_e - V_a) = \frac{dw_e}{dt} \frac{(V_e - V_a)}{g}$

In rockets, since there is no air intake:

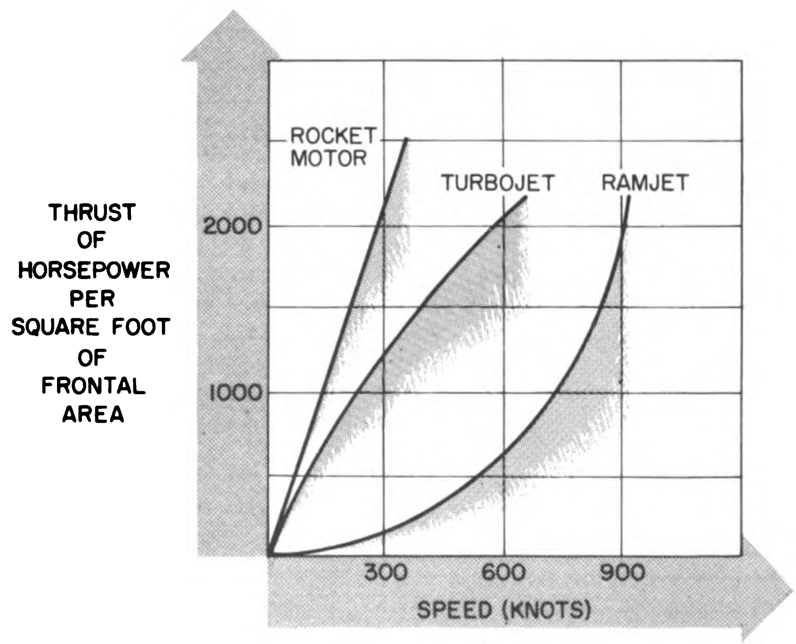
$$F_m = \frac{V_e}{g} \frac{dw_e}{dt}$$

Besides the thrust caused by changing the momentum of the working fluid, there is also a thrust which results from the difference in pressure between the jet stream and the surrounding medium, that is:

$$F_p = P_e A_e - P_o A_e = A_e (P_e - P_o)$$

Therefore, total thrust is expressed by:

$$F = F_m + F_p$$



relationship of thrust horsepower vs speed

application of thrust equations

thermal jet engine (in air)

Thrust equations can be applied to any reaction motor. For various types of motors operating in different mediums, the equations can be evaluated as follows:

As developed above:

$$F = \dot{W}_e \frac{V_e}{g} - \dot{W}_a \frac{V_a}{g} + (P_e - P_o) A_e$$

NOTE

The dot designation over a function denotes the derivative with respect to time of that function, for example

$$\dot{W}_e = \frac{dW_e}{dt}.$$

Since $\dot{W}_e = \dot{W}_a + \dot{W}_f$

$$F = \dot{W}_a \frac{V_e}{g} + \dot{W}_f \frac{V_e}{g} - \dot{W}_a \frac{V_a}{g} + (P_e - P_o) A_e$$

Let $\frac{\dot{W}_f}{\dot{W}_a}$ = fuel-air ratio = f

and $\frac{V_a}{V_e}$ = velocity ratio = v

$$F = \left(\frac{1+f}{v} - 1\right) \dot{W}_a \frac{V_a}{g} + (P_e - P_o) A_e$$

In a turbojet engine, the pressure thrust, $(P_e - P_o) A_e$, is insignificant when compared with momentum thrust.

Therefore $F = \left(\frac{1+f}{v} - 1\right) \dot{W}_a \frac{V_a}{g}$

Furthermore, since W_f is small enough so that

$$W_f/W_a = f = 0$$

then $F = \left(\frac{1}{v} - 1\right) \dot{W}_a \frac{V_a}{g}$

rocket motor (in air)

As previously stated, a rocket motor takes in no air; therefore W_a and $V_a = 0$ and the equation reduces to:

$$F = \dot{W}_e \frac{V_e}{g} + (P_e - P_o) A_e$$

In rockets, $W_e = W_o + W_f$ instead of $W_a + W_f$, where W_o represents the weight of the oxidizer.

To eliminate the term $(P_e - P_o) A_e$:

let $F = \dot{W}_e \frac{V_{eff}}{g}$

where $V_{eff} = V_e + \frac{g}{\dot{W}_e} (P_e - P_o) A_e$

V_{eff} is larger than V_e as long as P_e is larger than P_o. If there is no pressure differential, i.e., the exhaust gases are allowed to expand completely before they leave the nozzle so that P_e is equal to P_o, then V_{eff} is equal to V_e. Since F and W_e are measurable, effective velocity can be calculated.

rocket motor in vacuum

In the case of a rocket motor in vacuum, the term P_o is zero, so that the equation above becomes:

$$F = \frac{\dot{W}_e}{g} V_e + P_e A_e$$

rocket motor in water

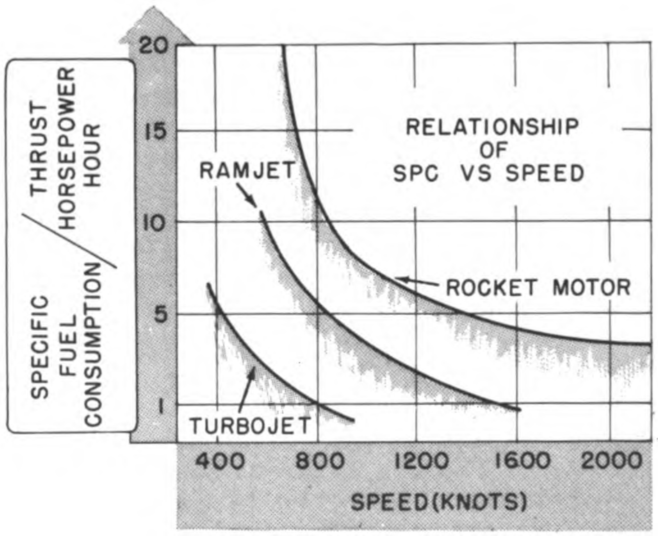
The thrust formula for the rocket motor in air applies directly: $F = \dot{W}_e \frac{V_e}{g} + (P_e - P_o) A_e$

where P_o = water pressure

hydrojet

The basic formula for the jet propulsion derived previously also applies to the hydrojet. In this case, $W_e = W_w$ (the weight of water taken in) i.e., exhaust equals intake and the thrust equals:

$$F = \frac{\dot{W}_e}{g} (V_e - V_a) + (P_e - P_o) A_e$$



propeller (in air)

In the ideal case, a propeller creates a slipstream. The ideal propeller may be replaced by a hypothetical actuator disc. Air with flight velocity V_1 approaches the propeller, flows through the bounded surfaces of the slipstream, and leaves the slipstream with velocity V_2. Since $P_1 = P_2$, the thrust is limited to the linear momentum change in the fluid flowing through the propeller disc. Since W_1, the input weight rate of flow, must equal W_2, the thrust equation becomes:

$$F = \frac{\dot{W}}{g} (V_2 - V_1)$$

If $v = V_1/V_2$

then $F = \frac{\dot{W}}{g} V_2 (1 - v)$

or $F = \frac{\dot{W}}{g} V_1 \left(\frac{1}{v} - 1\right)$

Note that the thrust equation for the turbojet engine is identical to this equation.

THERMAL JET ENGINE OPERATIONS

Any jet-propelled system that obtains oxygen from the surrounding atmosphere to support the combustion of its fuel is a thermal jet, air-breathing engine. Thermal jet engines may be classified into three basic types:

1. pulsejet engines
2. turbojet engines
3. ramjet engines

Obviously, the operation of these engines is limited by the amount of oxygen available and they can function only at altitudes where the oxygen content of the air is adequate. The upper limit of operation depends on the type and design of the particular engine. At the present time, pulsejets and turbojets have limited military application, as they do not operate efficiently at supersonic speeds.

pulsejet engines

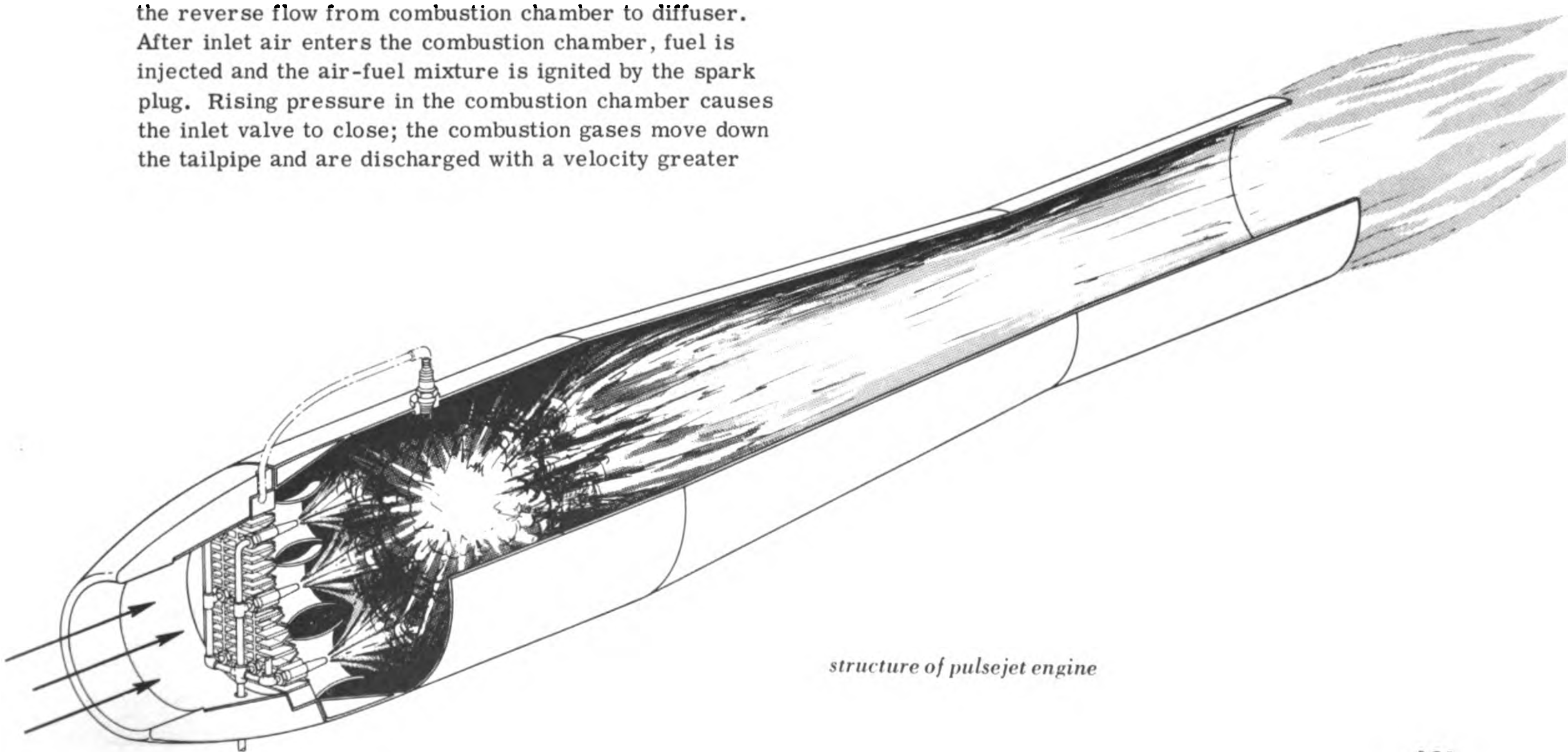
The pulsejet engine employs the forward motion of the missile, to compress the air and fuel vapor before combustion. The illustration shows the essential elements of the pulsejet engine.

It comprises an inlet diffuser, a grill assembly containing air valves and injectors so arranged that air can flow through the valves only in a downstream direction, fuel injector nozzles, the combustion chamber, the tailpipe, and a spark plug for initial ignition. In flight, air enters the diffuser and pressure builds up because of diffuser configuration. When the pressure exceeds that of the combustion chamber, the valve opens and air enters the combustion chamber. As indicated, the valves are arranged to act as spring-loaded shutters to prevent the reverse flow from combustion chamber to diffuser. After inlet air enters the combustion chamber, fuel is injected and the air-fuel mixture is ignited by the spark plug. Rising pressure in the combustion chamber causes the inlet valve to close; the combustion gases move down the tailpipe and are discharged with a velocity greater than that of the inlet air. The resulting pressure differential creates a thrust in the direction of flight.

The ejection of the gases from the combustion chamber reduces the pressure within the chamber. Because of the speed of the combustion gases leaving the chamber, a partial vacuum is created. The ram pressure in the diffuser acting on the inlet valve causes the latter to open, and a fresh supply of air enters the chamber. As a result of the partial vacuum, a portion of the hot exhaust gases are sucked back into the chamber and, as the temperature of the gas is high enough to ignite the air-fuel mixture, a new cycle begins. Note that the spark plug ignition is required only to start the engine; after starting, its combustion cycle is self sustaining.

It was this intermittent cycle (repetition rate of approximately 40-50 times per second) which gave the German weapon of World War II the name of "Buzz Bomb". The top speed a pulsejet may attain is a function of chamber pressure versus ram pressure developed in the diffuser. The cyclic action demands pressure gradients to allow the opening and closing of the intake valves. Thus, the speed of a pulsejet is limited to the low subsonic range by the fact that at higher speeds the ram pressure developed in the diffuser exceeds the chamber pressure at all times throughout the combustion cycle. The inlet valve cannot close, and the engine becomes inoperative.

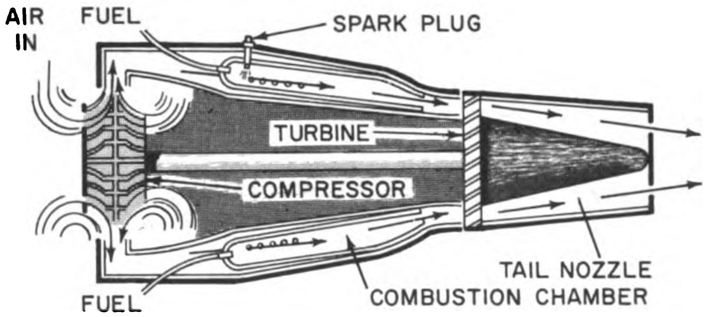
Another disadvantage is that at the instant of launch there is no air pressure in the diffuser unit. Therefore, pulsejets are incapable of developing static thrust for their own propulsion. They require help at launch, usually in the form of compressed air injected in the chamber along with the fuel, or of a catapult, or of booster rockets. Also, this type of engine has a low efficiency index because its fuel consumption rate is high.



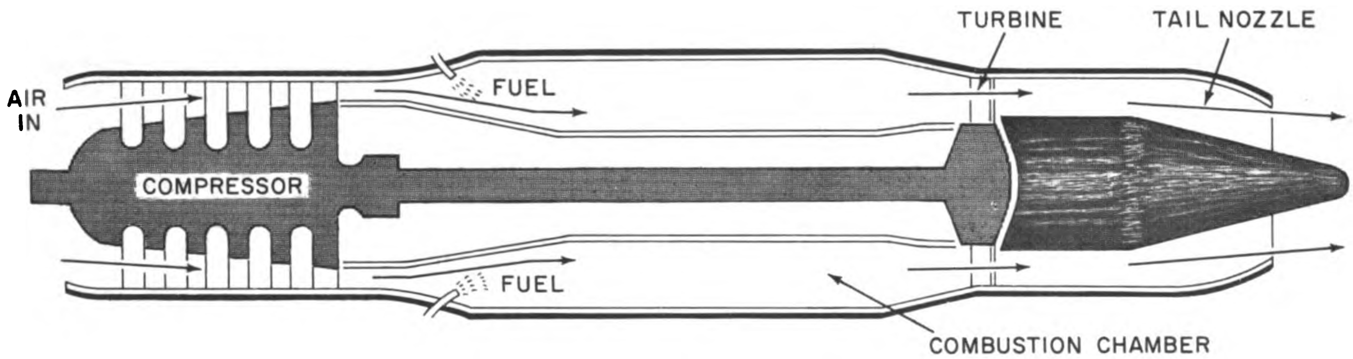
structure of pulsejet engine

turbojet engines

A turbojet engine derives its name from the fact that a portion of its exhaust output is diverted to operate a turbine which, in turn, drives the air compressor used to compress the input air flow. Turbojets are diveded into two types, depending on the type of compressor. These are centrifugal flow and axial flow turbojets.
A centrifugal compressor consists of a stator and a rotor or impeller. As the rotor (consisting of a series of blades extending radially from the axis of rotation) revolves, air is sucked in, whirled around by the blades and ejected by centrifugal force at high velocity. The stator, composed of diffuser-line vanes, compresses this air flow and directs it into the firing chamber.



centrifugal-flow turbojet



axial flow turbojet

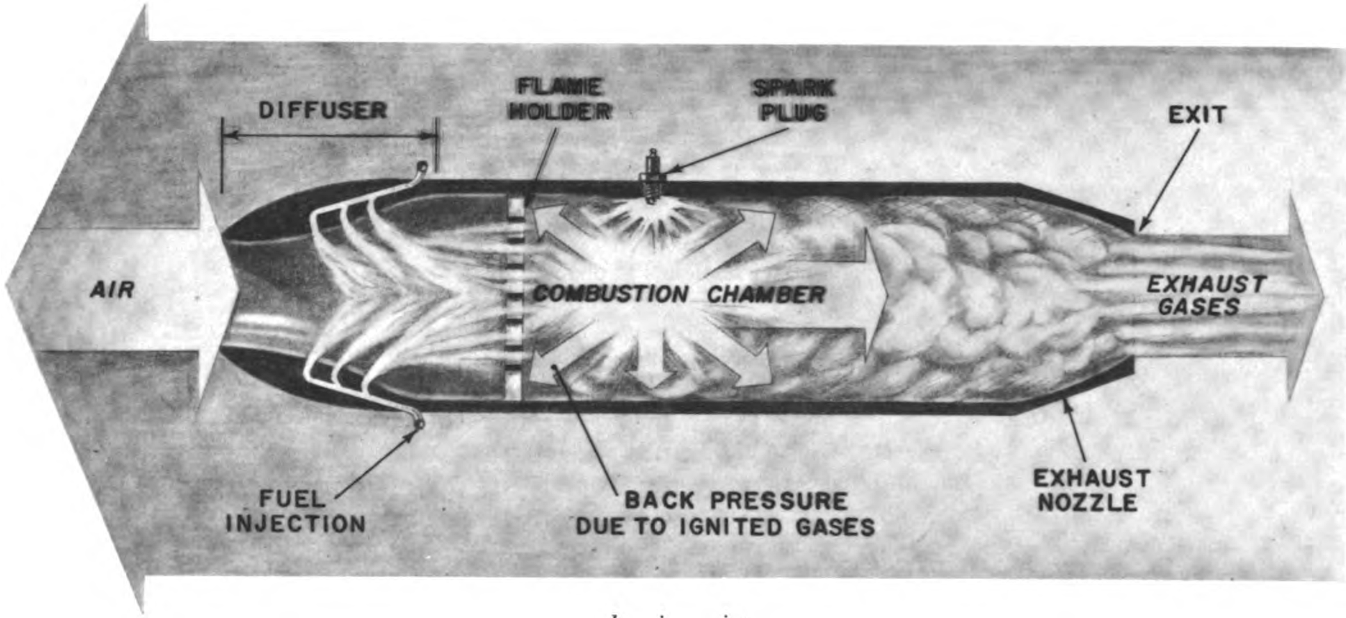
An axial compressor is similar in operation to a propeller. As the rotor of the axial compressor turns, the blades impart energy of motion in both a tangential and axial direction to the air entering the front of the engine. The stator is set in a fixed position with its blades preset at an angle such that the air thrown off the first stage rotor blades is redirected into the path of the second stage rotor blades. A number of rotors and stators comprise a multistage compressor.
The added velocity compresses the air, increasing its density and, as a result, its pressure potential. This cycle of events is repeated in each stage of the compressor. Therefore, by increasing the number of stages, the final pressure can be increased to almost any desired value.
The centrifugal compressor is simpler and has a high pressure ratio per stage. The axial flow compressor, however, has a higher per stage efficiency.
The combination of the air intake system, air compressor, combustion system and turbine is essentially an open-cycle gas turbine combined with a jet stream. In operation, the compressor driven by the gas turbine supplies air under high pressure to the combustion chamber, and the turbine absorbs only part of the jet energy, while the remainder is employed for thrust energy. Once started, combustion is continuous.
The turbojet is limited to less than the speed of sound. When it approaches mach 1 speed, shock waves develop on the compressor blades and interfere with the operation. A turbojet can develop large static thrust and carry its own fuel; its thrust is practically independent of speed.

ramjet engines

The most promising jet engine, from the standpoint of simplicity and efficiency at supersonic speed, is the ramjet, so called because of the ram action which makes possible its operation.

principles of operation

A ramjet engine has no moving parts. It consists of a cyclindrical tube open at both ends, with an interior fuel injection system.
As the ramjet moves through the atmosphere, air is taken in through the front or diffuser section, which is designed to convert high-speed, low-pressure gas flow into low-speed, high-pressure gas flow. Thus a pressure barrier is formed and the escape of combustion gases out of the front of the engine is prevented.
The high-pressure gas then mixes with the fuel, which is being sprayed continuously into the engine by the fuel injectors. Burning is initiated by spark plug action, after which it is uninterrupted and self supporting, i.e., no further spark plug assistance is required. The flame front is kept from extending too far toward the rear of the engine by a device called the flameholder. By restricting burning to the combustion chamber, the flameholder maintains the combustion chamber temperature at a point high enough to support combustion. The combustion gases bombard the sides of the diffuser and the ram air barrier, exerting a force in the forward direction. Since the gases are allowed to escape out of the rear through the exhaust nozzle, the force in the forward direction is unbalanced. The degree to which this force is unbalanced depends on the efficiency with which the exhaust nozzle can dispose of the rearward-moving combustion gases by converting their high-pressure energy into velocity energy.

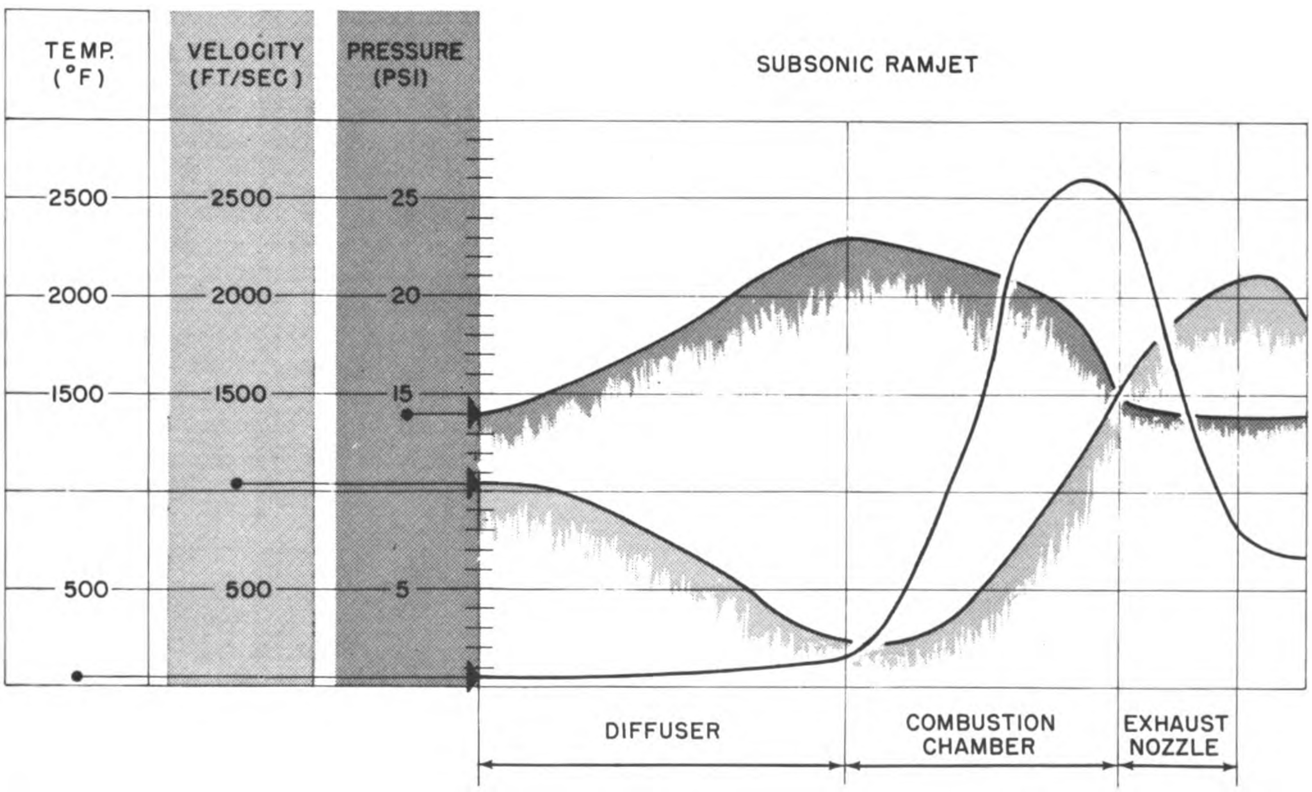


subsonic ramjet

limitations

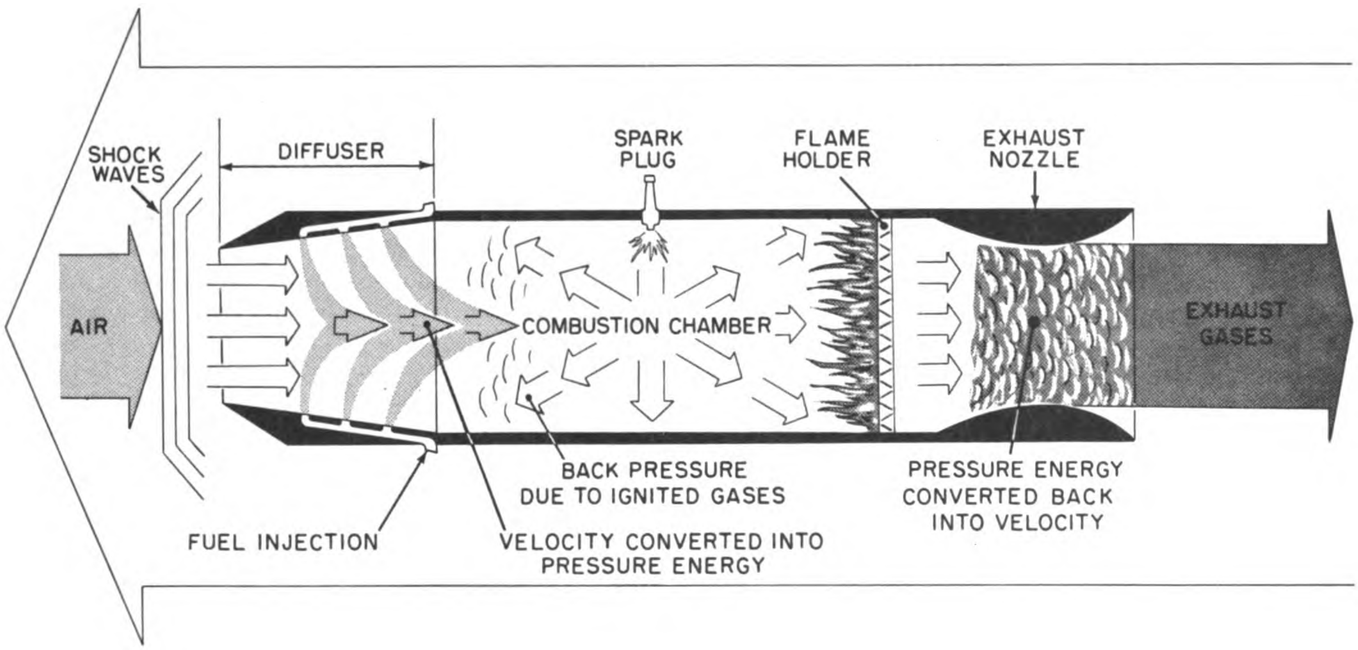
Ramjets operate in the subsonic, supersonic, and hypersonic ranges. Theoretically there is no limit to the speed they can attain. However, because of the intense heat generated by air friction, present-day materials are unable to withstand speeds in excess of mach 5.0 in the atmosphere.

The main disadvantage of ramjet engines lies in the fact that they are, by the nature of their operation, unable to develop static thrust. If fired at rest, high pressure combustion gases would escape from the front as well as the rear. Consequently, before a missile with a ramjet engine can function properly, it must first be boosted by some other propulsion system to a speed approximately equal to that at which it was designed to operate. Ramjets engines are also restricted to use in altitudes below approximately 90,000 feet, since they must have air to operate.



types of ramjets

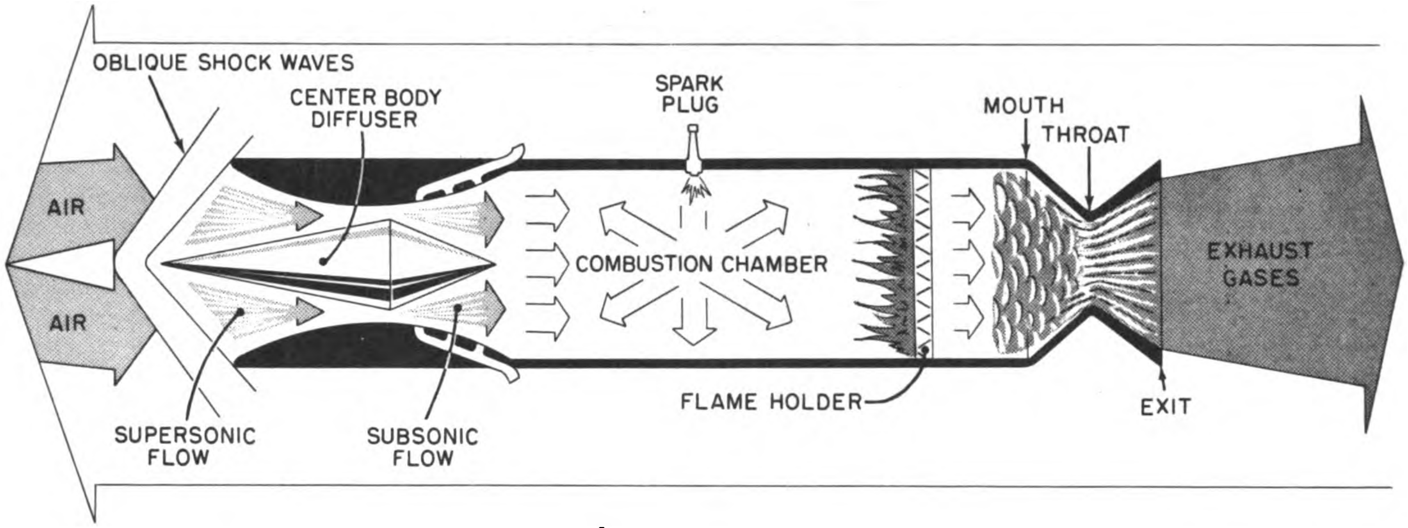
Ramjets engines are designed to operate best at some given speed and altitude. They are classified as subsonic, low supersonic (between mach 1 and mach 2), and hypersonic (in excess of mach 2). A subsonic engine follows most closely the general theory of ramjet operation while a low supersonic engine operates somewhat differently because of the shock wave which forms in front of the diffuser at supersonic speeds.



low-supersonic ramjet

Air passing through the shock wave experiences an abrupt reduction from a supersonic velocity relative to the diffuser to a subsonic velocity, with a corresponding increase in pressure. From this point, operation is essentially the same as in a subsonic jet engine. The diffuser and exhaust nozzle are designed for the higher speed and the design of the fuel injection system is correspondingly different.

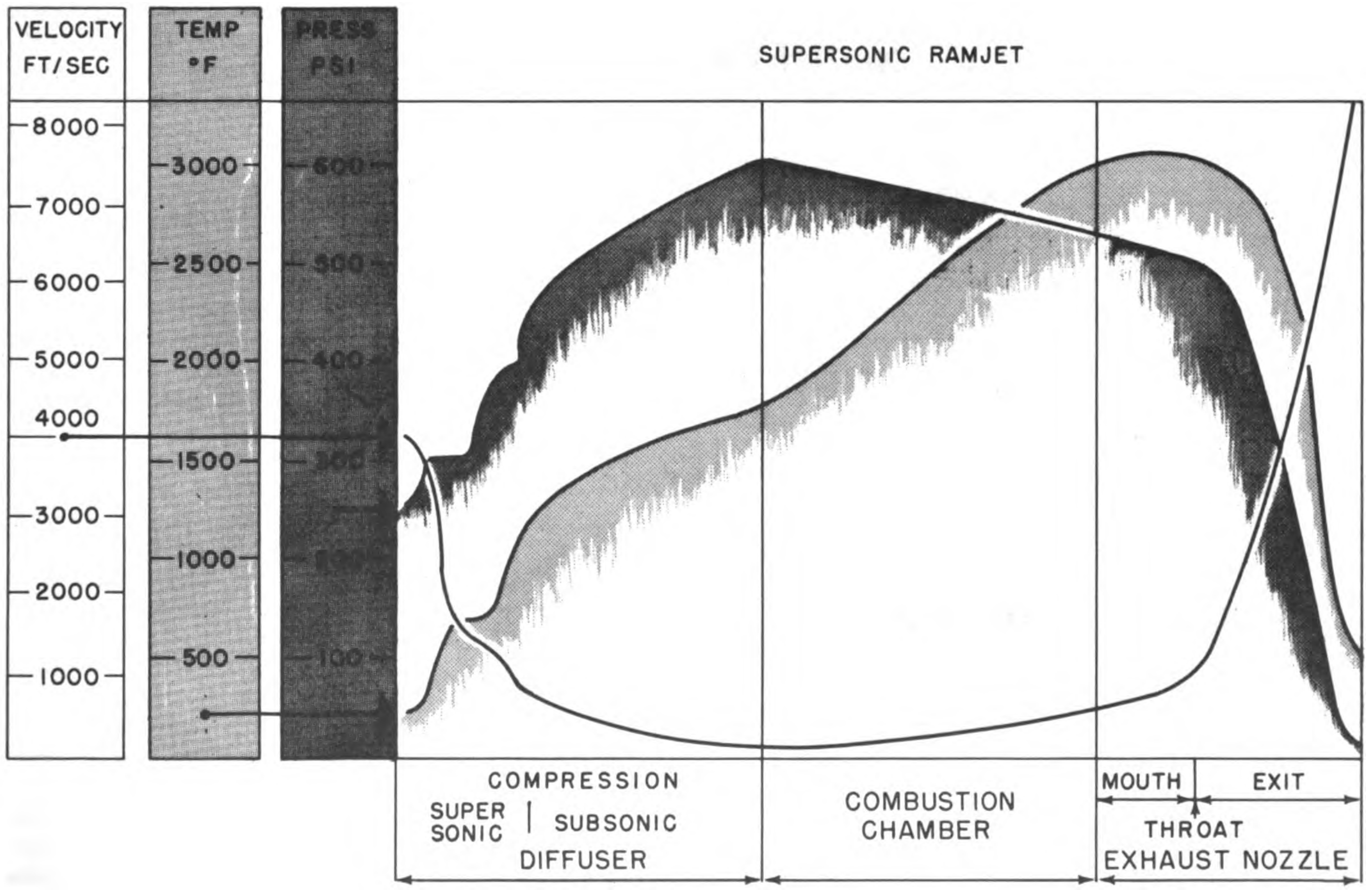
Ramjets operating at speeds greater than mach 2 create shock waves similar to those shown. Air passing through these shock waves, while slowed down, is still traveling at supersonic speed when it enters the diffuser. If the diffuser were of the divergent type used in both subsonic and supersonic ramjets, the air would be accelerated, rather than slowed down, because it is moving at supersonic speed. For this reason, a design similar to that illustrated on the next page is used.



hypersonic ramjet

The center body diffuser slows down the incoming air until a normal shock wave is formed. The air passing through this wave abruptly drops to some subsonic level. Past this point the diffuser diverges, and engine operation is the same as for lower speed ramjets. The exhaust nozzle in high supersonic ramjets is convergent-divergent and is designed to convert the high-pressure energy of the combustion gases into high-velocity energy.

The speed for which a ramjet is designed also affects the design of its fuel injection system and flameholder. The engineering problems thereby engendered can be formidable, but they are minimized by operating the ramjet at a constant altitude and speed as nearly as possible. At present, much research is being conducted to broaden the operating range in altitude and speed of ramjets.



ROCKET ENGINE OPERATIONS

A rocket is a missile propelled by the escape of gases produced by the firing of solid, liquid, or gaseous propellants completely contained within the missile. Rockets exhibit two characteristics of vital interest for supersonic and nonatmospheric flight. 1) The thrust developed is independent of the atmosphere, and thus the rocket may be propelled through empty space. 2) The propulsive force is unaffected by the velocity of the missile. The engines employed for propelling ballistic missiles are designated as chemical rocket engines, and they are divided into two main classes:

1. solid propellant rocket engines

which burn fuels that are in a SOLID STATE prior to combustion

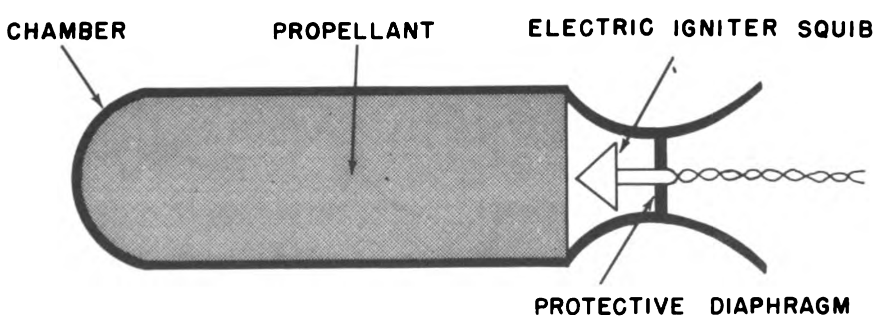
2. liquid propellant rocket engines

which burn fuels that are in a LIQUID STATE prior to combustion

Regardless of whether solid or liquid propellants are burned, the main objective is to produce a propulsion jet having the largest possible force.

solid fuel rockets

A solid fuel rocket engine produces its high-temperature, high-pressure gases by burning a solid propellant, the principal ingredients of which are a fuel and an oxidizer. The combustible elements may be contained either in the molecular structure of the solid propellant itself or as separate compounds that have been chemically combined or that are to be mixed within the combustion chamber. Solid propellants may be segregated into double-base propellants or composite heterogeneous propellants.



components of solid rocket motor

principles of operation

A solid-fuel rocket consists of a propellant, a combustion chamber, an igniter, and an exhaust nozzle. The combustion chamber of a solid-fuel rocket serves both as a storage area for the propellant and as a chamber in which burning takes place. The igniter is a device used to produce a temperature high enough to ignite the main rocket-propelling charge. It consists of a small charge of black powder or similar substance that can be easily ignited by either spark discharge or a hot wire.

The exhaust nozzle serves the same purpose as in any other jet-propulsion system.

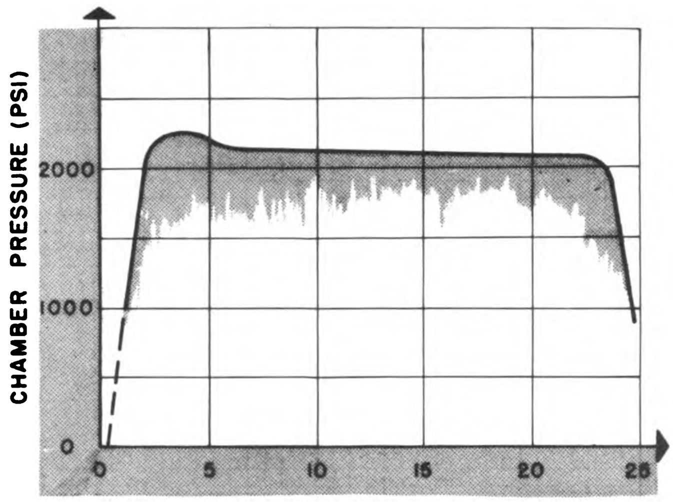
The operation of a solid-fuel rocket is relatively simple. Once attached to a missile, the rocket is ignited electrically by means of an ignition squib assembly which is blown out the exhaust nozzle after ignition takes place. The rocket then burns continuously until its fuel supply is exhausted.

pressure limitations

Ordinary solid propellants require pressures up to 2000 pounds per square inch in order to sustain combustion, and the exhaust gas temperatures reach 4000 to 5000 degrees F. Formerly, these high pressures and temperatures necessitated relatively thick motor walls to contain them. However, new lightweight materials such as fiberglass have done much to eliminate the need for relatively heavy castings. In addition, recent developments of slow internally burning grains and resulting low operating pressures have helped overcome this undesirable feature.

The manner in which a fuel burns under a given set of conditions establishes the pressure specification of the combustion chamber. The burning rate of a propellant is dependent upon the chamber pressure and increases as the pressure increases. The range of burning rates at pressures of 2000 psi for modern solid propellants is between 1 and 2 inches per second. Both the propellant and the motor design can be altered to give the desired performance.

Theoretically, the time-pressure curve in a rocket motor should be rectangular, but in practice a "plateau" is about the best shape attainable. The initial pressure rise within the rocket motor chamber is comparatively slow. Upon reaching its peak, it is maintained at a constant level of the order of 1000 to 5000 psi until the charge is completely consumed. The limitations on the maximum pressure are governed by the strength of the rocket tube and the maximum mass rate of discharge which can be permitted for a given end use.

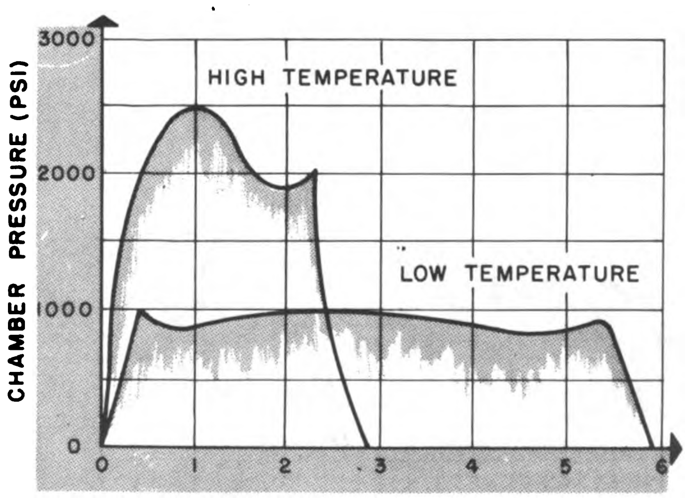


pressure-time curve for a rocket motor

temperature limitations

The rate at which solid fuel burns is affected by the temperature of the fuel. This change in burning property will vary with each formulation and, to a lesser degree, with the form of grain. To design a rocket motor properly, a knowledge of the change in burning rate with temperature is required. The pressure obtained within the rocket when it is fired varies directly with the temperature of the fuel just prior to firing. The illustration shows the pressure-time curves of a 3.5″ rocket fired at two different temperatures.

Excessive pressure at high temperatures and brittleness at low temperatures limit present solid fuel rockets to a temperature range of from approximately -20° F to +120° F.



pressure-time curve for a 3.5 rocket fired at a high and low temperature

liquid fuel rockets

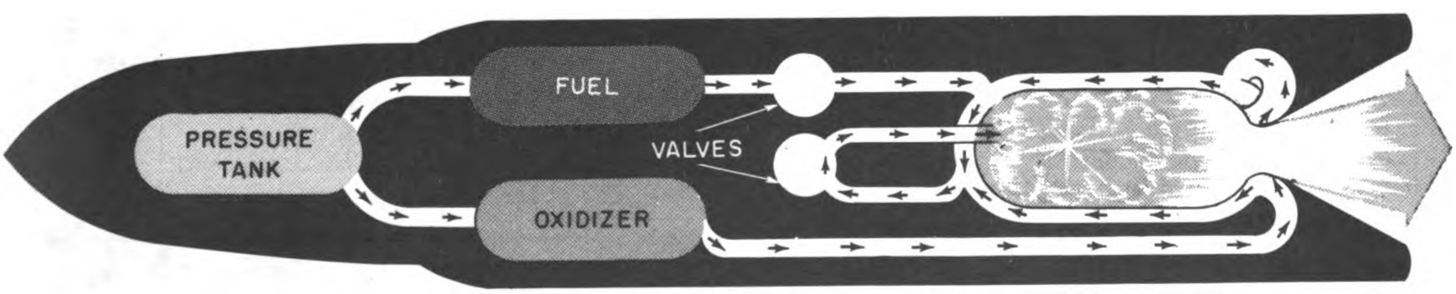
The burning duration of the liquid-fuel rocket is greater than that of the solid-fuel rocket. Furthermore, the design is relatively complicated, and intermittent operation is possible. The liquid-fuel rocket has been widely used for high-altitude, long-range missiles. The major components of a liquid-fuel rocket system are a fuel or propellant, a propellant feed system, a combustion chamber, an igniter, and an exhaust nozzle.

principles

In liquid-fuel rockets, burning of the fuel and oxidizer is usually initiated by a spark plug or a pyrotechnic device. Once started, combustion is self sustaining if the propellant is injected continuously. Many fuel-oxidizer combinations, called "hypergolic", are self igniting on mixing. The combustion chambers in liquid-fuel rockets are smaller than those in solid-fuel rockets, since they are not used to store fuel. The fuel and oxidizer are fed from their respective tanks to the combustion chamber by either a pressure-feed system or a pump-feed system. Most liquid-fuel rockets have regenerative cooling. Regeneratively cooled engines are built as a double shell, with separate openings for injection of the fuel and the oxidizer. The fuel enters the rear of the engine and flows between the walls, thereby cooling the inner surface and making possible the use of thin-walled combustion chambers. The fuel then enters the forward end of the combustion chamber. The fact that the engine is cooled permits longer burning than if the engine were not cooled. A further advantage of regenerative cooling is that the fuel is preheated before injection into the combustion chamber, with a resulting increase in the heat energy released on combustion. Other methods of cooling, such as film and sweat cooling, are also in use to keep down the weight and prolong the life of the chamber and nozzle.

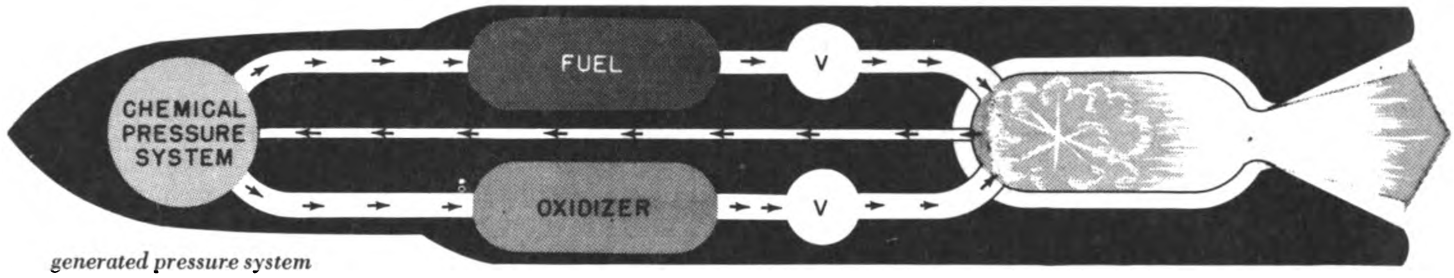
propellant feed systems

Commonly used are the pressure-feed system and the pump-feed system. Pressure-feed systems are subdivided into stored-pressure systems and generated-pressure systems. In the stored-pressure system, air or some other gas is stored under pressure within the missile before lauching and then injected into the propellant storage tanks, causing a pressurized flow toward the combustion chamber. In a generated-pressure system, components such as small chemical gas pressurization systems are carried within the missile to generate the high-pressure gas as needed. The gas is produced by chemically reacting a liquid or solid propellant at a steady, uniform rate.



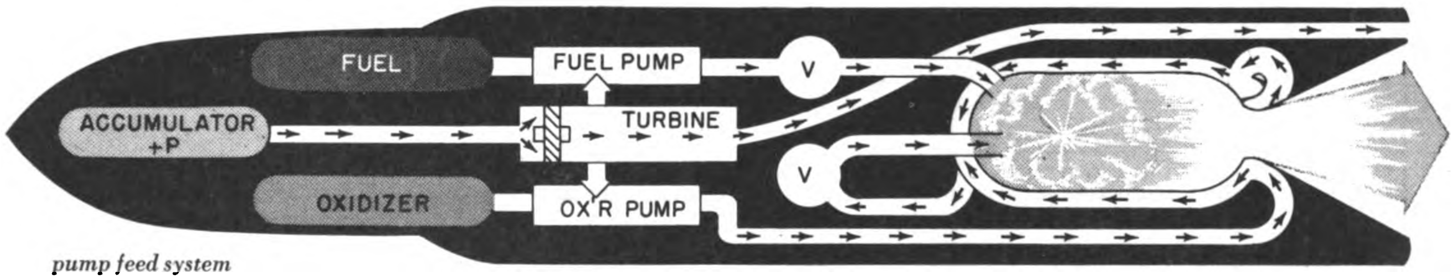
stored pressure feed system

Liquid-fuel rockets operate with a combustion chamber pressure from 250 to 500 psi; therefore, the fuel and oxidizer tanks must be pressurized to some greater value to insure a flow from the tanks to the combustion chamber. In order to withstand this pressure heavy tanks must be used. When the weight of an empty pressure-feed rocket is prohibitively large, 5 tons or more, pump-feed systems are used. A pump-feed system consists of a fuel pump and an oxidizer pump driven by a turbine wheel. If the power for driving this turbine wheel is gas from chemicals carried for that purpose within the missile, the system is referred to as a turbine-pump system. The diagram illustrates the major components of a liquid fuel rocket which uses a turbine pump-feed system. If the exhaust gases of the rocket motor drive the turbine wheel, it is called a turbopump system.



generated pressure system

The pump-feed system is essentially the same as the pressure-feed system except that pumps replace the pressure tanks to force the fuel and oxidizer into the combustion chamber. Pressure is felt only on the combustion chamber side of the pumps; consequently, the fuel and oxidizer tanks can be of lighter weight. The auxiliary devices and controls of this system are far more complicated than those of the stored-pressure system. For comparision, the table lists some of the advantages and disadvantages of the liquid-prcpellant feed systems.



pump feed system

COMPARISON OF LIQUID-PROPELLANT FEED SYSTEMS	
PRESSURE FEED	**PUMP FEED**
advantages	
Relatively simple construction	Relatively light weight
Relatively reliable	Can be stored fully fueled for longer periods
Very few components	More suitable for large rockets
Heavy tanks may be used as part of missile frame	May be operated intermittently
May be operated intermittently	
Better suited for small rockets (under 5 tons)	
Simple to check out at launching	
disadvantages	
Requires heavy tanks to withstand pressurization	Complexity of controls and auxiliary devices
Cannot be stored fully fueled for long periods of time	Less reliable because of complexity
	Complicated check out required

COMPARISON OF SOLID AND LIQUID FUEL ROCKETS

Solid-fuel rockets can be maintained in a condition of instant readiness for long periods of time; thus, they fulfill the important military requirements of rapid employment. Their simplicity, ease and safety of handling, and reliability make them ideally suited for use with a mobile military weapon. Solid-fuel rockets, particularly in smaller missiles, do not have the high shock of liquid-fuel rockets at starting, and, therefore, are more suited for shipboard launching. Solid-fuel rockets also cause much less vibration in the missile during operation in flight.

At present, the main disadvantage of solid-fuel rockets is the limited performance that has restricted their application in the past to use with short-range tactical missiles and intermediate-range ballistic missiles. However, modern rocket technology has produced solid-fuel rocket engines with over 1,500,000 pounds of thrust. Normally, liquid-fuel rockets are used for long-range warhead delivery. Because of the difficulties of fueling and handling, they are restricted to use at fixed launching sites, at present, and cannot be maintained in a state of instant readiness for prolonged periods of time. However, the development of liquid storables for use in ballistic missiles may well enable them to meet the military requirements of mobility and rapid employment. The characteristics of each type of rocket engine are summarized in the following table.

note

Some of the comparative advantages and disadvantages of solid-and liquid-fuel rockets may change as rocket technology advances.

LIQUIDS	SOLIDS
High energy values and better performance	Lower energy values and performance
Light overall powerplant weight	Heavier overall powerplant weight
Easy to control and cut off	Difficult to control and cut off accurately
Intermittent operation possible	Intermittent operation difficult
Propellants difficult, often dangerous, to handle	Fairly easy to handle and maintain, but sensitive to temperature changes
Elaborate plumbing system	No plumbing system
Complex system which breeds reduced reliability	Simple system with high reliability
Propellant sloshing must be controlled	Not susceptible to sloshing, therefore no weight needed for anti-sloshing device
Cannot be loaded and stored in ready-to-fire condition (except for storables)	Can be loaded and stored in ready-to-fire condition for long periods of time
High starting shock	Low starting shock
Susceptible to high vibration in flight	Low vibration in flight

REACTION MOTOR ADVANTAGES				
SOLID-FUEL ROCKET	LIQUID-FUEL ROCKET	PULSEJET	RAMJET	TURBOJET
Very simple	Relatively simple	Light weight	Very simple	Develops large static thrust
Unlimited speed	Practically unlimited speed	Economical and easy to construct	No wearing parts	Consumes fuel only; gets oxygen from air
Operates in any medium or in a vacuum	Operates in any medium or in a vacuum	Uses atmospheric oxygen, so needs no oxidizer	Light weight	Thrust practically independent of speed
No moving parts	Relatively few moving parts	Uses common fuels	Relatively inexpensive to construct and operate	Uses common fuels
Full thrust on take off	Develops full thrust on take off		Easy to build	
Requires no booster	Has less need for a booster than air-breathing engine		Uses common fuels	
Can be used in stages or clusters	Can be staged in combination with liquid or solid rockets		Efficient at high speeds and altitudes	
Can be stored fully fueled			Supersonic	
Ready to fire anytime				

REACTION MOTOR DISADVANTAGES				
SOLID-FUEL ROCKET	LIQUID-FUEL ROCKET	PULSEJET	RAMJET	TURBOJET
High rate of fuel consumption	High rate of fuel consumption	Speed subsonic; limited to about 450 m. p. h.	Must be boosted to high speed before it can operate	Limited to low supersonic speeds
Short burning time	Short burning time	Limited to atmospheric operation at relatively low altitude	Limited to operation in atmosphere	Complicated engine with many moving parts
Comparatively short range unless staged	Comparatively short range unless staged	High rate of fuel consumption	Speed presently limited to about 3,600 m. p. h.	Power limited by stresses on turbine blades
Fragile and sensitive to environmental conditions	Cannot be stored fully fueled for long periods of time			Limited to operation in Earth's atmosphere
	Long checkout procedure at launching			

REACTION MOTOR APPLICATIONS

Reaction motors can be used to propel missiles in several ways. A single reaction motor can serve as the sole mover of the missile, or multiple reaction motors can be employed in various combinations to produce the desired propulsive effects. The two common configurations for multiple-engine propulsion are the series arrangement, called staging, and the parallel arrangement, called clustering. Staging is used to increase velocity, while clustering furnishes higher thrust. A reaction motor (usually a rocket) is also used in series with a sustaining engine (such as the ramjet) to boost it to its operational starting velocity. Boosters are also used in parallel with a sustaining engine (such as the aircraft turbojet) to provide initial takeoff thrust. An example is the jet assist takeoff (JATO) unit.

series staging

A design aim for ballistic missiles is to achieve maximum burnout velocity, V_b (velocity after the consumption of fuel). In a rocket used for missile propulsion, the exhaust velocity, V_e, and the mass flow rate, m, remain constant during flight. Under these conditions, the missile will accelerate when the weight of the missile decreases and the force due to gravity decreases. The simplified equation for V_b in a vacuum, assuming complete exhaust expansion, is derived as follows:

Missile acceleration $a_x = \frac{F}{a_x} - g \sin \phi$

where $m = m_o - \dot{m}t$

and F equals the thrust

or $F = V_e\dot{m}$

$$V = \int_0^t a_x = \int_0^t \left[\frac{V_e\ \dot{m}}{m_o - \dot{m}t} - g \sin \phi\right] dt$$

$$V = V_e \left[\ln m_o - \ln (m_o - \dot{m}t)\right] - \int_0^t (g \sin \phi)\ dt$$

If ϕ is assumed constant:

$$V = V_e \ln \frac{m_o}{m_o - \dot{m}\ t_b} - (g \sin \phi)\ t$$

For the velocity at burnout (V_b):

$$V = V_e \ln m_o/m_b - (g \sin \phi)\ t_b$$

where t_b = time of total fuel consumption
m_b = mass at burnout

Since $V_e = Isp\ g$

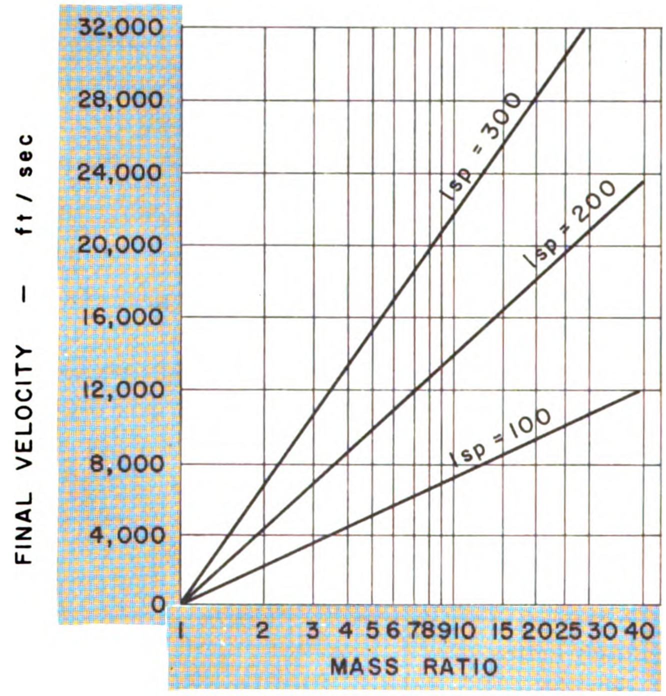
$$V_b = Isp\ g\ \ln MR - (g \sin \phi)\ t_b$$

where $MR = \text{mass ratio} = \frac{M_o}{M_b}$

If the mass ratio is made sufficiently large, the effect of the Earth's gravitational field((g Sin ϕ) becomes negligible and the equation for V_b becomes:

$$V_b = Isp\ g\ \ln MR$$

Ignoring the velocity loss due to gravity, the relationship among V_b, Isp, and mass ratio is shown.



final velocity vs mass ratio

The final weight, M_b, is made up by the payload, engine, and airframe. The major design effort for increasing V_b has been in decreasing M_b, particularly in the airframe. For singlestage missiles, the best achievements to date are Isp of 250 and MR of 9 to 10. This is a theoretical V_b of only 18,000 feet per second, which is insufficient for ICBM's. However, effective mass ratio can be increased by staging.

Staging is accomplished by consecutive firing of one or more rocket motors mounted on top of another rocket motor in series. The first rocket, or stage, commonly called a "booster", pushes all other stages to some speed, V_1. Then the first rocket is discarded, and the second rocket, or stage, pushes the remainder of the missile to some speed, V_2. It is then separated, and the third-stage rocket propels the remainder of the missile to the final required velocity. Four- and five-stage vehicles have been built, but most ballistic missiles require only two or three stages to reach the required speed. This is because the final velocity attainable by the last stage of a multistage missile is equal to the sum of the velocities developed by each of the stages; in this case:

$$V_f = V_1 + V_2 + V_3 = V_b$$

Using the basic rocket equation and neglecting the gravity terms for simplicity:

$$V_b = Isp\ g\ \ln MR_1 + Isp\ g\ \ln MR_2 + Isp\ g\ \ln MR_3$$

The general equation can be written as:

$$V_b = Isp\ g\ \ln MR_1 + Isp\ g\ \ln MR_2 - - - + Isp\ g\ MR_n$$

If the specific impulse of each stage is the same:

$$V_b = I_{sp}\ g\ (\ln MR_1 + \ln MR_2 - - - - - - + \ln MR_n)$$

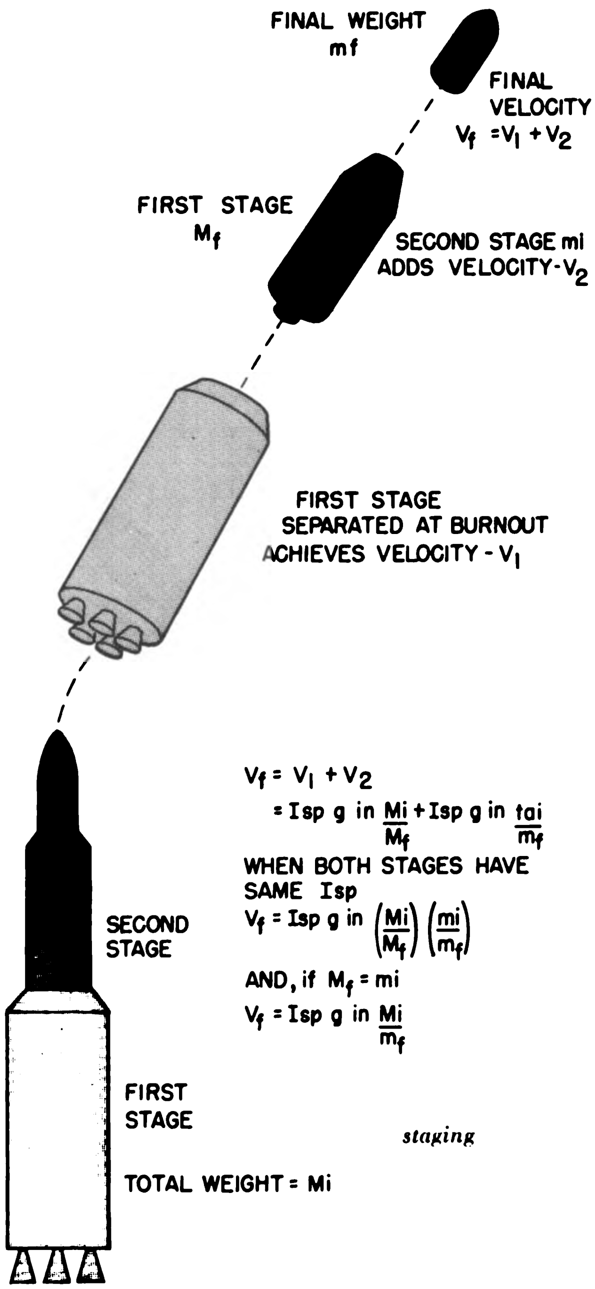
$$V_b = I_{sp}\ g\ \ln (MR_1)\ (MR_2) - - - - - - - - - (MR_n)$$

And if the mass ratios of all stages are equal:

$$V_b = I_{sp}\ g\ \ln MR^n$$

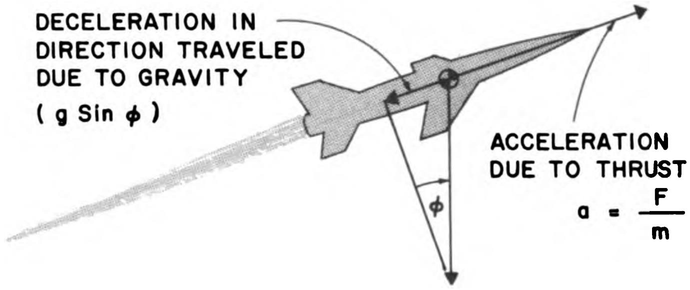
Then

$$V_b = I_{sp}\ g\ n\ \ln MR$$



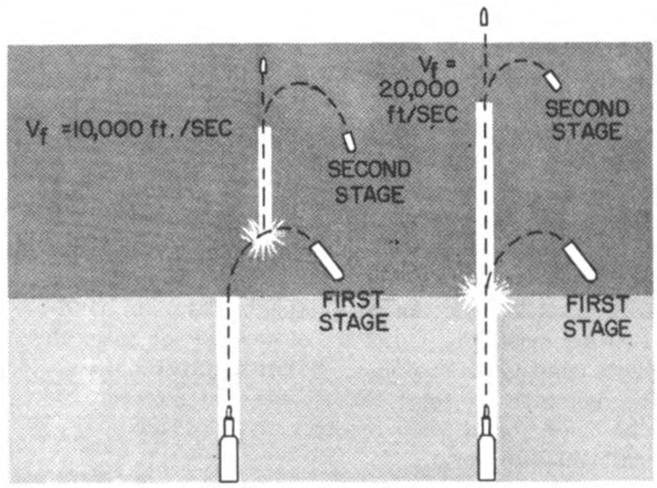
staging

The deceleration due to gravity is shown in the following illustration and can be seen to decrease as the angle ϕ decreases.



Staging is effective only because it increases the effective mass ratio by discarding the needless extra weight of the empty fuel containers. By means of staging, a much higher burnout velocity can be attained; at the same time, the initial weight and complexity of the total missile are increased considerably and its reliability is reduced. To obtain a final velocity of 20,000 feet per second, two stages of MR 3.5 and I_{sp} of 250 seconds each can provide a velocity of 10,000 feet per second. Since it is not difficult to build stages with a mass ratio of 3.5 and a specific impulse of 250 seconds, the required velocity can be easily achieved by staging.

To realize the maximum benefit from staging, the maximum velocity in increments from each stage must be obtained. This means that in the presence of gravity there will be a large velocity loss from coasting between stages. Velocity is continually lost due to the force of gravity. When a missile is coasting between stages, gravity is the prime operating force causing the missile to decelerate rapidly. Thus, neglecting certain practical considerations, the greatest final velocity will be attained with no interstage coasting.



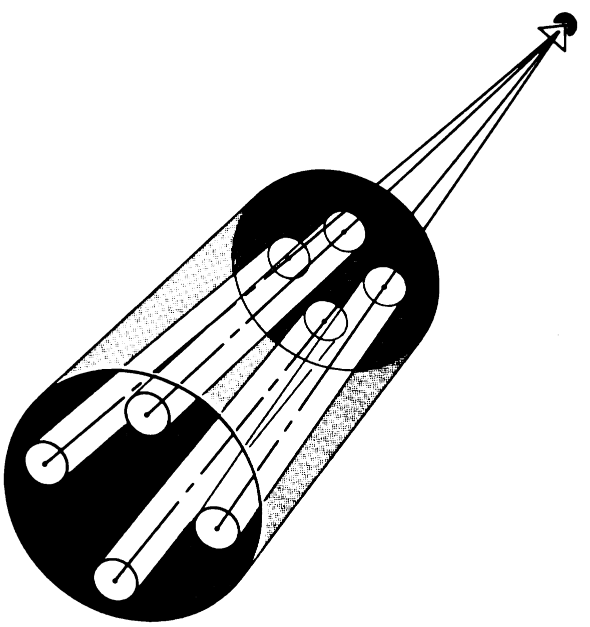
PEAK FIRING BURNOUT FIRING

Therefore, to obtain maximum final velocity, and thus maximum range, it is necessary to:

a. Maximize specific impulse
b. Maximize mass ratio by minimizing structural weight, and by using stages with equal mass ratio
c. Minimize interstage coasting time

parallel (clustering)

Rockets, arranged in parallel as shown, act as a single rocket, the thrust of which is the sum of the individual thrusts of the clustered rockets. When the available rocket engines lack sufficient thrust for the job required, a cluster of smaller rockets whose total thrust meets the job specification can be employed. Velocity losses due to gravity are reduced in clustered missiles as compared with staged or series missiles. All engines of a parallel cluster provide full thrust from launch to burnout, and do not waste energy in being boosted to high speed by preceding engines.



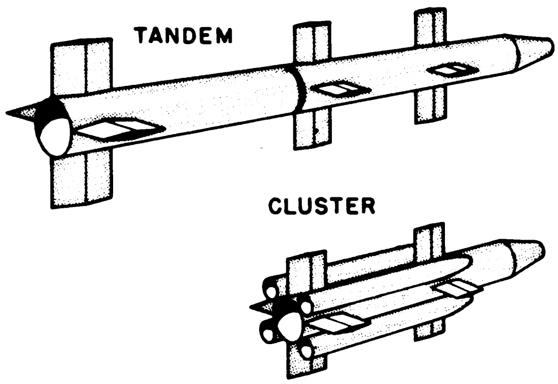
TOTAL THRUST = $T_1 + T_2 + T_3 + T_4$

Since Thrust = V_e m, the mass flow rate, m, is increased four times when there are four rockets in a cluster, and, since each rocket burns for t_b seconds, the total velocity loss due to gravity is g Sin ϕ t_b. If the four rockets were staged, each being fired successively, the total velocity loss due to gravity would be 4 g Sin ϕ t_b.

When solid-propellant rockets are used, clustering offers another advantage. Most solids tend to produce small errors in thrust vector alignment, which can be greatly reduced when using a cluster of solid-fuel engines, all pointing to a common center of gravity. Clustering reduces the overall length of the missile by some 10 to 15 percent, thereby reducing stowage volume requirements. This is of particular interest to shipboard missile installations. An interesting variation of solid-fuel rocket clustering is the use of several nozzles with a single combustion chamber. In this application there is no increase in total thrust, but the nozzles can be used for attitude control thus reducing overall missile length.

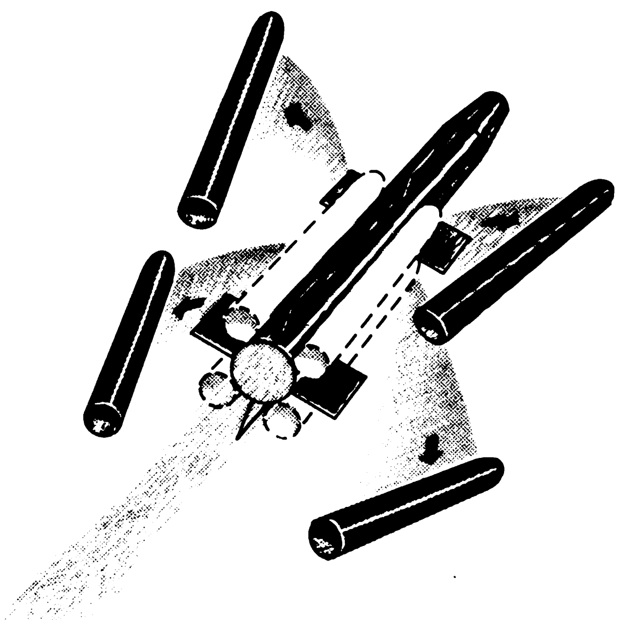
boosters

To bring the free-flight missile up to its flight speed, booster techniques are employed. The total weight required to transport a payload for a given distance can be minimized by the use of a booster, which is discarded when its purpose is accomplished.



In the case of ramjet propulsion, some sort of boost is essential to bring the missile up to the required flight speed to permit sufficient ram pressure for sustaining combustion. Boosters can be mounted in tandem as a single unit behind the missile, or clustered in a group around the body as illustrated. The tandem arrangement has the advantage of a single unit, which is an aid to stowage and quick assembly, but is considerably longer than the cluster type. Advantages of the cluster type are that they are compact and they present a minimum of aerodynamic stability problems. Placing boosters well forward on the missile airframe means that the missile fins may produce overall stability without the use of additional fins on the booster.

In the design of booster-missile configurations, arrangements must be made for effective, noninterfering separation after burnout of the booster.



Provisions must be made so that during the separation period (which may be from 0 to 0.2 second), the booster breaks clean, does not strike or reengage, does not interfere aerodynamically or with the next stage firing, and does not introduce unwanted momentum to the missile. Thus, a precise separation mechanism is required particularly for cluster separation.

GRAVITY-TYPE PROPULSION SYSTEMS

In order to determine the energy requirements of a gravity-type propulsion system, we must first develop an understanding of the projected missile's motion relative to the Earth. Since no additional forces are applied to the missile after initial launch, the only force affecting the motion of the missile is that of gravity. In a gravity-type propulsion system, propulsion is obtained solely from gravitational and inertial forces. The relationships among velocity, distance, and time are derived to provide a better understanding of gravity-type systems.

For the general case, assume that a gravity-type projectile traveling through a uniform medium has an initial velocity at angle θ at the instant of release. The velocity vector may be broken into components as follows:

v_{y_0}, the initial vertical component of velocity;
and v_{x_0}, the initial horizontal component.
K is also broken into similar components.

The coordinate system, having its origin at the release point, is positive downward and to the right.
Density and gravity are considered to be constant. According to Newton's Second Law, the summation of the forces acting on a projectile equal the product of mass times acceleration:

$$W - K = ma_y$$

where W is the weight of the bomb

K is the component of the force due to air resistance, assumed to be proportional to the velocity

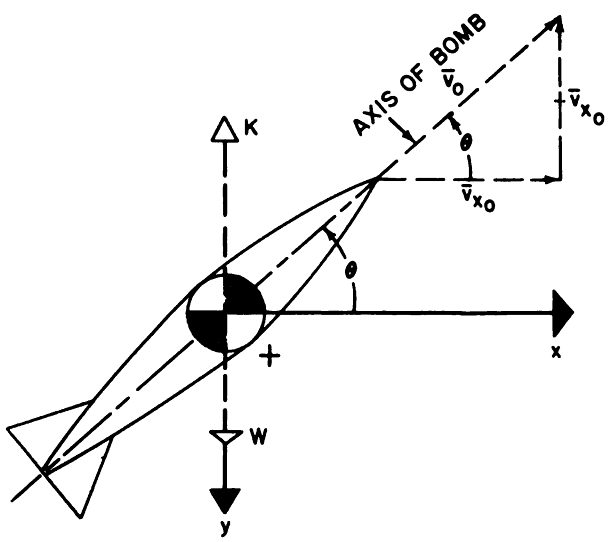
a_y is the vertical component of acceleration and equals dv_y/dt where v_y is the instantanoeus velocity

m is the mass and equals $\frac{W}{g}$

and $K_y = \alpha\, v_y$

where

α is the constant of proportionality or total drag coefficient and is dependent on the shape of the projectile and the viscosity of the medium through which it travels.



Further substitution leads to:

$$W - \alpha v_y = \frac{W}{g}\frac{dv_y}{dt}.$$

Integration leads to the velocity equation:

$$\int g\,dt = \int \frac{dv_y}{1 - \frac{\alpha}{W} v_y}$$

$$\ln C_1 + gt = -\frac{W}{\alpha}\left[\ln\left(1 - \frac{\alpha}{W} v_y\right)\right]$$

Solving for $\ln C_1$, using initial conditions $v_y = v_{y_0}$ at t=0, where $-v_{y_0}$ is the initial velocity in the vertical direction:

$$\ln C_1 = -\frac{W}{\alpha}\left[\ln\left(1 + \frac{\alpha}{W} v_{y_0}\right)\right]$$

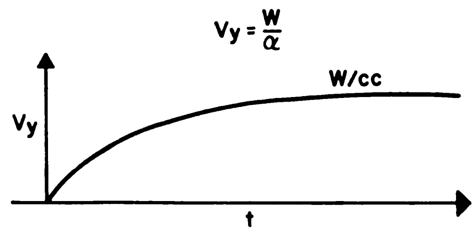
From this, the velocity equation becomes:

$$-\frac{W}{\alpha}\left[\ln\left(1 + \frac{\alpha}{W} v_{y_0}\right)\right] + gt = \frac{W}{\alpha}\left[\ln\left(1 - \frac{\alpha}{W} v_y\right)\right]$$

Transforming the equation into the form:

$$v_y = \frac{W}{\alpha}\left[1 - e^{\frac{-\alpha gt}{W}}\left(1 + \frac{\alpha}{W} v_{y_0}\right)\right]$$

and by plotting this equation, v_y can be seen to approach a constant value. Taking the limit as $t \to \infty$, the equation becomes: $v_y = \frac{W}{\alpha}$



Since $v_y = \frac{dy}{dt}$, where y is the distance traveled:

$$\frac{W}{\alpha}\left[1 - e^{\frac{-\alpha gt}{W}}\left(1 + \frac{\alpha}{W} v_{y_0}\right)\right] dt = dy$$

Upon integration of the velocity equation, the vertical distance equation is obtained:

$$C_2 + \frac{W}{\alpha} t + \left(\frac{W^2}{\alpha^2 g}\right) e^{\frac{-\alpha gt}{W}} + \left(v_{y_0}\right)\left(\frac{W}{\alpha g}\right) e^{\frac{-\alpha gt}{W}} = y$$

Knowing the initial conditions, y = 0 at t = 0, and, solving for C_2:

$$C_2 = -\frac{W}{\alpha g}\left(\frac{W}{\alpha} + v_{y_0}\right)$$

Substitution into the distance equation yields:

$$\frac{W}{\alpha g}\left[\left(gt - \frac{W}{\alpha} - v_{y_0}\right) + e^{\frac{-\alpha gt}{W}}\left(v_{y_0} + \frac{W}{\alpha}\right)\right] = y$$

The equations give instantaneous velocity and distance at any time. Also, if any one of the three parameters is known, the equations may be manipulated to find the other two parameters.

COMPARISON OF PROPULSION SYSTEMS

range

The impulse-propelled projectile has the shortest maximum attainable range among standard weapon propulsions (about 20 miles). The reaction-propelled ramjet can achieve a range of 5000 miles, whereas the range of rockets is practically unlimited when used in stages.

efficiency

The most efficient propulsion system is the impulse type. However, among the reaction systems, the rocket achieves the highest efficiency, with the liquid-fuel type surpassing the solid-fuel type. In all jet-propulsion units, efficiency increases with speed. It is interesting to note that about the highest overall efficiency is achieved by the turboprop engine at about 400 miles per hour; above that speed, however, its efficiency drops markedly.

environmental limitations

Impulse-type propulsion can be used in any medium, although its range is limited. Reaction propulsion in one form or another can be used in any medium. A rocket is the only form of reaction propulsion that can operate in a vacuum. Normally, changes in ambient conditions of temperature and pressure do not prevent the operation of a propulsion system, but proper operation can be affected. For instance, the burning rate of gunpowder and solid propellants for rockets varies directly with the temperature of the chemicals prior to firing. In addition, since jet propulsion thrust is a function of external pressure, changes in pressure due to barometric changes or variations in operating altitudes must be taken into account. Increased pressure decreases thrust, and is the reason why rockets operate more efficiently at higher altitudes. Air-breathing engines also operate more efficiently at higher altitudes, but the increase in thrust with altitude is offset by the decrease in available oxygen.

safety

The hazards of liquid fuels present a safety problem for personnel, particularly on shipboard. Most liquid fuels cannot be stored in the missile, but must be handled between storage tanks and missile. Solid fuels, although safer than liquids, present a shock hazard, and must be kept from extreme heat and handled carefully to prevent cracking. Safety precautions in submarines must be even more strigent. To protect personnel and equipment from the blast of jets, and rockets, special shields and precautions are usually required. Protection must also be provided at the rear of a recoilless gun.

reliability

In general, realiability is inversely related to complexity. It is a measure of the mean time or the quantity of units between faulty units. The most reliable of the reaction propulsion units is the solid-fuel rocket, which is free of moving parts. A solid-fuel rocket can be loaded and stored in ready-to-fire condition for long periods of time. Liquid-fuel rockets are more complex, and thus less reliable, although liquid-fuel rockets with pressure-feed systems are more reliable than those with pump feeds. The ramjet is also quite simple in design and is second in reliability only to the solid-fuel rocket. The most complex of standard jet-propulsion units is the turbojet, with its numerous engine parts. The reliability of a propulsion system in any medium is extremely important, especially for underwater use, since maintenance facilities in submarines are limited.

maintainability

As a general rule, maintainability is inversely related to complexity. However, even a reliable unit such as the solid-fuel rocket presents maintenance problems when used in clusters and stages. The mechanism for the separation of boosters presents maintenance and reliability problems. Certain environmental conditions, such as the corrosive atmosphere of salt air on shipboard, cause maintenance problems and increase preventive maintenance chores. Also, the location and ease of access to a given propulsion unit will affect its maintainability.

A measure of maintainabliity is the inverse of the mean "down" time required for maintenance and repair. A useful measure is:

$$D = RM$$

where

D = Dependability

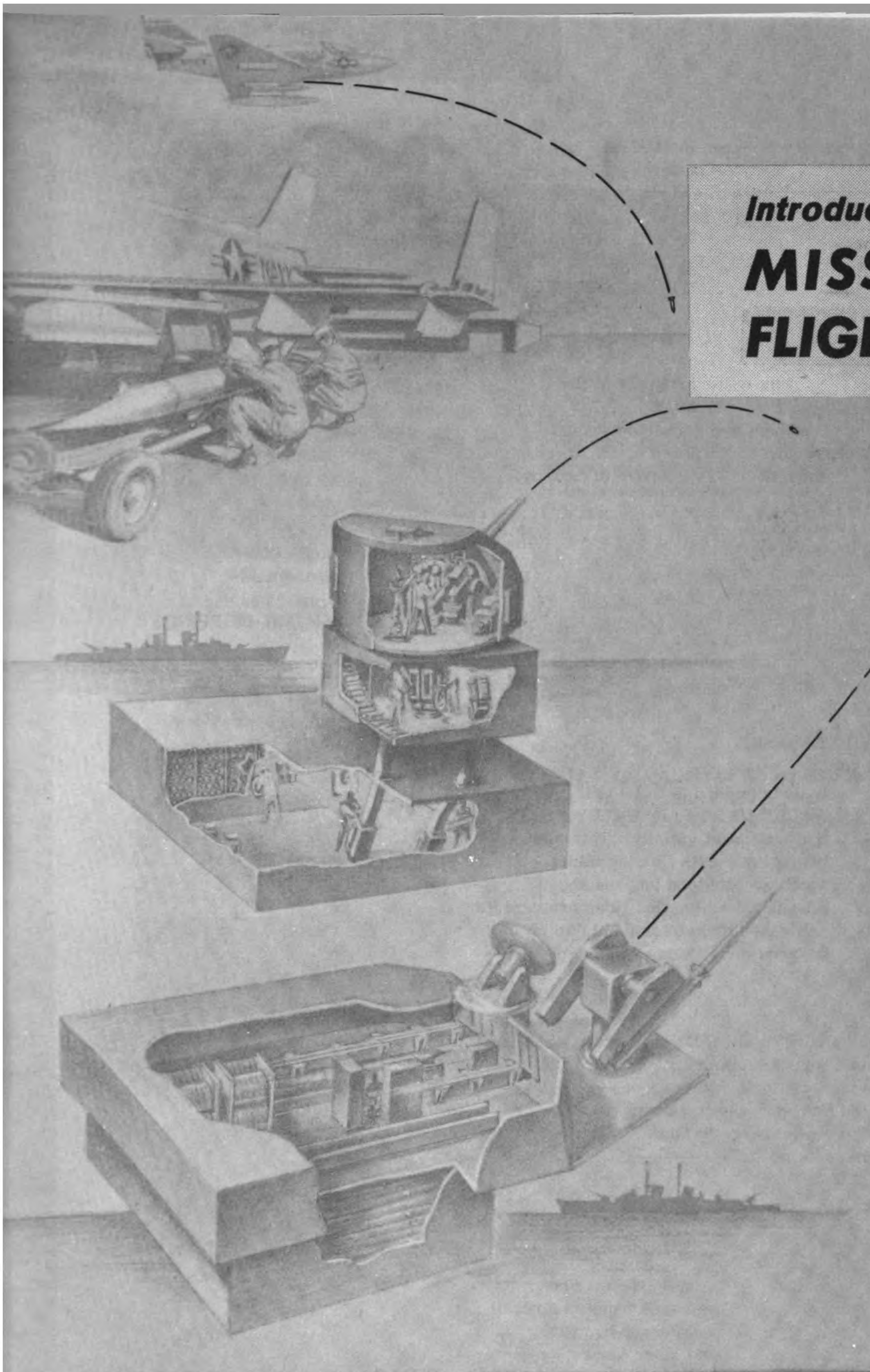
R = Reliability = mean time between failures and maintenance periods

M = Maintainability = mean down time

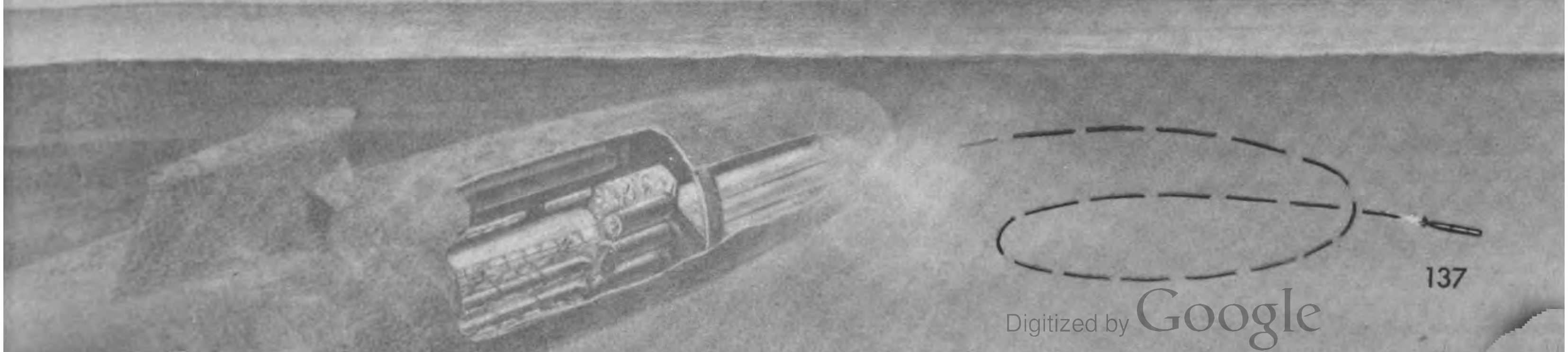
CHAPTER 4

Introduction to

MISSILE FLIGHT PATHS

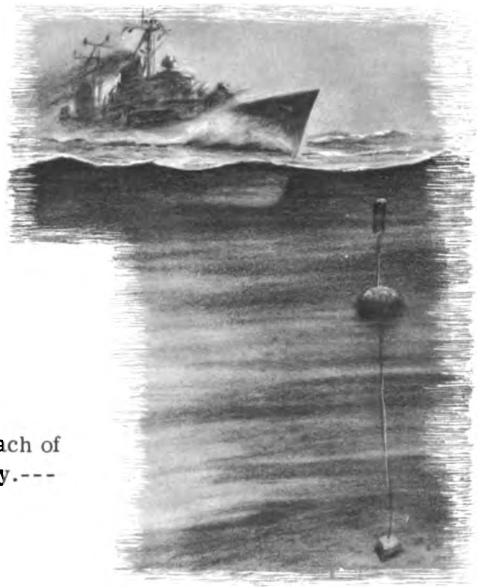


In addition to the material elements of a weapons system, warhead launcher, etc. attention must be given to the flight path the missile follows to the target since this path has a pronounced affect on the design of the material components. The medium or media through which the missile will pass greatly effects the choice of propulsion system and vehicle. The acceleration and velocity requirements of the flight path influence the selection of propulsion system, vehicle and weapon control systems and the type of flight path used is the most important consideration in the weapon control system. This chapter discusses the essential characteristics of the flight paths most frequently encountered.



one method of
bringing
a warhead and target
together
is to
LET THE TARGET
COME TO THE WARHEAD

this is the principle of the
MINE
which is a concealed warhead
whose fuze is triggered on the approach of
the object it is intended to destroy.---



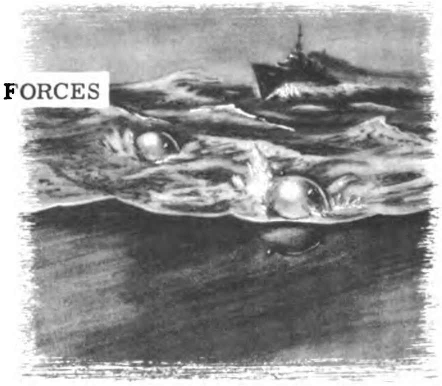
USE AGAINST LAND FORCES ----------

----------USE AGAINST SHIPPING

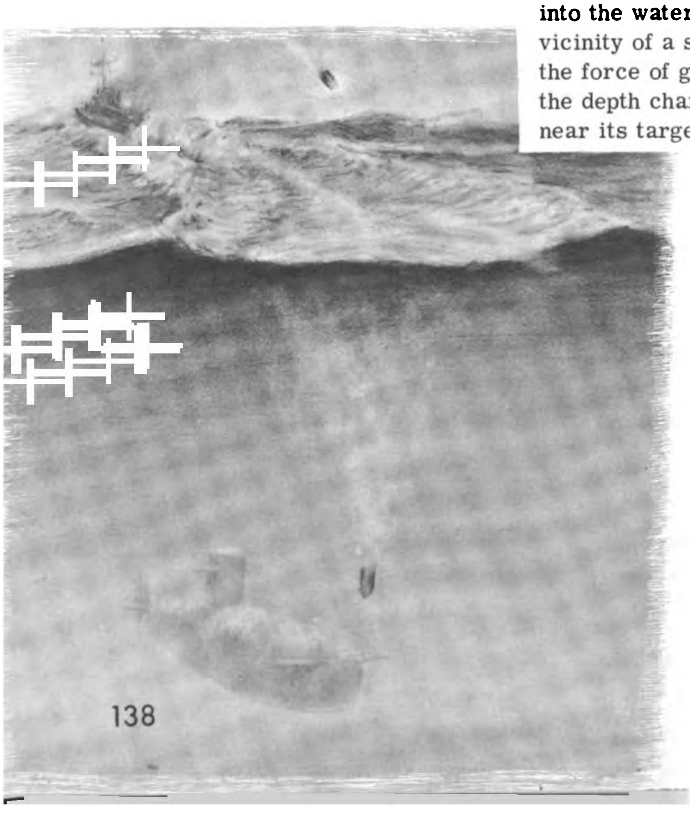
Various means may be adopted to
SEND A WARHEAD TO A TARGET.

One method is to have the
WARHEAD CARRIED TO TARGET BY PERSONNEL
A frogman, for instance, may carry a bomb
with a time fuze and attach it
to the side of an enemy ship. ----------

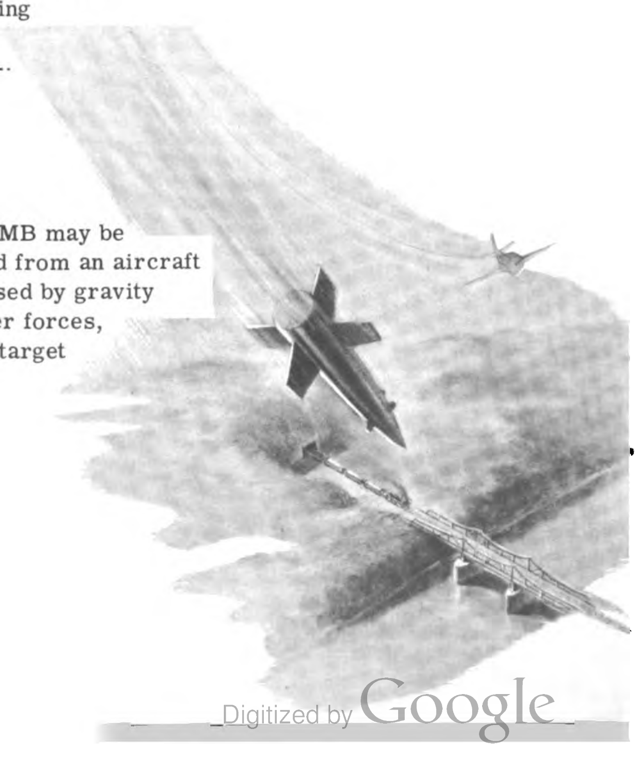
another method is to let the
WARHEAD BE CARRIED BY NATURAL FORCES
into the target vicinity. This method
is employed with floating mines,
which are dropped into the sea
in such a location that tides or currents
will cause them to drift into an area
frequented by enemy shipping ---------



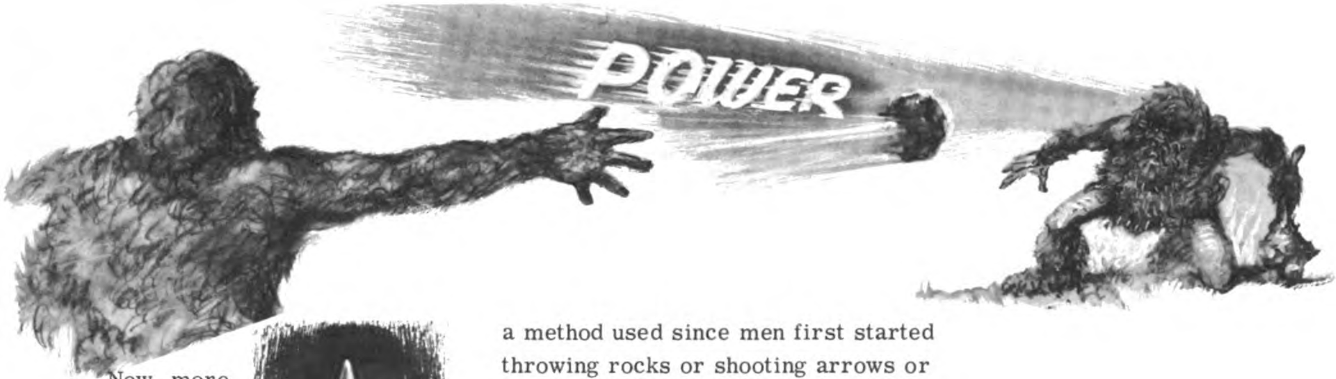
or a DEPTH CHARGE may be dropped
into the water in the
vicinity of a submarine ----------
the force of gravity causing
the depth charge to sink
near its target ---------



or a BOMB may be
released from an aircraft
and caused by gravity
and other forces,
to hit a target



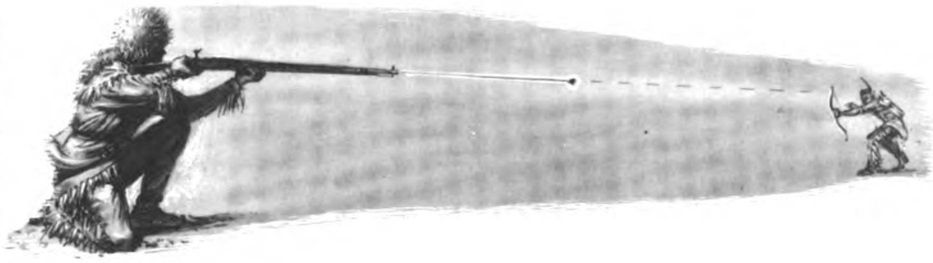
a common way
to bring a warhead and target together.
is to HURL OR LAUNCH
A MISSILE AT A TARGET BY-



a method used since men first started throwing rocks or shooting arrows or bullets at one another --------------

Now, more sophisticated, ballistic or guided missiles, which travel farther and faster, and have tremendous destructive power are used.

The warhead or missile follows a path determined by the forces acting upon it. If these forces have been correctly predicted, the warhead or missile will hit the target.



if the time of flight of the missile is very short, it may be thought of as A STRAIGHT LINE, which it closely approximates.

but if the time of flight is fairly long, the path is normally CURVED.

The characteristics of Flight Paths is the main subject of this section.

This introductory section begins by discussing the forces ---- nature and man-made ---- that act on missiles, and the manner in which these forces govern missile flight paths. This is followed by treatments of ballistic flight paths and guided flight paths. In the sections of this chapter which follow, flight paths will receive fuller treatment and some mathematical analysis.

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GENERAL

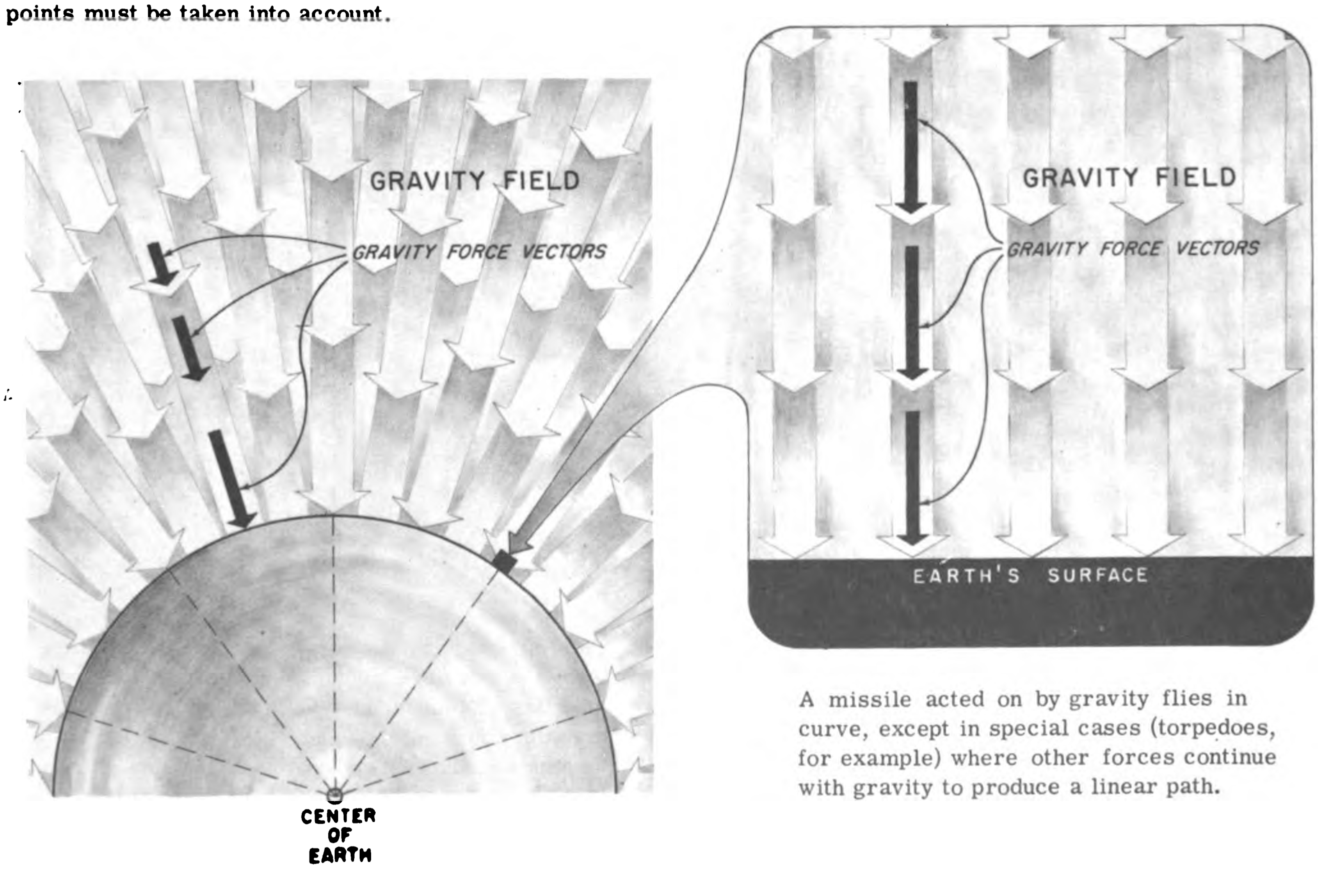
The flight path of a missile is determined by the forces acting on it. Some of these forces are due to nature; others are man-made. Nature forces include gravity, and those forces that are generated by the medium through which a missile is traveling (aerodynamic forces, for instance.) Nature forces are not fully controllable. Of course, their action on a missile can be modified by causing the missile to fly slower, reducing the aerodynamic forces acting on it; by making it fly higher, reducing the aerodynamic and the gravitational forces acting on it; and so on. But, if a missile at a given locality is moving with a given velocity vector, with its envelope at a given orientation, the nature forces acting on it are dictated by the ambient conditions. Thus, the nature forces, though not fully controllable, are to a considerable extent predictable. Man-made forces include propulsion through a gun barrel, jet thrust, and rocket thrust; also steering forces. Man-made forces may or may not require the presence of a medium (such as air, for rudders to act upon.) These forces can be fully controlled. At any given moment, a missile is acted on by either: (a) man-made and nature forces combined (gravity, at least, is always present). or (b) nature forces alone.

Different combinations of these forces may act on the missile during different "phases" of the flight. For example: a bullet in a gun barrel, or a ballistic missile before rocket burn-out, are acted on by man-made and nature forces. The bullet after leaving the barrel, and the ballistic missile after rocket burnout, are acted on by nature forces alone.

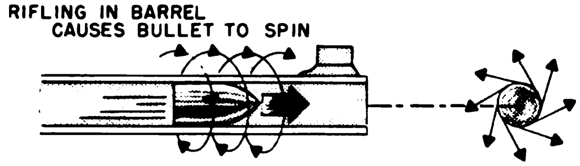
NATURE FORCES . . *GRAVITY*

Gravity acts on missiles that are launched from or near Earth. This force acts towards the center of Earth, and diminishes as the missile recedes from Earth, according to the law of inverse squares; that is, for points outside Earth. For long-range high-altitude missile flight, variations in the magnitude and direction of gravity at different points must be taken into account.

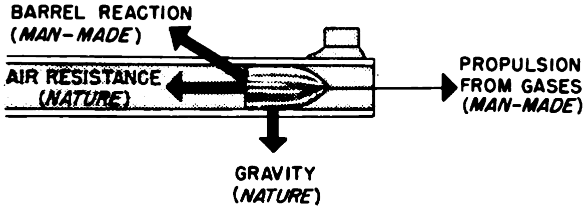
For missile flights of short range and low altitude we may pretend that the segment of Earth over which the missile travels is flat, and that gravitational force is vertical and has the same value throughout its flight.



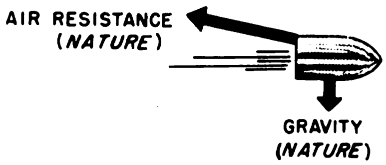
A missile acted on by gravity flies in curve, except in special cases (torpedoes, for example) where other forces continue with gravity to produce a linear path.



a. BULLET IN BARREL

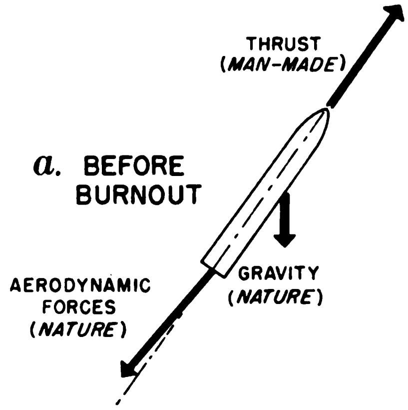


b. BULLET OUT OF BARREL

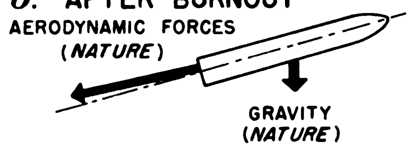


Man-made forces may exert — VELOCITY VECTOR CONTROL. For example, an aircraft can be made to have a given velocity in a given direction at any instant. This is also true of a ballistic missile before rocket burnout when it is receiving rocket thrust with gyro control of the direction of motion. Both these examples are receiving velocity vector control; the missile paths are being shaped as desired, at each instant. On the other hand, a rocket under pure thrust, uncontrolled in magnitude and with no direction control, is not considered as receiving velocity vector control.

a. BEFORE BURNOUT

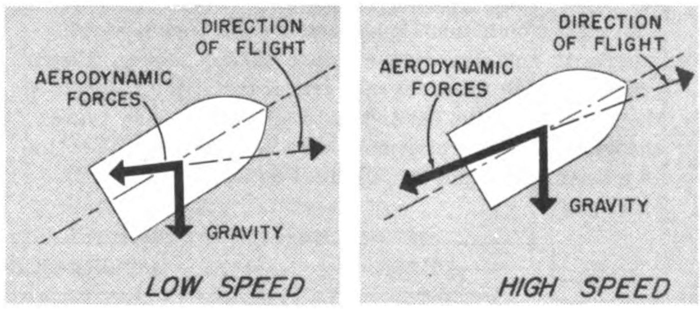


b. AFTER BURNOUT

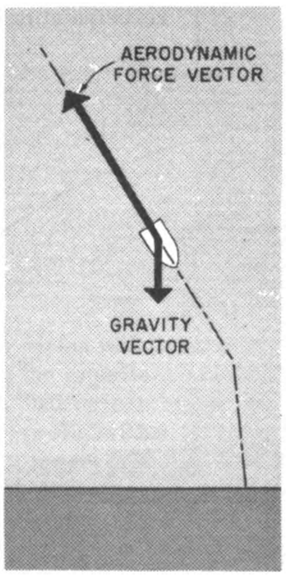


NATURE FORCES . . *DUE TO MEDIUM*

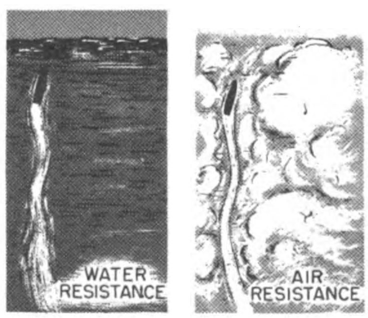
When a missile is traveling through a medium such as air or water, the medium exerts forces on the missile which affect its flight path. These forces increase with the density of the medium and the missile speed. The forces are also influenced by the size, shape, surface texture, and orientation of the missile (a streamlined missile encounters less resistance from the medium than a blunt missle, other conditions being equal.) When a missile increases its speed, the aerodynamic forces on it increase.



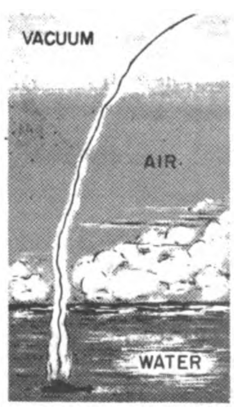
Consider, now, some effects of ballistic flight. A long-range ballistic missile at the crest of its flight is subject to almost no aerodynamic forces, because the density of Earth's atmosphere above 200 miles is negligible. At the other extreme, when the missile is re-entering Earth's atmosphere, its enormous speed is combined with increasing medium density. Therefore, aerodynamic forces on the missile are very great, causing high deceleration and consequent changes in the shape of its flight path.



Consider, also, an underwater missile whose path may be completely altered in character from what it would be if the water were not present. Air resistance modifies or distorts a missile path, but less so than does water resistance.



Some missiles pass through several different media in succession during a flight (water, air, vacuum.) Their paths may then become very complex.



MAN-MADE FORCES

The man-made forces that may act on a missile during flight include thrust, and directional controls.

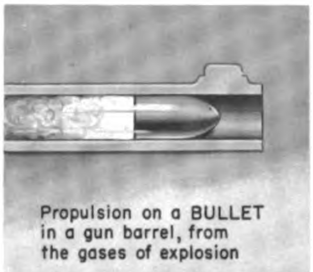
Thrust can take various forms. Some examples are given below.

Thrust can have various duration times. Some examples of thrust time are given below.

(1) Bullets: of the order of milliseconds (as compared to total flight time of several seconds).

(2) Ballistic missiles: 100 to 200 seconds (as compared to total flight time of 20 to 30 minutes).

(3) Guided missiles and torpedoes: equal to total flight time.



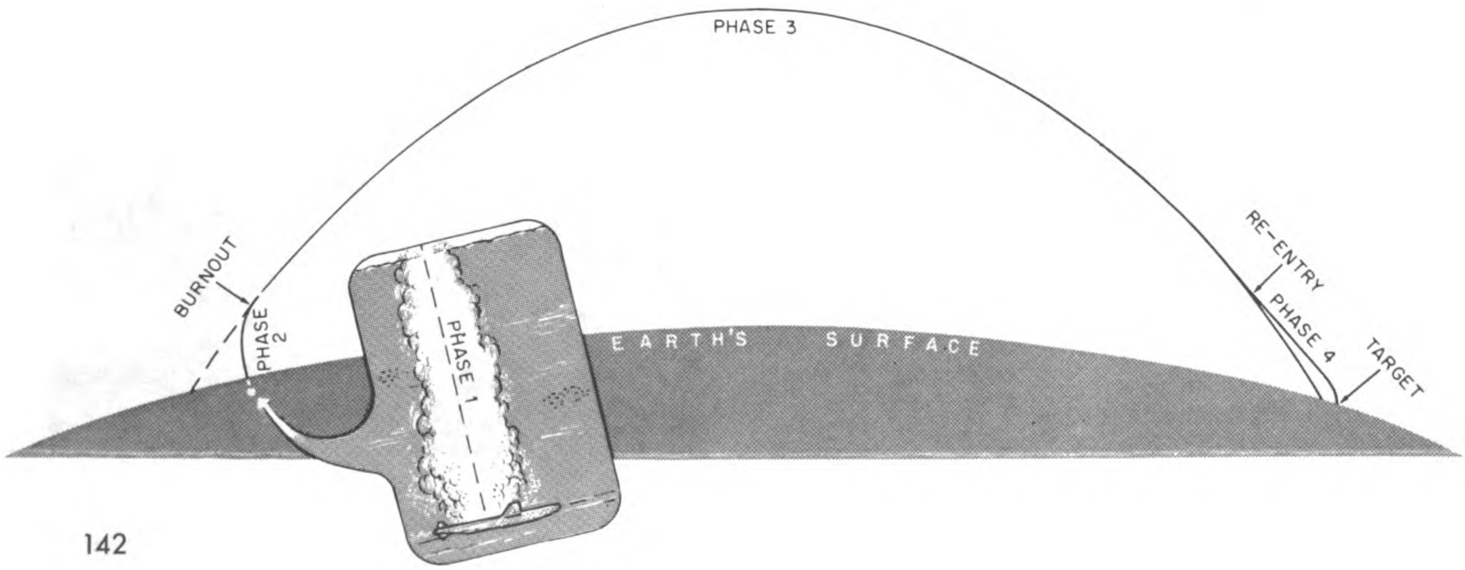
Thrust may or may not require the presence of a medium, such as air or water. Propellers and jets require a medium in which to operate; guns and rockets do not.

FLIGHT PATH PHASES

A missile in flight does not necessarily experience the same type of forces, nor travel in the same medium, all the time. During some portions of its flight, it may experience both man-made and nature forces; during others, it may experience nature forces alone. It may travel through water; it may travel through air, in which the density varies according to altitude; it may travel through rarified atmosphere or a vacuum. Each portion of a missile's flight is called a PHASE.

A guided missile generally experiences nature and man-made forces, with velocity vector control, throughout its flight. A ballistic missile or bullet experiences man-made forces with velocity vector control for a brief period at the beginning of its flight; after that, only nature forces are present. For instance, a long-range ballistic missile may be fired at a land target from a point under the sea. The phases of missile flight may then be:

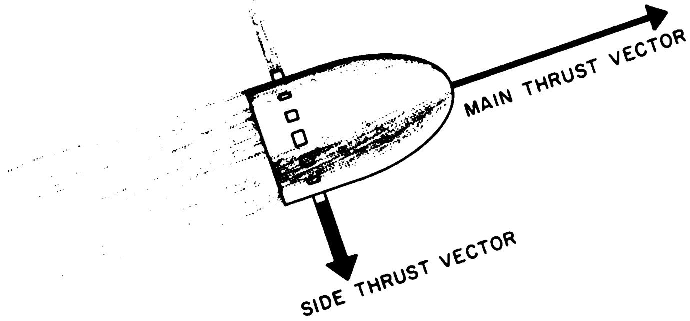
PHASE	NATURE FORCES	MAN-MADE FORCES	MEDIUM
1. Launch to surface	Gravitational, Hydrodynamic	Controlled rocket thrust	Water
2. Surface to burnout	Gravitational, Aerodynamic	Controlled rocket thrust	Air
3. Burnout to re-entry (ballistic)	Gravity	Controlled jet thrust for re-orientation	Partial Vacuum
4. Re-entry to impact (ballistic)	Gravitational, Aerodynamic	None	Air



A missile experiencing thrust may have its velocity vector controlled, even if the thrust is in itself invariant in magnitude or direction, by the conjunction of thrust with auxiliary devices so as to control the speed and direction of missile flight. Alternatively, the thrust itself may be controllable in magnitude and direction. We shall consider both of these types of systems as directional controls.

The process of directional control of a missile may be roughly broken down as follows:
A target is tracked, and its future position predicted by station outside the missile (at the launching site, for instance) or by the missile itself. Target data is converted to directional orders, either inside or outside the missile. In the case of bullets fired from guns, the directional orders position the gun barrel; the directional control forces are exerted on the bullet by the barrel.

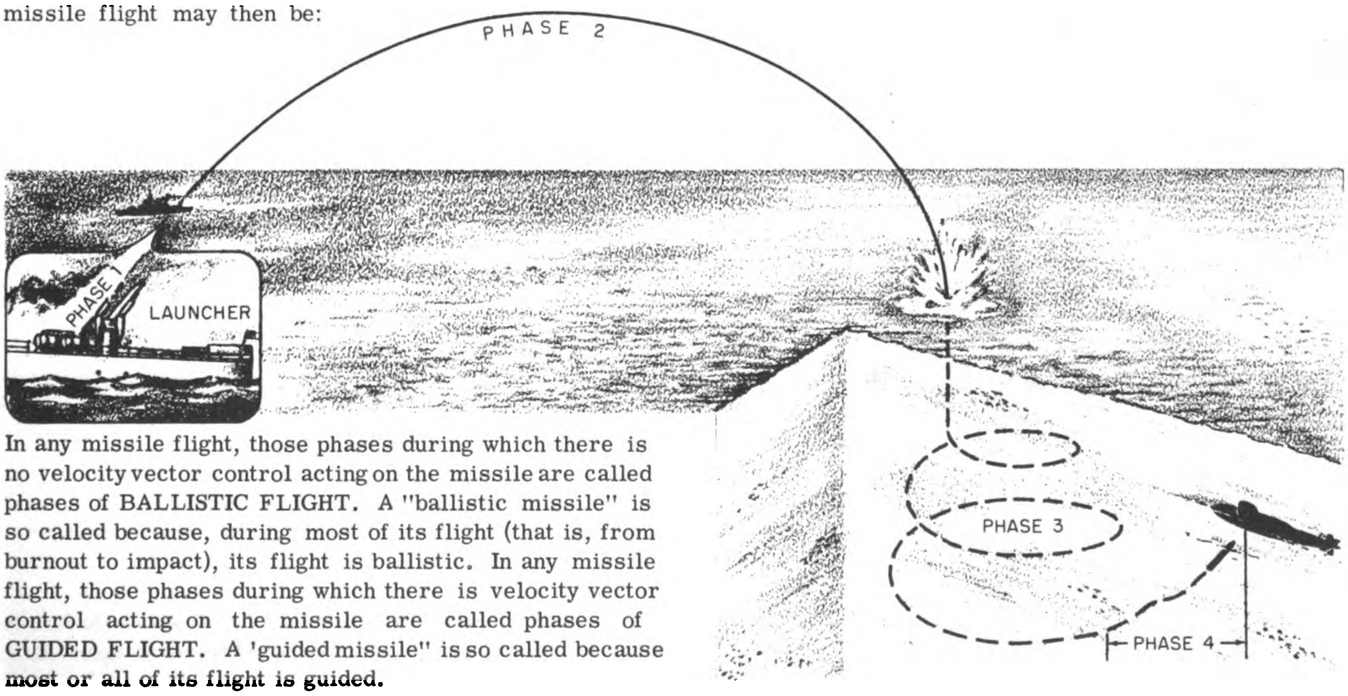
In the case of guided missiles, or ballistic missiles before rocket burnout, the directional orders are mechanized in the missile so as to produce directional control forces. This may be achieved by fins or rudders, by controlling the direction or magnitude of the main thrust, or by auxiliary side thrusts (rockets).



In the last example, there is a slight overlap between thrust and directional control; here directional control is achieved by means of thrust itself.

Or, to take another instance, a missile (internally mechanized — not a projectile) may be fired from a launcher on board a ship. It may fly ballistically a short range upward through the air to its maximum altitude, and descent in a curved path to the surface of the water in the known neighborhood of a submarine. It may then drop below the surface and "search" for the submarine until it picks it up, and finally "home" on the submarine. The phases of missile flight may then be:

PHASE	NATURE FORCES	MAN-MADE FORCES	MEDIUM
1. Passage through launcher	Gravitational, Aerodynamic	Impulse or reaction propulsion	Air
2. Launcher to water surface (ballistic)	Gravitational, Aerodynamic	None	Air
3. Water surface to target pick-up	Gravitational, Hydrodynamic	Controlled hydrodynamic	Water
4. Target pick-up to impact	Gravitational, Hydrodynamic	Controlled hydrodynamic	Water



In any missile flight, those phases during which there is no velocity vector control acting on the missile are called phases of BALLISTIC FLIGHT. A "ballistic missile" is so called because, during most of its flight (that is, from burnout to impact), its flight is ballistic. In any missile flight, those phases during which there is velocity vector control acting on the missile are called phases of GUIDED FLIGHT. A 'guided missile" is so called because most or all of its flight is guided.

BALLISTIC FLIGHT PATHS . .

A ballistic flight path is one during which a missile experiences only nature forces or those man-made forces that do not provide velocity vector control, such as non-controllable thrust. Usually a ballistic missile's flight path is ballistic except before rocket burnout, when it experiences rocket thrust that is controlled in magnitude and direction. Phases of ballistic flight under non-controllable thrust are also common. During ballistic flight a missile may be acted on by different combinations of the following forces: gravity, medium, and non-controllable thrust. Gravity is the only force that is always present.

ballistic missile in vacuum

When a ballistic missile files in a vacuum, the only force acting on it (after rocket burnout) is gravity. The resulting path is a conic section, as has long been established by Newtonian mechanics.

Long-range flight: The missile flight path is a close approximation to an ellipse. The slight deviation from a perfect ellipse is due to the following:

(a) Any vacuum in Earth's vicinity (even several hundred miles above Earth) is only partial.
(b) Earth's gravity field is not perfectly regular.
(c) Moon and Sun also exert slight gravitational forces on the missile.
(d) Newton's equations are subject to slight relativistic corrections.

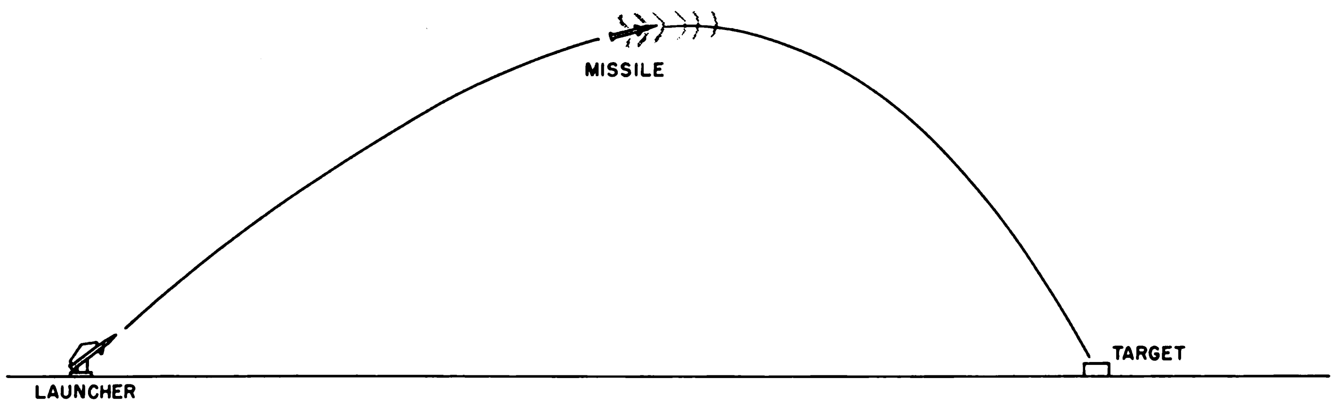
Short-range flight: The missile flight path is an ellipse of very high eccentricity, and is practically indistinguishable from a parabola. (As we are stipulating the absence of air, and this is not realizable on Earth at low altitudes, corrections must be added.

In computing short-range ballistic paths, it is customary to assume, for simplicity, that the terrain is flat, and that the gravity field is vertical and of constant magnitude. (Over short ranges and at low altitudes, these assumptions are nearly correct.) The flight path of a missile computed on these assumptions is a parabola.

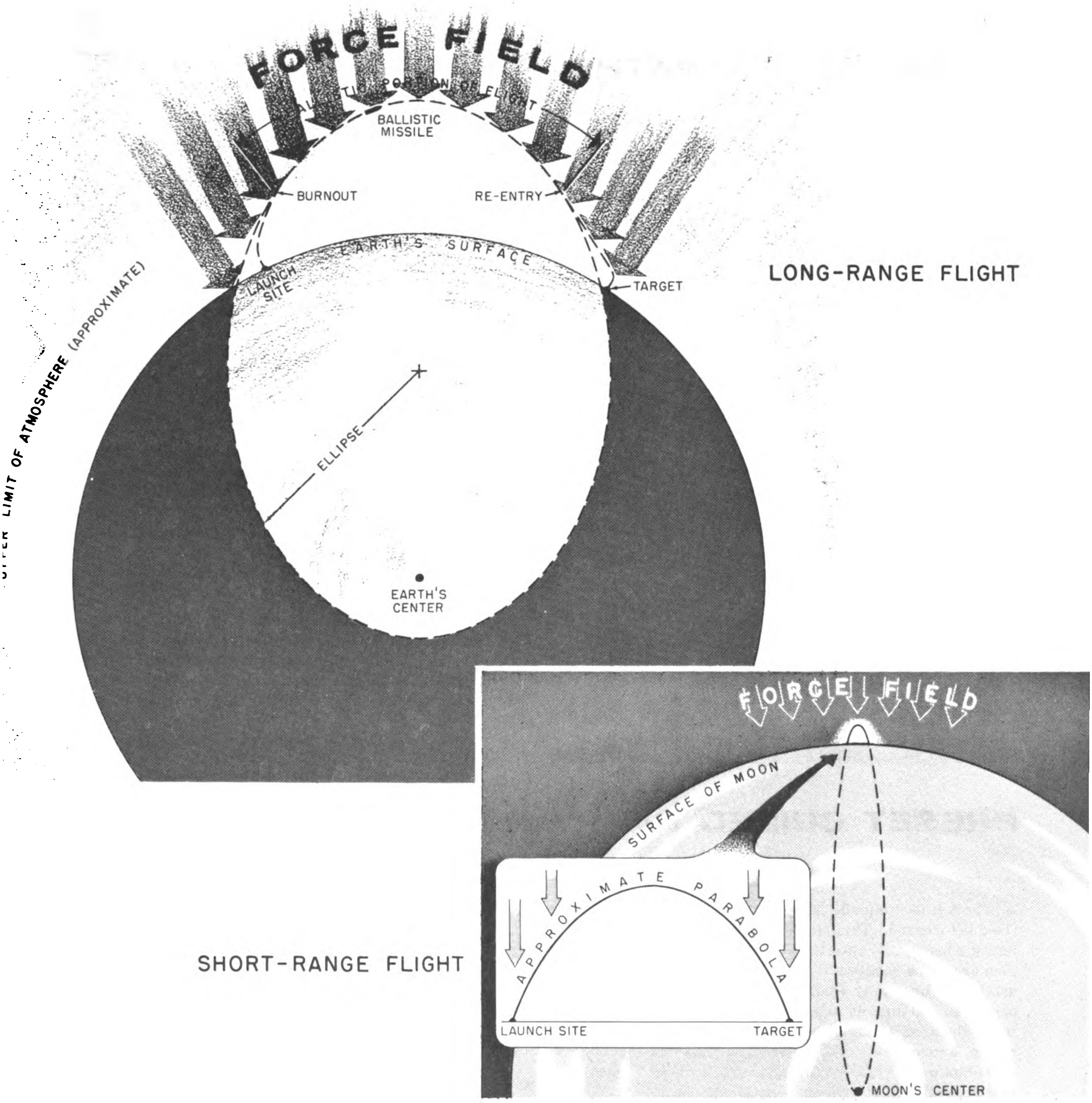
ballistic missile in a medium

A ballistic missile under gravitational and aerodynamic forces follows a path that is a distorted conic section. A short-range path is a distorted parabola; the last phase of a long-range path (re-entry) is a distorted ellipse.

In both cases the distortion is caused by the acceleration due to friction caused by movement of the missile through Earth's atmosphere.



A ballistic missile under water experiences hydrodynamic forces due to water friction which have a predominant effect upon its path; this rarely resembles an ellipse or a hyperbola. A depth charge dropped almost vertically soon falls vertically through the water. Other underwater missiles are usually guided, not ballistic, to counteract the effect of water friction on the missile path.



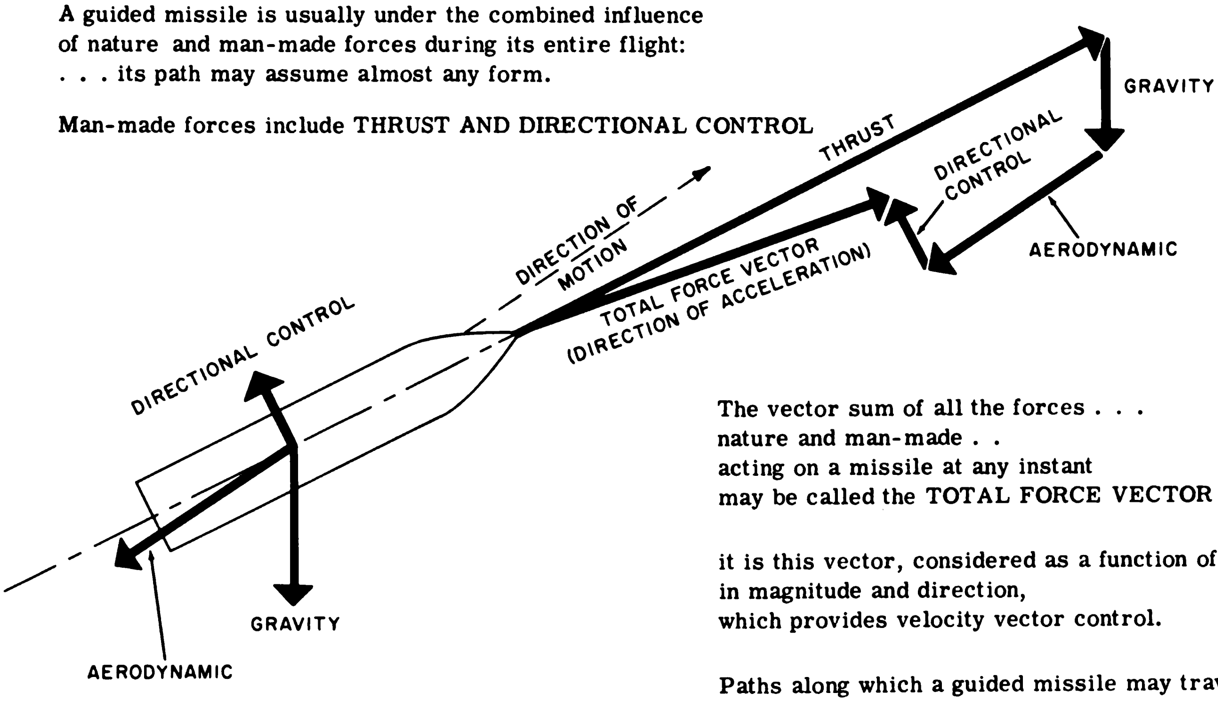
a missile under non-controllable thrust and gravity

A missile under non-controllable thrust and gravity follows a path that is neither parabolic nor elliptical. Rocket thrust introduces a logarithmic function into the path equation. This is due to the constantly diminishing weight resulting from the consumption of fuel during flight.

GUIDED FLIGHT PATHS . . . *GENERAL*

A guided missile is usually under the combined influence of nature and man-made forces during its entire flight: . . . its path may assume almost any form.

Man-made forces include THRUST AND DIRECTIONAL CONTROL



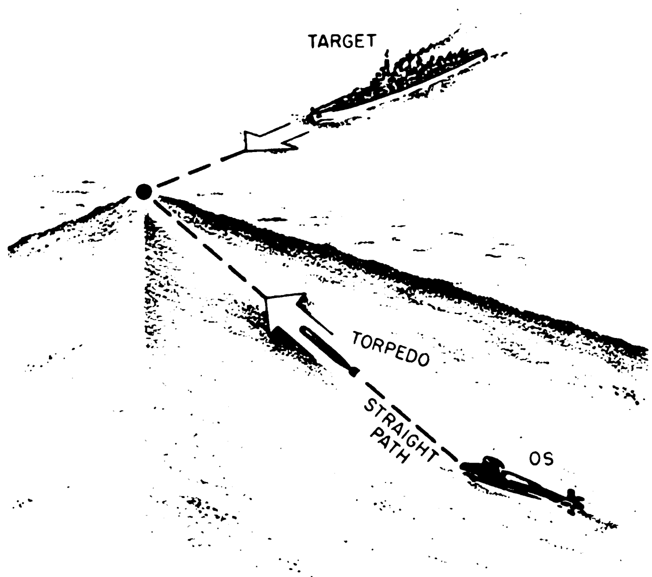
The vector sum of all the forces . . .
nature and man-made . .
acting on a missile at any instant
may be called the TOTAL FORCE VECTOR

it is this vector, considered as a function of time,
in magnitude and direction,
which provides velocity vector control.

Paths along which a guided missile may travel may be broadly classified as either preset or variable. The plan of a preset path cannot be changed in mid-flight; the plan of a variable path is altered according to conditions occuring during flight.

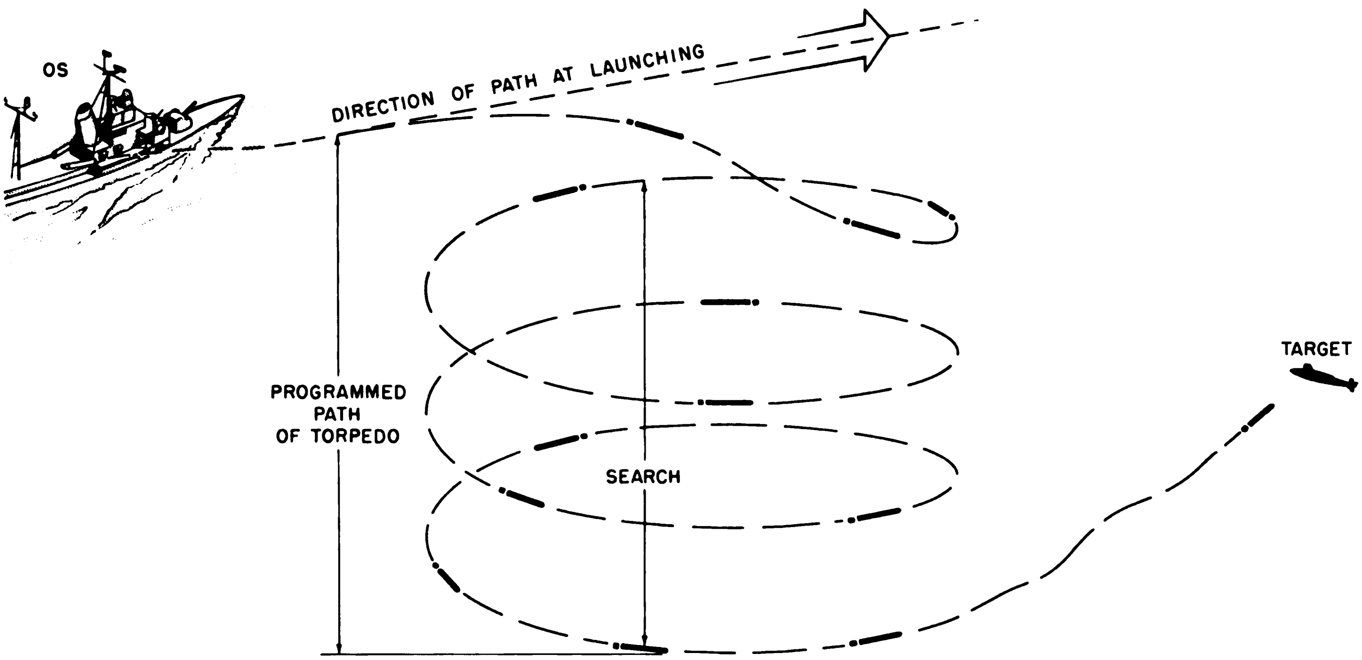
PRESET GUIDED PATHS . . . *CONSTANT*

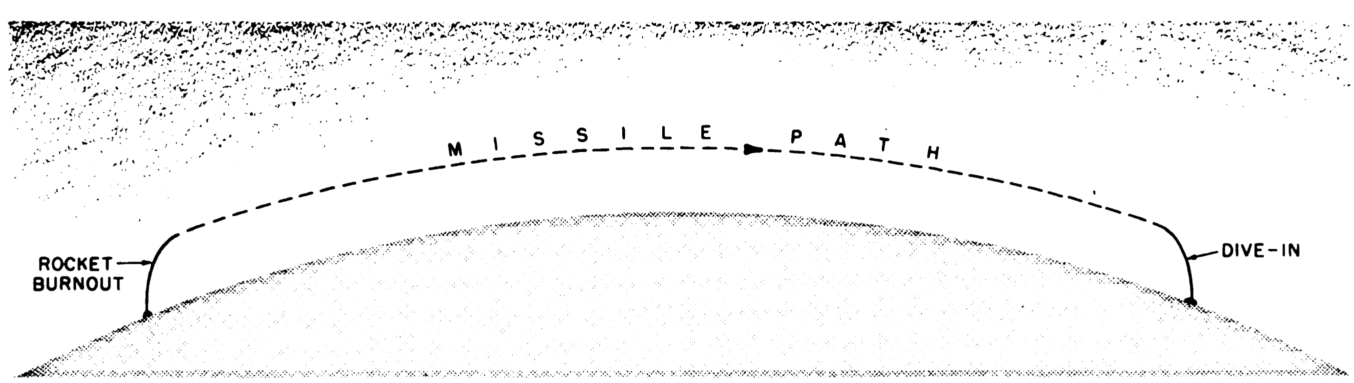
A preset guided missile path has a plan that has been fixed beforehand. This plan may include several different phases, but, once the missile is launched, the plan cannot be changed. The phases must follow one another as originally planned. The simplest type of preset guided missile path is the CONSTANT preset. Here, the missile flight has only one phase: for example, a torpedo executing a straight run. The term "constant preset" may be broadened to include flights that are constant after a brief launching phase that is different in character from the rest of the flight. During the main phase of a constant preset guided missile flight, the missile receives no control except that which has already been built into it. However, it receives this control throughout the guided phase of flight. Often it is powered all the way. The nature of a constant preset guided missile flight path depends on how it is powered, and the medium through which it travels. A torpedo, for instance, may describe a straight line.



PRESET GUIDED PATHS . . . *PROGRAMMED*

A missile could be guided in a preset path against a fixed target; the joint effect of missile power and gravity would then cause the path to be a curve. A missile following a preset path may be guided in various ways. Examples: by an autopilot, by loran (against a stationary target), or by inertial navigation. The means of propulsion may be motor, jet, or rocket. A more complex type of preset guided missile path is the PROGRAMMED preset. Here, the missile flight has several phases. For example: a torpedo executing an enabling run, with gyro angle, followed by a search pattern. During the first phase, the torpedo,having been launched in some initial direction other than the desired ultimate direction, gradually finds the desired direction by control mechanisms such as gyros and depth settings. The torpedo then maintains this direction for the remainder of this first phase, at the end of which it is pressumed to be in the neighborhood of a target. During the second phase, the torpedo executes a search pattern, possibly a circular path, or a helical path, etc.





Another type of programmed preset path may be followed by a long-range cruising missile. This may have a range as long as that of an intercontinental ballistic missile but, unlike the latter, is powered and guided throughout its entire journey to its target. Such a missile might proceed as follows:

(1) Receive guided rocket boost vertically upward.
(2) Proceed to a still greater height by ramjet power, its path inclining over to the horizontal.
(3) Proceed for several hundred or thousand miles horizontally, by ramjet.
(4) Dive in on the target, by ramjet. Guidance is programmed.

VARIABLE

GENERAL

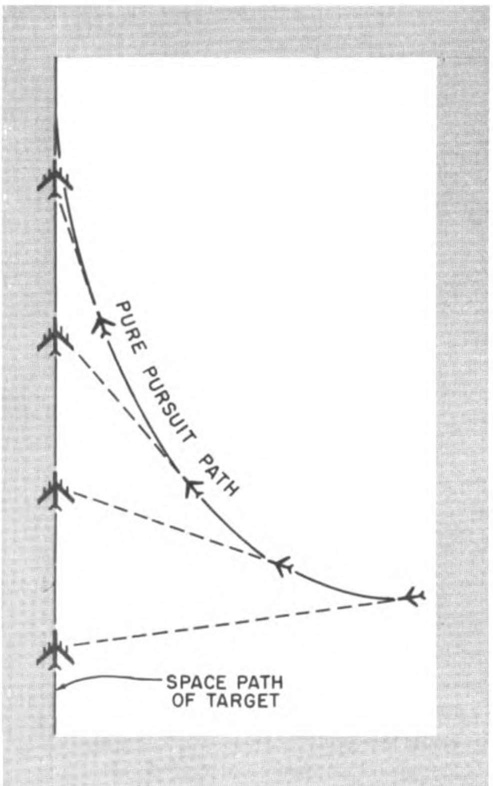
A variable guided missile path is one that can be changed during flight, either by new orders from a ground station, or in response to new target data received by the missile. Orders from a ground station may be communicated by a data link. When a missile path is determined by target data received directly from the target by the missile, the missile is said to be "homing". By convention, homing is divided into three categories:

(1) Active. The missile transmits energy to the target and guides itself by the echo signal received.

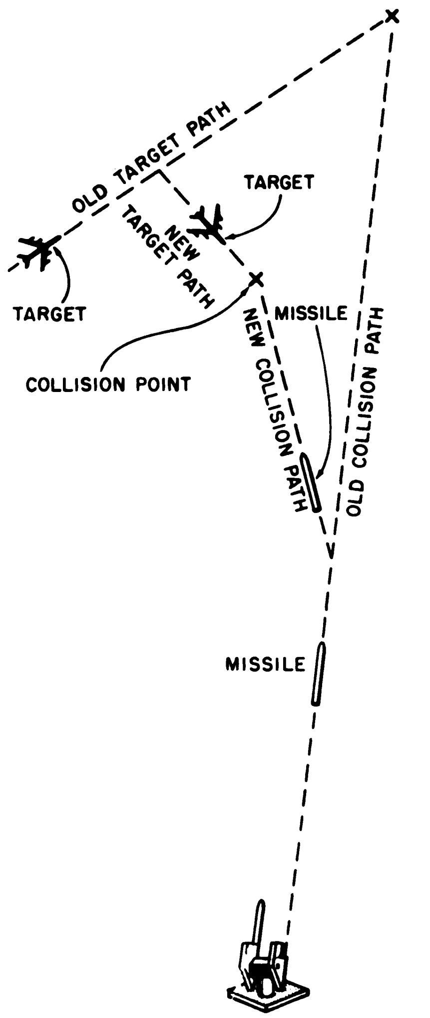
(2) Semi-active. The target receives energy from a source not

types of paths

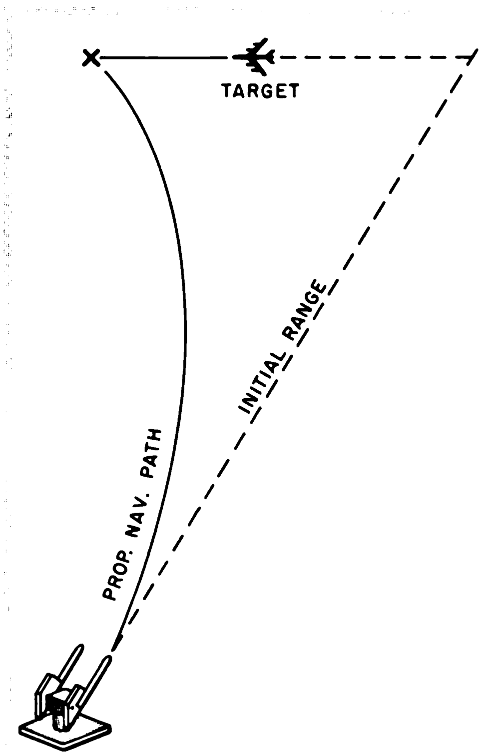
The simplest procedure for a guided missile directed at a moving target is to describe a PURSUIT path. The missile is then headed along the line-of-sight at every instant. Consequently, the rate of turn of the missile is always equal to the rate of turn of the line-of-sight. Pursuit paths are highly curved near the end of flight. (This is the PURE PURSUIT path; other types of pursuit path will be discussed later.)



At the opposite extreme to a pursuit path is a COLLISION path. The missile is aimed at such a point ahead of the target that both the missile and the target will reach that point at the same instant. The line-of-sight does not rotate relative to the missile. The missile path is as linear as a path can be when the missile is acted on by gravity and air resistance. If the target makes an evasive turn, a missile executing a variable collision path will compute a new collision point from the new target path, and will aim at that point.

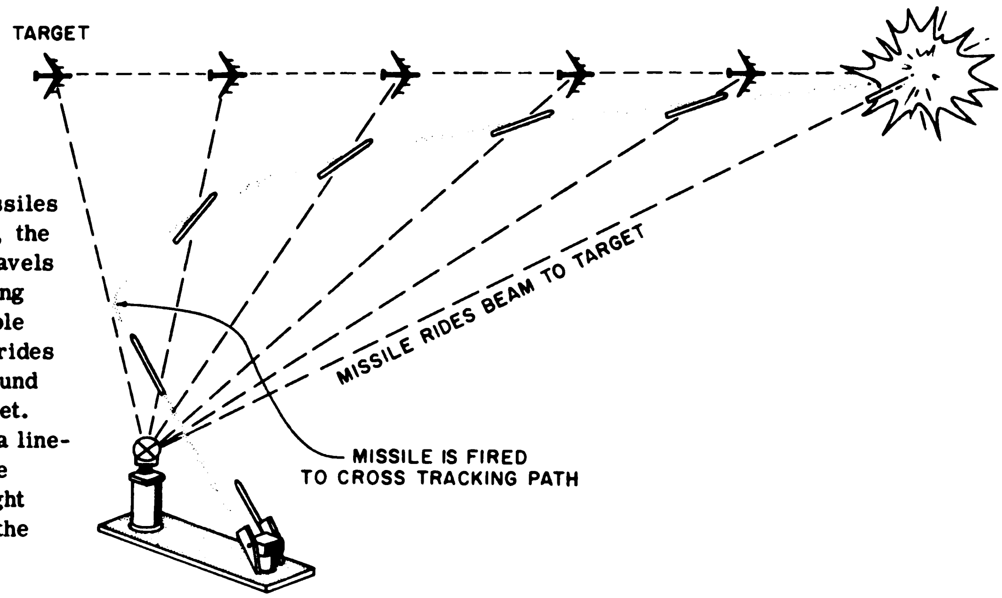


A missile may also fly a path which is intermediate between a pursuit path and a collision path and is known as PROPORTIONAL NAVIGATION. The missile travels in such a way that its own rate of turn is proportional to the rate of turn of the line-of-sight from missile to target at any instant. If the missile rate of turn equals the line-of-sight rate of turn, the flight path becomes a pursuit path (which is really a special case of the proportional navigation path). In general, in proportional navigation, the missile rate of turn is a fixed multiple of the line-of-sight rate of turn. Proportional navigation paths are less curved than pursuit paths, but more curved than collision paths.

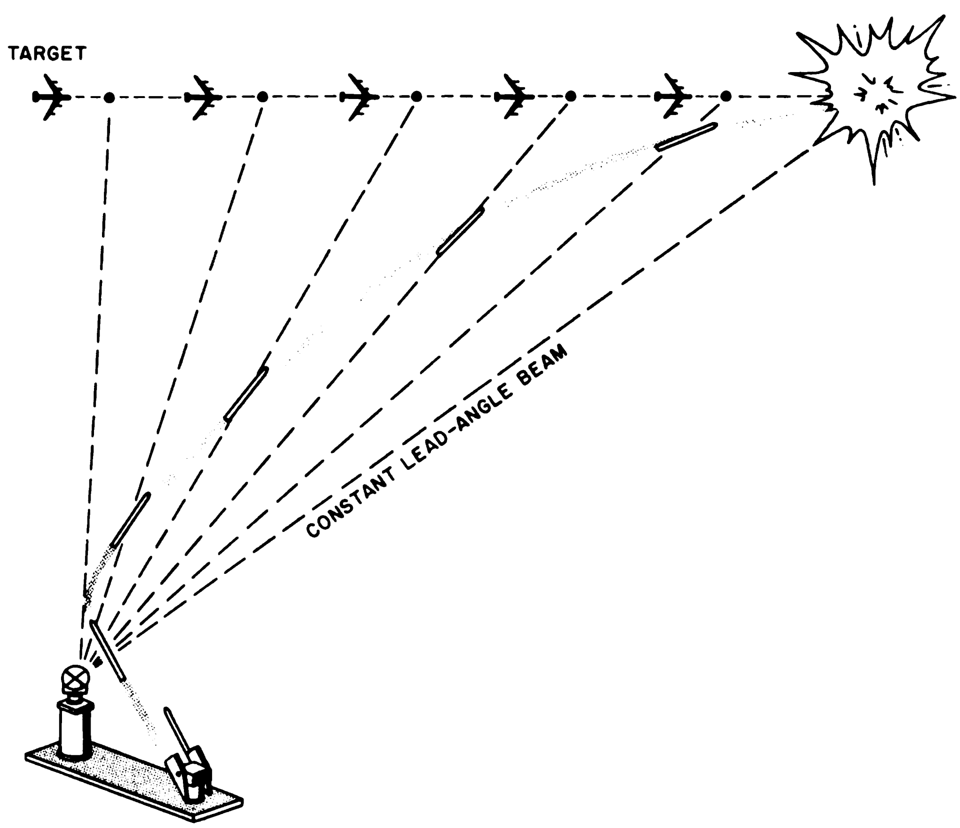


located in the missile — say, from a ground station. The missile picks up the echo signal and guides itself by the signals received. (3) Passive. The target receives no transmitted energy. The missile guides itself by radiation (heat, or infra-red) emitted from the target. There are several types of variable flight paths that a guided missile can fly; they are listed below and briefly described. Later, they will be discussed in detail, with their advantages and disadvantages.

Another path that can be flown by missiles is the LINE-OF-SIGHT path. Here, the missile is guided so that it always travels along the line-of-sight from launching station to target. This is an example of "beam riding", where the missile rides a beam that is radiated from a ground station and kept trained on the target. Note that a pursuit path also follows a line-of-sight: the line-of-sight from the missile. A beam-riding line-of-sight path follows the line-of-sight from the launching station.



An alternative form of beam riding path is the CONSTANT LEAD ANGLE path; here the beam is kept ahead of the line-of-sight by a constant offset.



TARGET CONSIDERATIONS

We can differentiate between missile path types by considering what assumptions are made about the motion of the target. Hitting a moving target - whether by bullet, ballistic missile, or guided missile - depends on the prediction of future target position. This prediction requires certain assumptions. When we launch a missile on a ballistic or preset guided path, we make the assumption that the target will either remain at rest during the flight of the missile, or preserve, during the flight of the missile, that state of motion which it was observed to have when tracked prior to the launching of the missile. It may seem that this assumption limits the usefulness of ballistic and preset guided missiles, and to some extent this is true. But such missiles have special uses. For instance, ICBM's are used primarily against stationary targets such as cities.

On the other hand, short-range ballistic missiles (including bullets) and short-range guided missiles usually have such short flight times that the assumption that a moving target will not change its motion during missile flight time is acceptable. A simple system must assume constant motion, or estimate how the motion will change. When we launch a missile on a variable guided path, we make no such assumption as far as the whole missile flight time is concerned. Rather, the tracking and computing devices make an assumption of constant target motion over small intervals of time; otherwise it could not predict. But the mechanism continuously reappraises its predictions; when it finds that they are wrong (because target motion has changed), it makes new predictions on the basis of the new motion of the target. This reappraisal and reprediction is what distinguishes variable guided missiles from ballistic and preset guided missiles, which make one prediction last for the whole missile flight.

summary

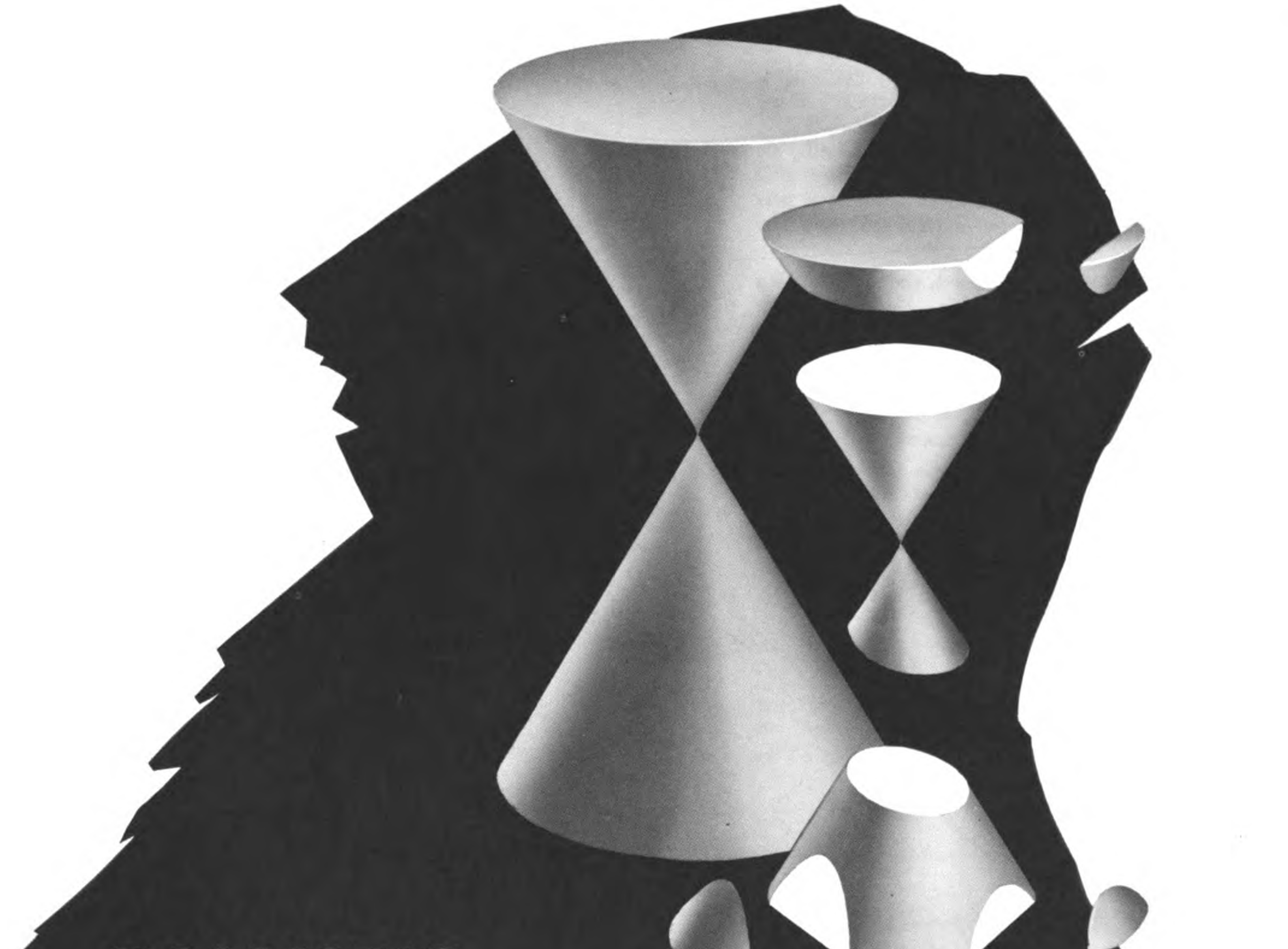
This introductory section discussed some of the basic conditions that determine flight paths, also some of the typical flight paths that these conditions cause a missile to describe in flight. A missile flies in a path that is determined by the forces acting on it. These forces may be due to nature (gravity, or the surrounding medium) or man-made (thrust or directional controls).

Man-made forces usually provide velocity vector control of a missile, that is, they determine its velocity in magnitude and direction at any instant. An exception is uncontrollable rocket thrust. This we do not consider as providing velocity vector control. Some methods of obtaining thrust (propellers, jets, rockets) and directional control (fins, rudders, side rockets) were discussed.

A missile in its flight may experience changes in the nature of the forces acting on it, and changes in the medium through which it travels. It is then convenient to divide the missile flight into "phases". We consider two basic phases of missile path: ballistic and guided. A ballistic phase we defined as one where the missile is under nature forces and no man-made force except uncontrollable rocket thrust; that is, a path where the missile does not experience velocity vector control. When only gravity is present, ballistic paths are conic sections. The presence of a surrounding medium, such as air, may change the shape of the curve. A guided phase is one in which the missile is under the influence of man-made forces that exercise velocity vector control.

Guided missile paths may themselves be classified as either preset or variable paths. Preset guided paths have plans that cannot be changed in mid-flight on the basis of new data received. A preset plan may be for a one-phase flight ("constant preset") or for a flight of several phases ("programmed preset"). Variable guided paths have plans that can be changed in mid-flight; thus they make possible the successful pursuit of a target that makes evasive maneuvers. Prediction of target position is almost continuously reappraised and missile course recomputed in the light of new target data. Some examples of variable guided paths are: pursuit, collision, proportional navigation, and beam-rider paths. The reader is reminded that bringing about the interception by a missile of a moving target depends on prediction of future target position, and this depends on certain assumptions. When using bullets, ballistic missiles, or preset guided missiles, we assume that the target motion measured while tracking will persist during missile flight. When using variable guided missiles, we assume that target motion, measured at any instant by almost continuous tracking, will persist over a short time interval.

SECTION 1



BALLISTIC FREE-FLIGHT TRAJECTORIES

A brief general survey of ballistic flight paths was made in the previous section. A detailed discussion of ballistic "free-flight trajectories" is the topic of this section. Ballistic missile flight paths may consist of several phases. Only the unpowered phase of the flight path is ballistic. The term "trajectories" is applied to this portion of the flight path to emphasize the fact that it is unpowered and unguided.

The ballistic "free flight trajectory" is covered in considerable detail because: 1) it represents the major portion of a ballistic missile flight-path, and 2) it is subject to simple mathematical analysis. Correction for the affect of the atmosphere will be added later.

All calculations are made in a geocentric, non-rotating frame (the origin of the coordinate frame is at the center of Earth but the frame does not rotate with Earth). In such a frame, the free-flight trajectory is a conic section in a plane through the center of Earth. The projection of the flight path on the surface of (non-rotating) Earth is an arc of a great circle.

The general equations of the path resulting from a given set of initial conditions are covered first. "Minimum energy paths" are then developed. These are important in determining maximum and minimum conditions, i.e., the maximum range for a given burnout velocity, optimum elevation angle for a given range, etc.

KINEMATIC EQUATIONS

Before we can compute the characteristics of any flight path, we must obtain the differential equation of a particle moving in a gravitational field that obeys the law of inverse squares, i.e., force is inversely proportional to distance squared. We shall first obtain kinematic equations for the velocities and accelerations of a moving particle, measured along, and perpendicular to, the radius vector from a fixed point to the particle.

Let P be the position of the particle at time t, and Q its position at time t + dt.

Let V_r and V_t be the velocities along OP and perpendicular to OP, respectively.

Then:

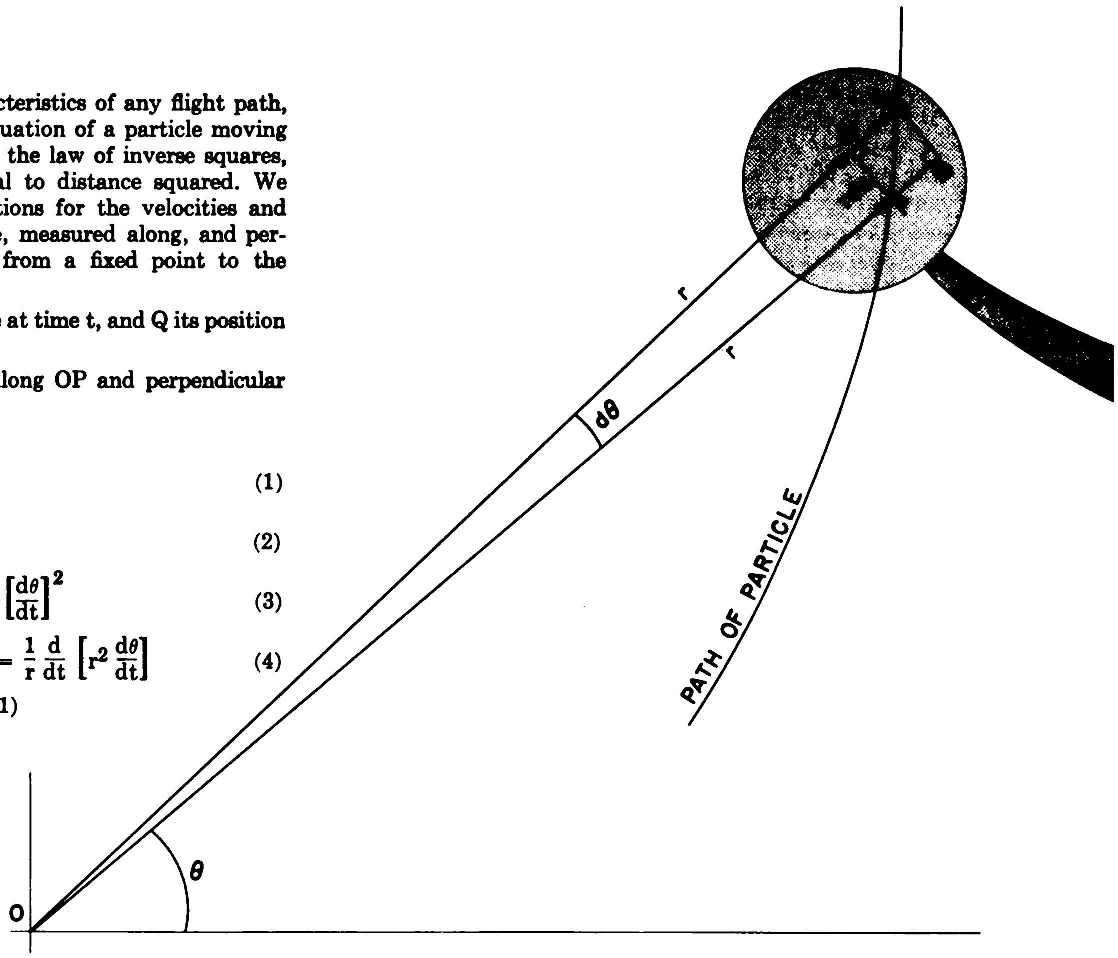
$$V_r = \frac{dr}{dt} \qquad (1)$$

$$V_t = \frac{rd\theta}{dt} \qquad (2)$$

$$\text{Accereration along OP} = \frac{d^2r}{dt^2} - r\left[\frac{d\theta}{dt}\right]^2 \qquad (3)$$

$$\text{Acceleration perpendicular to OP} = \frac{1}{r}\frac{d}{dt}\left[r^2\frac{d\theta}{dt}\right] \qquad (4)$$

(See Box 1)



1

Resolving distances in the direction of OP:

$$V_r = \frac{OM - OP}{dt} = \frac{(r + dr)\,\text{Cos}\,d\theta - r}{dt}$$

Since Cos $d\theta$ differs from 1 only by a second-order infinitesimal:

$$V_r = \frac{r + dr - r}{dt} = \frac{dr}{dt} \qquad (1\text{-}1)$$

Resolving all distances along a direction perpendicular to OP:

$$V_t = \frac{QM - O}{dt} = \frac{(r + dr)\,\text{Sin}\,\theta}{dt}$$

Putting $d\theta$ for Sin $d\theta$, and neglecting second-order infinitesimals:

$$V_t = \frac{rd\theta + dr\,d\theta}{dt} = r\frac{d\theta}{dt} \qquad (1\text{-}2)$$

Acceleration along OP =

$$\frac{\text{velocity along OP at time } (t + dt) - \text{velocity along OP at time } t}{dt}$$

Note that the vector $V_t + dV_t$ has a small component along OP, namely: $-(V_t + dV_t)$ Sin $d\theta$

Thus: acceleration along OP =

$$\frac{(V_r + dV_r)\,\text{Cos}\,d\theta - (V_t + dV_t)\,\text{Sin}\,d\theta - V_r}{dt}$$

Again, substitute $d\theta$ for Sin $d\theta$, and ignore second-order infinitesimals:

$$\text{Acceleration along OP} = \frac{dV_r - V_t\,d\theta}{dt}$$

$$= \frac{dV_r}{dt} - V_t\frac{d\theta}{dt}$$

Substitute for V_r, and V_t from (1-1) and (1-2):

$$\text{Acceleration along OP} = \frac{d}{dt}\left[\frac{dr}{dt}\right] - r\frac{d\theta}{dt}\frac{d\theta}{dt}$$

$$= \frac{d^2r}{dt^2} - r\left[\frac{d\theta}{dt}\right]^2 \qquad (1\text{-}3)$$

By similar reasoning:

Acceleration perpendicular to OP =

$$\frac{(V_t + dV_t)\,\text{Cos}\,d\theta + (V_r + dV_r)\,\text{Sin}\,d\theta - V_t}{dt}$$

$$= \frac{dV_t}{dt} + V_r\frac{d\theta}{dt}$$

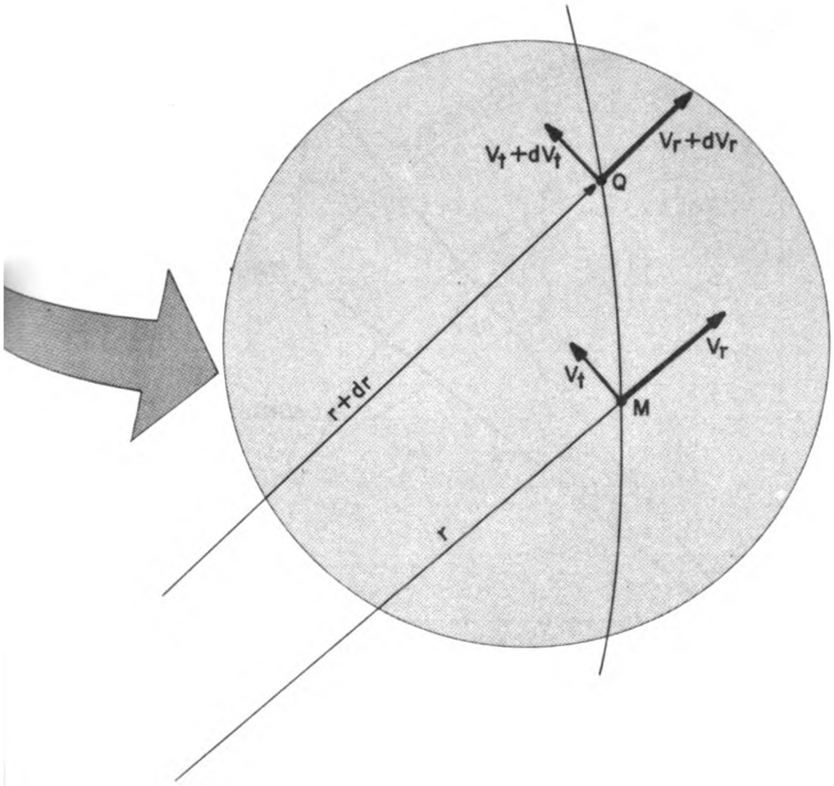
$$= \frac{d}{dt}\left[r\frac{d\theta}{dt}\right] + \frac{dr}{dt}\frac{d\theta}{dt}$$

$$= 2\frac{dr}{dt}\frac{d\theta}{dt} + r\frac{d^2\theta}{dt^2}$$

Acceleration perpendicular to OP =

$$\frac{1}{r}\frac{d}{dt}\left[r^2\frac{d\theta}{dt}\right] \qquad (1\text{-}4)$$

OF A PARTICLE



Now, suppose that our particle is a ballistic missile in free flight far above Earth where air resistance is negligible, and the only force is that of gravity acting toward the center of Earth (at point O), according to the law of inverse squares. Then:

$$\text{Force on missile toward O} = \frac{Gm_em}{r^2}$$

where m = mass of missile

m_e = mass of Earth

G = universal gravitational constant

$$\text{Acceleration toward O} = \frac{\text{force}}{m} = \frac{Gm_e}{r^2}$$

(Gm_e is a constant, equal to 14.05×10^{15} ft^3/sec^2.)

Then, applying equations (3) and (4) to this field of force, it can be shown that (taking for convenience $u = 1/r$):

$$\frac{d^2u}{d\theta^2} + u = \frac{Gm_e}{h^2} = \text{constant} \qquad (5)$$

$$\text{where } h = r^2\frac{d\theta}{dt} = \text{constant} \qquad (6)$$

(See Box 2)

Equation (5) is a special case of the general differential equation of a particle moving in an inverse force field that obeys the law of inverse squares. In this case, the force field is that of Earth's gravity.

2

$$\text{Acceleration along OP} = -\frac{Gm_e}{r^2} \qquad (2\text{-}1)$$

As the only force on the missile is the one toward O:

$$\text{Acceleration perpendicular to OP} = 0 \qquad (2\text{-}2)$$

Substitute (1-3) and (1-4) in (2-1) and (2-2):

$$\frac{d^2r}{dt^2} - r\left[\frac{d\theta}{dt}\right]^2 = -\frac{Gm_e}{r^2} \qquad (2\text{-}3)$$

$$\frac{1}{r}\frac{d}{dt}\left[r^2\frac{d\theta}{dt}\right] = 0 \qquad (2\text{-}4)$$

From (2-4): $\frac{d}{dt}\left[r^2\frac{d\theta}{dt}\right] = 0$

If $\frac{df}{dt} = 0$, f = constant, therefore:

$$r^2\frac{d\theta}{dt} = \text{constant} = h \qquad (2\text{-}5)$$

Equation (2-5) confirms the constancy of angular momentum.

We will now combine equations (2-3) and (2-5) to eliminate t and obtain a differential equation involving r and θ alone. Except for convenience, we shall use not r, but its reciprocal:

$$u = 1/r \qquad (2\text{-}6)$$

$$\text{Thus: } h = r^2\frac{d\theta}{dt} = \frac{1}{u^2}\frac{d\theta}{dt} \qquad (2\text{-}7)$$

From (2-6):

$$\frac{dr}{dt} = \frac{d}{dt}\frac{1}{u} = -\frac{1}{u^2}\frac{du}{dt}$$

$$= -\frac{1}{u^2}\frac{du}{d\theta}\frac{d\theta}{dt} = -h\frac{du}{d\theta} \qquad (2\text{-}8)$$

Substituting from (2-7):

$$\frac{d^2r}{dt^2} = \frac{d}{dt}\frac{dr}{dt} = \frac{d}{dt}\left[-h\frac{du}{d\theta}\right]$$

$$= -h\frac{d}{dt}\frac{du}{d\theta} = -h\frac{d}{d\theta}\left[\frac{du}{d\theta}\right]\frac{d\theta}{dt}$$

$$= -h\frac{d^2u}{d\theta^2}\frac{d\theta}{dt}$$

$$\text{But, from (2-7): } \frac{d\theta}{dt} = h\,u^2 \qquad (2\text{-}9)$$

Substituting for $\frac{d\theta}{dt}$:

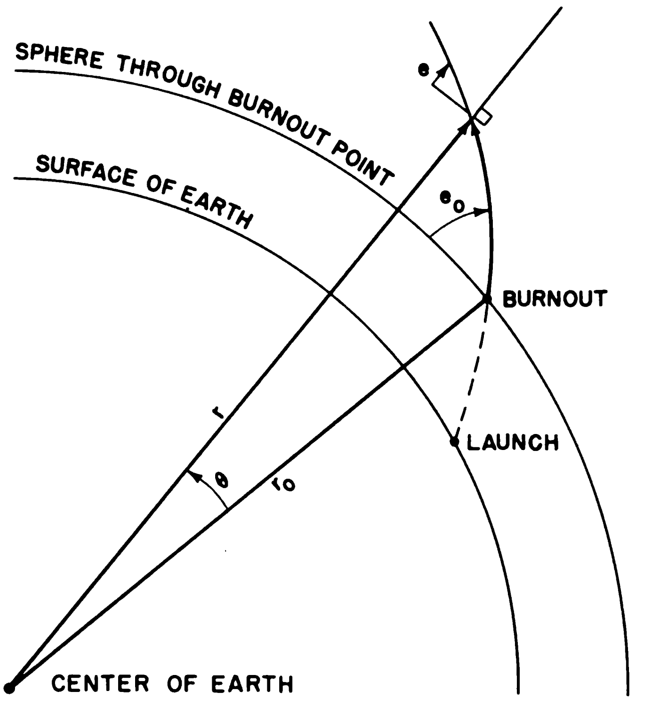
$$\frac{d^2r}{dt^2} = -h^2u^2\frac{d^2u}{d\theta^2} \qquad (2\text{-}10)$$

Substitute (2-6), (2-10), and (2-9) in (2-3):

$$-h^2u^2\frac{d^2u}{d\theta^2} - \frac{1}{u}h^2u^4 = -Gm_e u^2$$

$$\frac{d^2u}{d\theta^2} + u = \frac{Gm_e}{h^2} = \text{constant} \qquad (2\text{-}11)$$

PARAMETRIC EQUATIONS



We are now ready to investigate the path of a ballistic missile projected at burnout from a given height above Earth, in vacuum.

Let r = distance from center of Earth
θ = angular displacement from burnout point
e = elevation of path above horizontal
V = velocity
r_o = value of r at burnout
e_o = value of e at burnout
V_o = value of V at burnout
g_o = acceleration due to gravity at burnout

Before solving equation (5), a small change in notation is convenient:

Force on missile at burnout $= \frac{Gm_e m}{r_o^2}$

Also: Force on missile at burnout $= mg_o$

$$\frac{Gm_e m}{r_o^2} = mg_o \qquad (7)$$

$$Gm_e = g_o r_o^2 \qquad (8)$$

Throughout most of this analysis we shall use $g_o r_o^2$ instead of Gm_e. Note that g_o and r_o are constants for given burnout conditions, but would be different constants for other burnout conditions. On the other hand, $g_o r_o^2$ is the same for all burnout situations, because it equals Gm_e, a universal constant.

Substituting (8) in (5):

$$\frac{d^2u}{d\theta^2} + u = \frac{g_o r_o^2}{h^2} \qquad (9)$$

where $u = \frac{1}{r}$

and $h = r^2\, d\theta/dt$ = constant

Solving (9) (see box 3), we obtain

$$\frac{1}{r} = f - k \operatorname{Cos}(\theta - p) = \frac{1 - (k/f) \operatorname{Cos}(\theta - p)}{1/f} \qquad (10)$$

where $f = \frac{g_o}{V_o^2 \operatorname{Cos}^2 e_o}$ = constant, (11)

and k and p are constants, not yet evaluated.

The polar equation for a conic section (elipse, hyperbola or parabola) with one focus at the origin is:

$$\frac{1}{r} = \frac{1 - \epsilon \operatorname{Cos}(\theta - p)}{slr} \qquad (12)$$

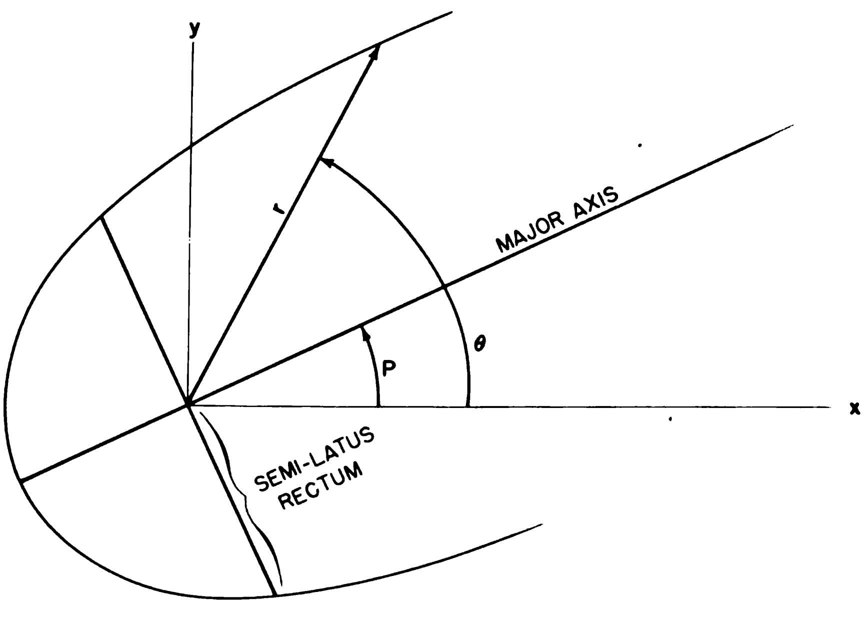
where: ϵ = eccentricity
slr = semi-latus rectum
p = upward tilt of major axis

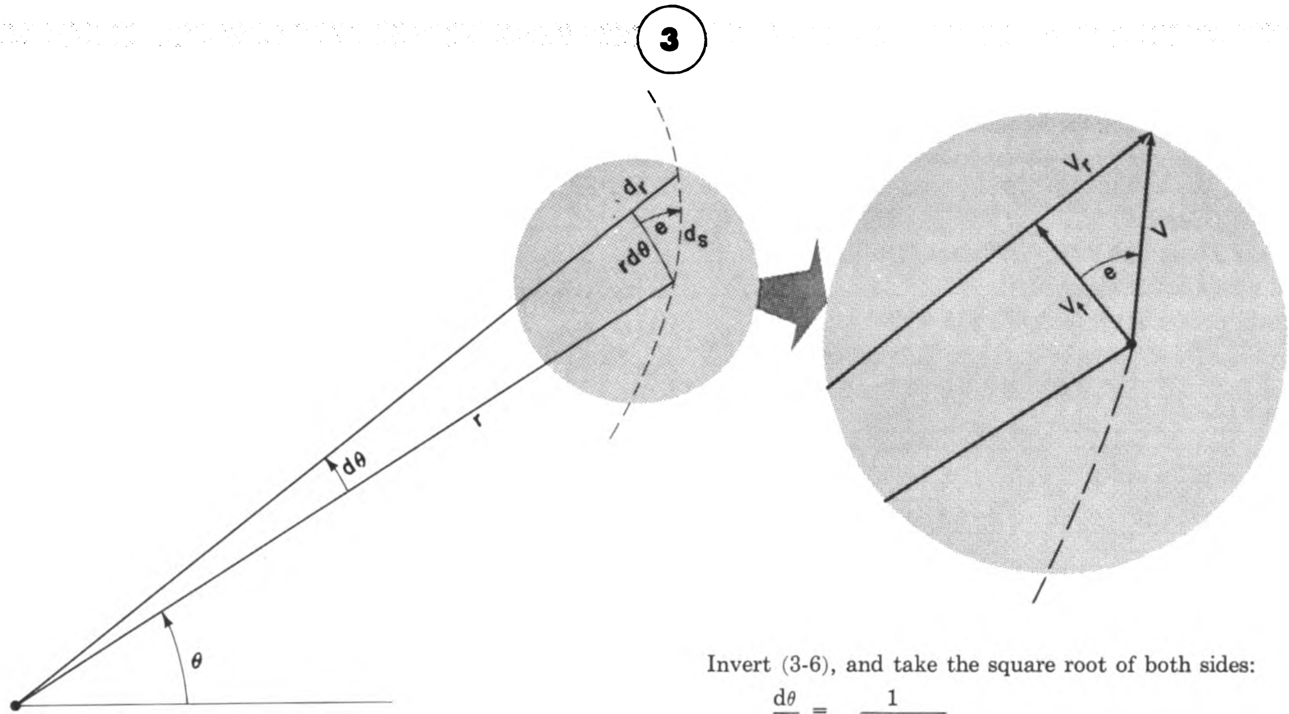
Comparing like terms of (10) and (12) shows that the flight path is a conic section, and

$$\epsilon = k/f \qquad (13)$$

$$slr = 1/f \qquad (14)$$

$$= V_o^2 \operatorname{Cos}^2 e_o / g_o \qquad (15)$$





From diagrams of infinitesimal increments:

$$V_r = V \text{ Cos } e$$

$$r \frac{d\theta}{dt} = V \text{ Cos } e$$

$$h = r^2 \frac{d\theta}{dt} = rV \text{ Cos } e$$

Since h is a constant, we may substitute the values of r, V, and e at burnout:

$$h = r_o V_o \text{ Cos } e_o \tag{3-1}$$

$$\frac{g_o r_o^2}{h^2} = \frac{g_o r_o^2}{r_o V_o^2 \text{ Cos}^2 e_o} = \frac{g_o}{V_o^2 \text{ Cos}^2 e_o}$$

$$= f \text{ (a new constant)} \tag{3-2}$$

We can now write (9) in a simpler form:

$$\frac{d^2u}{d\theta^2} + u = f \tag{3-3}$$

$$\text{Let } y = u - f \tag{3-4}$$

From (3-4):

$$\frac{d^2y}{d\theta^2} = \frac{d^2u}{d\theta^2}$$

$$= f - u \text{ (from 3-3)}$$

$$\frac{d^2y}{d\theta^2} = -y \text{ (from 3-4)} \tag{3-5}$$

This is analogous to the well-known differential equation of simple harmonic motion, which is customarily solved as follows:

Multiply both sides of (3-5) by $2\frac{dy}{d\theta}$:

$$2\frac{dv}{d\theta}\frac{d^2y}{d\theta^2} = -2y\frac{dy}{d\theta}$$

Integrate both sides with respect to θ:

$$\left[\frac{dy}{d\theta}\right]^2 = -\int 2y\frac{dy}{d\theta}\,d\theta$$

$$= -\int 2y\,dy$$

$$= -y^2 + k^2 \tag{3-6}$$

where k^2 is an unknown constant. The reason for using a squared quantity for the constant will be apparent later.

Invert (3-6), and take the square root of both sides:

$$\frac{d\theta}{dy} = \frac{1}{\sqrt{k^2 - y^2}} \tag{3-7}$$

$$\theta = \int \frac{dy}{\sqrt{k^2 - y^2}}$$

This is a standard integral, which can be looked up in any book of tables. However, in order to obtain the final equation for a missile path in the most convenient form, perform the integration step-by-step, as follows:

$$\text{Let } y = -k \text{ Cos } \beta \tag{3-8}$$

$$dy = +k \text{ Sin } \beta\, d\beta \tag{3-9}$$

Substitute (3-8) and (3-9) in (3-7):

$$\theta = \int \frac{k \text{ Sin } \beta\, d\beta}{\sqrt{k^2 - k^2 \text{ Cos}^2 \beta}}$$

$$= \int \frac{k \text{ Sin } \beta\, d\beta}{k \text{ Sin } \beta}$$

$$= \int d\beta$$

$$\theta = \beta + p \tag{3-10}$$

where p is another unknown constant.

From (3-8): $\text{Cos } \beta = -\frac{y}{k}$

$$\beta = \text{Cos}^{-1}\left[\frac{-y}{k}\right] \tag{3-11}$$

Substitute (3-11) in (3-10):

$$\theta = \text{Cos}^{-1}\left[-\frac{y}{k}\right] + p$$

$$\text{Cos}^{-1}\left[-\frac{y}{k}\right] = \theta - p$$

$$-\frac{y}{k} = \text{Cos } (\theta - p)$$

$$y = -K \text{ Cos } (\theta - p) \tag{3-12}$$

From (3-4):

$$\frac{1}{r} = u = f + y$$

Substitute (3-12):

$$\frac{1}{r} = f - k \text{ Cos } (\theta - p) \tag{3-13}$$

trajectory

We shall now evaluate ϵ, slr, and p, in terms of burnout conditions. At this point, a certain change in nomenclature is useful:

The quantity V_o^2/g_or_o plays an important part in the ensuing mathematics, which we simplify by substituting a new symbol:

$$\rho = V_o^2/g_or_o \tag{16}$$

The value of ρ has much more significance than V_o.

We shall subsequently show that:

(1) ρ = the square of the ratio of V_o to orbital velocity.

(2) ρ = twice the square of the ratio of V_o to escape velocity.

Also, ρ is "dimensionless," and therefore independent of the system of units used.

Note, also, that ρ is proportional to the kinetic energy (K.E.) of the missile at burnout:

$$\rho = \frac{V_o^2}{g_or_o} = \frac{mV_o^2}{mg_or_o} = \frac{2\ \text{K.E.}}{mr_o}$$

To convert from ρ to V_o:

$$V_o = 4.18\sqrt{\rho} \qquad \text{(V in nautical mi/sec)} \tag{17}$$

$$V_o = 25{,}900\sqrt{\rho} \qquad \text{(V in ft/sec)} \tag{18}$$

To convert from V_o to ρ:

$$\rho = \frac{V_o^2}{15.5} \text{(V in nautical mi/sec)}$$

$$\rho = \frac{V_o^2}{6.73 \times 10^8} \text{ (V in ft/sec)} \tag{20}$$

We can now prove (see Box 4) that:

$$\text{Cot}\ \rho = \text{Cot}\ e^o \left[\frac{\sec^2 e_o}{\rho} - 1\right] \tag{21}$$

$$\epsilon = \sqrt{1 - \rho\,(2 - \rho)\,\text{Cos}^2\,e_o} \tag{22}$$

$$\text{slr} = \rho r_o\,\text{Cos}^2\,e_o \tag{23}$$

Now we have ρ, ϵ, and slr in terms of the initial burnout conditions. (We need not obtain k.)

Using (21), (22) and (23), we could substitute these values of p, ϵ, and slr in (12), the equation of the path. However, this would be rather cumbersome; it is more convenient to keep (21) and (23) on the side, as adjuncts to (12).

4

At burnout, $\theta = 0$ and $r = r_o$; then (12) becomes:

$$\frac{1}{r_o} = \frac{1 - \epsilon\,\text{Cos}\,(-\,p)}{\text{slr}} \tag{4-1}$$

$$= \frac{1 - \epsilon\,\text{Cos}\,p}{\text{slr}}$$

Differentiating (12):

$$\frac{d}{d\theta}\left[\frac{1}{r}\right] = +\frac{\text{Sin}\,(\theta - p)}{\text{slr}}$$

At burnout, $\theta = 0$, so:

$$\frac{d}{d\theta}\left[\frac{1}{r}\right] = \frac{\text{Sin}\,(-\,p)}{\text{slr}} \tag{4-2}$$

$$= -\frac{\text{Sin}\,p}{\text{slr}}$$

Differentiating (12) in another way:

$$\frac{d}{d\theta}\left[\frac{1}{r}\right] = \frac{d}{dr}\left[\frac{1}{r}\right]\frac{dr}{d\theta}$$

$$= -\frac{1}{r^2}\frac{dr}{d\theta}$$

$$\frac{d}{d\theta}\left[\frac{1}{r}\right] = -\frac{1}{r}\frac{dr}{d\theta} \tag{4-3}$$

From infinitesimal increments:

$$\frac{1}{r}\frac{dr}{d\theta} = \text{Tan}\ e \tag{4-4}$$

Substitute (4-4) in (4-3):

$$\frac{d}{d\theta}\left[\frac{1}{r}\right] = -\frac{1}{r}\,\text{Tan}\ e \tag{4-5}$$

At burnout:

$$\frac{d}{d\theta}\left[\frac{1}{r}\right] = -\frac{1}{r_o}\,\text{Tan}\ e_o \tag{4-6}$$

Now equate (4-2) and (4-6), both of which give values of $\frac{d}{d\theta}\left[\frac{1}{r}\right]$ at burnout:

$$-\frac{\epsilon\,\text{Sin}\,p}{\text{slr}} = -\frac{1}{r_o}\,\text{Tan}\ e_o \tag{4-7}$$

From (4-1):

$$\frac{\epsilon}{\text{slr}}\,\text{Cos}\,p = \frac{1}{\text{slr}} - \frac{1}{r_o} \tag{4-8}$$

From (4-7):

$$\frac{\epsilon}{\text{slr}}\,\text{Sin}\,p = \frac{\text{Tan}\ e_o}{r_o} \tag{4-9}$$

Divide (4-8) by (4-9):

$$\text{Cot}\,p = \left[\frac{1}{\text{slr}} - \frac{1}{r_o}\right] \Big/ \left[\frac{\text{Tan}\ e_o}{r_o}\right]$$

$$= \text{Cot}\ e_o\left[\frac{r_o}{\text{slr}} - 1\right] \tag{4-10}$$

Substituting from (15):

$$\text{Cot}\,p - \text{Cot}\ e_o\left[\frac{r_og_o}{V_o^2}\,\text{Sec}^2\,e_o - 1\right] \tag{4-11}$$

So we have obtained p, the "tilt angle", in terms of known quantities.

Substituting (16) in (4-11):

$$\text{Cot}\,p = \text{Cot}\ e_o\left[\frac{\text{Sec}^2\,e_o}{p} - 1\right] \tag{4-12}$$

To obtain the eccentricity of the path, (ϵ), re-write equations (4-8) and (4-9), make substitution from (15) and (16), square both sides, and add:

$$\text{Cos}\,p = 1 - \frac{\text{slr}}{r_o} = 1 - \frac{V_o^2\,\text{Cos}^2\,e_o}{g_or_o} = 1 - \rho\,\text{Cos}^2\,e_o$$

$$\text{Sin}\,p = \frac{\text{slr Tan}\ e_o}{r_o} = \frac{V_o^2\,\text{Cos}^2\,e_o}{g_or_o}\,\text{Tan}\ e_o = \rho\,\text{Sin}\ e_o\,\text{Cos}\ e_o$$

$$\epsilon^2 = 1 - 2\rho\,\text{Cos}^2\,e_o + \rho^2\,\text{Cos}^4\,e_o + \rho^2\,\text{Sin}^2\,e_o\,\text{Cos}^2\,e_o$$

$$= 1 - 2\rho\,\text{Cos}^2\,e_o + \rho^2\,\text{Cos}^4\,e_o + \rho^2\,\text{Cos}^2\,e_o\,(1 - \text{Cos}^2\,e_o)$$

$$= 1 - 2\rho\,\text{Cos}^2\,e_o + \rho^2\,\text{Cos}^2\,e_o$$

$$= 1 - \rho\,(2 - \rho)\text{Cos}^2\,e_o$$

$$\epsilon = \sqrt{1 - \rho(2 - \rho)\,\text{Cos}^2\,e_o} \tag{4-13}$$

Now transform (15) into the new notation:

$$\text{slr} = \frac{V_o^2\,\text{Cos}^2\,e_o}{g_o}$$

$$= \frac{V_o^2}{g_or_o}r_o\,\text{Cos}^2\,e_o$$

$$= \rho r_o\,\text{Cos}^2\,e_o \text{ (from 16)} \tag{4-14}$$

parameters

Let us now find out what kind of a conic section the missile follows, under given initial burnout conditions. We know that the path will be an ellipse if $\epsilon^2 < 1$. In that event:

$1 - \rho\,(2 - \rho)\,\mathrm{Cos}^2\, e_o < 1$ (see 22)

$\rho\,(2 - \rho)\,\mathrm{Cos}^2\, e_o$ is a positive quantity;

$2 - \rho$ is a positive quantity; $\rho < 2$.

Similarly, the path will be a parabola if $\rho = 2$, and a hyperbola if $\rho > 2$.

Summarizing, regardless of the value of e_o, the path will be:

An ellipse, if $\rho < 2$; $(V_o^2 < 2\,g_o r_o)$

A parabola, if $\rho = 2$; $(V_o^2 = 2\,g_o r_o)$

A hyperbola, if $\rho < 2$; $(V_o^2 > 2\,g_o r_o)$

A special case of the elliptical path is also of interest.

If $e_o = 0$ and $\rho = 1$:

$\epsilon = \sqrt{1 - \rho\,(2 - \rho)\,\mathrm{Cos}2\, e_o = 0}$

The path is then circular. Also, since $\rho = 1$:

$V_o = \sqrt{g_o r_o}$: orbital velocity.

We shall encounter this case again.

We have proved a well-known law of ballistics—a particle projected in a radial gravitational field will describe an open curve (parabola or hyperbola) and never return to the Earth, if its velocity at projection equals or exceeds escape velocity; that is, $\sqrt{2g_o r_o}$ at the point of projection, regardless of the direction of projection. If the velocity at projection is less than escape velocity, the path will be a closed curve (an ellipse, or in certain special cases, a circle). Escape velocity at the surface of Earth, neglecting air resistance, is about 6.03 nautical miles (6.95 statute miles) per second. At typical ICBM burnout altitudes, escape velocity may be about 5.95 nautical miles per second. Escape velocity diminishes as altitude increases (see Box 5).

5

Let V_e be escape velocity at Earth's surface, r_e be Earth's radius (assumed to be spherical and homogeneous), while g is, as usual, acceleration due to gravity at Earth's surface.

$$V_e^2 = 2gr_e$$

$$V_o^2 = 2g_o r_o$$

$$\frac{V_o^2}{V_e^2} = \frac{2g_o r_o}{2gr_e} \qquad (5\text{-}1)$$

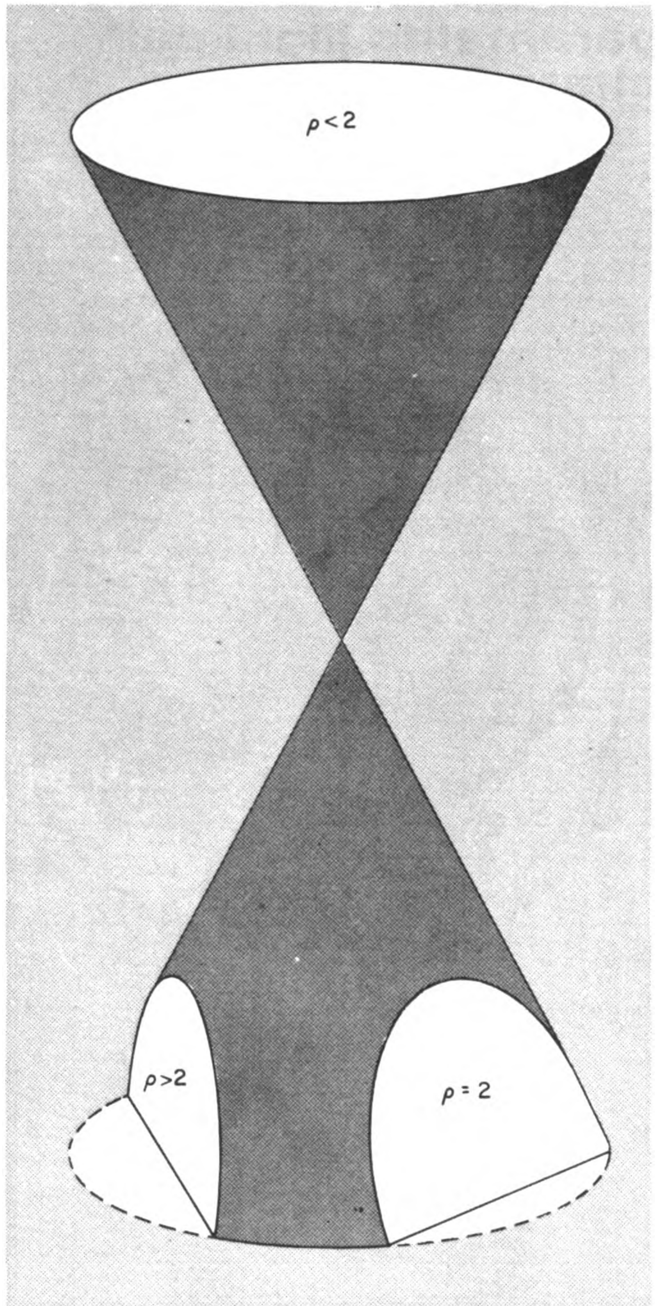
By the Law of Inverse Squares:

$$\frac{g_o}{g} = \frac{r_e^2}{r_o^2} \qquad (5\text{-}2)$$

Substitute (5-2) in (5-1):

$$\frac{V_o^2}{V_e^2} = \frac{r_e^2 r_o}{r_o^2 r_e} = \frac{r_e}{r_o}, \text{ which is less than 1.}$$

Therefore, $V_o < V_e$.



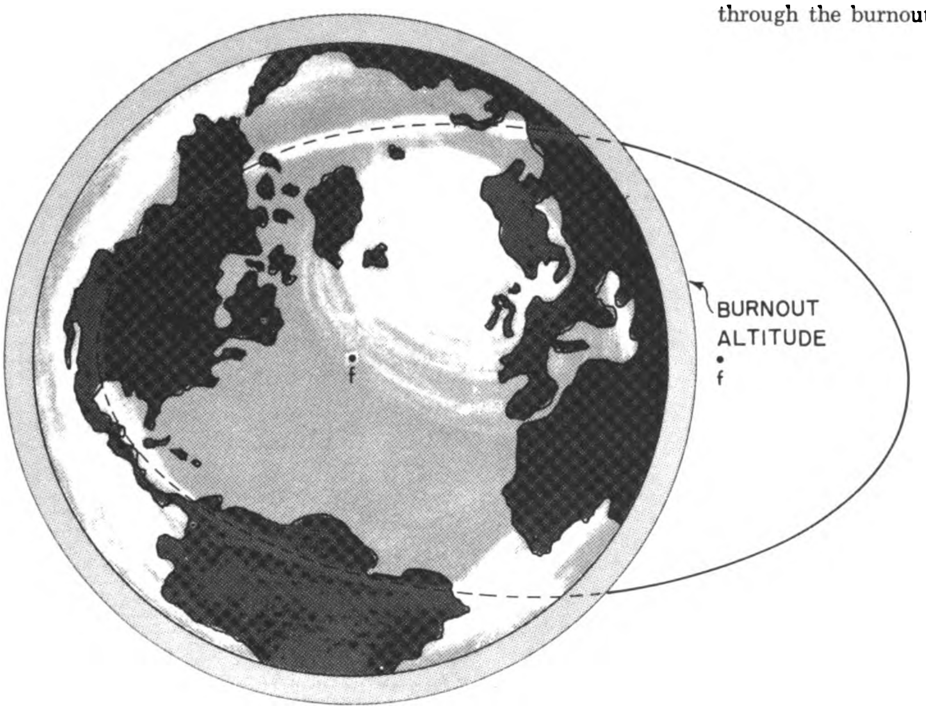
Note that the conic section described by the missile has a focus at the "orgin," which, in this case, is the center of Earth, confirming Kepler's First Law. These flight paths are all relative to the center of Earth.

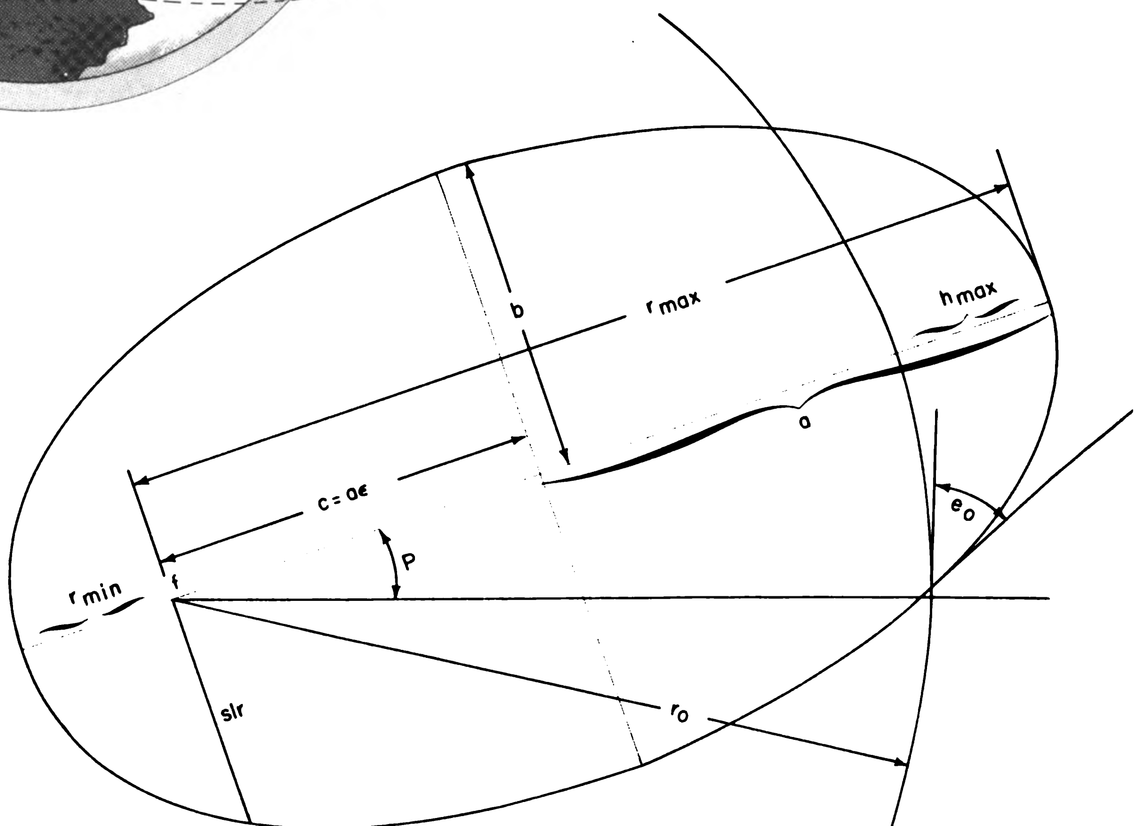
In launching ballistic missiles, it is necessary to select an ellipse, following which the missile will hit the target. For this, we select the required velocity vector at burnout (V_o and e_o).

Note that we have been assuming an isolated force field due to Earth alone. A missile fired with a velocity exceeding $\sqrt{2g_o r_o}$ would escape, but it would not follow a hyperbola forever. It would come under the influence of Sun's gravity field and probably end up in a elliptic orbit around Sun.

parametric flight path characteristics

In this section on missile ballistic flight paths, we are concerned only with elliptic paths that intersect the "re-entry sphere." We define the re-entry sphere as a sphere concentric with Earth and slightly larger, outside of which atmospheric pressure is considered negligible. For simplicity, and to obtain a symmetrical flight path, we assume that the re-entry sphere is identical with the "burnout sphere"; that is, the sphere through the burnout point.





First let us establish some general laws concerning elliptical ballistic paths. Returning to the path equation:

$$\frac{1}{r} = \frac{1 - \epsilon \operatorname{Cos}(\theta - p)}{slr}$$

The maximum value of r occurs when 1/r is a minimum; that is:

$$\operatorname{Cos}(\theta - p) = 1$$
$$\theta - p = 0$$
$$\theta = p$$

(This is evident from the diagram.)

$$\text{Thus, } \frac{1}{r_{max}} = \frac{1 - \epsilon}{slr}$$

$$r_{max} = \frac{slr}{1 - \epsilon} \qquad (24)$$

The minimum value of r occurs when 1/r is maximum; that is:

$$\operatorname{Cos}(\theta - p) = -1$$
$$\theta - p = 180°$$
$$\theta = p + 180°$$

(This, also, is evident from the diagram.)

$$\text{Thus, } \frac{1}{r_{min}} = \frac{1 + \epsilon}{slr}$$

$$r_{min} = \frac{slr}{1 + \epsilon} \qquad (25)$$

From maximum and minimum r we can evaluate the constants of the ellipse: semi-major axis (a), semi-minor axis (b), and semi-focal distance (c). We have already evaluated the semi-latus rectum (slr). From the diagram:

$$2a = r_{max} + r_{min}$$
$$= \frac{slr}{1 - \epsilon} + \frac{slr}{1 + \epsilon}$$
$$= slr \frac{2}{1 - \epsilon^2}$$

$$a = \frac{slr}{1 - \epsilon^2} \qquad (26)$$

$$= \frac{r_o \operatorname{Cos}^2 e_o}{(2 - \rho) \operatorname{Cos}^2 e_o} \qquad \text{from (22) and (23)}$$

$$= \frac{r_o}{2 - \rho}$$

A simple way of expressing the major axis (a) is to give its ratio to r_o (burnout radius):

$$\frac{a}{r_o} = \frac{1}{2 - \rho} \qquad (27)$$

Note that the value of a depends on p, but is independent of e_o; thus:

ALL FREE-FLIGHT ELLIPSES HAVING THE SAME POINT AND SPEED OF PROJECTION AT BURNOUT HAVE EQUAL MAJOR AXLES.

Note that if $\rho = 2$, major axis (a) is infinite; this accords with our previous finding for the limiting condition for an elliptical path.

Note that the tilt angle (p) is half the range angle: $p = \theta_r/2$.

The other constants of the ellipse (c, the semi-focal distance, and b, the semi-minor axis) are readily obtained from a:

$$\frac{c}{r_o} = \frac{a\epsilon}{r_o} = \frac{\sqrt{1 - \rho(2 - \rho) \operatorname{Cos}^2 e_o}}{2 - \rho} \qquad (28)$$

$$\frac{b^2}{r_o^2} = \frac{a^2}{r_o^2} - \frac{c^2}{r_o^2}$$
$$= \frac{1}{(2-\rho)^2} - \frac{1 - \rho(2 - \rho) \operatorname{Cos}^2 e_o}{(2 - \rho)^2}$$
$$= \frac{\rho(2 - \rho) \operatorname{Cos}^2 e_o}{(2 - \rho)^2}$$
$$= \frac{\rho \operatorname{Cos}^2 e_o}{2 - \rho}$$

$$\frac{b}{r_o} = \operatorname{Cos} e_o \sqrt{\frac{\rho}{2 - \rho}} \qquad (29)$$

Now let us compute h_{max}, the maximum height above burnout reached by the missile. In accordance with our present policy, we shall compute the ratio h_{max}/r_o, which is more basic than h_{max} and gives us a simpler equation.

From the diagram:

$$h_{max} = a + c - r_o$$
$$\frac{h_{max}}{r_o} = \frac{a + c}{r_o} - 1$$
$$= \frac{a}{r_o}(1 + \epsilon) - 1$$
$$\frac{h_{max}}{r_o} = \frac{1 + \epsilon}{2 - \rho} - 1$$

This is a useful, compact, and easy-to-remember equation for h_{max}/r_o; but it includes ϵ, which must be computed. If we wish to obtain h_{max}/r_o in trems of initial burnout conditions, we must proceed as follows:

$$\frac{h_{max}}{r_o} = \frac{a}{r_o} + \frac{c}{r_o} - 1$$

Substitute from (27) and (28).

$$\frac{h_{max}}{r_o} = \frac{1}{2 - \rho} + \frac{\sqrt{1 - \rho(2 - \rho) \operatorname{Cos}^2 e_o}}{2 - \rho} - 1$$

$$\frac{h_{max}}{r_o} = \frac{1 + \sqrt{1 - \rho(2 - \rho) \operatorname{Cos}^2 e_o}}{2 - \rho} - 1 \qquad (31)$$

From a practical standpoint, what we need to know is maximum height above the ground. This is easily obtained from h_{max}:
Maximum height above ground =
h_{max} + height of burnout point
$= h_{max} + r_o - r_e$

We should also like to compute the time of free flight of the missile between burnout and re-entry.

It can be shown that time of free flight (t_t) is given by:

$$t_t = (\phi + \epsilon \operatorname{Sin} \phi) \frac{2a^{3/2}}{r_o - g_o} \qquad (32)$$

$$\text{or } t_t = (\phi + \epsilon \operatorname{Sin} \phi)\, 2\sqrt{\frac{r_o}{g_o}}(2 - \rho)^{3/2} \qquad (33)$$

$$\text{where } \operatorname{Cos} \phi = (2 - \rho) \operatorname{Cos} p - \epsilon \qquad (34)$$

$$= (2 - \rho) \operatorname{Cos} p - \sqrt{1 - \rho(2 - \rho) \operatorname{Cos}^2 e_o} \qquad (35)$$

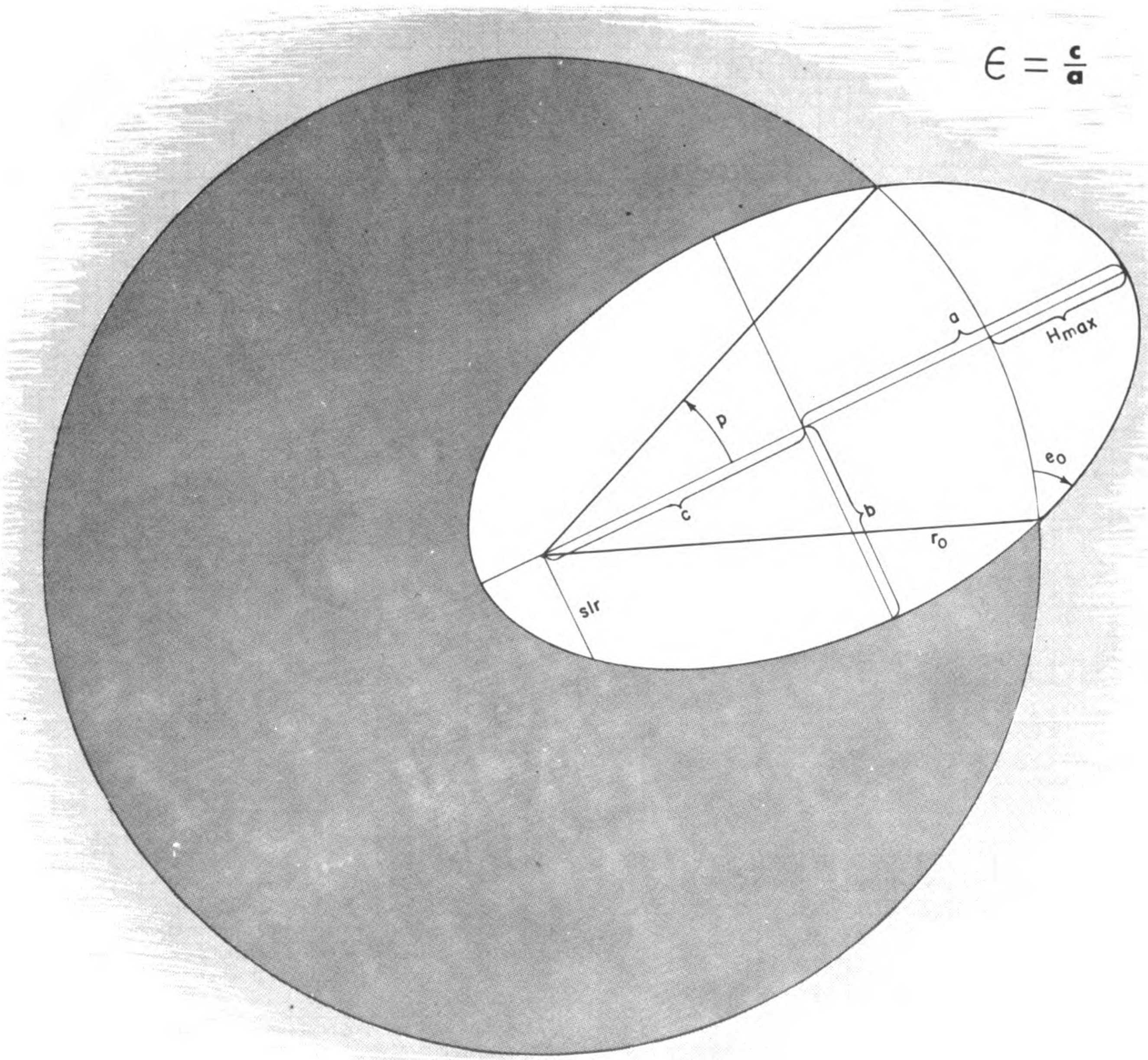
Combining equations (32) and (33) into a single, explicit equation for t_t would be cumbersome. It is better to compute a and ϵ separately and substitute ϵ in (34) to obtain ϕ; then substitute a, ϵ, and ϕ in (32), to obtain quick results. An explicit equation for t_t (cumbersome, but in standard use) can be obtained in terms of p and ϵ. We shall derive this equation when we evaluate free-flight path characteristics in terms of range angle and eccentricity.

Let us first tabulate and summarize free-flight path characteristics in terms of the initial burnout conditions.

CHARACTERISTICS OF FREE-FLIGHT PATH IN TERMS OF INITIAL BURNOUT CONDITIONS:

		From equation:
Half range angle:	$p = \mathrm{Cot}^{-1}\left[\frac{1}{2\rho \,\mathrm{Sin}\, \frac{1}{2} e_o} - \mathrm{Cot}\, e_o\right]$	(21)
Ellipse semi-major axis:	$a = \frac{1}{2-\rho} r_o$	(27)
Ellipse eccentricity:	$\epsilon = \sqrt{1-\rho\,(2-\rho)\mathrm{Cos}^2\, e_o}$	(22)
Ellipse semi-focal distance:	$c = \frac{\epsilon^*}{2-\rho} r_o$	(28)
Ellipse semi-minor axis:	$b = \mathrm{Cos}\, e_o \sqrt{\frac{\rho}{2-\rho}}\, r_o$	(29)
Ellipse semi-latus rectum:	$slr = r_o \rho \mathrm{Cos}^2\, e_o$	(23)
Max. height above burnout:	$h_{max} = \left[\frac{1+\epsilon^*}{2-\rho} - 1\right] r_o$	(30)
Time of free flight:	$t_f = (\phi + \epsilon^* \,\mathrm{Sin}\, \phi) \frac{2a}{r_o g_o}^{3/2}$	(32)
	where $\mathrm{Cos}\, \phi = (2-\rho)\, \mathrm{Cos}\, p - \epsilon^*$	(34)

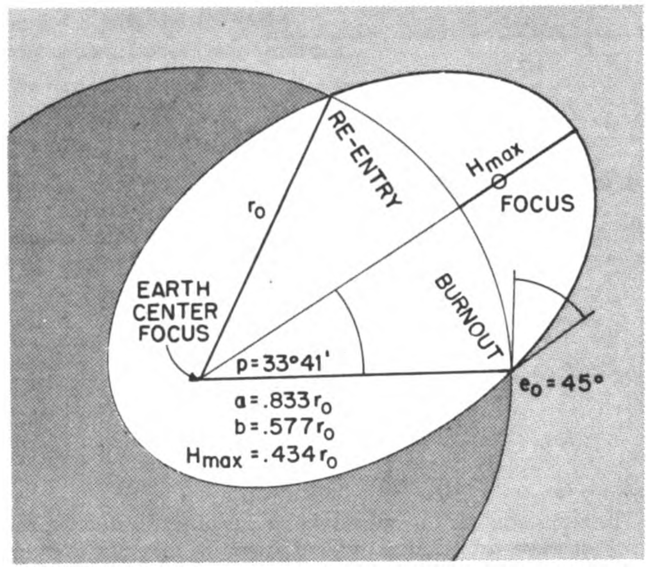
*Substitute for ϵ from (22)



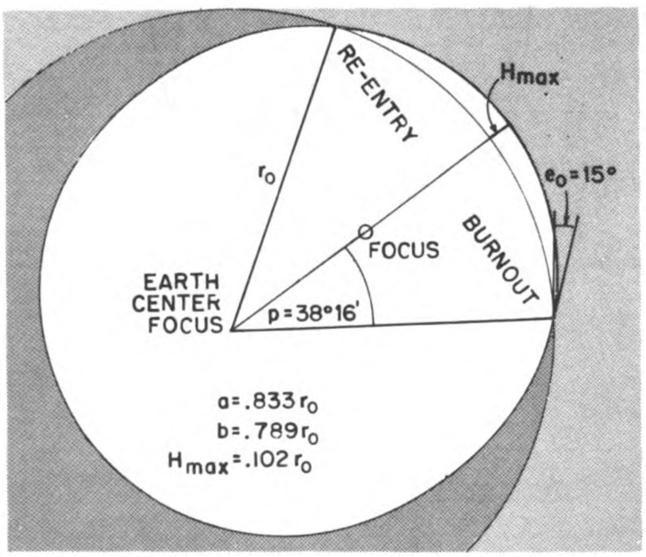
We have now obtained equations for flight path characteristics p, a, ϵ, c, b, slr, and h_{max} in terms of initial burnout conditions and e_0. Here we show four numerical examples of flight paths all having the same ρ ($\rho = 0.8$; i.e. $V_0 = \sqrt{0.8 \times g_0 r_0}$ = approx. 13,700 nm/hr). They have different values for e_0, namely: 15°, 30°, 45°, and 60°, respectively. Two interesting facts are apparent:

(1) All the completed ellipses have equal major axes, in accordance with (27).

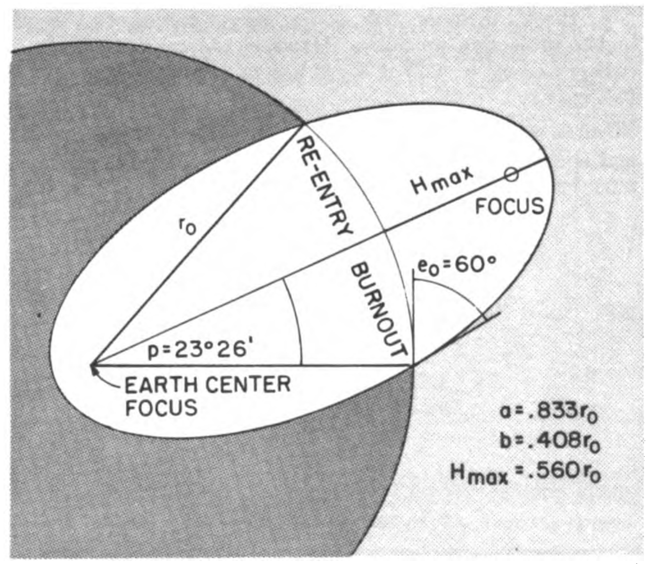
(2) Range is greatest for the path with $e_0 = 30°$. Paths with higher or lower values for e_0 have smaller ranges. Evidently there is an "optimum e_0". We shall return to this question when we discuss "minimum energy paths".



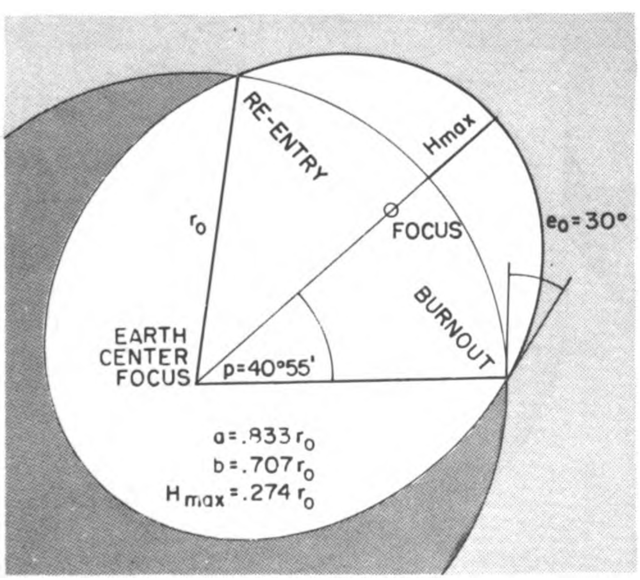
$e_0 = 45°$



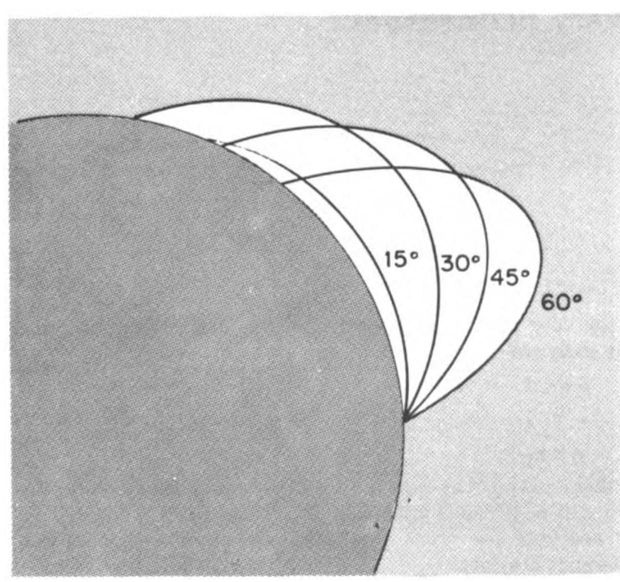
$e_0 = 15°$



$e_0 = 60°$



$e_0 = 30°$



composite view

Now we will apply the table to two interesting special cases: the antipodal target (farthest target on Earth), and the very near target.

antipodal target

If the portions of the path before burnout and after re-entry are ignored, the following path characteristics apply: (see box 6)

$\rho = \text{Sec}^2\, e_o \; (V_o = \text{Sec}\, e_o \sqrt{g_o r_o})$

$\epsilon = \text{Tan}\, e_o$

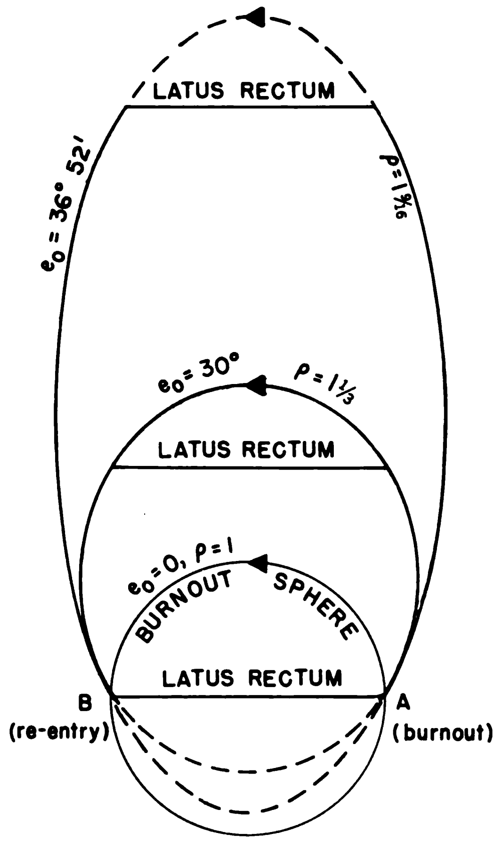
$a = \frac{r_o}{2}\,(\text{Sec}\, 2e_o + 1)$

$b = r_o\sqrt{\frac{\text{Sec}\, 2e_o + 1}{2}}$

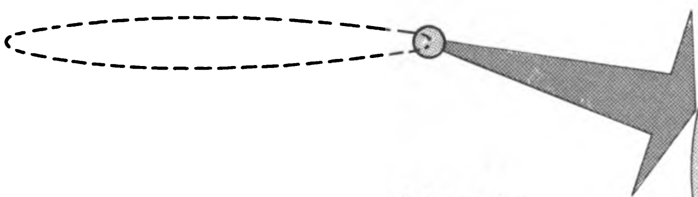
$slr = r_o$

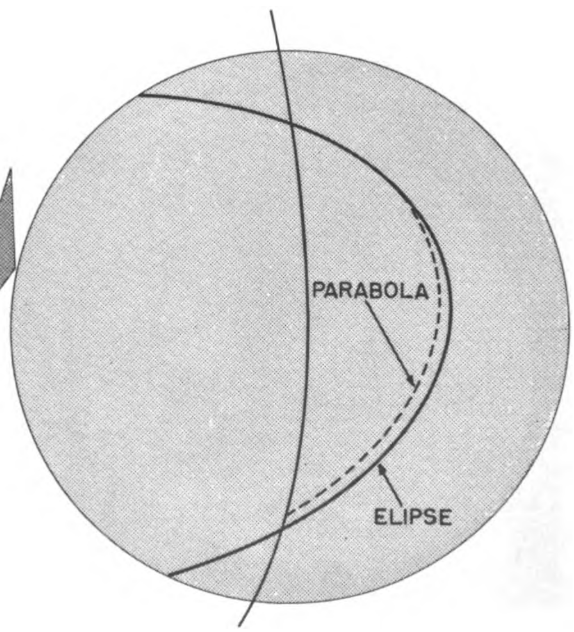
The semi-latus rectum equals the radius of the burnout sphere for all these orbits. It is evident from the diagram that this must be the case, since the line AOB is a latus rectum of each ellipse. The result, therefore, is a family of ellipses corresponding to different values of e_o, all having a common latus rectum. Note the V_o increase with e_o, i.e., larger values of e_o require higher projection velocities. However, e_o cannot exceed 45° (where $\text{Sec}\, e_o = \sqrt{2}$) if V_o is not to exceed escape velocity ($\sqrt{2\, g_o\, r_o}$).

When e_o is 0, V_o is equal to $\sqrt{g_o\, r_o}$, this is the "orbital velocity" and is equal to escape velocity divided by $\sqrt{2}$ (about 15,300 n m /hr).



very near target





For the very near target, $p \approx o$, and the characteristics of the flight path are:

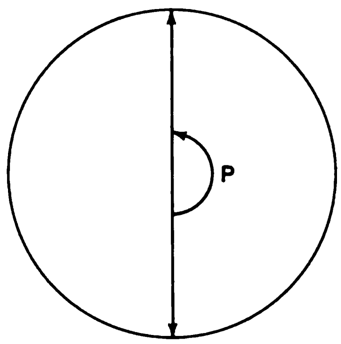
$\rho << 1$

$\epsilon \approx 1$

$a \approx r_o/2.$

The flight path is the extreme tip of a very narrow ellipse, the major axis of which is approximately equal to r_o. The curve is very nearly a parabola; a well-known approximation for a short-range trajectory.

ANTIPODAL TARGET



$p = 90°$, and (21) becomes:

$$0 = \text{Cot } e_o \left[\frac{\text{Sec}^2 e_o}{\rho} - 1\right]$$

If e_o does not equal 90°:

$$\frac{\text{Sec}^2 e_o}{\rho} - 1 = 0$$

$$\rho = \text{Sec}^2 e_o \qquad (6\text{-}1)$$

from (22): $\epsilon = \sqrt{1 - \text{Sec}^2 e_o\,(2 - \text{Sec}^2 e_o)\,\text{Cos}^2 e_o}$

$$= \sqrt{1 - 2 + \text{Sec}^2 e_o}$$

$$\epsilon = \text{Tan } e_o \qquad (6\text{-}2)$$

Therefore, e_o must not exceed 45°, or ϵ will exceed 1, and the missile will "escape".

From (27) and (6-1):

$$\frac{a}{r_o} = \frac{1}{2 - \text{Sec}^2 e_o}$$

$$= \frac{\text{Cos}^2 e_o}{2\,\text{Cos}^2 e_o - 1} \qquad (6\text{-}3)$$

$$= \frac{(1/2)\,(1 + \text{Cos}2e_o)}{\text{Cos}2e_o}$$

$$= \frac{\text{Sec}2e_o + 1}{2} \qquad (6\text{-}4)$$

As e_o approaches 45°, $\text{Sec}2e_o$ approaches infinity; therefore, a approaches infinity.

We can also obtain b, the semi-minor axis:

$$b^2 = a^2 - a^2 \epsilon^2$$

$$\frac{b^2}{r_o^2} = \frac{a^2}{r_o^2} - \frac{a^2 \epsilon^2}{r_o^2}$$

$$= \frac{a^2}{r_o^2}\,(1 - \epsilon^2)$$

$$= \frac{\text{Cos}^4 e_o}{(2\,\text{Cos}^2 e_o - 1)^2}\,(1 - \text{Tan}^2 e_o)$$

$$= \frac{\text{Cos}^2 e_o\,(\text{Cos}^2 e_o - \text{Sin}^2 e_o)}{(\text{Cos}^2 e_o - \text{Sin}^2 e_o)}$$

$$= \frac{\text{Cos}^2 e_o}{\text{Cos}^2 e_o - \text{Sin}^2 e_o}$$

$$= \frac{1 + \text{Cos } 2e_o}{2\,\text{Cos } 2e_o}$$

$$= \frac{\text{Sec } 2e_o + 1}{2}$$

$$\frac{b}{r_o} = \sqrt{\frac{\text{Sec } 2e_o + 1}{2}} \qquad (6\text{-}5)$$

Remember that a and b are both greater than r_o in these orbits, except for the limiting case, discussed further on, where $e_o = 0$, and both a and b equal r_o. We can also obtain the semi-latus rectum, slr.

From (26): $\text{slr} = a\,(1 - \epsilon^2)$

Substitute from (6-2) and (6-3):

$$\frac{\text{slr}}{r_o} = \frac{\text{Cos}^2 e_o}{2\,\text{Cos}^2 e_o - 1}\,(1 - \text{Tan}^2 e_o)$$

$$= \frac{\text{Cos}^2 e_o - \text{Sin}^2 e_o}{2\,\text{Cos}^2 e_o - 1} = \frac{2\,\text{Cos}^2 e_o - 1}{2\,\text{Cos}^2 e_o - 1}$$

$$\frac{\text{slr}}{r_o} = 1 \qquad (6\text{-}6)$$

Another limiting case is when $e_o = 0$.

Then, from (6-1): $\rho = \text{Sec}^2\,0 = 1$

from (6-2): $\epsilon = \text{Tan } e_o = 0$

from (6-4): $\frac{a}{r_o} = \frac{\text{Sec } 0 + 1}{2} = 1$

So, the orbit is a circle on the surface of the burnout sphere.

Note that since $\rho = 1$:

$$V_o^2 = g_o r_o$$

$$V_o = \sqrt{g_o r_o}$$

VERY NEAR TARGET

Here, logic tells us that ρ must be very small. This is confirmed by (21):

$$\text{Cot } e_o \left[\frac{\text{Sec}^2 e_o}{\rho} - 1\right] = \text{Cot p}$$

= cotangent of a very small angle

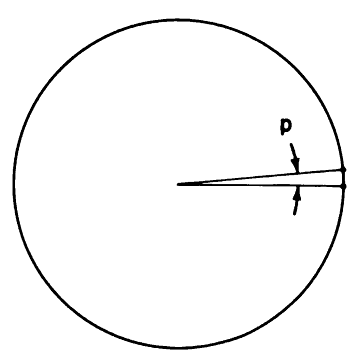
= a very large quantity

Therefore, ρ is very small.

From (22): $\epsilon \approx 1$.

From (27): $a \approx r_o/2$.

$2a \approx r_o$.



RANGE CURVES

We have obtained range angle as a function of the initial burnout conditions, ρ and e_o. Using the cotangent of p, the half range angle, as giving the most convenient expression, we have:

$$\text{Cot } p = (\text{Cot } e_o)\left(\frac{\text{Sec}^2 e_o}{\rho} - 1\right) \qquad \text{from (21)}$$

An interesting set of graphs is obtained if we plot V_o against e_o for different values of p; that is, if we see how V_o varies with e_o for a given range, and do this for several different ranges. The graphs so obtained are called range curves.

To draw a range curve, we must transform (21) to express e_o as a function of V_o, with p considered as a constant:

$$\frac{\text{Sec}^2 e_o}{\rho} - 1 = \text{Cot } p \text{ Tan } e_o$$

$$\text{Sec}^2 e_o = \rho + \rho \text{ Cot } p \text{ Tan } e_o$$

$$1 + \text{Tan}^2 e_o = \rho + \rho \text{ Cot } p \text{ Tan } e_o$$

$$\text{Tan}^2 e_o - \rho \text{ Cot } p \text{ Tan } e_o + (1-\rho) = 0$$

$$\text{Tan } e_o = \frac{\rho \text{ Cot } p + \sqrt{\rho^2 \text{ Cot}^2 p - 4(1-\rho)}}{2} \qquad (37)$$

This quadratic equation gives two real solutions for Tan e_o, provided

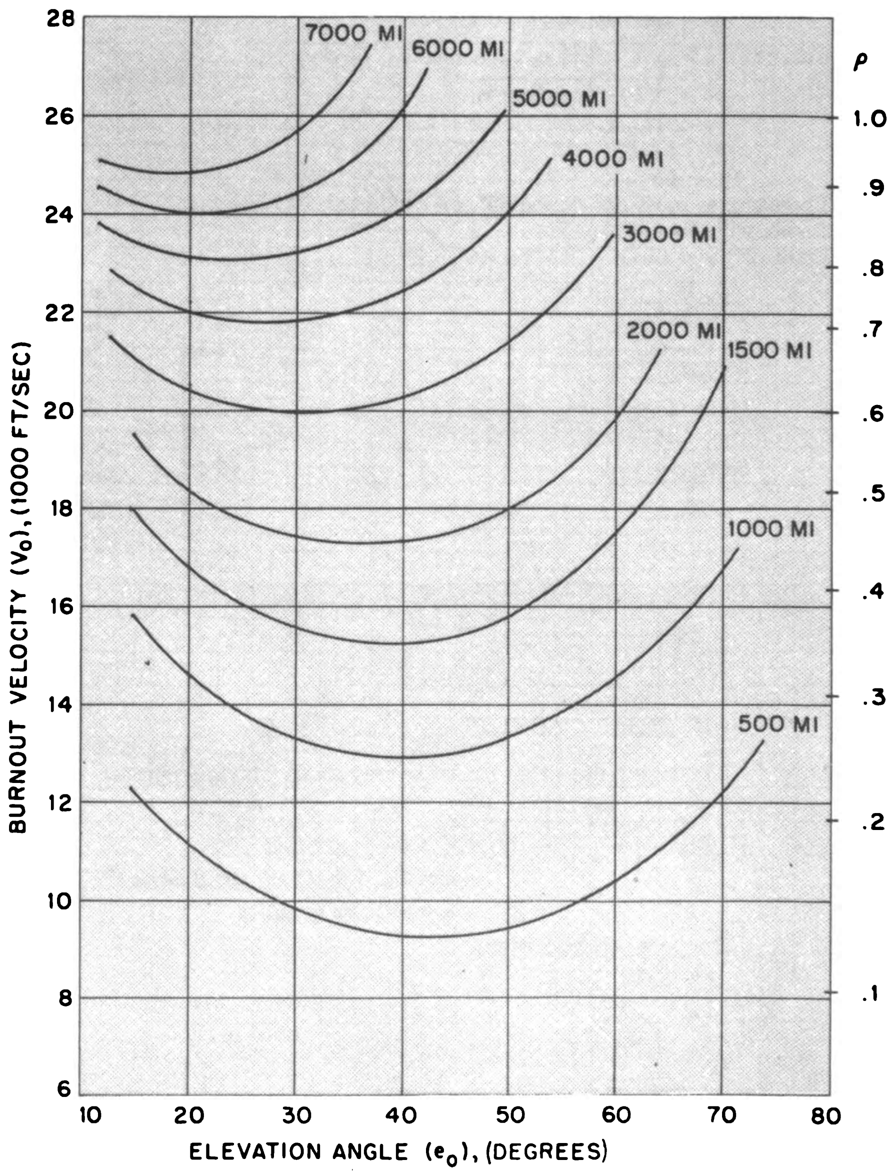
$$\rho^2 \text{ Cot}^2 p - 4(1-\rho) > 0$$

Some range curves are illustrated. Any point on a range curve represents a velocity vector (V_o and e_o) that will cause a missile to traverse the free-flight range indicated on that curve. Note that these curves are for free-flight only. The total range consists of free-flight range plus the ranges of powered flight and re-entry.

Examination of these curves reveals some important points. If elevation angle (e_o) is fixed, the burnout velocity (V_o) for a given range has a fixed value. But the converse is not true. If velocity is fixed, then the elevation angle for a given range has two possible values. For example, the 18,000 ft/sec. velocity line intersects the 2000-mile range at two points: e_o = 23° and e_o = 51°. Thus, with a burnout velocity of 18,000 ft/sec., a missile will travel 2000 miles if it has an elevation angle of 23° or 51°. If e_o is between these values, the range will exceed 2000 miles; if e_o is less than 23° or greater than 51°, the range will be less than 2000 miles. It is common to call the trajectory defined by the smaller elevation angle the "low " trajectory, and the other the "high" or "lofted" trajectory.

Note that increasing the range by changing the elevation angle has its limitations. Suppose that the burnout velocity is 18,000 ft/sec. The greatest range we can obtain (when e_o = 38°) is about 2300 miles. To increase the range further, we must increase the burnout velocity. For very long ranges, very high burnout velocities are required, sometimes exceeding 10,000 mph.

These curves have a special interest when applied to "minimum-energy paths" further on.



explicit flight path characteristics

We shall now compute some new equations, giving the characteristics of a ballistic free-flight path in terms of p (half range angle) and ϵ (ellipse eccentricity).

From (12): $r = \frac{slr}{1-\epsilon \operatorname{Cos}(\theta - p)}$

At burnout, $r = r_o$, and $\theta = 0$:

$$r_o = \frac{slr}{1-\epsilon \operatorname{Cos} p}$$

$$\text{From (26): } r_o = \frac{a\,(1-\epsilon^2)}{1-\epsilon \operatorname{Cos} p} \qquad (40)$$

$$\text{Transposing: } a = \frac{r_o\,(1-\epsilon \operatorname{Cos} p)}{1-\epsilon^2} \qquad (41)$$

$$\text{Therefore: } c = a\epsilon = \frac{r_o\epsilon\,(1-\operatorname{Cos} p)}{1-\epsilon^2} \qquad (42)$$

$$b^2 = a^2 - e^2 = a^2\,(1-\epsilon^2)$$

$$b = \frac{r_o\,(1-\epsilon \operatorname{Cos} p)}{\sqrt{1-\epsilon^2}} \qquad (43)$$

$$\text{From (26): } slr = a\,(1-\epsilon^2) = r_o\,(1-\epsilon \operatorname{Cos} p) \qquad (44)$$

$$\frac{h_{max}}{r_o} = \frac{a + c - r_o}{r_o}$$

$$= \frac{a\,(1+\epsilon)}{r_o} - 1$$

$$\text{From (41): } \frac{a}{r_o} = \frac{1-\epsilon \operatorname{Cos} p}{1-\epsilon^2}$$

$$\text{Substituting: } \frac{h_{max}}{r_o} = \frac{1-\epsilon \operatorname{Cos} p}{1-\epsilon} - 1$$

$$= \frac{\epsilon\,(1-\operatorname{Cos} p)}{1-\epsilon} \qquad (45)$$

We also need to know what velocity and angle of projection at burnout will produce a free-flight path of given range and eccentricity. That is, we must obtain ρ and e_o in terms of p and ϵ.

$$\text{From (27): } \frac{1}{2-\rho} = \frac{a}{r_o}$$

$$= \frac{1-\epsilon \operatorname{Cos} p}{1-\epsilon} \qquad \text{from (40)}$$

$$= \frac{1-2\epsilon \operatorname{Cos} p + \epsilon^2}{1-\epsilon \operatorname{Cos} p} \qquad (46)$$

$$\text{From (23): } \operatorname{Cos}^2 e_o = \frac{slr}{r_o\rho}$$

$$= \frac{1-\epsilon \operatorname{Cos} p}{\rho} \qquad \text{from (44)}$$

$$\text{From (46): } \operatorname{Cos}^2 e_o = \frac{(1-\epsilon \operatorname{Cos} p)^2}{1-2\epsilon \operatorname{Cos} p+\epsilon^2} \qquad (47)$$

The computation of t_f is more elaborate. The following equation can be proved:

$$t_f = 2\sqrt{\frac{r_o^3}{G_{me}}}\left[\frac{1-\epsilon \operatorname{Cos} p}{1-\epsilon^2}\right]^{3/2}\left\{\frac{\epsilon\sqrt{1-\epsilon^2}\operatorname{Sin} p}{1-\epsilon \operatorname{Cos} p} + 2\operatorname{Tan}^{-1}\left[\frac{1+\epsilon}{1-\epsilon}\operatorname{Tan}\frac{p}{2}\right]\right\} \qquad (48)$$

Let us now tabulate and summarize what we have learned as to free-flight path characteristics in terms of half range angle and eccentricity.

FREE-FLIGHT PATH CHARACTERISTICS IN TERMS OF HALF RANGE ANGLE AND ECCENTRICITY

		From equation
Semi-major axis:	$a = \frac{1-\epsilon \operatorname{Cos} p}{1-\epsilon^2} r_o$	(41)
Semi-focal distance:	$c = \frac{\epsilon\,(1-\epsilon \operatorname{Cos} p)}{1-\epsilon^2} r_o$	(42)
Semi-minor axis:	$b = \frac{1-\epsilon \operatorname{Cos} p}{\sqrt{1-\epsilon^2}} r_o$	(43)
Semi-latus rectum:	$slr = (1-\epsilon \operatorname{Cos} p)\, r_o$	(44)
Max. height above burnout:	$h_{max} = \frac{\epsilon\,(1-\operatorname{Cos} p)}{1-\epsilon} r_o$	(45)
Required burnout velocity:	$V_o = \sqrt{\left(\frac{1-2\epsilon \operatorname{Cos} p + \epsilon^2}{1-\epsilon \operatorname{Cos} p}\right) g_o r_o}$	(46)
Required elevation angle:	$\operatorname{Cos} e_o = \frac{1-\epsilon \operatorname{Cos} p}{\sqrt{1-2\epsilon \operatorname{Cos} p + \epsilon^2}}$	(47)
Time of free flight:	$t_f = 2\sqrt{\frac{r_o^3}{Gm_e}}\left[\frac{1-\epsilon \operatorname{Cos} p}{1-\epsilon^2}\right]^{3/2}\left\{\frac{\epsilon\sqrt{1-\epsilon^2}\operatorname{Sin} p}{1-\epsilon \operatorname{Cos} p} + 2\operatorname{Tan}^{-1}\left[\sqrt{\frac{1+\epsilon}{1-\epsilon}}\operatorname{Tan}\frac{p}{2}\right]\right\}$	(48)

MINIMUM ENERGY

In discussing range curves, we establish the fact that there are, in general, two values of elevation angle (e_o) required to reach a given range of half-angle p. These values correspond to the intersections of a range curve by a straight line at a height corresponding to the value of e_o. These values also correspond to the two roots of the quadratic equation:

$$\text{Tan } e_o = \frac{\rho \text{ Cot p} \pm \sqrt{\rho^2 \text{ Cot}^2\text{p} + 4\rho - 4}}{2} \qquad \text{from (37)}$$

The optimum e_o occurs where the square root in (37) equals zero and the equation has "equal roots."

If $\rho^2\text{Cot}^2\text{p} + 4\rho - 4 = 0$, it can be shown that:

$$\rho = 2 \text{ Tan p (Sec p} - \text{Tan p)} \qquad \text{see (38)}$$

Since, in (37), the square root = 0.

$$\text{Tan } e_o = \frac{\rho \text{ Cot p}}{2} \qquad (49)$$

Substituting (38) in (49)

$$\begin{aligned} \text{Tan } e_o &= \frac{\text{Cot p}}{2} \, 2 \text{ Tan p (Sec p} - \text{Tan p)} \\ &= \text{Sec p} - \text{Tan p} \\ &= \frac{1 - \text{Sin p}}{\text{Cos p}} \end{aligned}$$

Let $x = 90° - p$:

$$\begin{aligned} \text{Tan } e_o &= \frac{1 - \text{Cos x}}{\text{Sin x}} \\ &= \text{Tan } \frac{x}{2} \\ e_o &= x/2 \\ e_o &= 45° - \frac{p}{2} \end{aligned} \qquad (50)$$

So the optimum angle of elevation of projection at burnout is obtained by subtracting ¼ of the range angle from 45°.

When elevation angle has this optimum value (45° minus one quarter of the range angle), then, for a given range, required burnout velocity has a minimum value. Hence the term: "minimum-energy path."

Likewise, when elevation angle has the optimum value, then, for a given burnout velocity, the missile has a maximum range. We could just as well speak of a "maximum range path."

For instance, a missile having a burnout velocity of 20,000 ft/sec. would have the maximum range (3000 miles) when elevation angle was about 33 degrees (see range curves).

Note that the value of optimum elevation angle decreases with range. For very small ranges, the optimum angle is 45 degrees. For the opposite extreme (an antipodal target, when p = 90°) the optimum elevation angle is zero. (The missile then describes a circular half-orbit.) Also we can obtain a relation between optimum e_o and ρ:

From (50): $p = 90° - 2e_o$

From (49):

$$\begin{aligned} \text{Tan } e_o &= \frac{\rho \text{ Cot p}}{2} \\ &= \frac{\rho \text{ Tan } 2e_o}{2} \\ \rho &= \frac{2 \text{ Tan} e_o}{\text{Tan } 2e_o} \\ &= 2 \text{ Tan } e_o \, \frac{1 \text{ Tan}^2 e_o}{2 \text{ Tan } e_o} \end{aligned}$$

$$\text{So: } \rho = 1 - \text{Tan}^2 e_o \qquad (51)$$

From (38) and Box (7): $\rho = \dfrac{2 \text{ Sin p}}{1 + \text{Sin p}}$

Conversely:

$$\text{Sin p} = \frac{\rho}{2 - \rho} \qquad (52)$$

(This gives maximum range with a given burnout velocity) (see Box 7)

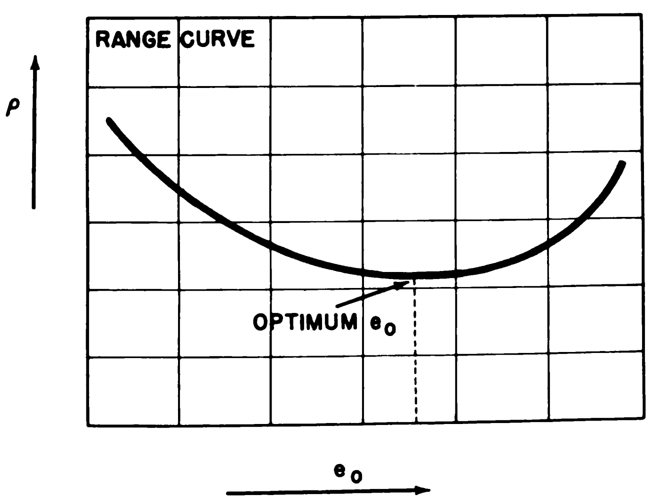
$$= \frac{V_o^2}{2 \, g_o r_o - V_o^2}$$

From (50):

$$p = 90° - e_o \qquad (53)$$

(This gives maximum range with a given e_o)

We shall now proceed to obtain the characteristics including time of free flight of the completed ellipse of a minimum energy path, in terms of range;

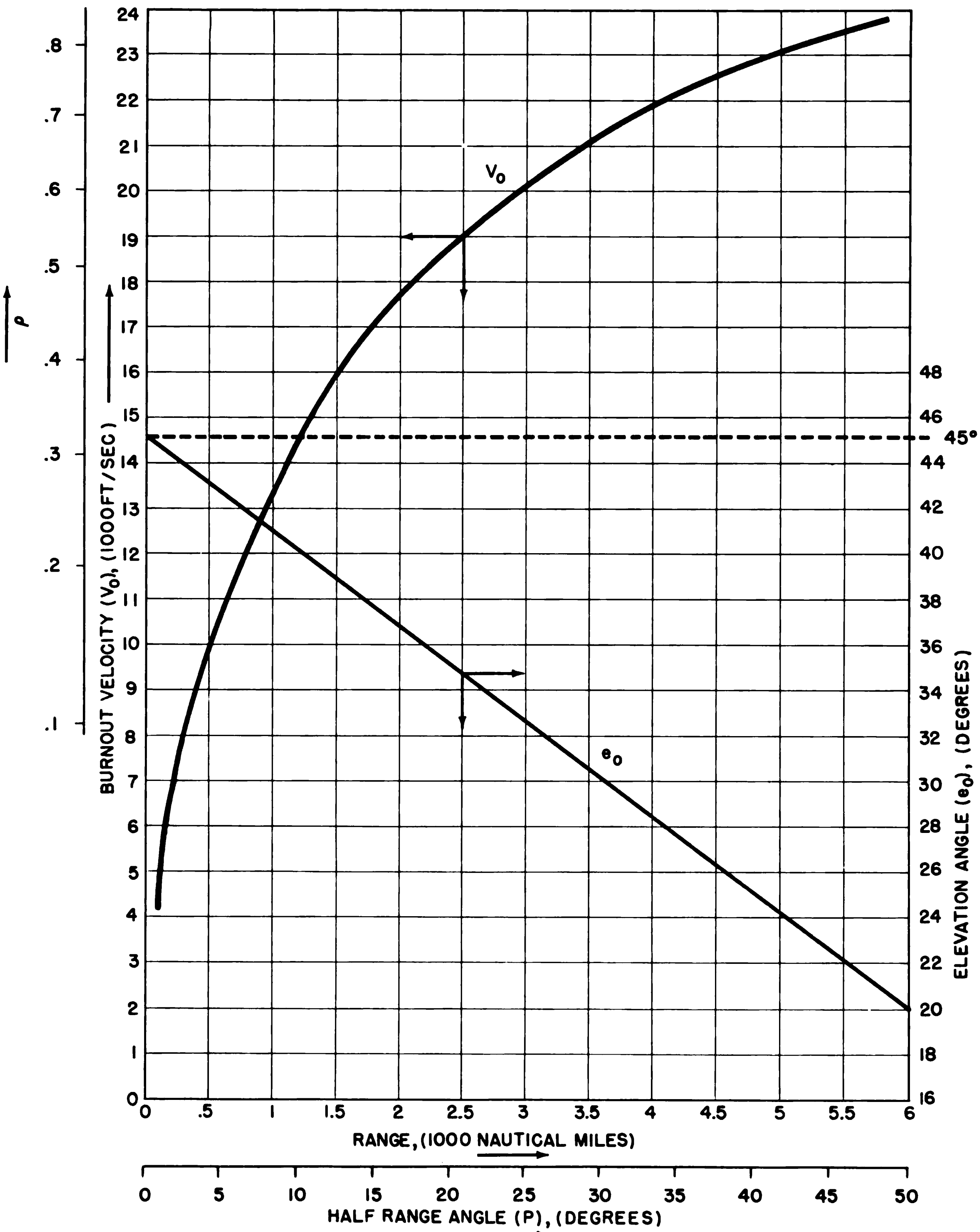


(7)

From (38):

$$\begin{aligned} \rho &= 2 \text{ Tan p (Sec p} - \text{Tan p)} \\ &= \frac{2 \text{ Sin p}}{\text{Cos p}} \left[\frac{1}{\text{Cos p}} - \frac{\text{Sin p}}{\text{Cos p}} \right] \\ &= 2 \text{ Sin p} \left[\frac{1 - \text{Sin p}}{\text{Cos}^2 \text{ p}} \right] \\ &= 2 \text{ Sin p} \left[\frac{1 - \text{Sin p}}{1 - \text{Sin } 2\text{p}} \right] \\ &= \frac{2 \text{ Sin p}}{1 + \text{Sin p}} \end{aligned} \qquad (7\text{-}1)$$

$$\begin{aligned} \rho + \rho \text{ Sin p} &= 2 \text{ Sin p} \\ \text{Sin p } (2 - \rho) &= \rho \\ \text{Sin p} &= \frac{\rho}{2 - \rho} = \frac{V_o^2}{2 \, g_o r_o - V_o^2} \end{aligned} \qquad (7\text{-}2)$$



burnout velocity and elevation angle vs. range and half-range angle for minimum energy paths

elipse characteristics

From (27) $\frac{a}{r_o} = \frac{1}{2-\rho}$

$= \frac{1}{1 + \text{Tan}^2 e_o}$ from (51)

$= \text{Cos}^2 e_o$

$= \frac{1 + \text{Cos } 2e_o}{2}$

From (53), Cos $2e_o$ = Sin p. Thus:

$$\frac{a}{r_o} = \frac{1 + \text{Sin p}}{2} \quad (54)$$

From (22): $\epsilon^2 = 1-\rho\,(2-\rho)\,\text{Cos}^2 e_o$

From which we prove: $= \frac{1-\text{Sin p}}{\text{Cos p}} = \text{Sec p} - \text{Tan p}$ (55)

$= \text{Tan } e_o$ (56)

(See box 8)

Also: $\frac{c}{r_o} = \frac{a\epsilon}{r_o}$

From (54) and (55): $\frac{c}{r_o} = \left[\frac{1 + \text{Sin p}}{2}\right]\left[\frac{1 - \text{Sin p}}{\text{Cos p}}\right]$

$= \frac{1 - \text{Sin}^2 p}{2\,\text{Cos p}}$

$= \frac{\text{Cos}^2 p}{2\,\text{Cos p}}$

$$\frac{c}{r_o} = \frac{\text{Cos p}}{2} \quad (57)$$

Also: $\frac{b^2}{r_o^2} = \frac{a^2}{r_o^2} - \frac{c^2}{r_o^2}$

$= \frac{(1 + \text{Sin p})^2}{4} - \frac{\text{Cos}^2 p}{4}$

$= \frac{1 + 2\,\text{Sin p} + \text{Sin}^2 p - \text{Cos}^2 p}{4}$

$= \frac{2\,\text{Sin p} + 2\,\text{Sin}^2 p}{4}$

$$\frac{b}{r_o} = \sqrt{\frac{\text{Sin p}\,(1 + \text{Sin p})}{2}}$$

Also:

From (26): $\frac{slr}{r_o} = \frac{a\,(1-\epsilon^2)}{r_o} = \text{Sin p}$ (59)

(See box 9)

Also: $\frac{h_{max}}{r_o} = \frac{a + e - r_o}{r_o}$

$= \frac{1 + \text{Sin p}}{2} + \frac{\text{Cos p}}{2} - 1$

$= \frac{1 + \text{Sin p} + \text{Cos p} - 2}{2}$

$$\frac{h_{max}}{r_o} = \frac{\text{Sin p} + \text{Cos p} - 1}{2} \quad (60)$$

It will be observed that the expressions for a, c, b, slr, and h_{max} for minimum energy paths are simpler than for the general case. This is partly because we only need one parameter, p.

8

$\epsilon^2 = 1-\rho\,(2-\rho)\,\text{Cos}^2 e_o$

$= 1-(1-\text{Tan}^2 e_o)\,(1+\text{Tan}^2 e_o)\,\text{Cos}^2 e_o$

$= 1-(1-\text{Tan}^2 e_o)\,\text{Sec}^2 e_o\,\text{Cos}^2 e_o$

$= 1-(1-\text{Tan}^2 e_o)$

$= \text{Tan}^2 e_o, \left[\epsilon = \text{Tan } e_o\right]$

$= \frac{1-\text{Cos } 2\,e_o}{1+\text{Cos } 2\,e_o}$

$= \frac{1-\text{Sin p}}{1+\text{Sin p}}$ from (53)

$\epsilon^2 = \frac{(1-\text{Sin p})^2}{1-\text{Sin}^2 p} = \frac{(1-\text{Sin p})^2}{\text{Cos}^2 p}$

$\epsilon = \frac{1-\text{Sin p}}{\text{Cos p}} = \text{Sec p}-\text{Tan p}$

9

From (26), (54) and (55):

$\frac{slr}{r_o} = \frac{a}{r_o}\,(1-\epsilon^2) = \frac{1+\text{Sin p}}{2}\left[1-\frac{1-2\,\text{Sin p}+\text{Sin}^2 p}{\text{Cos}^2 p}\right]$

$= \left[\frac{1+\text{Sin p}}{2}\right]\left[\frac{\text{Cos}^2 p-1+2\,\text{Sin p}-\text{Sin}^2 p}{\text{Cos}^2 p}\right]$

$= \left[\frac{1+\text{Sin p}}{2}\right]\left[\frac{2\,\text{Sin p}-2\,\text{Sin}^2 p}{\text{Cos}^2 p}\right]$

$= \left[\frac{1+\text{Sin p}}{2}\right]\left[\frac{2\,\text{Sin p}\,(1-\text{Sin p})}{\text{Cos}^2 p}\right]$

$= \text{Sin p}\,\frac{(1-\text{Sin}^2 p)}{\text{Cos}^2 p}$

$\frac{slr}{r_o} = \text{Sin p}$

time-of-flight

It remains to compute t_f, the time of free flight. Attempts to combine the special data for minimum energy paths with the explicit equation (48) are unfruitful. On the other hand, equations (32) and (34), for minimum energy paths, lend themselves very well toward obtaining a quick solution for t_f (see box 10):

Neglecting altitude of burnout point above Earth:

$$t_f = \frac{84.5}{\pi}\,\text{Cos}^3 e_o\,(\phi + \epsilon\,\text{Sin }\phi) \text{ min.} \quad (61)$$

where $\text{Cos }\phi = \text{Tan } e_o$ (62)

and $\epsilon = \text{Tan } e_o$ (See 56)

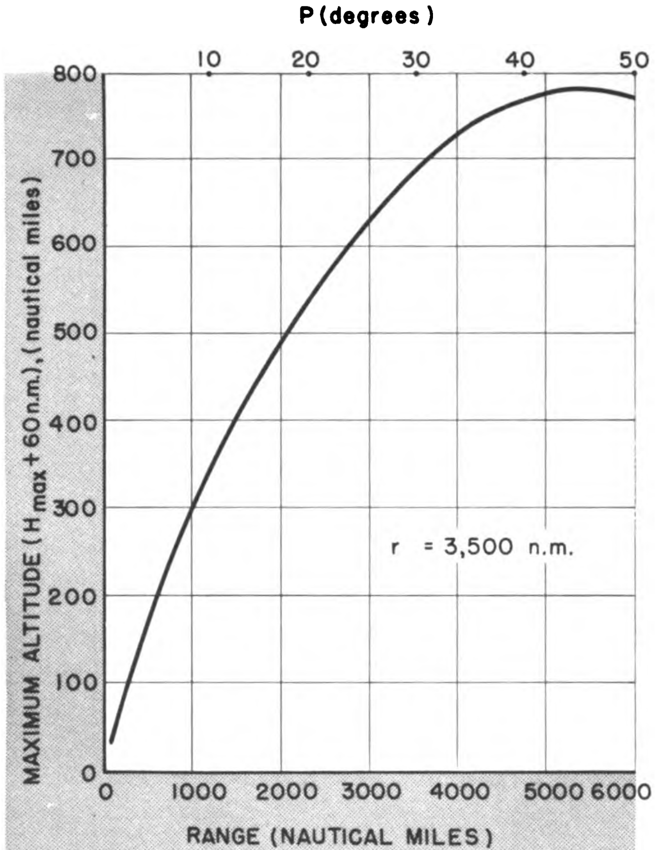
When the correct value for e_o is substituted, these three equations give a much quicker solution for t_f than any explicit equation could. Note the simplification we have achieved by using e_o instead of p. This is justified in minimum energy paths because e_o is instantly obtainable from range; see (50):

$$e_o = 45° - \frac{p}{2}$$

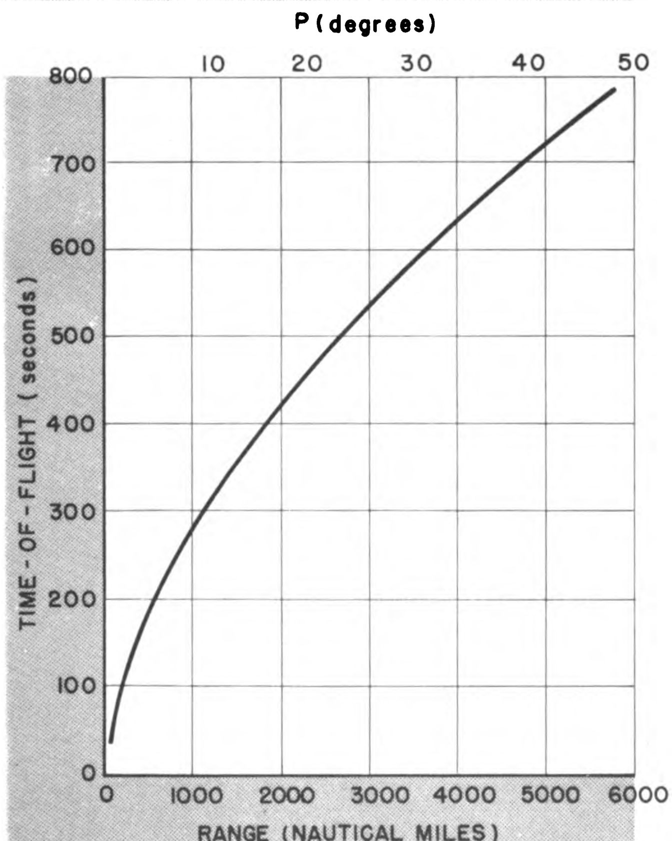
When the burnout point is so high that the difference between r_o and r_e cannot be ignored, then (referring to 32) it is evident that we can use equation (61) only if we multiply the value of t_f so obtained by

$$\left[\frac{r_o}{r_e}\right]^{3/2}$$

A curve representing free-flight time for minimum energy trajectories vs range is shown



maximum in altitude above ground for minimum energy paths



time of flight for minimum energy paths

10

$$t_f = (\phi + \sin\phi)\,\frac{2a^{3/2}}{r_o\sqrt{g_o}} \qquad \text{from (32)}$$

$$= (\phi + \sin\phi)\,\frac{2a^{3/2}}{Gm_e} \qquad \text{see (7)} \qquad (10\text{-}1)$$

where $\cos p = (2-\rho)\cos p - \epsilon$ from (34)

We will first transform (10-1) slightly:

For minimum energy paths:

$$\frac{a}{r_o} = \frac{1 + \sin p}{2} \qquad \text{see (54)}$$

$$a^{3/2} = \left[\frac{1 + \sin p}{2}\right]^{3/2} r_o^{3/2} \qquad (10\text{-}2)$$

Here, it is convenient to re-introduce e_o.

From (53): $\dfrac{1 + \sin p}{2} = \dfrac{1 + \cos 2e_o}{2}$

$$= \cos^2 e_o$$

$$\left[\frac{1 + \sin p}{2}\right]^{3/2} = \cos^3 e_o \qquad (10\text{-}3)$$

Substitute (10-2) and (10-3) in (10-1):

$$t_f = (\phi + \sin\phi)\,\frac{2\,r_o^{3/2}}{\sqrt{Gm_e}}\cos^3 e_o \qquad (10\text{-}4)$$

A still further simplification can be made in the computation of ϕ:

From (51): $\rho = 1 - \tan^2 e_o$

$2-\rho = 1 + \tan^2 e_o = \sec^2 e_o$

From (53): $\cos p = \sin 2e_o$

From (56): $\epsilon = \tan e_o$ (10-5)

Substitute these last expressions for $2-\rho$, $\cos p$, and ϵ in (34):

$$\cos\phi = \sec^2 e_o \sin 2e_o - \tan e_o$$

$$= 2\tan e_o - \tan e_o$$

$$= \tan e_o \qquad (10\text{-}6)$$

Now, let $e_o = 0$. Then, $p = 90°$; that is, range angle $= 180°$. This is the case, already discussed, of "orbital flight" parallel to the surface of Earth to an antipodal target.

Substituting $e_o = 0$ in (10-5) and (10-6):

$\epsilon = 0$

$\cos\phi = 0$

$\phi = \frac{\pi}{2}$ radians

Substitute these values of ϵ and ϕ in (10-4):

$$t_f = (\tfrac{\pi}{2} + 0)\,\frac{2r_o^{3/2}}{\sqrt{Gm_e}}$$

$$= \frac{\pi\, r_o^{3/2}}{\sqrt{Gm_e}} \qquad (10\text{-}7)$$

This is the time of a half-orbit. If the burnout point is not too high, we can equate r_o with the radius of Earth (r_e). Then the time can readily be shown to be approximately 42.25 min.

$$\frac{\pi\, r_o^{3/2}}{\sqrt{Gm_e}} = 42.25 \text{ min}$$

$$\frac{2\, r_o^{3/2}}{\sqrt{Gm_e}} = \frac{84.5}{\pi} \qquad (10\text{-}8)$$

Now, substitute (10-8) in (10-2) and we have:

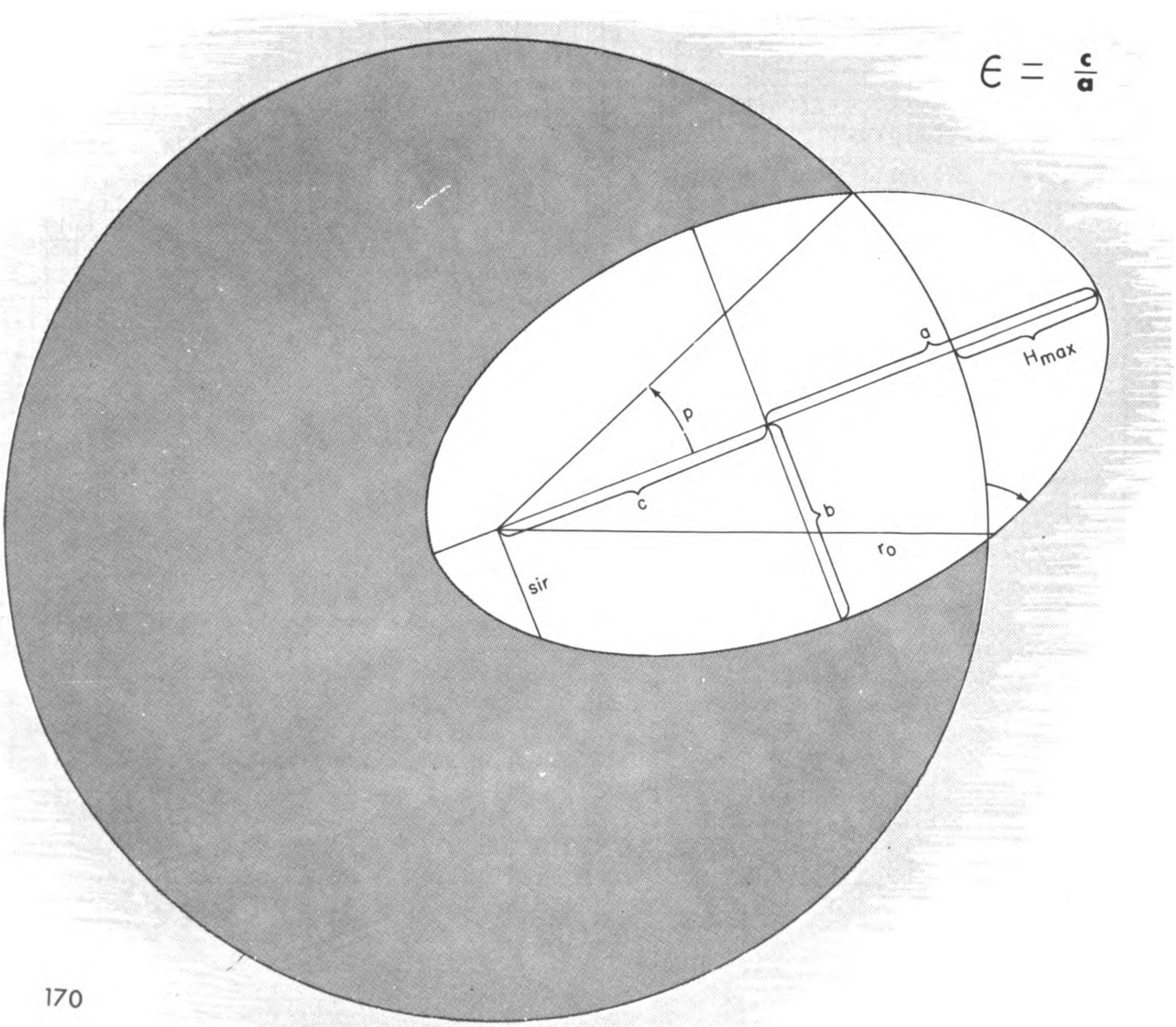
$$t_f = \frac{84.5}{\pi}\cos^3 e_o\,(\phi + \epsilon\sin\phi) \text{ minutes} \qquad (10\text{-}9)$$

where $\cos\phi = \tan e_o$ see (10-6)

and $\epsilon = \tan e_o$

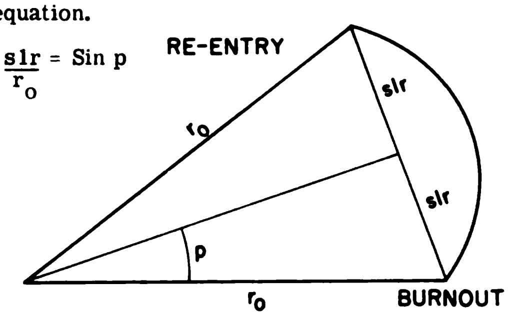
CHARACTERISTICS OF MINIMUM-ENERGY FREE-FLIGHT PATHS

		From equation
Minimum ρ for a given range:	$\rho = \dfrac{2 \operatorname{Sin} p}{1 + \operatorname{Sin} p}$	(38)
	$\rho = 1 - \operatorname{Tan}^2 e_o$	(51)
Maximum range with a given ρ:	$\operatorname{Sin} p = \dfrac{\rho}{2 - \rho}$	(52)
Minimum burnout velocity for a given range:	$V_o = \sqrt{g_o\, r_o\, (1 - \operatorname{Tan}^2 e_o)}$	
Maximum range with a given burnout velocity:	$\operatorname{Sin} p = \dfrac{V_o^2}{2\, g_o\, r_o - V_o^2}$	
Optimum elevation angle:	$e_o = 45° - \dfrac{p}{2}$	(50)
Maximum range with a given elevation angle:	$p = 90° - 2\, e_o$	(53)
Semi-major axis:	$a = \dfrac{1 + \operatorname{Sin} p}{2}\, r_o$	(54)
Eccentricity:	$\epsilon = \dfrac{1 - \operatorname{Sin} p}{\operatorname{Cos} p}$	(55)
Semi-latus rectum:	$slr = r_o \operatorname{Sin} p$	
Maximum height:	$h_{max} = \dfrac{\operatorname{Sin} p + \operatorname{Cos} p - 1}{2}\, r_o$	(60)
Time of free flight:	$t_f = \dfrac{84.5}{\pi} \operatorname{Cos}^3 e_o\, (\phi + \epsilon \operatorname{Sin} \phi) \left[\dfrac{r_o}{r_e}\right]^{3/2}$ min.	
	where $\operatorname{Cos} \phi = \operatorname{Tan} e_o$	

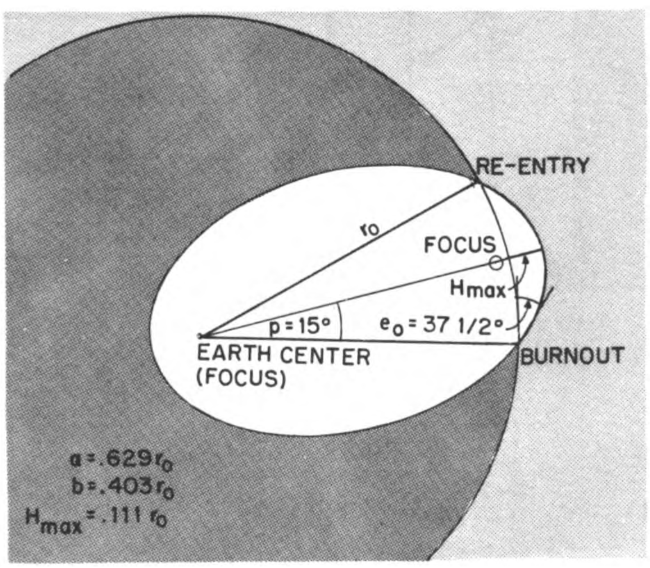


Some minimum energy paths are shown for range angles of 30°, 60°, 90°, and 120°.
Note that in each case the burnout and re-entry points lie at the extremities of a latus rectum. This follows from the equation.

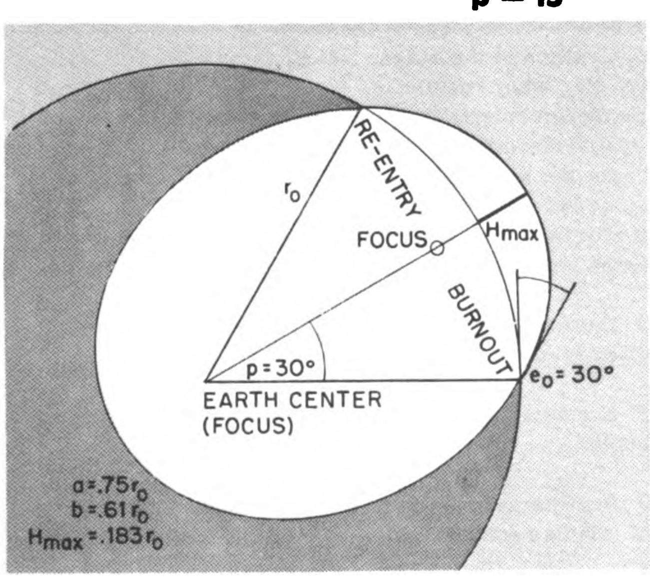
$$\frac{slr}{r_o} = \text{Sin } p$$



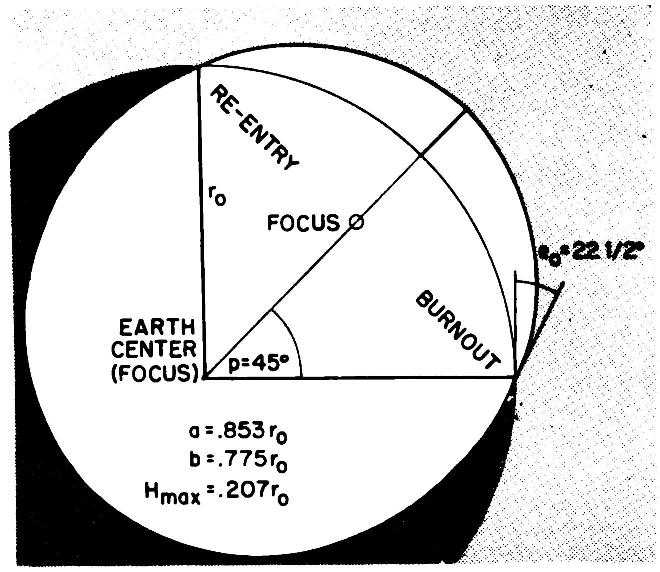
In other words, the second focus of the ellipse is half-way between the burnout point and the re-entry point, making minimum energy paths easy to construct geometrically. It also will be noticed that the path with a 90-degree range angle (p=45°) goes higher than any of the others.



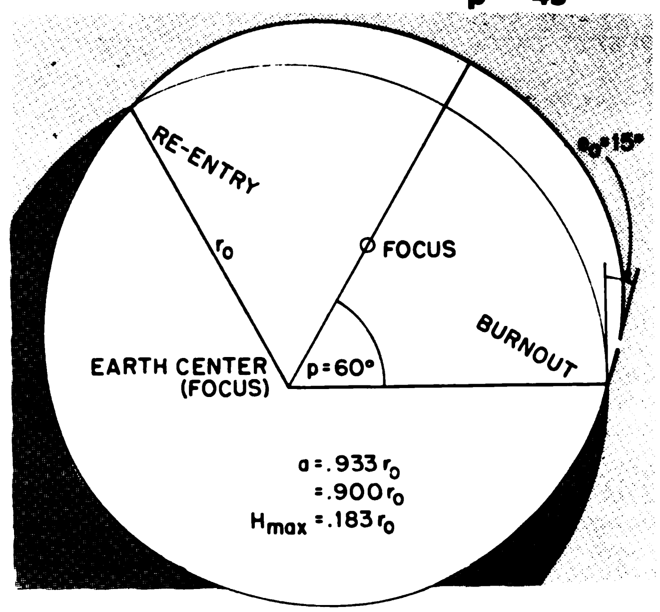
p = 15°



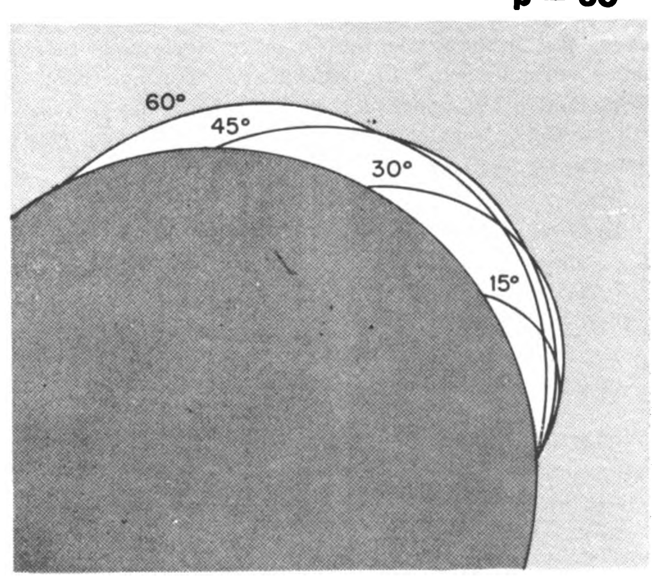
p = 30°



p = 45°



p = 60°



composite view

712-591 O - 63 - 12

The advantage of using a minimum energy flight path to direct a ballistic missile to a given destination is obvious: economy of fuel. However, there are other considerations. If we look again at the range curves, we see that the "operating point" for a minimum energy path is the point at the bottom of a range curve, which gives us no margin for error. Suppose that a range of 3000 nautical miles is desired; that is, a range angle of 50° (1 nautical mile = 1 minute of arc).
Then:

$$p = 25°$$

$$e_0 = 45° - \frac{25°}{2} = 32.5°$$

$$\text{Required } \rho = 1 - \tan^2 32.5°$$

$$= .594$$

$$\text{Required } V_0 = 25884 \sqrt{.594}$$

$$= 19{,}930 \text{ ft/sec.}$$

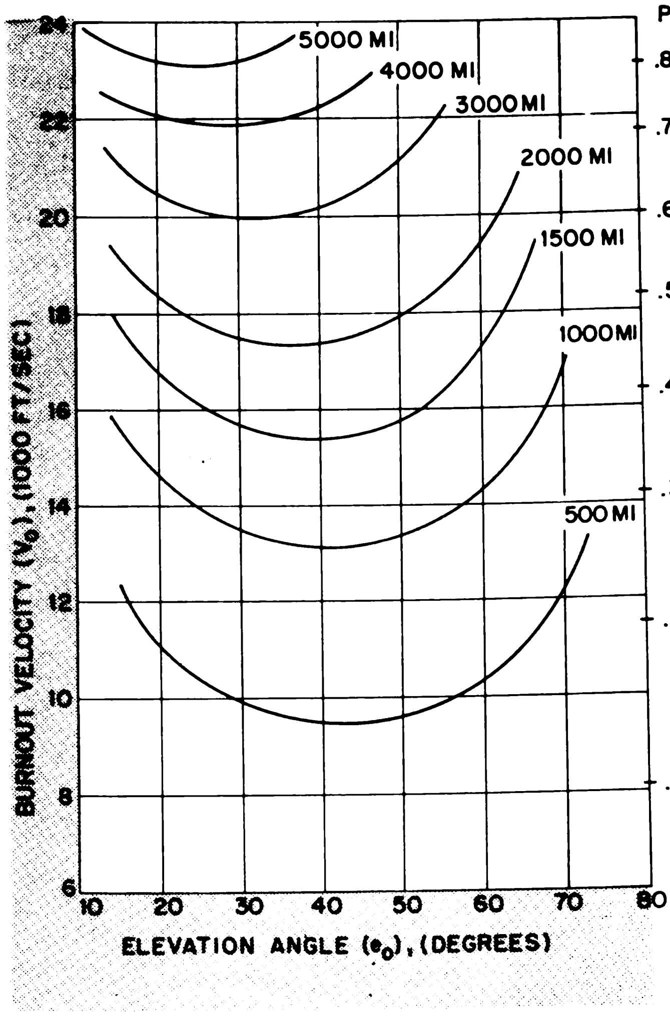
These values of e_0 and V_0 are confirmed by the graph shown. Now, suppose that the missile (aimed upward at 32.5°) fails to attain this velocity, and achieves a velocity of only 16,000 ft/sec. at burnout. Then it would travel only about 1700 miles, falling short of the target.

Consider a missile with a free-flight range of 1500 nautical miles. By computation similar to the above:
For a minimum energy path:

$$\text{Required } e_0 = 38.75°$$

$$\text{Required } V_0 = 15{,}450 \text{ ft/sec.}$$

For a more practical path, we select a burnout velocity a little greater than the required minimum: V_0 = 16,000 ft/sec. At this velocity a horizontal line on the graph intersects the 1500-mile range curve in two places. This means that we can obtain the same range with two angles of elevation: 26.5° and 52° (corresponding to the two roots of the quadratic equation for Tan e_0). The 52-degree path carries the missile further out of Earth's atmosphere, resulting in less aerodynamic drag. The 26-degree path is shorter and has a shorter horizon, providing less time for the enemy to take counter-measures. But on the whole, the "high" trajectory is the more advantageous. Its comparative freedom from drag, and its steeper reentry path (resulting in less "dispersion") outweigh the advantages of the shorter horizon of the "low" trajectory.



summary

The ballistic free flight path, i.e., that followed by a ballistic missile travelling in vacuum, is a portion of an ellipse with on focus at the center of Earth. The characteristics of the ellipse (axes, focal distance, eccentricity, latus rectum, as well as time-of-flight and maximum height) have been given in terms of: 1) velocity and elevation angle at burnout, and 2) half-range angle and eccentricity. All free-flight paths with the same burnout velocity have equal major axes.
Analysis of the "minimum energy" or "maximum range" paths show that three important laws govern these paths:

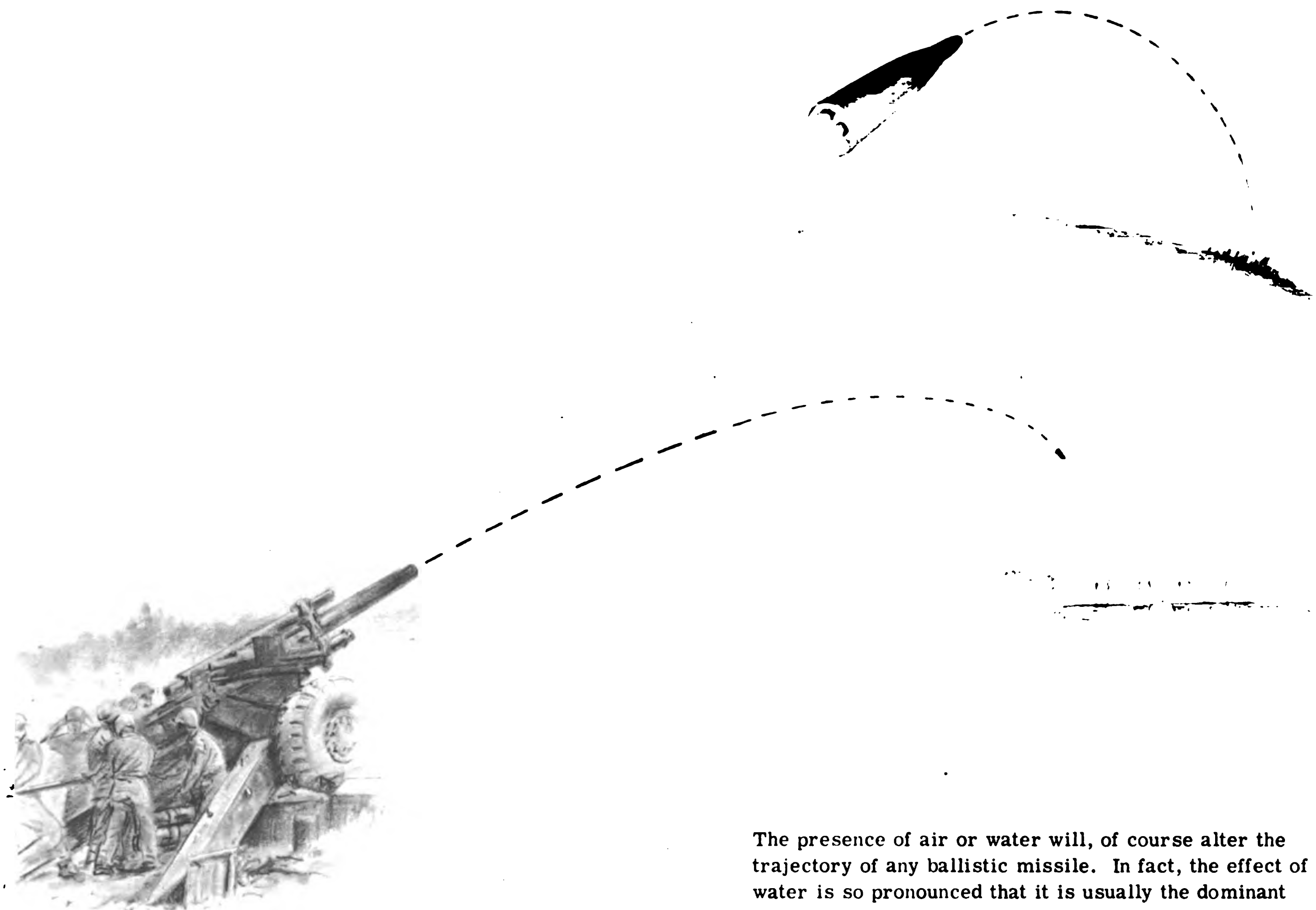
1 Burnout elevation angle equals 45 degrees less one quarter of the range angle.

2 Eccentricity equals the tangent of burnout elevation angle.

3 Burnout and re-entry points are at the extremities of a latus rectum; that is, the second focus lies halfway between them.

SECTION 2

BALLISTIC TRAJECTORIES IN THE ATMOSPHERE



The presence of air or water will, of course alter the trajectory of any ballistic missile. In fact, the effect of water is so pronounced that it is usually the dominant force. This section, therefore, is limited to ballistic trajectories in air.

Two particular problems are covered: the ballistic missile re-entry flight path, and the short range ballistic missile in the atmosphere (gun projectile).

In all cases, methods of approximation, rather than exact calculations, are given.

Little space is devoted to the gun projectile problem beyond the basic approximations. The many refinements of the basic equations, including aerodynamic effects other than drag, are covered in detail in many books on gunnery.

RE-ENTRY FLIGHT PATHS

The objective in launching a ballistic missile is not to reach a given re-entry point, but to hit a target which is presumably on or near the ground. Therefore, the portion of the trajectory which is within the atmosphere must be considered along with the free flight trajectory.

Any calculation of a ballistic flight path must be worked backwards. First the free flight and re-entry trajectories must be known, so that the burnout point, and the flight path to the burnout point, can be calculated. Therefore, the re-entry path will be discussed first. The curvature of the earth has only a minor effect over the ranges involved and will be ignored for the sake of simplicity.

The beginning of the re-entry path is arbitrarily chosen at the same altitude as burnout. Thus, the re-entry velocity and re-entry angle are the same as the burnout velocity and burnout angle. Once the missile re-enters the atmosphere, aorodynamic forces of lift and drag and gravitational attraction act on it. These forces can be resolved into components along the velocity vector, and perpendicular to the velocity vector. For the small angle of attack normally assumed by a re-entering ballistic missile, the coefficient of lift (C_l) is very small. Tests have shown that the lift remains nearly equal to the perpendicular component of gravitational attraction throughout the early part of the re-entry flight path; the trajectory is a straight line.

Above 200,000 feet the density of the atmosphere is so low (less than 3/10,000 of the density at the surface of Earth) that the effect of drag on the trajectory is very slight. However, there is sufficient drag to reorient the missile so that the body axis is parallel to the velocity vector. This reorientation is necessary since, while travelling in a vacuum, the missile atitude will remain the same as at burnout. Reorientation is usually accomplished by the use of skirts at the rear of the missile, or by side jets which rotate the missile during free flight.

To determine the flight path at lower altitudes, a knowledge of the behavior of the drag is required (see box). During the early phase of re-entry, the drag remains small and nearly constant under the conflicting effects of decreasing altitude and decreasing velocity. As the altitude decreases, the atmospheric density increases, tending to increase the drag. At the same time, the velocity decreases, tending to reduce the drag. From the time of re-entry until the time the missile falls below 80,000 feet, these effects counterbalance one another, and the drag remains very low. Below 80,000 feet the atmosphere rapidly becomes denser, and the drag becomes very great. The increased drag causes the velocity to fall off sharply to a point where the drag begins to lessen. At this point the gravitational attraction becomes the dominant force, and the missile begins to fall at a steeper angle.

The equation for aerodynamic drag is:

$$\text{Drag} = \frac{C_d A}{2}\,\rho\, V^2$$

The atmospheric density (ρ), as a function of altitude (h) is:

$$\rho \approx \rho_0\, e^{-\sigma h} \qquad \sigma = 41 \times 10^{-6}/\text{ft.}$$

ρ_0 = Density at the surface of Earth

The velocity as a function of altitude can be approximated by:

$$V = V_0 \sqrt{\frac{h}{h_0}}$$

This rough approximation will be used only for a qualitative study of the drag.

The various constants can be combined to produce a single constant (K).

$$\text{Drag} = \frac{C_d A}{2}\,\rho_0 e^{-\sigma h}\left(V_0\sqrt{\frac{h}{h_0}}\right)^2$$

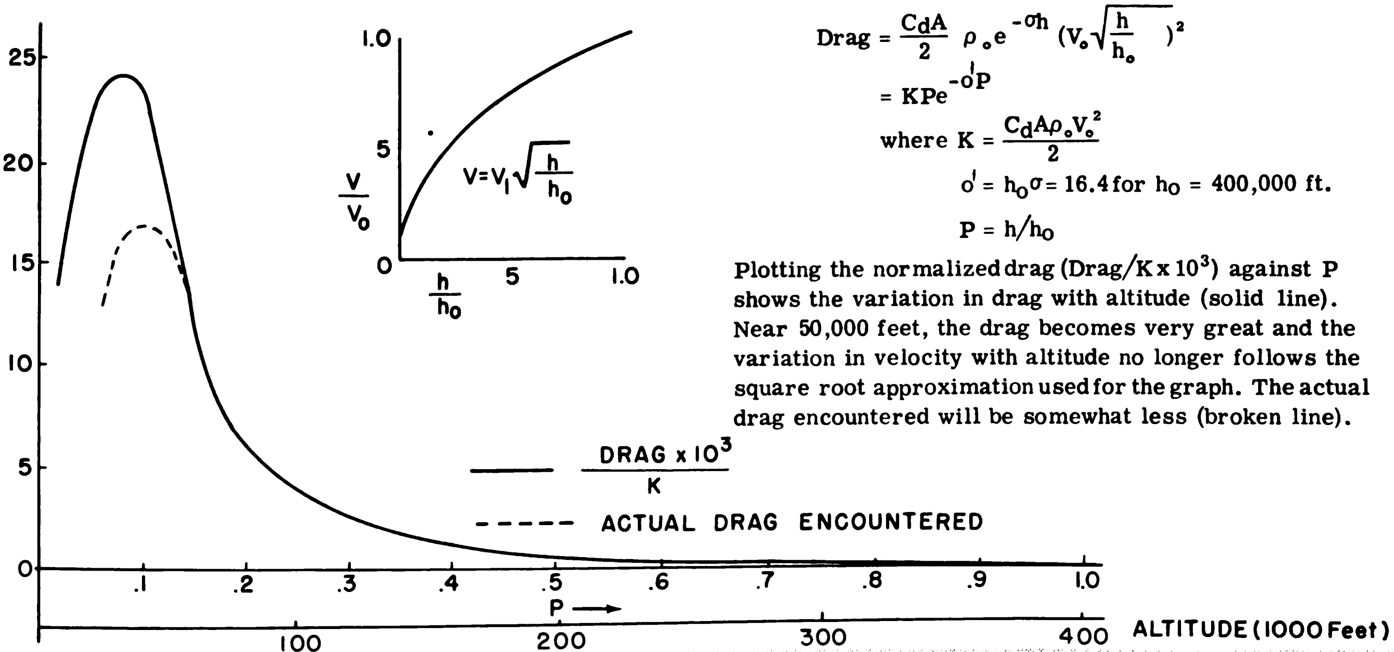
$$= K P e^{-\sigma' P}$$

$$\text{where } K = \frac{C_d A \rho_0 V_0^2}{2}$$

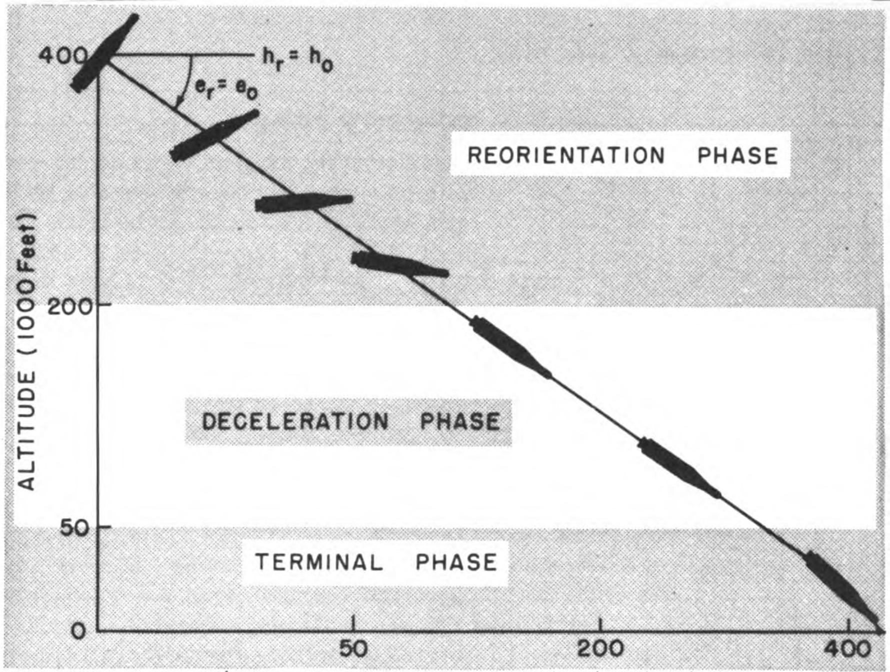
$$\sigma' = h_0 \sigma = 16.4 \text{ for } h_0 = 400{,}000 \text{ ft.}$$

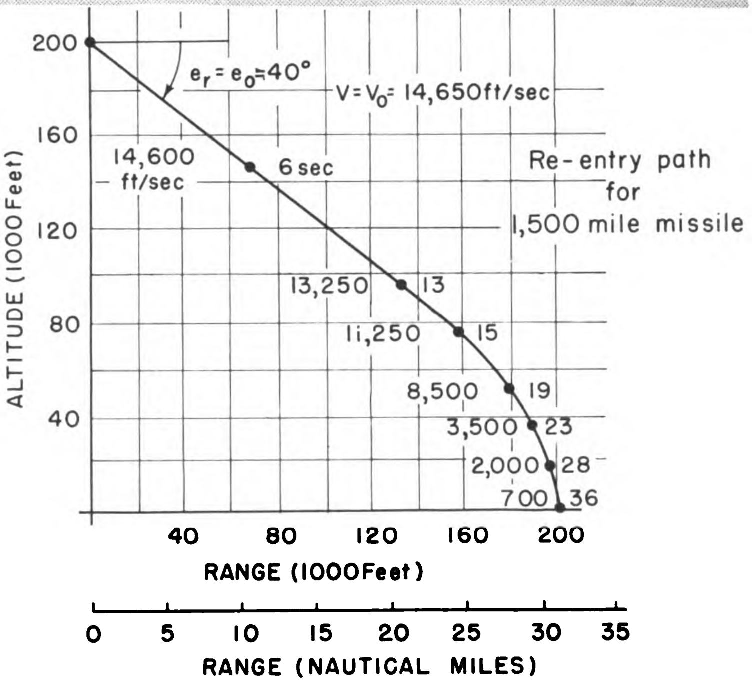
$$P = h/h_0$$

Plotting the normalized drag (Drag/K x 10^3) against P shows the variation in drag with altitude (solid line). Near 50,000 feet, the drag becomes very great and the variation in velocity with altitude no longer follows the square root approximation used for the graph. The actual drag encountered will be somewhat less (broken line).



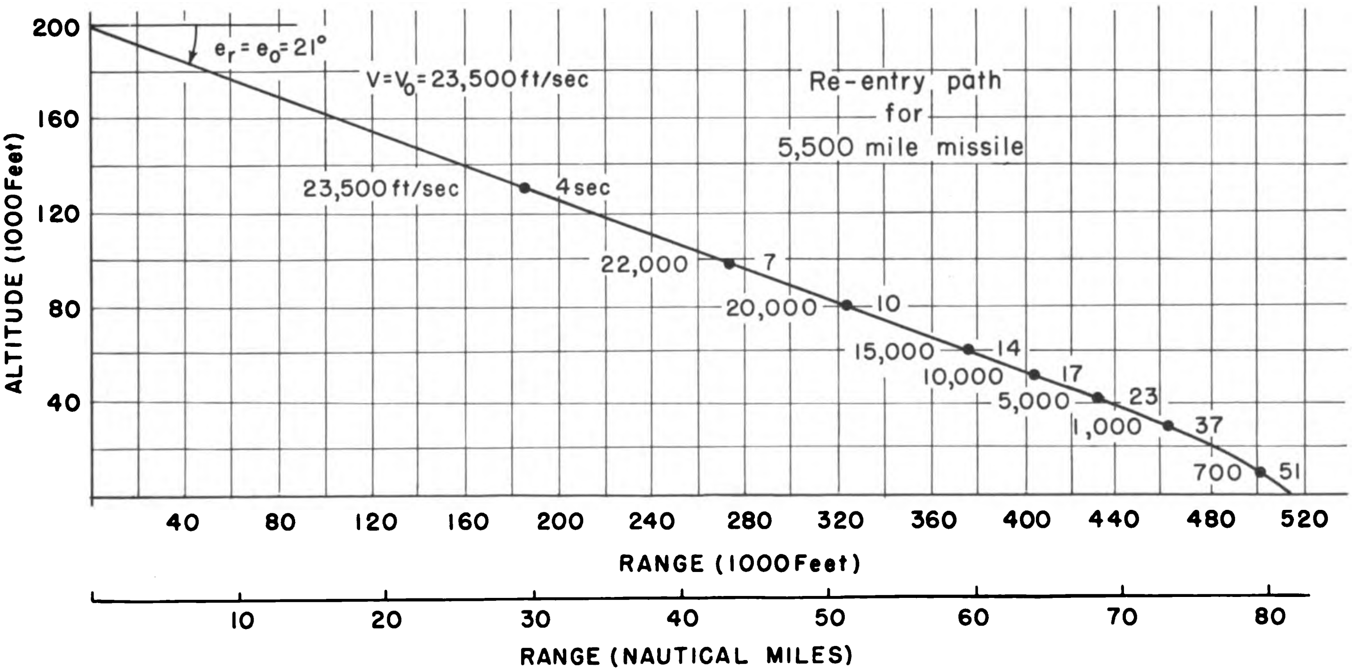
The re-entry path can be considered in three parts. The first part is the reorientation phase (from the re-entry point to 200,000 ft.) during which time the velocity remains the same as at the re-entry point, and the path is a straight line. The second part is the deceleration phase (200,000 to 50,000 ft.) where the path remains a straight line, and the missile loses most of its kinetic energy and undergoes intense heating. The third part is the terminal phase (50,000 feet to impact) where the missile departs from a straight line path and begins to cool off.





The re-entry path for the first phase is simply a straight line at constant velocity. The re-entry path below 200,000 feet is illustrated for two situations, with elapsed time (from 200,000 feet) and velocity superimposed at several points.

The exact calculation of the missile position at every instant of time is a very complex problem. The two important quantities, re-entry range (R_r) and re-entry time-of-flight (t_r) can be approximated with little difficulty.



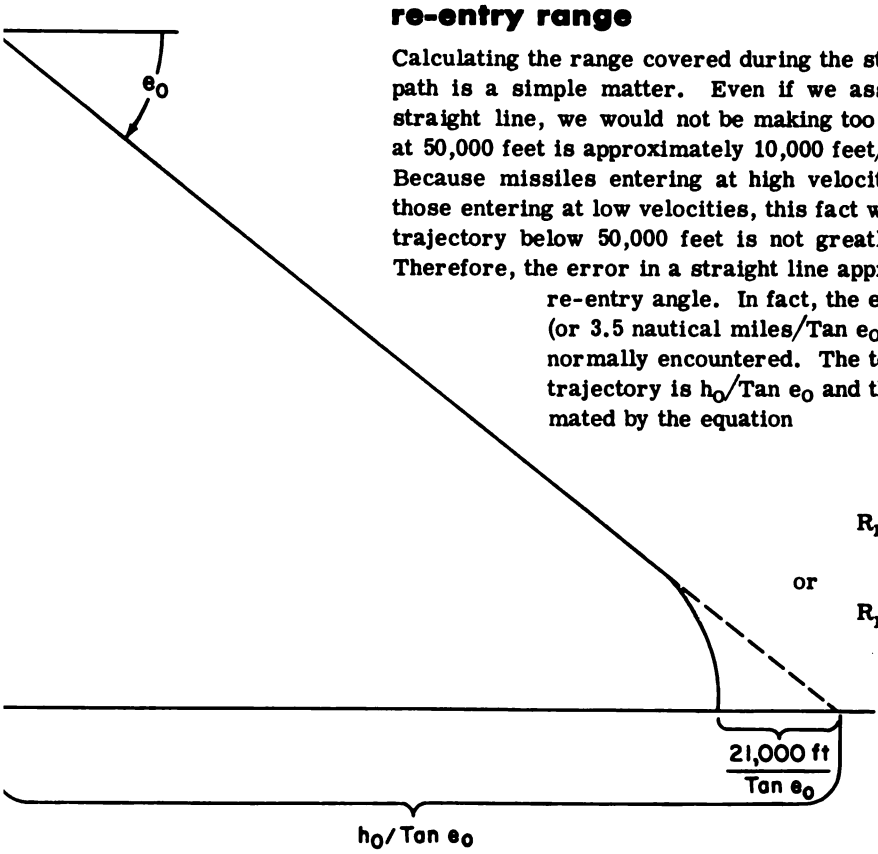
re-entry range

Calculating the range covered during the straight line portion of the re-entry flight path is a simple matter. Even if we assume that the entire trajectory were a straight line, we would not be making too great an error. Note that the velocity at 50,000 feet is approximately 10,000 feet/sec. in both the situations illustrated. Because missiles entering at high velocities undergo higher accelerations than those entering at low velocities, this fact will be nearly true for all missiles; the trajectory below 50,000 feet is not greatly dependent on the re-entry velocity. Therefore, the error in a straight line approximation will be dependent only on the re-entry angle. In fact, the error is fairly close to 21,000 ft./Tan e_o (or 3.5 nautical miles/Tan e_o) for the range of re-entry velocities normally encountered. The total range covered by a straight line trajectory is h_o/Tan e_o and the true range can be closely approximated by the equation

$$R_T = \frac{h_o}{\text{Tan } e_o} - \frac{21{,}000 \text{ ft}}{\text{Tan } e_o} = \frac{h_o - 21{,}000 \text{ ft}}{\text{Tan } e_o},$$

or

$$R_T = \frac{h_o}{\text{Tan } e_o} - \frac{3.5 \text{ n.m.}}{\text{Tan } e_o} = \frac{h_o - 3.5 \text{ n.m.}}{\text{Tan } e_o}$$



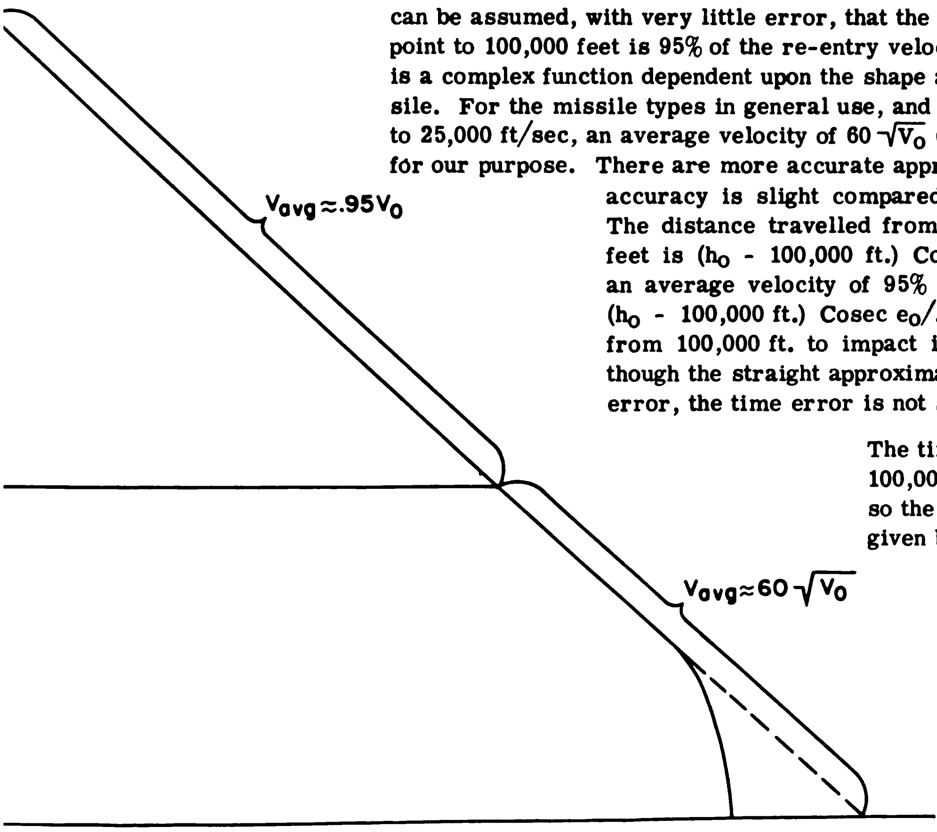
re-entry time-of-flight

The re-entry time-of-flight depends primarily on the average velocity of the missile. At 100,000 feet, the velocity is not much less than the re-entry velocity. It can be assumed, with very little error, that the average velocity from the re-entry point to 100,000 feet is 95% of the re-entry velocity. Below this point the velocity is a complex function dependent upon the shape and the effective area of the missile. For the missile types in general use, and for re-entry velocities from 10,000 to 25,000 ft/sec, an average velocity of $60\sqrt{V_o}$ (in ft/sec) is sufficiently accurate for our purpose. There are more accurate approximations, but the increase in accuracy is slight compared with the increase in complexity. The distance travelled from the re-entry point to 100,000 feet is (h_o - 100,000 ft.) Cosec e_o, and the time (assuming an average velocity of 95% of the re-entry velocity) is (h_o - 100,000 ft.) Cosec e_o/.95 V_o. The distance travelled from 100,000 ft. to impact is 100,000 ft x Cosec e_o. (Although the straight approximation gives a significant range error, the time error is not significant.)

The time in seconds is equal to 100,000 ft x Cosec $e_o/60\sqrt{V_o}$, so the total time-of-flight for re-entry is given by the equation

$$t = \frac{h_o - 100{,}000 \text{ ft}}{.95 \text{ Sin } e_o \, V_o} + \frac{1670}{\text{Sin } e_o}\sqrt{V_o},$$

where h_o is in feet, V_o in ft/sec, and t in seconds.



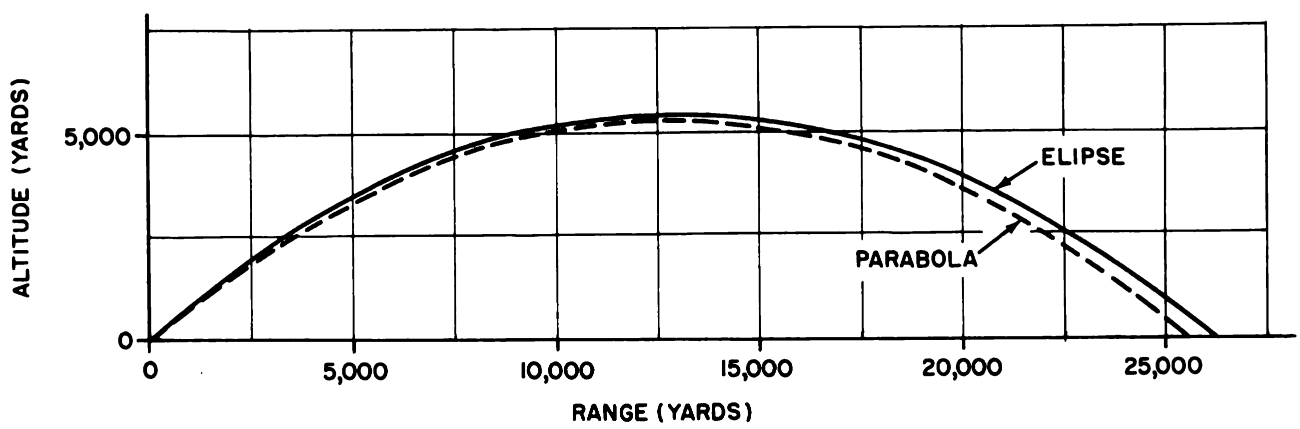
SHORT RANGE BALLISTIC TRAJECTORIES

In calculating the trajectory of the short-range ballistic missile in air (gun projectile), two complicating factors are encountered which are not found in the same problem in a vacuum. The first factor is the presence of drag. Drag is proportional to the density of the atmosphere (a function of altitude and temperature), and the drag coefficient of the projectile. Neither variable is wholly predictable. The second factor is the dependence upon aerodynamic and launching effects, which are also difficult to predict accurately (because of erosion, launcher motion, etc.). Therefore, the flight paths are usually determined by experiment. Since any calculation is compromised by unavoidable approximations, a few approximations are deliberately made for simplicity. Three types of approximation are usually made: the flat Earth approximation, the constant air density approximation, and various approximations to the velocity dependence of drag. These approximations, and the results of the better ones, are discussed in this section.

FLAT EARTH APPROXIMATION

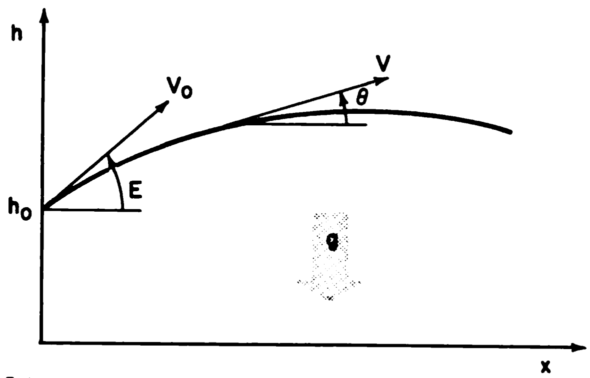
The approximation having the greatest effect in simplifying calculations is the assumption of a flat earth. The gravity field in the vicinity of a flat earth would be constant, and therefore, the dependence of g on altitude is eliminated. Cartesian coordinates can be used instead of spherical coordinates also, further simplifying the equations. The assumption is justified because the curvature has little effect over a short range. Therefore, the error due to the approximation will be greater for long-range trajectories than for short-range trajectories. Assuming a flat earth, and neglecting the effect of the atmosphere leads to a parabolic flight path (see box).

The parabolic trajectory obtained for a 26,000 yard shot from the equations shown is illustrated, along with the true elliptical trajectory for the same problem. It is easily seen that there is very little error introduced on the flat earth assumption in the plane of the parabola. The error in time-of-flight for the case illustrated is one-half of one percent.



The basic equations for the ballistic trajectory in the constant gravity field (g) of a flat Earth, ignoring drag, are obtained from Newton's second law (F = ma) for each component.

$$m\ddot{x} = 0 \qquad (1)$$
$$m\ddot{h} = -mg \qquad (2)$$



Integrating each equation twice:

$$\dot{x} = A \qquad (3)$$
$$x = At + B \qquad (4)$$
$$\dot{h} = -gt + C \qquad (5)$$
$$h = -\frac{1}{2}gt^2 + Ct + D \qquad (6)$$

where A, B, C and D are constants of integration. The initial conditions at t = 0 are:

$$x = 0$$
$$\dot{x} = V_o \text{ Cos } E$$
$$h = h_o$$
$$\dot{h} = V_o \text{ Sin } E$$

where V_o is the initial velocity and E is the initial value of θ. Substituting These values in equations (3) through(6)

$$V_o \text{ Cos } E = A$$
$$0 = B$$
$$V_o \text{ Sin } E = C$$
$$h_o = D$$

Therefore, from (4) and (6)

$$\dot{x} = V_o \text{ Cos } E \qquad (7)$$
$$x = V_o t \text{ Cos } E \qquad (8)$$
$$\dot{h} = -gt + V_o \text{ Sin } E \qquad (9)$$
$$h = -\frac{1}{2}gt^2 + V_o t \text{ Sin } E + h_o \qquad (10)$$

These are parametric equations of a parabola. Time may be eliminated to obtain the explicit equation by solving (8) for t and substituting in (10).

$$h = -\frac{1}{2}g\left[\frac{x}{V_o \text{ Cos } E}\right]^2 + V_o \text{ Sin } E\left[\frac{x}{V_o \text{ Cos } E}\right] + h_o$$
$$= -\frac{g x^2}{2 V_o^2 \text{ Cos}^2 E} + x \text{ Tan } E + h_o \qquad (11)$$

The calculation of time of flight can be simplified by using the knowledge of the symmetry of the parabola. The apogee occurs at h = 0.

from (9)
$$0 = -g t_a + V_o \text{ Sin } E$$
$$t_a = \frac{V_o \text{ Sin } E}{g} \qquad (12)$$

from (8) and (12):
$$x_a = \frac{V_o^2 \text{ Sin } E \text{ Cos } E}{g}$$

Since $\text{Sin } \theta \text{ Cos } \theta = \frac{1}{2} \text{ Sin } 2\theta$:
$$x_a = \frac{V_o^2 \text{ Sin} 2E}{2g} \qquad (13)$$

from (10) and (12):
$$h_a = -\frac{1}{2}g\left[\frac{V_o \text{ Sin } E}{g}\right]^2 + V_o \text{ Sin } E\left[\frac{V_o \text{ Sin } E}{g}\right] + h_o$$
$$= \frac{V_o^2 \text{ Sin}^2 E}{2g} + h\,o \qquad (14)$$

The time-of-flight to a point at the same altitude as h_o is equal to $2t_a$ and the range is $2x_a$. The time-of-flight to a point below h_o (h_f) is equal to $2t_a$ plus the time required to fall a distance of $h_o - h_f$ with an initial downward velocity of V_o Sin E. From (10)

$$h_f = -\frac{1}{2}g t_f^2 = V_o t_f \text{ Sin } E + h_o$$
$$t_f = \frac{-V_o \text{ Sin } E \pm \sqrt{V_o^2 \text{ Sin}^2 E + 2g(h_o - h_f)}}{g}$$
$$= \frac{V_o \text{ Sin } E}{g}\left[1 \pm \sqrt{1 + \frac{2g(h_o - h_f)}{V_o^2 \text{ Sin}^2 E}}\right] \qquad (15)$$

The range as a function of h_f and E is obtained from (8) and (15).

$$x_f = V_o t_f \text{ Cos } E$$
$$= \frac{V_o^2 \text{ Sin } E \text{ Cos } E}{g}\left[1 \pm \sqrt{\quad}\right]$$
$$x_f = \frac{V_o^2 \text{ Sin} 2E}{2g}\left[1 \pm \sqrt{1 + \frac{2g(h_o - h_f)}{V_o^2 \text{ Sin } E}}\right] \qquad (16)$$

The velocity is most easily obtained use of the fact that the total energy is constant.

$$\text{K. E.} + \text{P. E.} = \text{Constant}$$
$$\frac{mV^2}{2} + mgh = E$$
$$\frac{V_o^2}{2} + g h_o = \frac{V^2}{2} + g h = \frac{E}{m} \qquad (17)$$
$$V = \sqrt{V_o^2 + 2g(h_o - h)} \qquad (18)$$

CONSTANT AIR DENSITY APPROXIMATION

The most effective approximation in simplifying the calculations is the approximation of constant air density. As pointed out in the re-entry discussion, the atmospheric density (ρ) is given very closely by the formula:

$$\rho = \rho_0 e^{-\sigma h}$$

where ρ_0 = ground level density
h = altitude
σ = 4.15×10^{-5}/ft

Since drag is equal to

$$\frac{C_d \ \rho \ A}{2} V^2 ,$$

the exponential form for ρ complicates the equations considerably. A constant air density, therefore, is usually assumed. The simplest approximation uses the density at the launch point. A better approximation, however, uses the density at the launch point minus one-third the difference between the launch-point density and the density at the apogee, i.e.,

$$\rho = \rho_0 - \frac{1}{3} (\rho_0 - \rho_a)$$

where ρ_0 = launch-point density
ρ_a = apogee density

The best results are obtained when the range of altitudes covered is small, i.e., when the initial elevation angle (E) is small.

DRAG APPROXIMATIONS

As pointed out before, drag is proportional to velocity squared. The use of a V^2 term in the equations, however, leads to results which are time consuming to evaluate. Therefore, several approximation to the V^2 dependance are made.

D = KV Approximation

The simplest approximation treats drag as a linear function of velocity, i.e., Drag = K V,

where $K = \sqrt{\frac{C_d \ \rho_0 \ mg \ A}{2}}$

This approximation gives poor results when the range of altitudes or velocity is large.

D = K′V$\dot{x}$ Approximation

Since V will not differ greatly from x when the trajectory is fairly flat, Vx can be substituted for V^2. The drag can, therefore, be approximated by:

$$D = K' V\dot{x}$$

where $K' = \frac{C_d \rho_0 A}{2}$

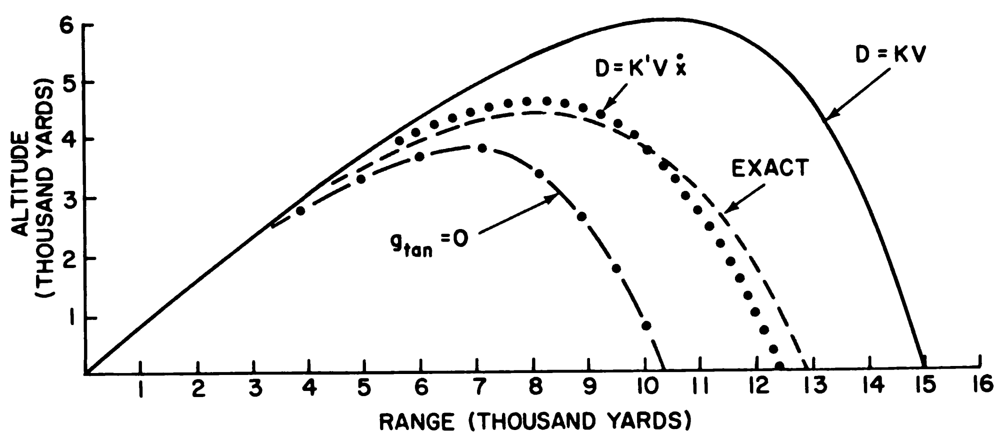
The equations of motion are obtained by straight forward integration of the acceleration equations (see box)

g/tan = 0 Approximation

Another approximation can be made to simplify the equations which follow the D = K′ V^2 assumption. This involves obtaining a differential equation for velocity, and solving it by neglecting the component of gravitational force along the velocity vector (see box).

Trajectories calculated using the three drag approximations discussed are illustrated. The "exact" trajectory, accounting for the variation in air density with altitude and for the curvature of Earth, obtained by numerical integration of the equations of motion, is also shown. It can be seen that the D = K′ V$\dot{x}$ approximation is closest to the "exact" trajectory for the situation illustrated. The range error in this approximation, for the situation illustrated, is four percent, and the error in time of flight is less than one percent. As the effect of changing air density increases, either by higher velocities or larger launch angles, the results can be expected to become less accurate. If the initial velocity in the case illustrated were doubled, the range error would increase to sixteen percent and the time-of-flight error to seven percent.

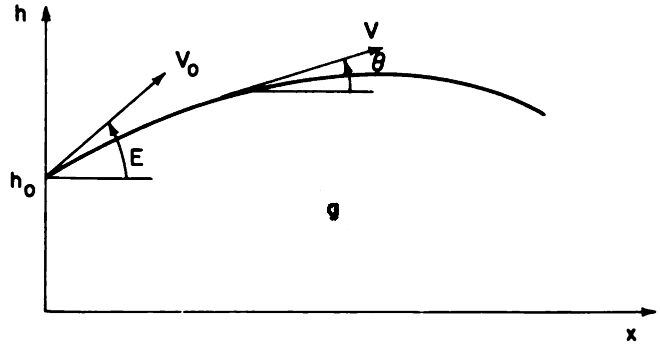
The error can be reduced, however, by using a more refined value for air density. The average density throughout the flight will be somewhat less than the density at the launch point. By subtracting one-third of the difference in the apogee density and the launch point density from the launch point density, a better approximation can be made. In the case just mentioned, this refinement will reduce the range and time-of-flight errors from sixteen percent and seven percent to four percent and one percent respectively.



The basic equations for the ballistic trajectory, using the flat-earth approximation and accounting for drag, are:

$$m\ddot{x} = -D \cos\theta \qquad (1)$$

$$m\ddot{h} = -D \sin\theta - mg \qquad (2)$$



As a first approximation, the drag (D) is taken as D = KV, where:

$$K = \frac{C_d \rho A\, mg}{2}$$

$\rho = \rho_0$ (a constant)

Since $V = \dot{x}/\cos\theta$ and $V = \dot{h}/\sin\theta$, equations (1) and (2) become:

$$\ddot{x} = -\frac{k}{m}\dot{x} \qquad (3)$$

$$\ddot{h} = -\frac{k}{m}\dot{h} - g \qquad (4)$$

Substituting p for k/m, and integrating each equation twice:

$$\dot{x} = Ae^{-pt} \qquad (5)$$

$$x = -\frac{A}{p}e^{-pt} + B \qquad (6)$$

$$\dot{h} = \left[C + \frac{g}{p}\right]e^{-pt} - \frac{g}{p} \qquad (7)$$

$$h = -\frac{1}{p}\left[C + \frac{g}{p}\right]e^{-pt} - \frac{g}{p}t + D \qquad (8)$$

where A, B, C and D are constants of integration. Evaluating the constants by substituting the initial conditions:

$t = 0$, $V = V_0$, $\dot{x} = V_0 \cos E$, $\dot{h} = V_0 \sin E$, $x = 0$ and $h = h_0$ gives:

$$\dot{x} = V_0 \cos E\, e^{-pt} \qquad (9)$$

$$x = \frac{V_0 \cos E}{p}\left[1 - e^{-pt}\right] \qquad (10)$$

$$\dot{h} = \frac{1}{p}\left[V_0 \sin E + \frac{g}{p}\right]\left[1 - e^{-pt}\right] - \frac{g}{p}t + h_0 \qquad (11)$$

$$h = \frac{1}{p}\left[V_0 \sin E + \frac{g}{p}\right]\left[1 - e^{-pt}\right] - \frac{g}{p}t + h_0 \qquad (12)$$

For the second method of approximation, drag is assumed equal to $K' V \dot{x}$ instead of $K' V^2$, where

$$K' = \frac{C_d \rho A}{2}$$

Equations (1) and (2) become:

$$m\ddot{x} = -K'\dot{x}^2 \qquad (13)$$

$$m\ddot{h} = -K'\dot{h}\dot{x} \qquad (14)$$

Integrating each equation twice, and substituting the same initial conditions as before:

$$\dot{x} = \frac{V_0 \cos E}{1 + q t V_0 \cos E} \qquad (15)$$

$$x = \frac{1}{q}\ln\left[1 + \frac{V_0 t \cos E}{q}\right] \qquad (16)$$

$$\dot{h} = \frac{V_0 \sin E - \frac{gt}{2}}{1 + q t V_0 \cos E} - \frac{gt}{2} \qquad (17)$$

$$h = \frac{1}{q}\left[\operatorname{Tan} E + \frac{g}{2 p V_0^2 \cos^2 E}\right] \ln\left[1 + q t V_0 \cos E\right] - \frac{gt}{2q V_0 \cos E} - \frac{g}{4}t^2 + h_0 \qquad (18)$$

where

$$q = \frac{K'}{m} = \frac{C_d \rho A}{2\,m}$$

The equations are considerably more complex than those obtained by the D = KV approximation. The results, however, will be much more exact, especially when the trajectory is nearly flat and $\dot{x}$ is very nearly equal to V throughout the trajectory.

The third method of approximation involves the assumption that the component of gravitational force tangent to the path ($g \sin\phi$) is zero. In this case, the sum of the forces along the velocity vector is:

$$ma = m\dot{V} = -K'V^2 \qquad (19)$$

Integrating twice, and substituting V_0 for V at t = 0:

$$V = \frac{V_0}{1 + q V_0 t} \qquad (20)$$

Substituting (20) in (1) and (2), and using $K'V^2$ for D:

$$\ddot{x} = -\frac{q V_0 \dot{x}}{1 + q V_0 t} \qquad (21)$$

$$\ddot{h} = -\frac{q V_0 \dot{h}}{1 + q V_0 t} - g \qquad (22)$$

Where K' and q have the same values as in the $D = K'V\dot{x}$ approximation.

Integrating these equations yields:

$$\dot{x} = \frac{V_0 \cos E}{1 + q V_0 t} \qquad (23)$$

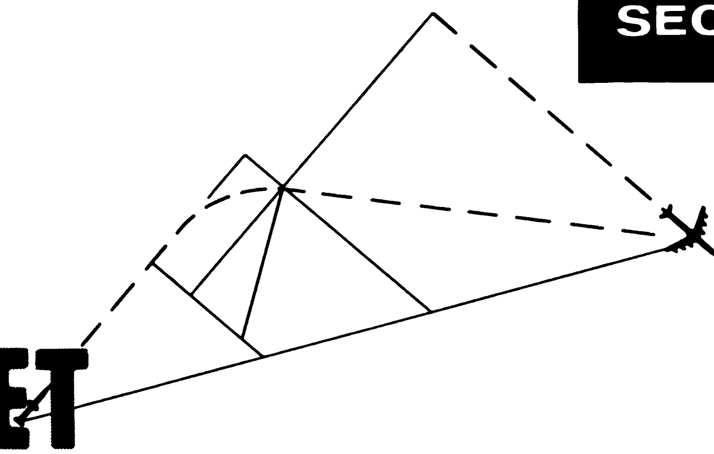
$$x = \frac{\cos E}{q}\ln\left[1 + q V_0 t\right] \qquad (24)$$

$$\dot{h} = \frac{V_0 \sin E - \frac{gt}{2}}{1 + q V_0 t} - \frac{1}{2}g t \qquad (25)$$

$$h = \frac{1}{q}\left[\sin E + \frac{g}{2 q V_0^2}\right] \ln(1 + q V_0 t) - \frac{gt}{2 q V_0} - \frac{1}{4}g t^4 + h_0 \qquad (26)$$

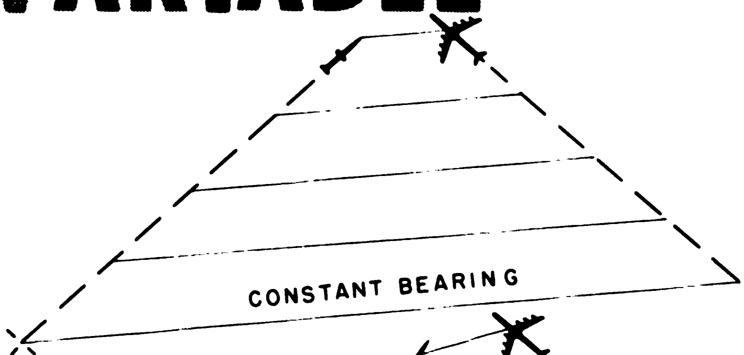
Note that these equations are very much like the equations obtained by the $D = K' V \dot{x}$ approximation.

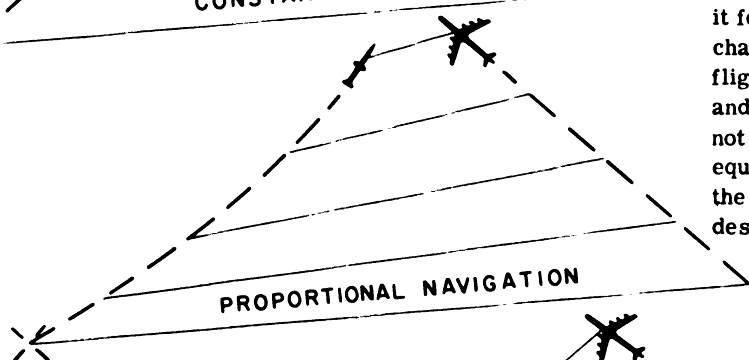
SECTION 3

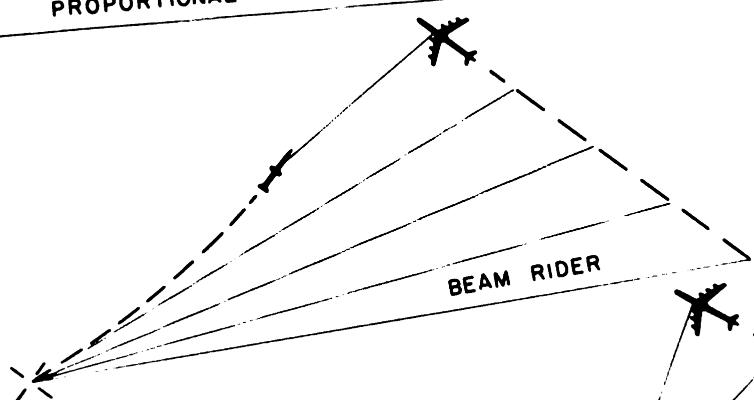


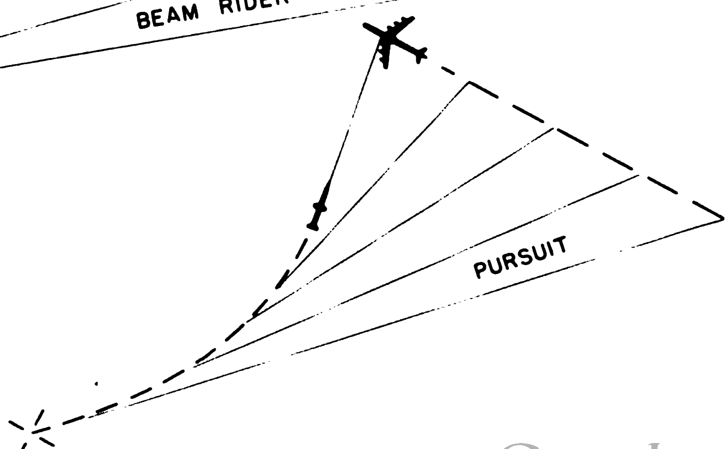
PRESET VARIABLE GUIDED FLIGHT PATHS

The next topic of discussion is guided flight paths, i.e., those paths resulting when a missile is made, through a control system, to follow a prescribed course, instead of being allowed to travel in a course determined by the natural forces acting on the missile. This necessarily implys the existance of some man-made method of applying forces to the missile in order to make it follow the desired path. Since this chapter is concerned only with the flight paths, the nature of these forces and the methods of applying them are not covered. It is simply assumed that equipment is available and will cause the missile to follow the flight path desired.









PRESET

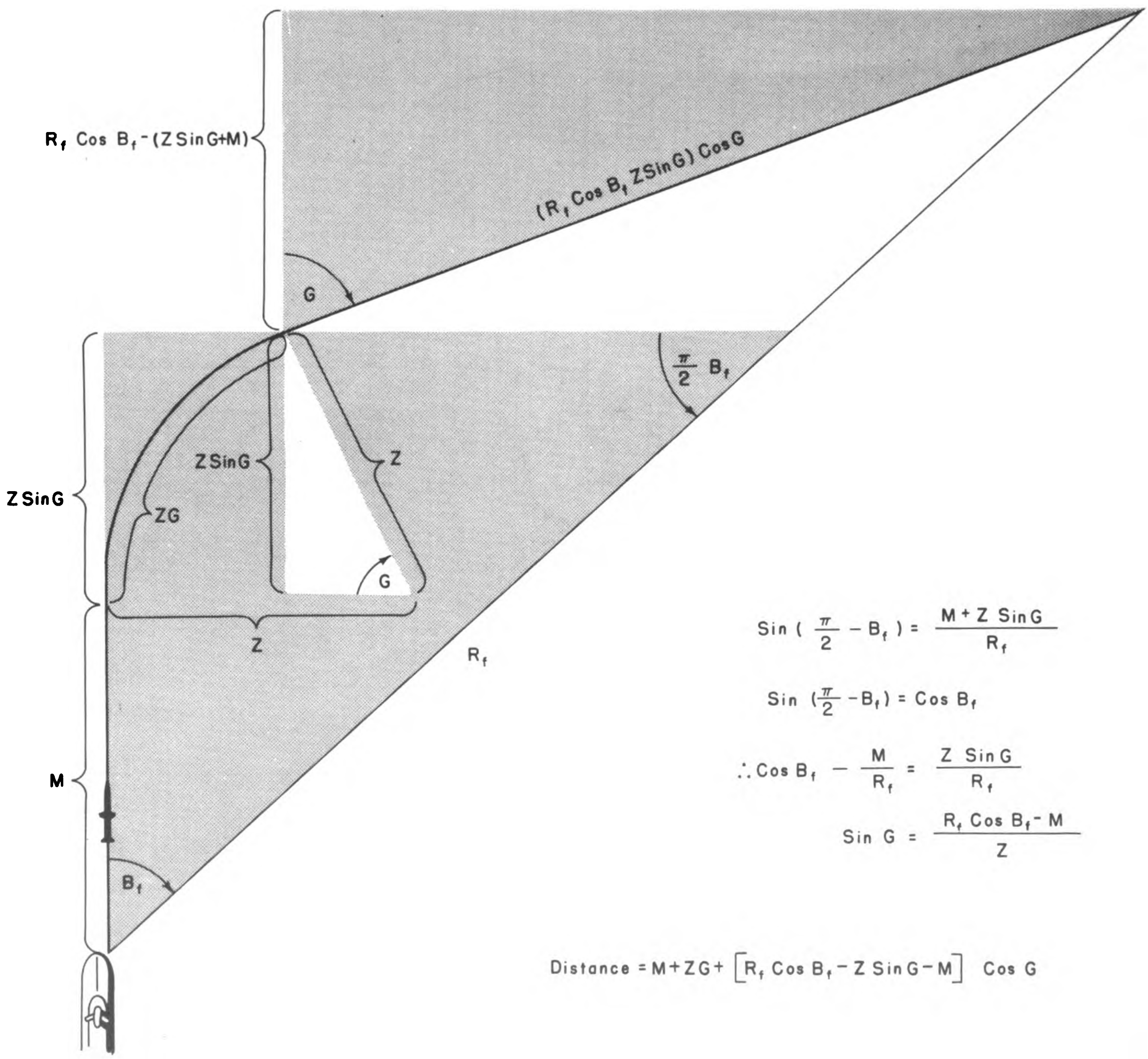
The category of preset flight paths includes the ballistic trajectories covered in the preceding section, and a number of guided flight paths as well.

The simplest preset flight paths are those described by weapons which simply maintain some constant parameter, such as the constant position of the tethered mine or the constant heading of some torpedoes. No discussion of these paths is necessary.

The category of preset flights also includes a large variety of more complex paths. The case of a torpedo fired from a constrained launcher is one of the simplest examples.

Common practice in such cases is to preset a gyro angle before launching. The torpedo is programed to travel a fixed distance (M) in a straight line, then execute a constant radius turn through the preset angle and resume a straight line course.

The analysis of the path to an impact point at future range R_f and future bearing B_f is shown in the illustration. The gyro angle (G), and the distance traveled are obtained as functions of the turning radius (Z), the future range and bearing, and the constant distance (M).



$$\mathrm{Sin}\left(\frac{\pi}{2} - B_f\right) = \frac{M + Z\,\mathrm{Sin}\,G}{R_f}$$

$$\mathrm{Sin}\left(\frac{\pi}{2} - B_f\right) = \mathrm{Cos}\,B_f$$

$$\therefore \mathrm{Cos}\,B_f - \frac{M}{R_f} = \frac{Z\,\mathrm{Sin}\,G}{R_f}$$

$$\mathrm{Sin}\,G = \frac{R_f\,\mathrm{Cos}\,B_f - M}{Z}$$

$$\text{Distance} = M + ZG + \left[R_f\,\mathrm{Cos}\,B_f - Z\,\mathrm{Sin}\,G - M\right]\,\mathrm{Cos}\,G$$

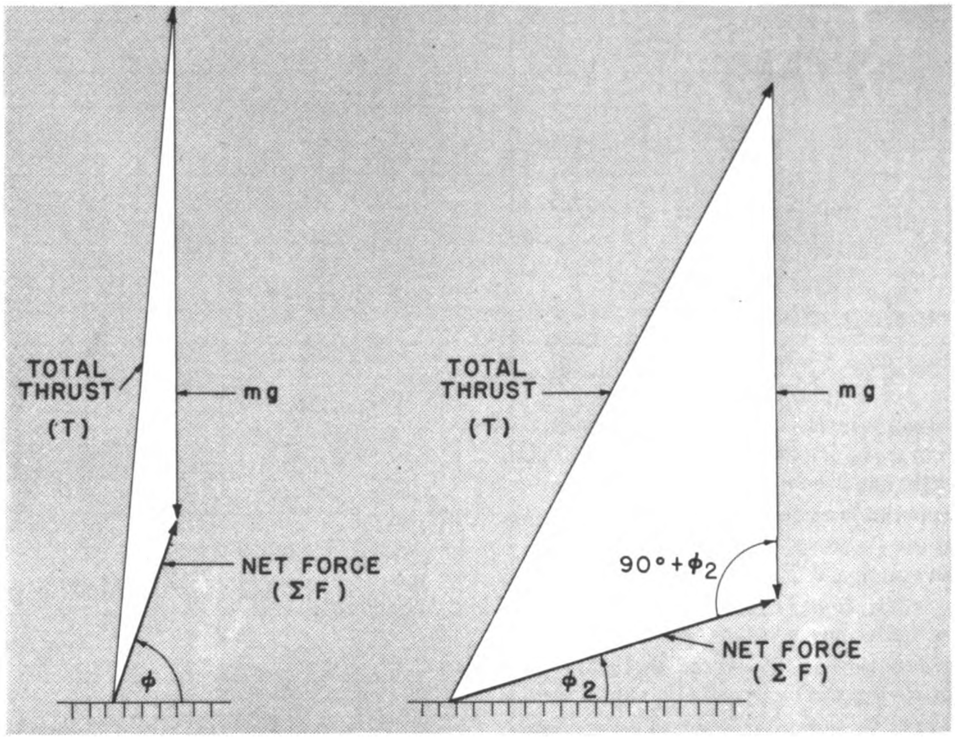
FLIGHT PATHS

long range cruising missiles

The long range cruising missile is an excellent example of a weapon with a complex preset flight path. This type of missile usually has a control system which enables it to follow a particular course from launch impact. The nature of the flight path might be one of constant altitude and direction for a fixed range, and then a steep dive at the target.
In other cases the missile may be launched at a steep angle, and follow a trajectory similar to that of a ballistic missile until it gradually levels off at a preset altitude. In other cases the missile may be programed to fly at the speed and altitude which gives the best fuel economy until it reaches a point near the target, and then fly at a low altitude and at maximum speed to minimize the chance of interception. Many variations are possible, but all have the essential characteristic that the programed path cannot be altered once the missile is launched. In all cases, three essential requirements must be present. The weapon must have a programed course, it must have some method of determining its actual course and position, and it must have some means of adjusting its course to conform to the programed course.

ballistic missile powered flight

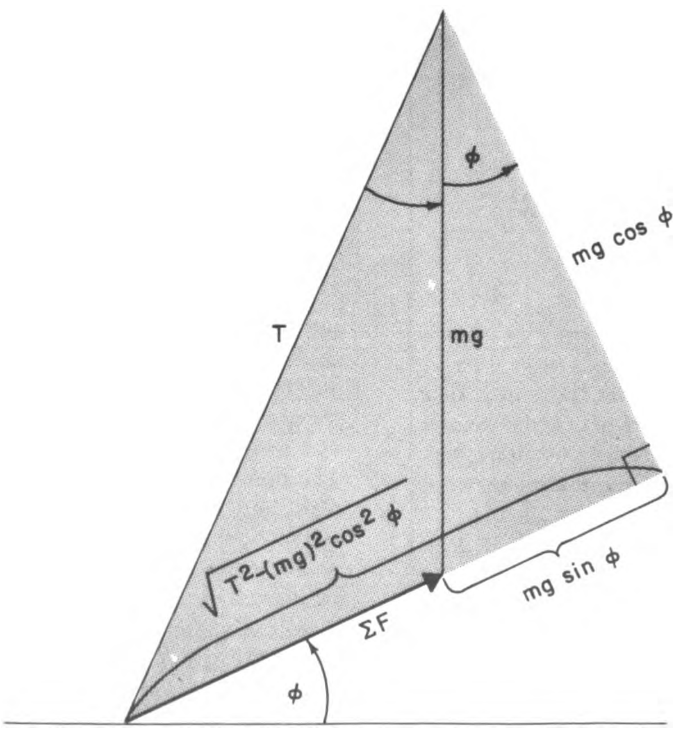
The powered portion of ballistic missile flight path is a similar situation. The optimum flight path from launch to burnout would be the one which permits attaining the necessary burnout velocity with the minimum expendure of energy. Practical missile design factors impose limitations on the choice of trajectories, however. For obvious reasons it is desirable to keep the structural weight of the missile as low as possible. Since increasing strength requires greater weight, ballistic missile airframes usually have adequate axial (lengthwise) strength but very little lateral strength. Therefore they cannot be subjected to large forces except along the longitudinal axis. Any angle of departure other than a vertical launch would impose transverse aerodynamic forces on the missile, so the initial phase of flight is almost always vertical.
The missile must eventually be tilted to the burnout angle (e_o). The most economical way to accomplish this is the "zero - lift" or "gravity" turn. The missile is subjected to a brief period of rotational force due to tilting the direction of thrust for an instant, or firing side jets. This causes the missile to rotate at a constant rate. By repeating this process several times the missile is slowly tilted to the required burnout angle.



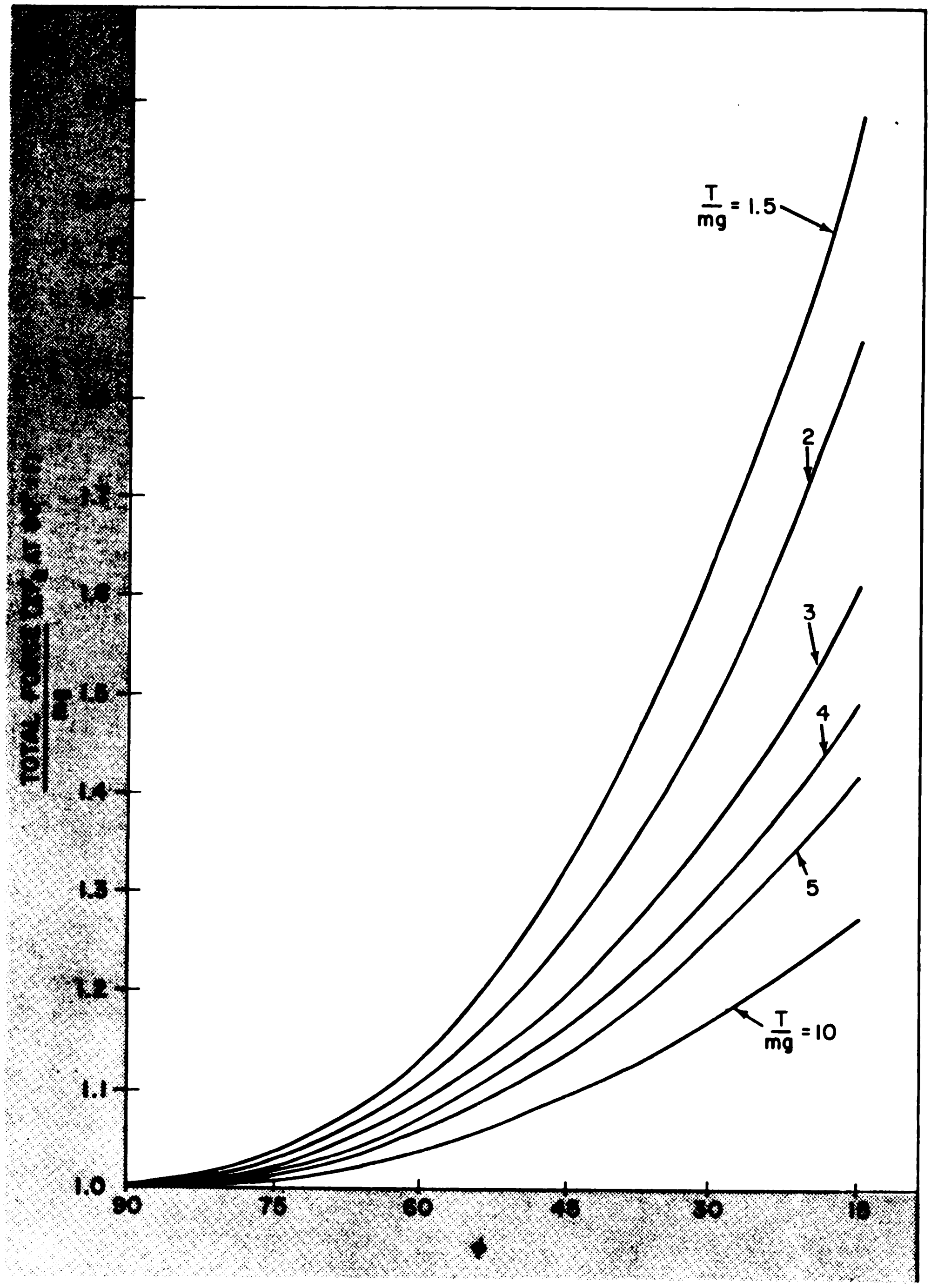
The illustration shows the thrust and gravitational forces on a missile at a near vertical angle (ϕ_1) and at a very shallow angle (ϕ_2). The thrust and gravitational forces are the same in both cases. The net force on the missile (ignoring aerodynamic forces for the moment) is the vector sum of these two forces. It is evident that the total force is greater for the smaller angle. By resolving forces along the net force line:

$$F = \sqrt{T^2 - (mg)^2 \cos^2 \phi} - mg \sin \phi$$

$$= mg \left(\sqrt{\left(\frac{T}{mg}\right)^2 - \cos^2 \phi} - \sin \phi \right)$$



The function T/mg is the thrust to weight ratio of the missile, which must be greater than 1 if the missile is to move at all. At launching, a typical thrust to weight ratio is 1.5. As the weight is reduced due to the consumption of fuel, the ratio increases, usually to a factor of 3 to 6 for single stage missiles, and 5 to 10 for multistage missiles. The net force as a function of ϕ is shown for various thrust to weight ratios (normalized so that the net force is 1 at $\phi = 90°$).



The graph shows that there is a significant increase in total force at shallow angles when the thrust to weight ratio is low, and a lesser increase when the thrust to weight ratio is high. The aerodynamic forces have been ignored up to this point. Since a missile launched at a shallow angle reaches higher velocities at a low altitude, it encounters more drag than a missile launched vertically. The amount of drag depends upon the acceleration and shape of the missile so no general statements can be made as to the exact value of drag. However, it is usually enough to significantly reduce, and in some cases completely nullify, the advantages of a non-vertical launch. The drag becomes much less important at high altitudes, but the thrust to weight ratio is so high that the advantage of a non-vertical flight path is slight.

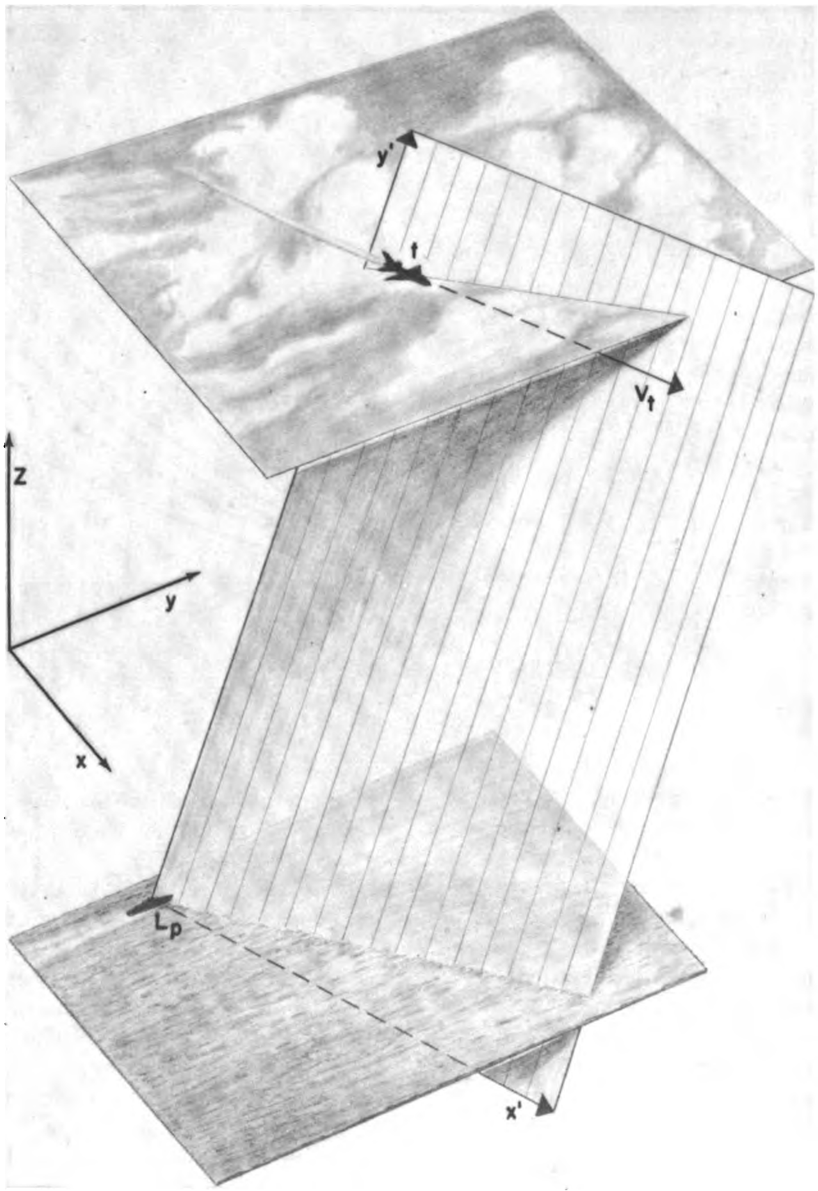
The problem is further complicated when the center of thrust is not at the center of gravity. If a non-vertical straight-line flight path is attempted, the thrust vector must be offset from the velocity vector by an amount sufficient to counteract the gravitational attraction. This causes a torque about the center of gravity which tends to rotate the missile.

The flight path from launch to burnout, therefore, is determined primarily by the guidance and loading factors, and not by fuel economy.

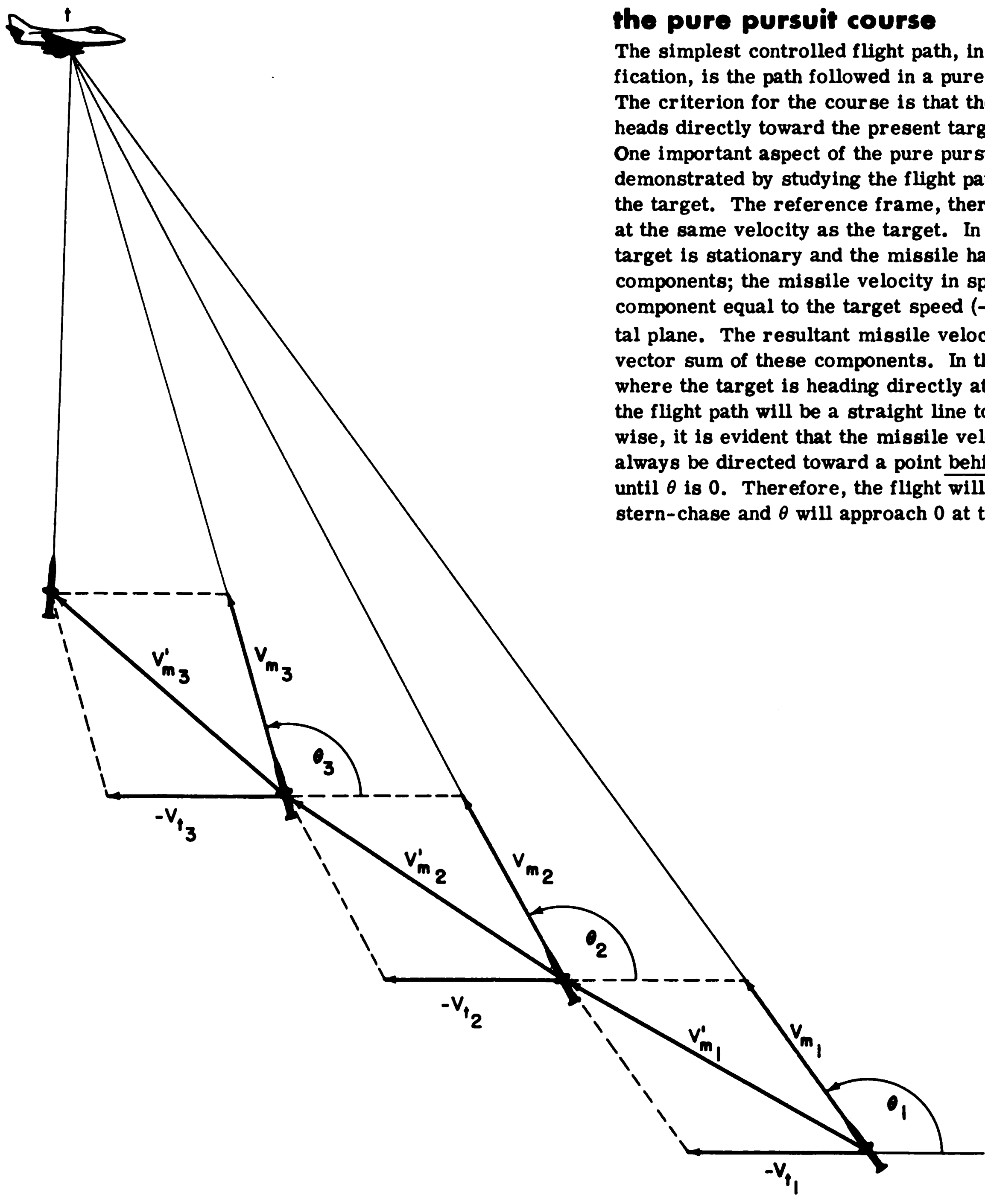
VARIABLE

The guided flight paths of greatest interest are those which can vary during flight. In general, the heading of the weapon is a function of the position and velocity of the target. The four basic types of flight paths in common use were described briefly in the introduction. These flight paths are analyzed here for a constant speed missile attacking a constant speed target moving in a straight line. The assumption of constant speed is reasonably close to the situations most frequently encountered. The assumption of straight line target motion is not valid in most cases. However, the analysis can be amended to account for target maneuvers and most of the important details can be gathered from a study of straight line target motion.

The flight paths are analyzed in the "plane of action", i.e. the plane defined by the target velocity vector and the launch point. It is also assumed that the target track is parallel with the x-axis. This is general since such a coordinate system can always be found.



PURSUIT COURSES



the pure pursuit course

The simplest controlled flight path, in terms of classification, is the path followed in a pure pursuit course. The criterion for the course is that the missile always heads directly toward the present target position.

One important aspect of the pure pursuit course is best demonstrated by studying the flight path as seen from the target. The reference frame, therefore, is moving at the same velocity as the target. In this system, the target is stationary and the missile has two velocity components; the missile velocity in space (V_m) and a component equal to the target speed ($-V_t$) in the horizontal plane. The resultant missile velocity (V_m') is the vector sum of these components. In the special case where the target is heading directly at the launch point, the flight path will be a straight line to the target. Otherwise, it is evident that the missile velocity (V_m') will always be directed toward a point behind the target until θ is 0. Therefore, the flight will always become a stern-chase and θ will approach 0 at the impact point.

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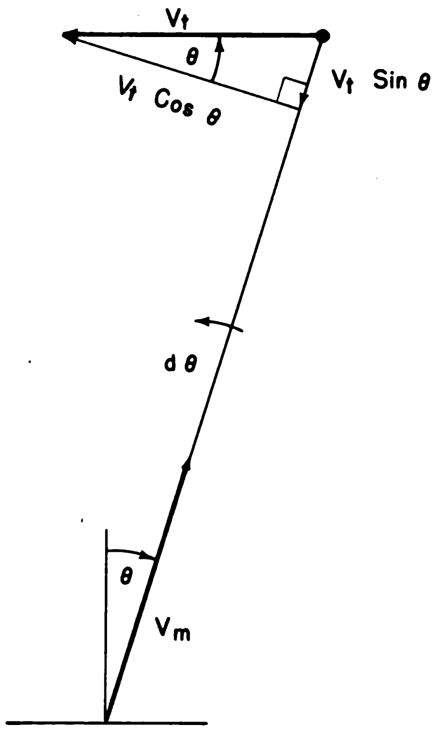


$$\dot{R} = V_t \operatorname{Cos}\theta - V_m \tag{1}$$

$$R\dot{\theta} = -V_t \operatorname{Sin}\theta \tag{2}$$

$$(1)\div(2) \quad \frac{\dot{R}}{R\dot{\theta}} = \frac{V_t \operatorname{Cos}\theta - V_m}{-V_t \operatorname{Sin}\theta} = -\operatorname{Cot}\theta + \frac{V_m}{V_t}\operatorname{Csc}\theta \tag{3}$$

$$\frac{\dot{R}}{R} = (h \operatorname{Csc}\theta - \operatorname{Cot}\theta)\,\dot{\theta}, \quad h = \frac{V_m}{V_t} \tag{4}$$

$$\int (4)\,dt \qquad \ln R \Big|_{R_i}^{R} = h \ln \operatorname{Tan} \tfrac{1}{2}\theta - \ln \operatorname{Sin}\theta \Big|_{\theta_i}^{\theta} \tag{5}$$

$$\ln\frac{R}{R_i} = h \ln \frac{\operatorname{Tan}\frac{1}{2}\theta}{\operatorname{Tan}\frac{1}{2}\theta_i} - \ln\frac{\operatorname{Sin}\theta}{\operatorname{Sin}\theta_i} \tag{6}$$

Where ln = $\log_e$, and R_i and θ_i are the initial values of R and θ.

$$e^{(6)} \quad R = K\left\{\frac{\operatorname{Tan}\frac{1}{2}\theta}{\operatorname{Tan}\frac{1}{2}\theta_i}\right\}^h \frac{\operatorname{Sin}\theta_i}{\operatorname{Sin}\theta} \tag{7}$$

$$\text{where } K = R_i \frac{\operatorname{Sin}\theta_i}{(\operatorname{Tan}\frac{1}{2}\theta_i)^h}$$

When $\theta = \pi/2$, R = K. Therefore, K is the range when V_m is perpendicular to V_t.

$$\text{Since } \operatorname{Tan}\tfrac{1}{2}\theta = \operatorname{Sin}\theta/(1+\operatorname{Cos}\theta) \tag{8}$$

$$R = K\frac{(\operatorname{Sin}\theta)^{h-1}}{(1+\operatorname{Cos}\theta)^h} \tag{9}$$

The turning rate ($\dot{\theta}$) can be obtained by substituting equation (9) for R in equation (2).

$$\dot{\theta} = -\frac{V_t}{K}(\operatorname{Sin}\theta)^{2-h}(1+\operatorname{Cos}\theta)^h \tag{10}$$

Since θ always approaches 0 as R approaches 0

$$\lim_{R\to 0}\dot{\theta} = \lim_{\theta\to 0}\dot{\theta} = \pm\frac{2^h V_t}{K}(\operatorname{Sin}\theta)^{2-h} \tag{11}$$

The $\pm$ sign adapts the equation to the case where V_t is negative. The signs could be retained throughout the mathematics, but since the sense of the various quantities can easily be determined from the geometry, the signs are ignored in the following development.

For h less than 2, the limit of equation (11) is 0 as θ approaches 0. When h is greater than 2, 2-h is negative and equation (11) becomes:

$$\pm\frac{2^h V_t}{K(\operatorname{Sin}\theta)^{h-2}}$$

which is infinite at $\theta = 0$. When h = 2, equation (11) becomes

$$\pm\frac{4V_t}{K}$$

Therefore

$$\lim_{R\to 0}\dot{\theta} = 0, \qquad h < 2 \tag{12}$$

$$\lim_{R\to 0}\dot{\theta} = \pm\frac{4V_t}{K}, \qquad h = 2 \tag{13}$$

$$\lim_{R\to 0}\dot{\theta} = \pm\infty, \qquad h > 2 \tag{14}$$

To find the maximum value of $\dot{\theta}$ in the interval $1 \leq h \leq 2$, set $d\dot{\theta}/dt$ equal to 0.

$$\frac{d\dot{\theta}}{dt} = -\frac{V_t}{K}\left\{(2-h)(\operatorname{Sin}\theta)^{1-h}\operatorname{Cos}\theta(1+\operatorname{Cos}\theta)^h - (\operatorname{Sin}\theta)^{2-h}h(1+\operatorname{Cos}\theta)^{h-1}\operatorname{Sin}\theta\right\} = 0 \tag{15}$$

$$(2-h)\operatorname{Cos}\theta(1+\operatorname{Cos}\theta) = h\operatorname{Sin}^2\theta \tag{16}$$

Since $\operatorname{Sin}^2\theta = 1 - \operatorname{Cos}^2\theta$

$$2\operatorname{Cos}\theta - h\operatorname{Cos}\theta + 2\operatorname{Cos}^2\theta - h\operatorname{Cos}^2\theta = h - h\operatorname{Cos}^2\theta \tag{17}$$

$$2\operatorname{Cos}^2\theta + (2-h)\operatorname{Cos}\theta - h = 0 \tag{18}$$

$$\operatorname{Cos}^2\theta + \left(1-\frac{h}{2}\right)\operatorname{Cos}\theta - \frac{h}{2} = 0 \tag{19}$$

$$\left(\operatorname{Cos}\theta - \frac{h}{2}\right)(\operatorname{Cos}\theta + 1) = 0 \tag{20}$$

$$\operatorname{Cos}\theta = \frac{h}{2}, -1 \tag{21}$$

If $\operatorname{Cos}\theta = -1$, $\theta = \pi$. From the geometry, it can be seen that θ will never equal π unless the initial value is π. This is the special case of a target heading directly at the launch point, in which case the flight path will be a straight line.

Otherwise, the maximum turning rate will occur at $\theta = \operatorname{Cos}^{-1}h/2$.

Substituting h/2 for $\cos\theta$, and $[1-(h/2)^2]^{1/2}$ for $\sin\theta$ in equation (10):

$$\dot{\theta}\,\max = -\frac{V_t}{K}\left[1-\frac{h^2}{4}\right]^{1-h/2}\left[1+\frac{h}{2}\right]^{h} \tag{22a}$$

for V_t in the direction assumed in the initial diagram. Note that this reduces to equation (13) at h = 2.

For V_t in the opposite direction, the maximum turning rate is given by:

$$\dot{\theta}\,\max = \frac{V_t}{K}\left[1-\frac{h^2}{4}\right]^{1+h/2}\left[1-\frac{h}{2}\right]^{-h} \tag{22b}$$

obtained by substituting $-V_t$ for V_t and $-h$ for h.

The time of flight is more difficult to obtain.

Multiply (1) by $\cos\theta$, and (2) by $\sin\theta$:

$$\dot{R}\cos\theta = V_t\cos^2\theta - V_m\cos\theta \tag{23}$$

$$R\dot{\theta}\sin\theta = -V_t\sin^2\theta \tag{24}$$

(23)—(24) $$\dot{R}\cos\theta - R\dot{\theta}\sin\theta = V_t(\cos^2\theta+\sin^2\theta) - V_m\cos\theta \tag{25}$$

From (1): $\cos\theta = \dfrac{\dot{R}+V_m}{V_t} = \dfrac{\dot{R}}{V_t} + h$

$$\dot{R}\cos\theta - R\dot{\theta}\sin\theta = V_t - h\dot{R} - hV_m \tag{26}$$

$$\dot{R}(\cos\theta+h) - R\dot{\theta}\sin\theta = V_t - hV_m \tag{27}$$

$\int$(26) dt $$\int\frac{dR}{dt}(\cos\theta+h)\,dt - \int R\frac{d\theta}{dt}\sin\theta\,dt = \int(V_t - hV_m)\,dt \tag{28}$$

$$\int(\cos\theta+h)\,dR - \int R\sin\theta\,d\theta = \int(V_t-hV_m)\,dt \tag{29}$$

let $u = (\cos\theta + h)$, $du = -\sin\theta\,d\theta$

let $v = R$, $dv = dR$

$$\int u\,dv + \int v\,du = \int(V_t - V_m)\,dt \tag{30}$$

$$u\,v = \int(V_t - hV_m)\,dt \tag{31}$$

$$(\cos\theta+h)\,R\Big|_{R_i,\theta_i}^{R,\theta} = (V_t - hV_m)\,t \tag{32}$$

$$t = \frac{(\cos\theta+h)R - (\cos\theta_i+h)R_i}{V_t - hV_m} \tag{33}$$

The total time of flight is t at R = 0.

$$t_f = \frac{R_i(\cos\theta_i + h)}{V_t - hV_m} \tag{34}$$

The distance travelled is given by

$$S = V_m t_f = \frac{hR_i(\cos\theta_i+h)}{1-h^2} \tag{35}$$

ACCELERATION CIRCLES

One important consideration in a pursuit course is determining if the missile is capable of the acceleration necessary to maintain the course. The lateral acceleration is equal to $V_m\dot{\theta}$. Equation (14) shows that infinite acceleration is required to maintain a pursuit course if the missile velocity is greater than twice the target velocity. This does not prevent the use of a pursuit course in such cases because it is often not necessary that the missile actually hit the target. If the missile has a proximity fuse, for example, it may get close enough before infinite acceleration is required. Or if the pursuer is an interceptor aircraft, the pursuit path need only be followed until the target is within range of the interceptors' weapons.

From equation (2), it can be seen that the acceleration (A) is given by:

$$A = V_m\dot{\theta} = \frac{-V_m V_t \sin\theta}{R} \tag{36}$$

or $$R = \frac{-V_m V_t \sin\theta}{A} \tag{37}$$

This is the equation for a family of circles of diameter $V_m V_t/A$. The circles are tangent to the target track at the target position.

From the diagram, it is evident that the missile can get close to the target on a pursuit course without requiring excessive acceleration. Note that if a missile is launched from a point ahead of the target, the acceleration required is greater than if the missile is launched from a point behind the target.

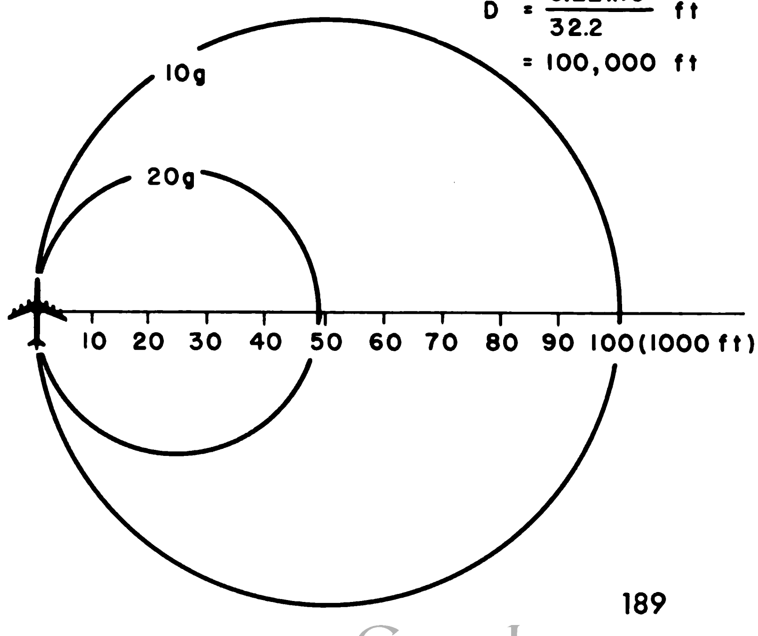
Even if a hit is required, it is possible to use a pursuit course when h > 2. The analysis applies to points, or to the center of mass of the missile and of the target. Since missiles and targets are not points, it is possible for part of the missile to strike part of the target.

Note that much greater acceleration is needed to hit a target which is approaching the missile than a target which is receding. This gives a manueverable target a simple means of evasion by turning toward the approaching missile.

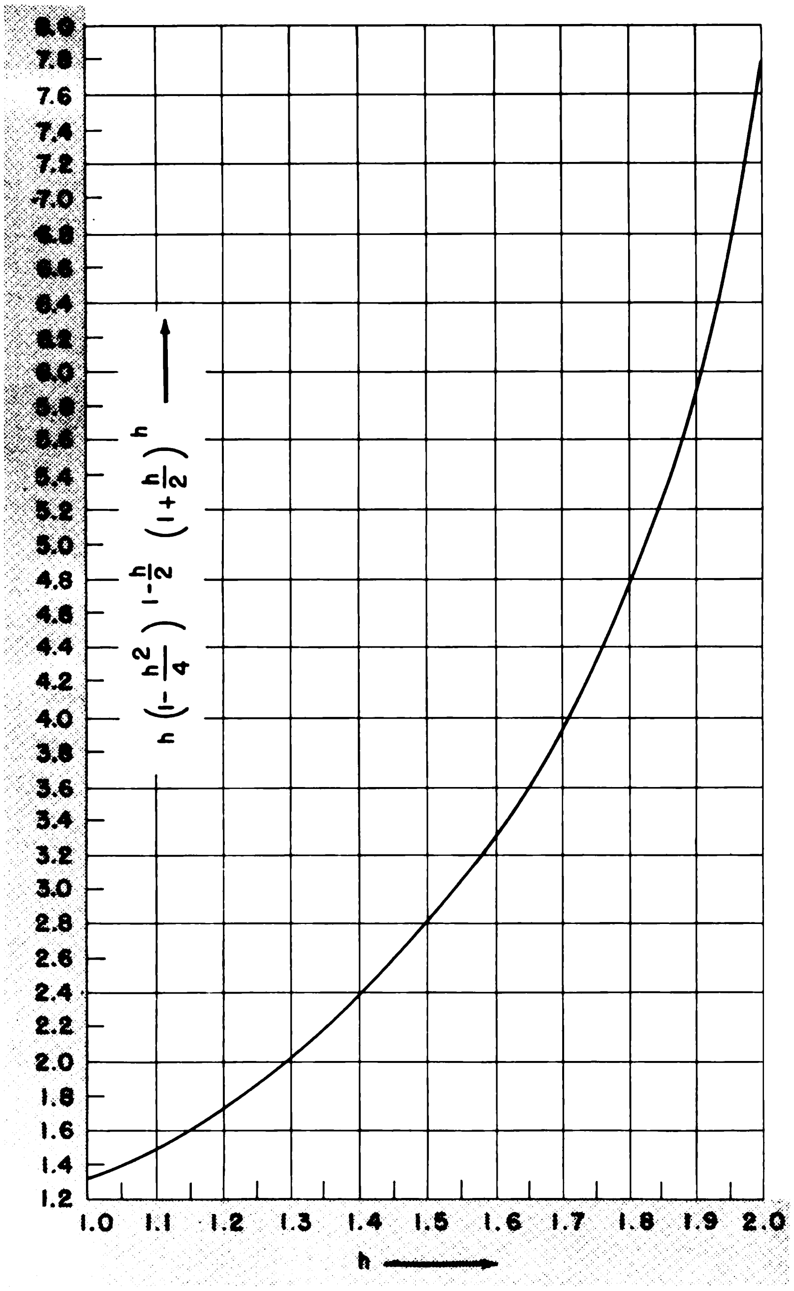
V_m = 2000 ft / sec

V_t = 1,610 ft / sec

$D = \dfrac{3.22\times10^6}{32.2}$ ft = 100,000 ft



For most situations which require that the missile comes close to the target on a pursuit path, missile-to-target velocity ratios less than 2 are called for. The maximum acceleration in this case occurs when $\theta = \text{Cos}^{1} h/2$. The maximum acceleration can be calculated from equation (22). The factor $h\left[1-\frac{h^2}{4}\right]^{1-\frac{h}{2}}\left[1+\frac{h}{2}\right]^{h}$ is plotted as a function of h below.



Note that the acceleration required increases rapidly as h increases. The magnitude of $1/K$ also depends on h in a manner which makes the increase in acceleration even more pronounced for large values of h. A typical situation will serve to illustrate this point.

V_t = 1,000 ft/sec (593 knots) $\theta_i = 135°$
V_m = 1,500 ft/sec (889 knots) R_i = 35,000 ft
h = 1.5

$$V_m \dot{\theta}_{max} = \frac{-V_m V_t}{K}\left[1-\frac{h^2}{4}\right]\left[1+\frac{h}{2}\right]$$

$$= \frac{-1.5 \times 10 \ \text{ft}^2/\text{sec}^2 \ (\text{Tan } 67.5°)\ (.437)\ (1.75)}{3.4 \times 10 \ \text{ft} \quad (\text{Sin } 135°) \quad 32.2 \ \text{ft}/\text{sec2}/\text{g}}$$

= 1150 g's

This is obviously unattainable. If the missile velocity is only 1100 ft/sec (h = 1.1), the same situation requires an acceleration of

$$\frac{-1.1 \times 10 \ (\text{Tan } 67.5°)\ (.6975)\ (1.55)}{3.5 \times 10 \ (\text{Sin } 135°)\ 32.2} = 5.0 \text{ g's}$$

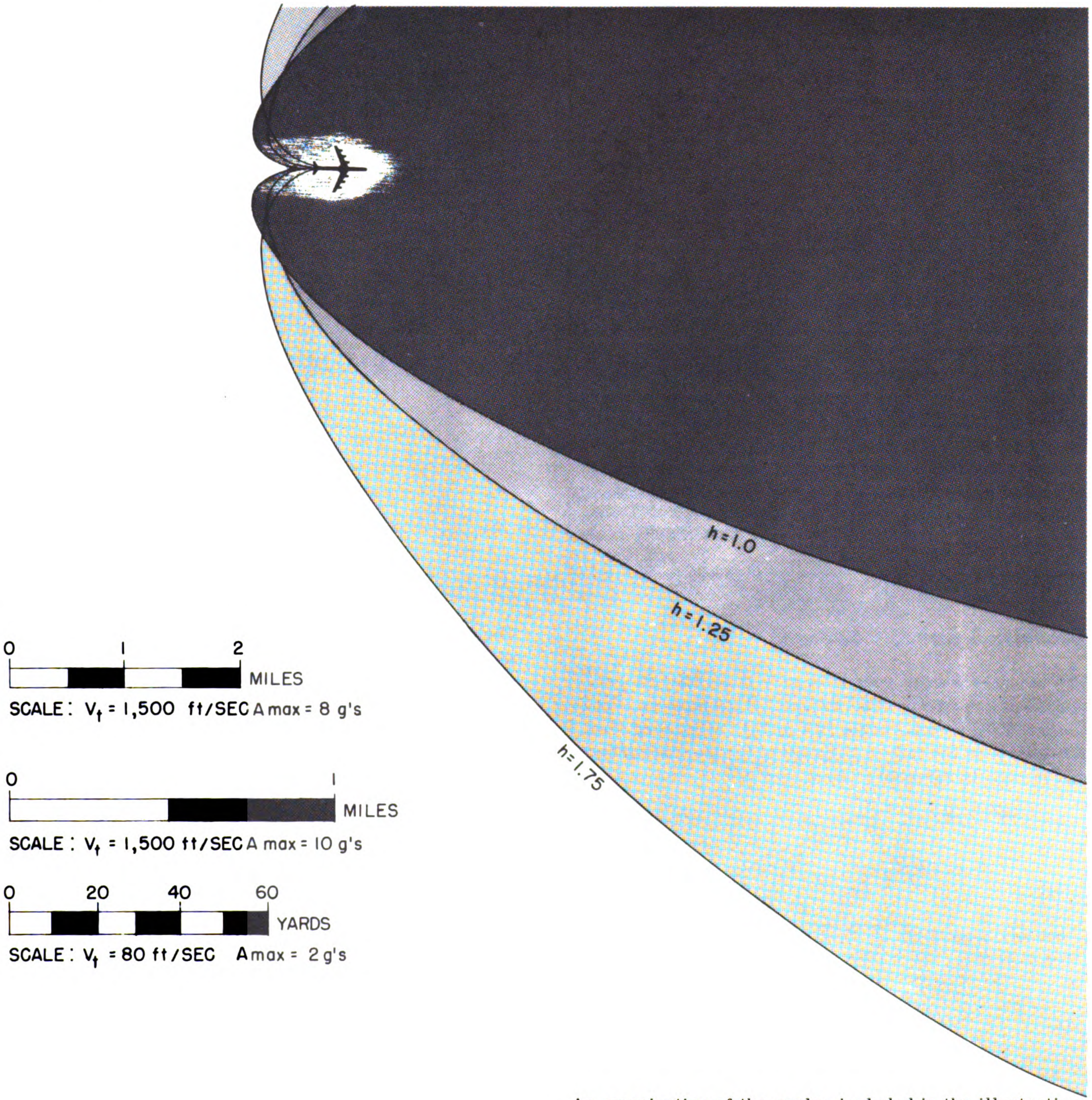
This is a much more reasonable situation. The time of flight in the second case, from equation (34), is

$$T = \frac{-3.5 \times 10 \ \text{ft}\ (.707 + 1.1)}{(10^{'} - 1.2 \times 10\ )\ \text{ft}/\text{sec}} = 301 \text{ sec},$$

or 5 minutes. The target will travel almost 6 miles in this time; an excessive distance in many cases.

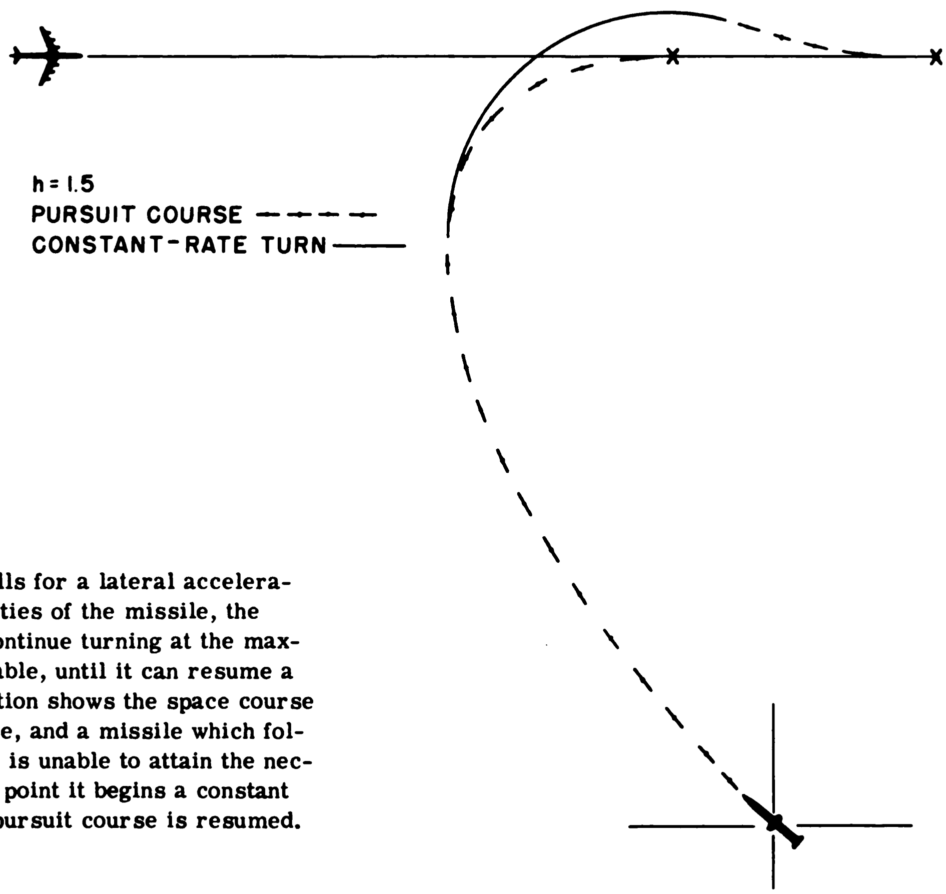
For a missile capable of a particular maximum acceleration, and for a particular value of h, there are only a limited number of launch points with respect to the target position, which will permit a pursuit course. From equations (22) and (2) the boundary of the area of acceptable launch points is given by:

$$\frac{Ri\ (\sin\theta_i)^{h-1}}{(1+\cos\theta_i)^{h}} = \frac{V_m\ V_t\left[1-\frac{h}{4}\right]^{2}\ 1\text{-}h/2\ \left[1+\frac{h}{2}\right]^{h}}{A\ max} = \frac{-h\ V_t^{2}\left[1-\frac{h^{2}}{4}\right]^{1-h/2}\left[1+\frac{h}{2}\right]^{h}}{A\ max}$$



The factor $h\ \left[1-\frac{h^2}{4}\right]^{1-h/2}\ \left[1+\frac{h}{2}\right]^{h}$ can be obtained from the graph illustrated earlier. Each value of h will yield a different curve but different values of V_t or A max will only change the scale. The area of acceptable launch points for H = 1.75, h = 1.2 and h = 1 are illustrated.

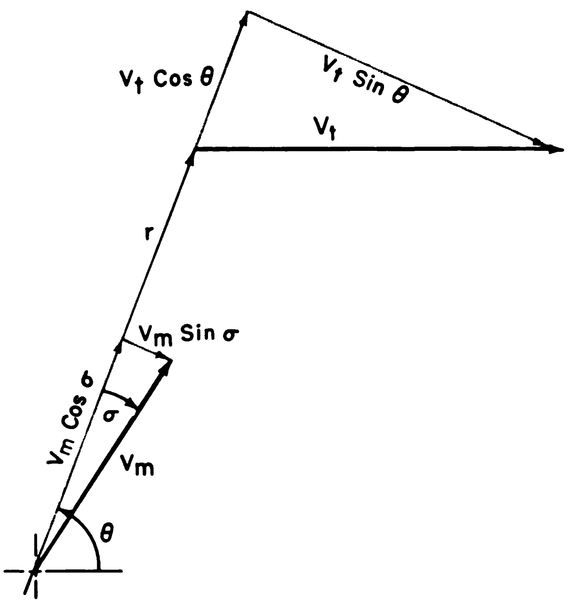
An examination of the scales included in the illustration shows that the area of acceptable launch points is severely limited when the target velocity is high, but not very restricted when the target velocity is low. This is not unexpected since maximum acceleration varies as V_t^2. At typical aircraft speeds, the pure pursuit course is most useful for missiles launched from a point behind the target, as is common for air-to-air missiles.



If the pure pursuit course calls for a lateral acceleration greater than the capabilities of the missile, the missile can be designed to continue turning at the maximum rate of which it is capable, until it can resume a pursuit course. The illustration shows the space course for both a pure pursuit course, and a missile which follows a pursuit course until it is unable to attain the necessary acceleration. At this point it begins a constant rate turn to a point where a pursuit course is resumed.

deviated pursuit course

The deviated pursuit course is very much like the pure pursuit course except that the missile heading (α) leads the target by a small angle (σ) instead of heading directly toward the target. The angle (σ) may be variable, and in fact, is variable in most cases.

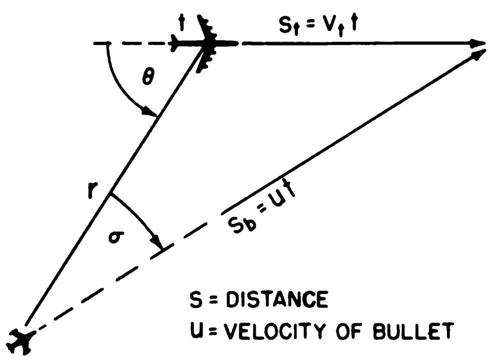


The most common application of this course occurs when the missile is an aircraft. The angle (σ) is normally chosen so that the pursuer's guns lead the target by the proper amount to secure a hit on the target. Therefore, the time of flight of the bullet must be equal to the time of flight of the target for the distances (S_b, S_t) shown on the diagram.

$$\frac{S_t}{V_t} = \frac{S_b}{u}$$

Since: $$\frac{S_t}{\sin\sigma} = \frac{S_b}{\sin(180° - \theta)} = \frac{S_b}{\sin\theta}$$

$$\sin\sigma = \frac{V_t}{u}\sin\theta$$



The equation for the deviated pursuit course is derived for this situation (see box) with the approximation: $\cos\sigma = 1$. A typical ratio of V_t/u is 0.1. The worst case value of $\cos\sigma$ in this case would occur when $\sin\theta = 0.1$.

$$\cos\sigma = \sqrt{1-\sin^2\theta} = \sqrt{1-0.1^2} = .995$$

The approximation, therefore, is a very good one.

From the vector diagram:

$$\dot{R} = V_t \cos\theta - V_m \cos\sigma \qquad (1)$$

$$R\dot{\theta} = V_t \sin\theta + V_m \sin\sigma \qquad (2)$$

$$(1)\div(2) \quad \frac{\dot{R}}{R} = \frac{V_t\cos\theta - V_m\cos\sigma}{V_t\sin\theta + V_m\sin\sigma}\dot{\theta} \qquad (3)$$

For a particular situation (see text)

$$\sin\sigma = \frac{V_t}{u}\sin\theta \qquad (4)$$

$$\cos\sigma = \sqrt{1-\sin^2\sigma} = \frac{V_t}{u}\sqrt{\frac{u^2}{V_t^2} - \sin^2\theta} \qquad (5)$$

Substitution in equation (3) gives:

$$\frac{\dot{R}}{R} = \frac{V_t\cos\theta - \frac{V_m V_t}{u}\sqrt{\frac{u^2}{V_t^2} - \sin^2\theta}}{-V_t\sin\theta + \frac{V_m V_t}{u}\sin\theta}\dot{\theta} \qquad (6)$$

$$\frac{\dot{R}}{R} = \frac{\epsilon\cos\theta - h\sqrt{\epsilon^2 - \sin^2\theta}}{(h-\epsilon)\sin\theta}\dot{\theta} \qquad (7)$$

where $h = \frac{V_m}{V_t}$ **and** $\epsilon = {}^{u}/V_t$

$$\frac{dR}{R} = \frac{1}{h-\epsilon}\left\{\epsilon\cot\theta - h\sqrt{\epsilon^2\csc^2\theta - 1}\right\}d\theta \qquad (8)$$

This can be integrated, although with considerable difficulty. The result is:

$$R = K'\left\{\frac{\cos\sigma - \cos\theta}{\cos\sigma + \cos\theta}\right\}^{\frac{h\epsilon}{2(\epsilon-h)}}\left\{\frac{\epsilon\cos\sigma + \cos\theta}{\epsilon\cos\sigma - \cos\theta}\right\}^{\frac{h}{2(\epsilon-h)}}(\sin\theta)^{\frac{h\epsilon}{\epsilon-h}}$$

Where K′ includes all the constants of integration and is equal to the range when $\theta = 90°$, just as in the pure pursuit course. This equation is much too cumbersome for the purposes of this volume so it will not be derived or used.

Substituting 1 for $\cos\sigma$ in equation (3) gives:

$$\frac{\dot{R}}{R} = \frac{V_t\cos\theta - V_m}{-V_t\sin\theta + \frac{V_m}{V_t}u\sin\theta}\dot{\theta} \qquad (9)$$

instead of equation (6),

$$\frac{\dot{R}}{R} = \frac{h\cos\theta - 1}{(\epsilon-h)\sin\theta}\dot{\theta} \qquad (10)$$

$$\frac{dR}{R} = \frac{\csc\theta - h\cot\theta}{h-\epsilon}d\theta \qquad (11)$$

$$\ln\frac{R}{R_i} = \frac{(\ln\tan\tfrac{1}{2}\theta - h\ln\sin\theta) - (\ln\tan\tfrac{1}{2}\theta_i - h\sin\theta_i)}{h-\epsilon} \qquad (12)$$

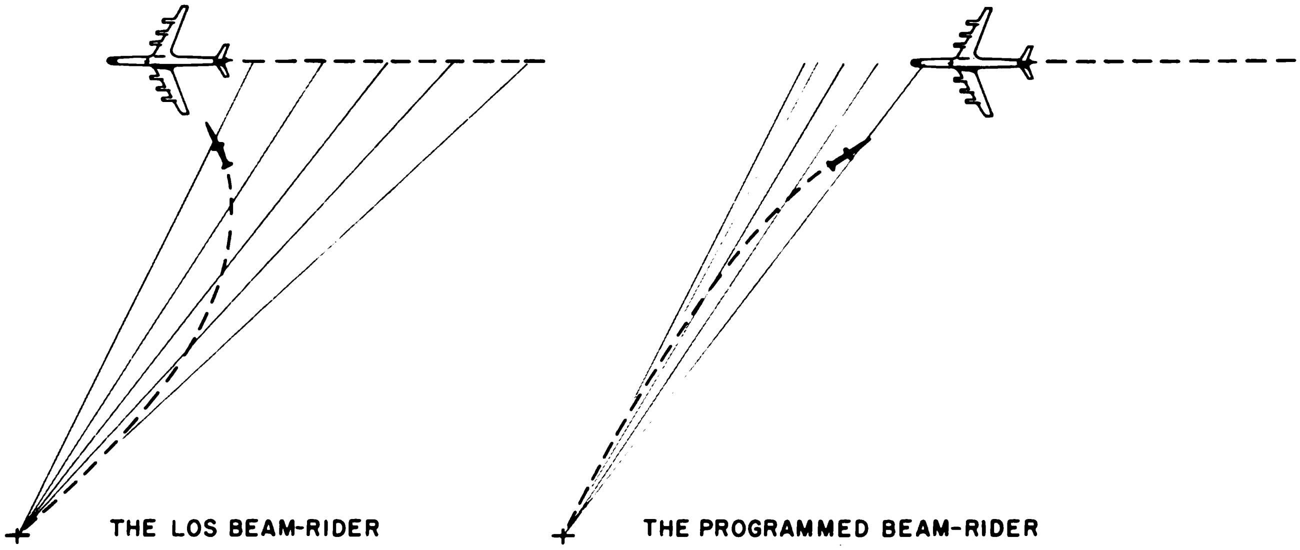
$$R = K'\left\{\frac{\tan\tfrac{1}{2}\theta}{(\sin\theta)^{1/h}}\right\}^{\frac{h}{1-h/\epsilon}} \qquad (13)$$

where K′ includes all the constants of integration. The constant (K′) is equal to the range when $\theta = 90°$, just as in the pure pursuit course. Equation (13) can be rewritten as:

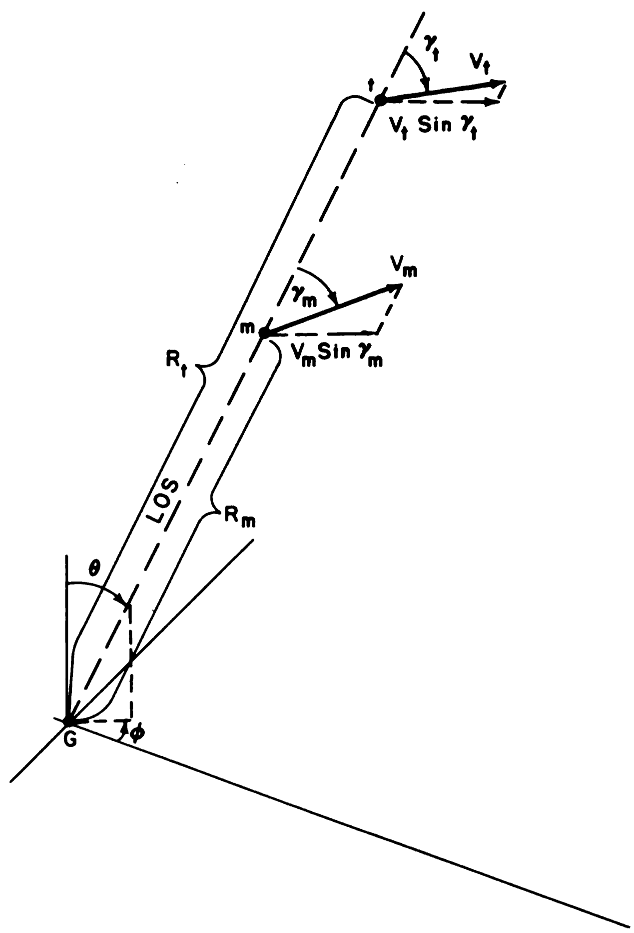
$$R = \left\{K\frac{(\tan\tfrac{1}{2}\theta)^h}{\sin\theta}\right\}^{\frac{1}{1-h/\epsilon}} \qquad (14)$$

The expression in the bracket is the same as the equations for a pure pursuit course.

BEAM RIDER COURSE



The Beam Rider course is followed by a missile which moves along a line, eminating from a guiding point. In practice, the guiding point is normally an antenna which transmits an energy beam at the target. The missile moves so that it always lies within this beam. The term "beam rider" is, therefore, frequently used to describe this flight path. The energy beam may establish the line of sight between sensor and target, or it may be directed into space along a programmed target intercept. This permits missiles to reach high altitudes rapidly-where their propulsion systems can operate more efficiently, thereby achieving greater speeds and greater ranges. It also makes enemy countermeasures more difficult. In this "up-and-over" type of flight path, the missile is still a beam rider but the flight path is quite different from the conventional LOS type of beam rider path. In this chapter, only the simple beam rider flight path, where the missile follows the LOS from launch site to target, will be examined. The illustration shows the notation used. The guiding point is taken as the origin. Primed notations are measurements taken with respect to the missile, rather than the origin.
At first glance, the beam rider course may seem the same as the pure pursuit course. The velocity vector of a missile following a pure pursuit course lies along the line-of-sight from missile to target, and rotates with it.



There is a component of acceleration perpendicular to the LOS, but no component of velocity. The missile following a beam rider course, however, has a component of velocity perpendicular to the LOS, and the dynamic properties are somewhat different.

The basic equations for the beam rider course follow directly from the fact that both the missile and the target may lie in the line-of-sight.

$$\theta_m = \theta_t = \theta \tag{1}$$

$$\phi_m = \phi_t = \phi \tag{2}$$

$$\dot{\theta}_m = \dot{\theta}_t = \dot{\theta} \tag{3}$$

$$\dot{\phi}_m = \dot{\phi}_t = \dot{\phi} \tag{4}$$

$$\dot{\theta}_m = \frac{V_t \operatorname{Sin} \gamma_t}{R_t} = \frac{V_m \operatorname{Sin} \gamma_m}{R_m} \tag{5}$$

$$k \operatorname{Sin} \gamma_t = h \operatorname{Sin} \gamma_m \tag{6}$$

where $k = \frac{R_m}{R_t}$ and $h = \frac{V_m}{V_t}$

Also

$$\operatorname{Cos} \gamma_m = \sqrt{1-\operatorname{Sin}^2 \gamma_m} = \sqrt{\frac{h^2-k^2 \operatorname{Sin}^2 \gamma_t}{h}} \tag{7}$$

The components of V_m are shown in the illustration.

$$V_m^2 = (R_m)^2 + (R_m \theta)^2 + (R_m \operatorname{Sin} \theta \, \phi)^2 \tag{8}$$

In the case of straight line target motion, the axes can always be chosen so $\phi = 0$ and $\gamma_t = \frac{\pi}{2} - \theta$ (target moving parallel to x axis).

In this case, from the diagram.

$$(\Delta S)^2 = (\Delta R_m)^2 + (R_m \Delta \theta)^2 \tag{9}$$

$$\left[\frac{ds}{d\theta}\right]^2 = \left[\frac{dR_m}{d\theta}\right]^2 + R_m^2 \tag{10}$$

Since: $\frac{ds}{d\theta} = \frac{ds}{dt}\frac{dt}{d\theta} = V_m \frac{dt}{d\theta} = \frac{V_m R_t}{V_t \operatorname{Cos} \theta}$

$$\left[\frac{ds}{d\theta}\right]^2 = \frac{h^2 R_t^2}{\operatorname{Cos}^2 \theta} \tag{11}$$

$$\left[\frac{dR_m}{d\theta}\right]^2 + R_m^2 = h^2 R_t^2 \operatorname{Sec}^2 \theta \tag{12}$$

The missile turning rate is equal to $d\alpha/dt$. From the diagram:

$\operatorname{Tan} \alpha = \frac{dx}{dy}$

$$y = R_m \operatorname{Cos}\theta, \quad dy = R_m' \operatorname{Cos}\theta \, d\theta - R_m \operatorname{Sin}\theta \, d\theta \tag{13}$$

$$x = R_m \operatorname{Sin}\theta, \quad dx = R_m' \operatorname{Sin}\theta \, d\theta + R_m \operatorname{Cos}\theta \, d\theta \tag{14}$$

where the primed notation indicates a derivative with respect to θ.

$$\operatorname{Tan}\alpha = \frac{R_m' \operatorname{Sin}\theta + R_m \operatorname{Cos}\theta}{R_m' \operatorname{Cos}\theta - R_m \operatorname{Sin}\theta} \tag{15}$$

since $\operatorname{Sec}^2 \alpha = 1 + \operatorname{Tan}^2 \alpha$,

$$\operatorname{Sec}^2 \alpha = \frac{R_m'^2 \operatorname{Cos}^2\theta + R_m^2 \operatorname{Sin}^2\theta - R_m R_m' \operatorname{Cos}\theta \operatorname{Sin}\theta + R_m'^2 \operatorname{Sin}^2\theta + R_m^2 \operatorname{Cos}^2\theta + R_m R' \operatorname{Sin}\theta \operatorname{Cos}\theta}{(R_m' \operatorname{Cos}\theta - R_m \operatorname{Sin}\theta)^2} \tag{16}$$

$$= \frac{R_m'^2 + R_m^2}{(R_m' \operatorname{Cos}\alpha - R_m \operatorname{Sin}\alpha)^2} \tag{17}$$

Differentiating equation (12) with respect to θ.

$$\operatorname{Sec}^2 \alpha \, \alpha' = \frac{2R_m'^2 + R_m^2 + R_m R_m''}{(R_m' \operatorname{Cos}\theta - R_m \operatorname{Sin}\theta)^2} \tag{18}$$

Since: $\frac{d\alpha}{dt} = \frac{d\alpha}{d\theta}\frac{d\theta}{dt} = \alpha' \dot{\theta}$, the turning rate ($\dot{\alpha}$) can be obtained from equations (5), (17) and (18)

$$\dot{\alpha} = \alpha' \dot{\theta} = \frac{2R_m'^2 + R_m^2 + R_m R_m''}{R_m'^2 + R_m^2} \times \frac{V_t \operatorname{Sin}\gamma_t}{R_t} \tag{19}$$

From the diagram,

$$\operatorname{Sin}\gamma_t = \operatorname{Cos}\theta \tag{20}$$

From equation (12)

$$\begin{aligned} R_m'^2 &= h^2 R_t^2 \operatorname{Sec}^2\theta - R_m^2 \\ &= h^2 R_t^2 \operatorname{Cos}^2\theta \operatorname{Sec}^4\theta - R_m^2 \end{aligned} \tag{21}$$

$$\begin{aligned} R_t \operatorname{Cos}\theta &= \gamma_t = \text{constant.} \\ 2 R_m' R_m'' &= h^2 R_t^2 \operatorname{Cos}^2\theta [4 \operatorname{Sec}^4\theta \operatorname{Tan}\theta] - 2R_m R_m' \end{aligned} \tag{22}$$

Therefore, equation (19) becomes:

$$\dot{\alpha} = \frac{2V_t \operatorname{Cos}\theta}{R_t}\left[1 - \frac{k \operatorname{Sin}\theta}{\sqrt{h^2 - R^2 \operatorname{Cos}^2\theta}}\right] \tag{23}$$

where $k = R_m/R_t$
$h = V_m/V_t$

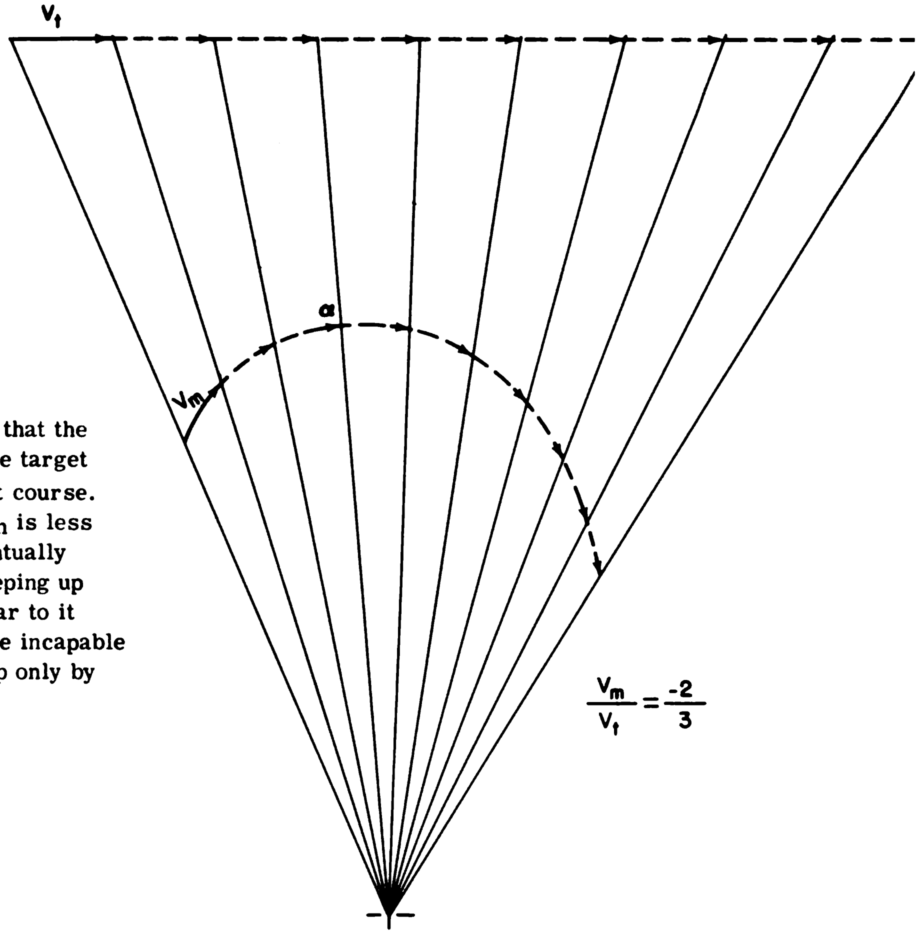
The lateral acceleration of the missile is simply:

$$A = V_m \dot{\alpha} \tag{24}$$

For a particular target velocity and range, the acceleration is a maximum when $k = 1$ and $\theta = 0$.

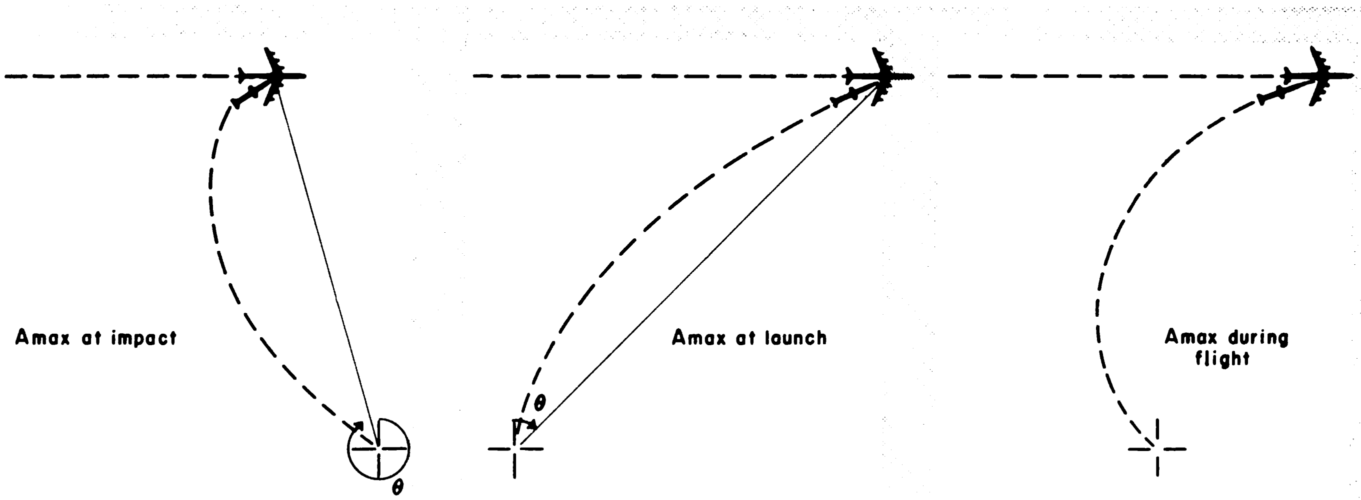
At this point

$$A_{max} = \frac{2V_m V_t}{R_t} \tag{25}$$



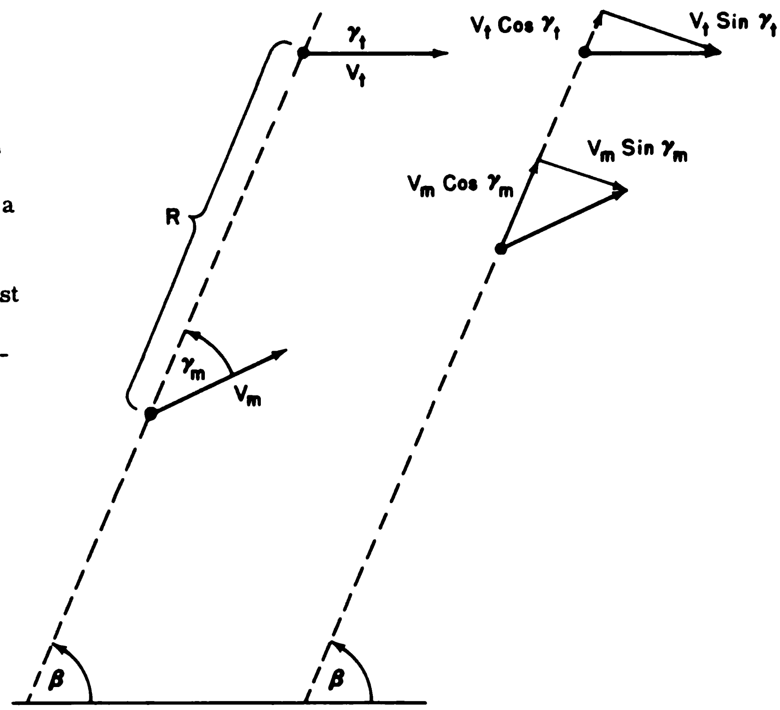
An important point in a beam-rider course is that the missile velocity (V_m) must be greater than the target velocity (V_t), just as in the case of the pursuit course. The illustration shows a situation in which V_m is less than V_t. In such a case, the missile may eventually reach a point at which it is just capable of keeping up with the beam, even when moving perpendicular to it (point a on the diagram). Thereafter, it may be incapable of keeping up with the beam, or it may keep up only by turning in an attack on the guiding point.

The characteristics of the missile acceleration can be obtained from equation (23), (24) and (25). As shown, the maximum acceleration occurs when $\theta = 0$ and $h = 1$. In general, these two conditions will not occur simultaneously and the maximum acceleration will be less than $2V_mV_t/R_t$. For a situation in which θ is always in the fourth quadrant (an approaching target intercepted before the point of closest approach), the maximum acceleration will occur at impact (maximum h). For a situation in which θ is always in the first quadrant the maximum acceleration will occur at launching (minimum θ). If θ passes through 0, the maximum acceleration will occur during flight. It is evident that the beam-rider flight path does not create the severe acceleration problems encountered with a pure pursuit course.



CONSTANT BEARING COURSE

The constant bearing course is followed by a missile when the line-of-sight from missile to target maintain constant direction in space, i.e. β = constant.
This is often referred to as a collision course. Since a collision is the ultimate result in all the flight paths discussed, the term constant bearing course is more descriptive. It is evident that the missile velocity must be greater than the target velocity. For the case of straight line target motion, since $\dot{\gamma}_m = \dot{\gamma}_t = 0$, the missile undergoes no acceleration and the equations of motion are simple.
For a maneuvering constant speed target, the missile acceleration never exceeds the target acceleration. This fact makes the constant bearing course of particular value against high performance targets.
From the geometry, and the fact that β is a constant:



$$\dot{R} = V_t \operatorname{Cos} \gamma_t - V_m \operatorname{Cos} \gamma_m \quad (1)$$

$$\dot{B} = 0 = V_t \operatorname{Sin} \gamma_t - V_m \operatorname{Sin} \gamma_m \quad (2)$$

Since γ_m and γ_t are constants:

$$R = (V_t \operatorname{Cos} \gamma_t - V_m \operatorname{Cos} \gamma_m)\, t + R_o \quad (3)$$

where R_o = R at t = o

From (2) $\operatorname{Sin} \gamma_m = \frac{V_t}{V_m} \operatorname{Sin} \gamma_t \quad (4)$

$$\operatorname{Cos} \gamma_m = \sqrt{1 - \left(\frac{V_t}{V_m} \operatorname{Sin} \gamma_t\right)^2}$$

$$= \sqrt{\frac{h^2 - \operatorname{Sin}^2 \gamma_t}{h^2}} \quad (5)$$

where $h = \frac{V_m}{V_t}$

$$R = \left[V_t \operatorname{Cos} \gamma_t - \frac{V_m}{h}\sqrt{h^2 - \operatorname{Sin}^2 \gamma_t}\right] t + R_o \quad (6)$$

The time of flight is obtained by setting R = 0.

$$t_f = \frac{R_o/V_t}{\operatorname{Cos} \gamma_t - \sqrt{h^2 \operatorname{Sin}^2 \gamma_t}} \quad (7)$$

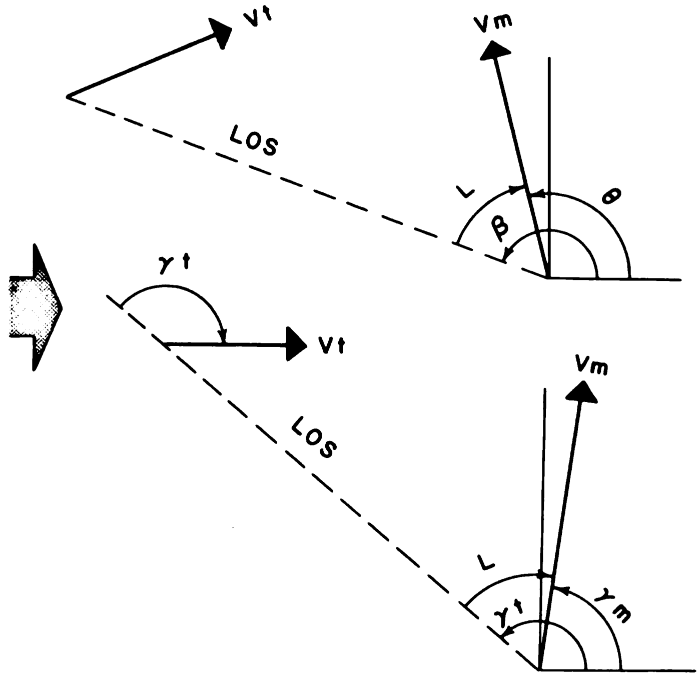
The distance traveled by the missile follows directly.

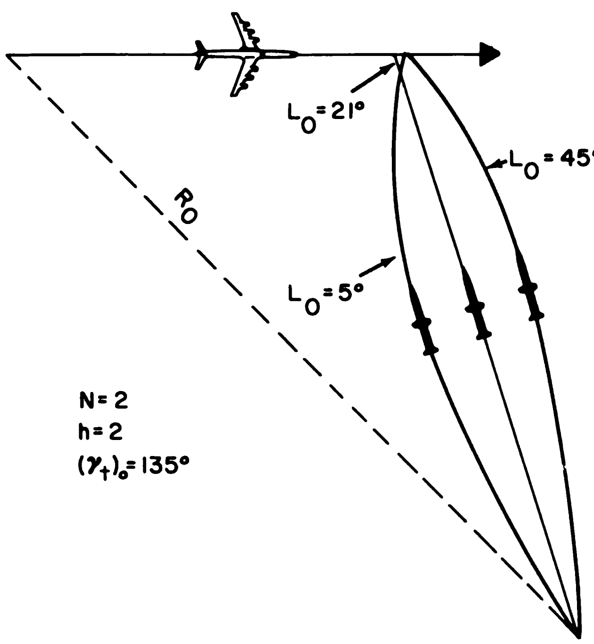
$$D_m = V_m t_f = \frac{hR_o}{\operatorname{Cos} \gamma_t - \sqrt{h^2 - \operatorname{Sin}^2 \gamma_t}} \quad (8)$$

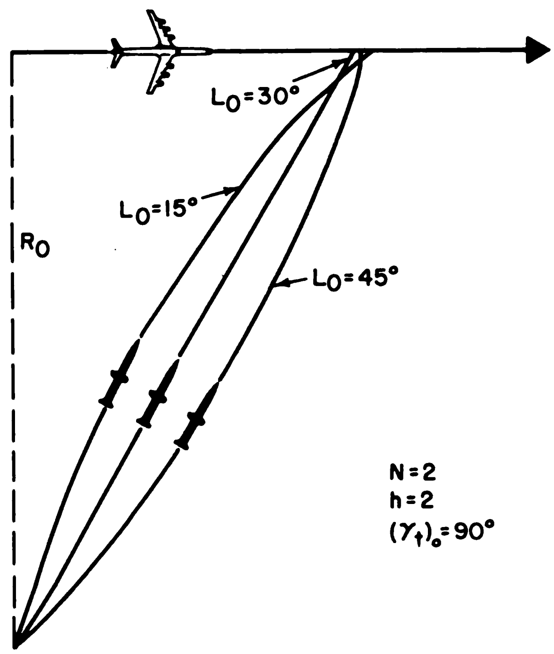
PROPORTIONAL NAVIGATION COURSES

Proportional navigation (or partial navigation) is a practical method of approximating a constant bearing course to a maneuvering target. The perfect constant bearing course requires an instantaneous change in missile heading in response to any change in the target velocity vector. This, of course, is an impossible achievement. In a proportional navigation course, the turning rate of the missile is made proportional to the turning rate of the line of sight, i.e., $\dot{\theta} = N\dot{\beta}$, where N is the proportional navigation ratio. In the constant velocity target situation, with V_t along the horizontal axis, this becomes $\dot{\gamma}_m = N\,\dot{\gamma}_t$. A constant bearing course to a constant velocity target, however, is not a major problem, and the proportional navigation method in this case simply permits correction for error in the initial lead angle.

If the lead angle is chosen to meet the requirements of a constant bearing course, i.e., $V_m \cos \dot{\gamma}_m = V_t \cos \dot{\gamma}_t$, the line of sight will not rotate and the flight path will be a straight line constant bearing course. The proportional navigation courses following three choices of lead angle are illustrated.







The basic equation for the proportion navigation course is:

$$\dot{\gamma}_m = N\,\dot{\gamma}_t \tag{1}$$

$$\dot{\gamma}_m = N\dot{\gamma}_t + \text{constant} \tag{2}$$

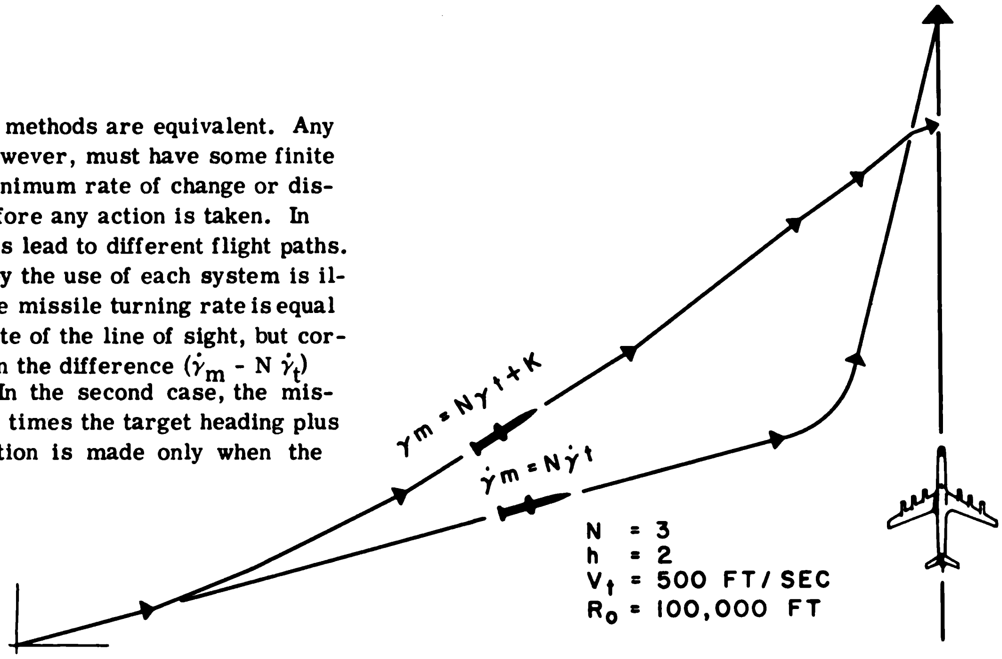
$$\dot{\gamma}_m = N\dot{\gamma}_t + (\dot{\gamma}_m)_0 - N\,(\dot{\gamma}_t)_0 \tag{3}$$

where $(\dot{\gamma}_m)_0$ and $(\dot{\gamma}_t)_0$ are initial values of the angles.

$$\dot{\gamma}_t = \frac{\dot{\gamma}_m - (\dot{\gamma}_m)_0}{N} + (\dot{\gamma}_t)_0 \tag{4}$$

The basic equations for the constant velocity target are easily found (see box). They reveal a fundamental dependence on the type of control system used, since a guidance system could be based on equation (1) or on equation (2). A system based on equation (1) would sense a motion of the line of sight and control the missile velocity to correct any deviation. A system based on equation (2) would sense a displacement of the line of sight bearing from the initial bearing and act on this information. (Any practical system would almost certainly be based on a combination of both parameters).

In the ideal case, the two methods are equivalent. Any actual control system, however, must have some finite sensitivity. A certain minimum rate of change or displacement must occur before any action is taken. In this case, the two methods lead to different flight paths. One flight path obtained by the use of each system is illustrated. In one case the missile turning rate is equal to N times the turning rate of the line of sight, but correction is made only when the difference ($\dot{\gamma}_m$ - N $\dot{\gamma}_t$) reaches 2° per second. In the second case, the missile heading is equal to N times the target heading plus a constant, and a correction is made only when the difference reaches 2°.



A detailed analysis of missile acceleration, time of flight and existence of courses which result in interception of the target would be tedious, since the general equations can be solved in closed form only when N = 2. Some general results can be stated, however.

SUITABLE NAVIGATION RATIOS

If the navigation ratio is less than 2, several undesirable consequences result. Flight paths spiral around the target and missile acceleration or time of flight is excessive. When the unavoidable lag resulting from lack of infinite control-system sensitivity is considered, values of N in the order of 2 to 4 are usually most favorable.

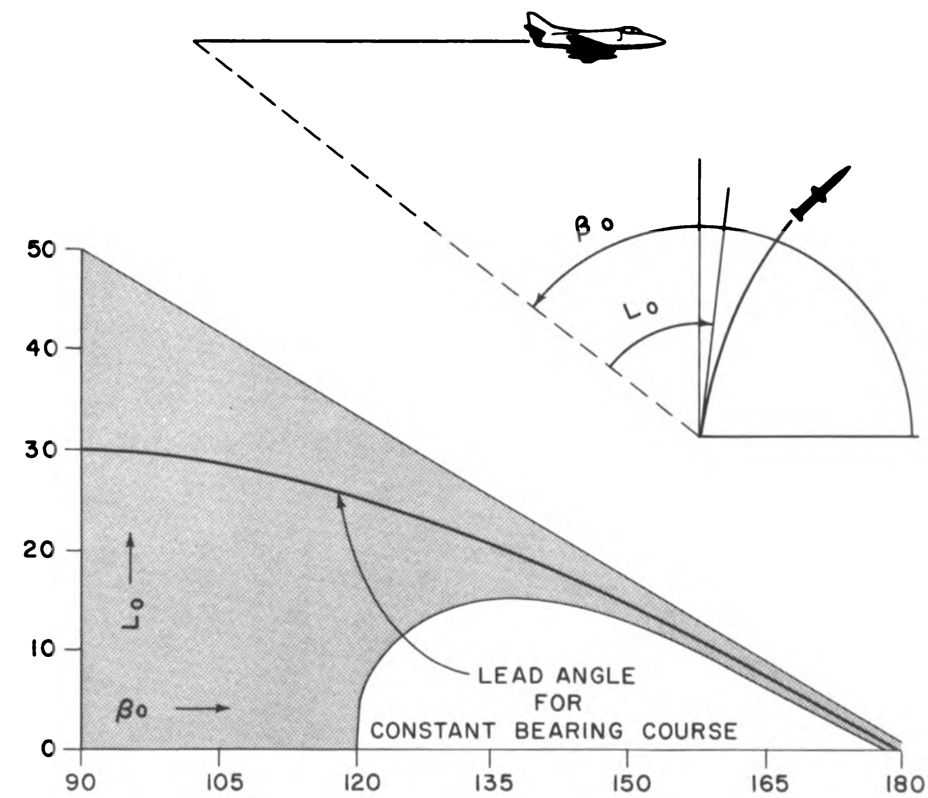
ACCELERATION

For the particular case in point, in which a constant velocity target is attacked by a missile whose velocity exceeds that of the target, a lead angle which results in a flight path requiring no acceleration always exists. This is the lead angle for the constant bearing course.

There is an area of acceptable lead angles about the constant bearing course angle for which a given acceleration will not be exceeded before the missile reaches the vicinity of the target. This point is illustrated for approaching targets. The illustration shows for each initial target bearing angle, the lead angles (shaded area) which do not require acceleration in excess of five g's before the missile is within 50 feet of the target. The solid line represents the constant bearing lead for which the acceleration is zero. Note that the proportional navigation course is similar to the pursuit course in this respect; the limitations on launching become more severe as the initial target bearing approaches 180°.

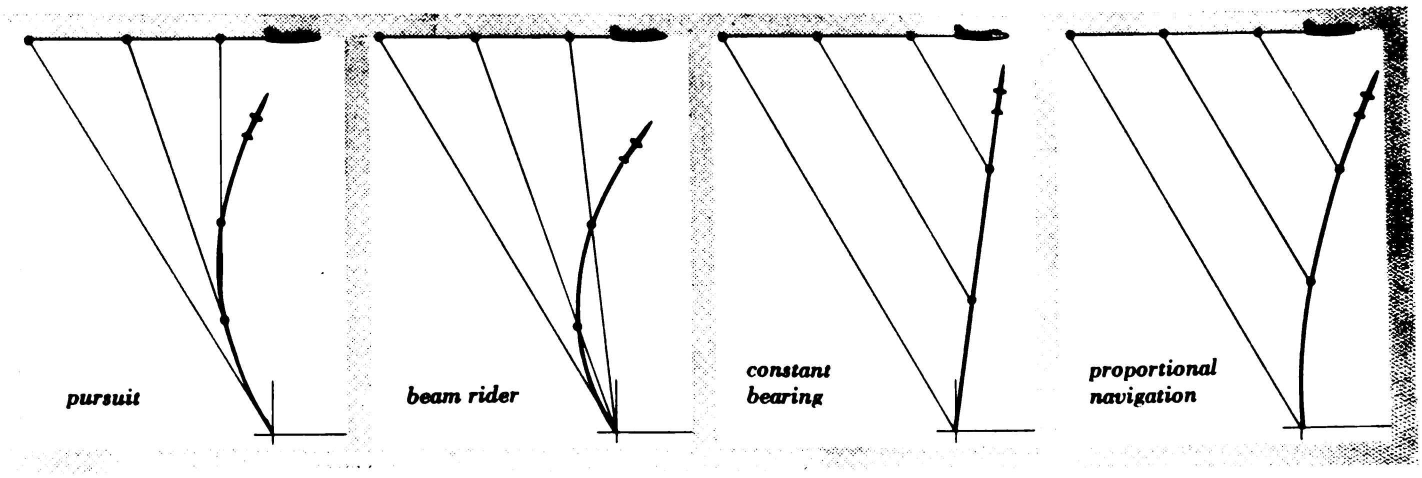
ACCELERATION WITH MANEUVERING TARGET

The case of the maneuvering target presents several complications which rule out any detailed discussion here. However, it can be shown that with appropriate navigation ratios and missile-to-target velocity ratios, the proportional navigation course will require missile turning rates only slightly in excess of the target turning rate under most conditions. The basic limiting conditions are an initial target bearing not too close to 180° and a launch angle which is close to that for a constant bearing course.



summary

All the flight paths analyzed here are idealized cases. They assume point missiles and targets as well as perfect control system performance. Any complete analysis must account for the spacial extent of both missile and target, errors in the guidance system, and the dynamic characteristics of the guidance system (stiffness, lag, etc.). This analysis was further simplified by the restriction to constant velocity targets and constant speed missiles. Since most missiles which employ these flight paths do not vary their speed once operational velocity is attained, this restriction degrades the results very little. However, the guided flight path is of greatest importance against a maneuvering target; the constant velocity target is a lesser challange. Nevertheless, the results of the derivations given here illustrate most of the salient features of the flight paths covered, even though restricted to a special case. It can be seen from the development given that the most significant parameter is the rotation of the missile-to-target line of sight. The course with the least rotation will require less missile acceleration and a shorter time of flight. The line-of-sight rotation is greatest for the pursuit course and least for the constant bearing course. The others lie between, with the proportional navigation course exhibiting less rotation than the beam rider when the initial lead angle is correctly chosen.



PURSUIT COURSES

The pure pursuit course is defined as that described by a missile which always heads directly toward the target. The most important parameter is the missile-to-target velocity ratio, which must be greater than one and less than two in the ideal case. Values greater than two require infinite missile acceleration as the missile approaches the target, while values less than one require infinite time of flight. In general, for a missile with a given upper limit of acceleration, there is a region of permissable launch points which approaches closest to the target at the rear. The most favorable application of this type of course is against slow moving targets, or for missiles launched from a point to the rear of the target.

BEAM RIDER COURSES

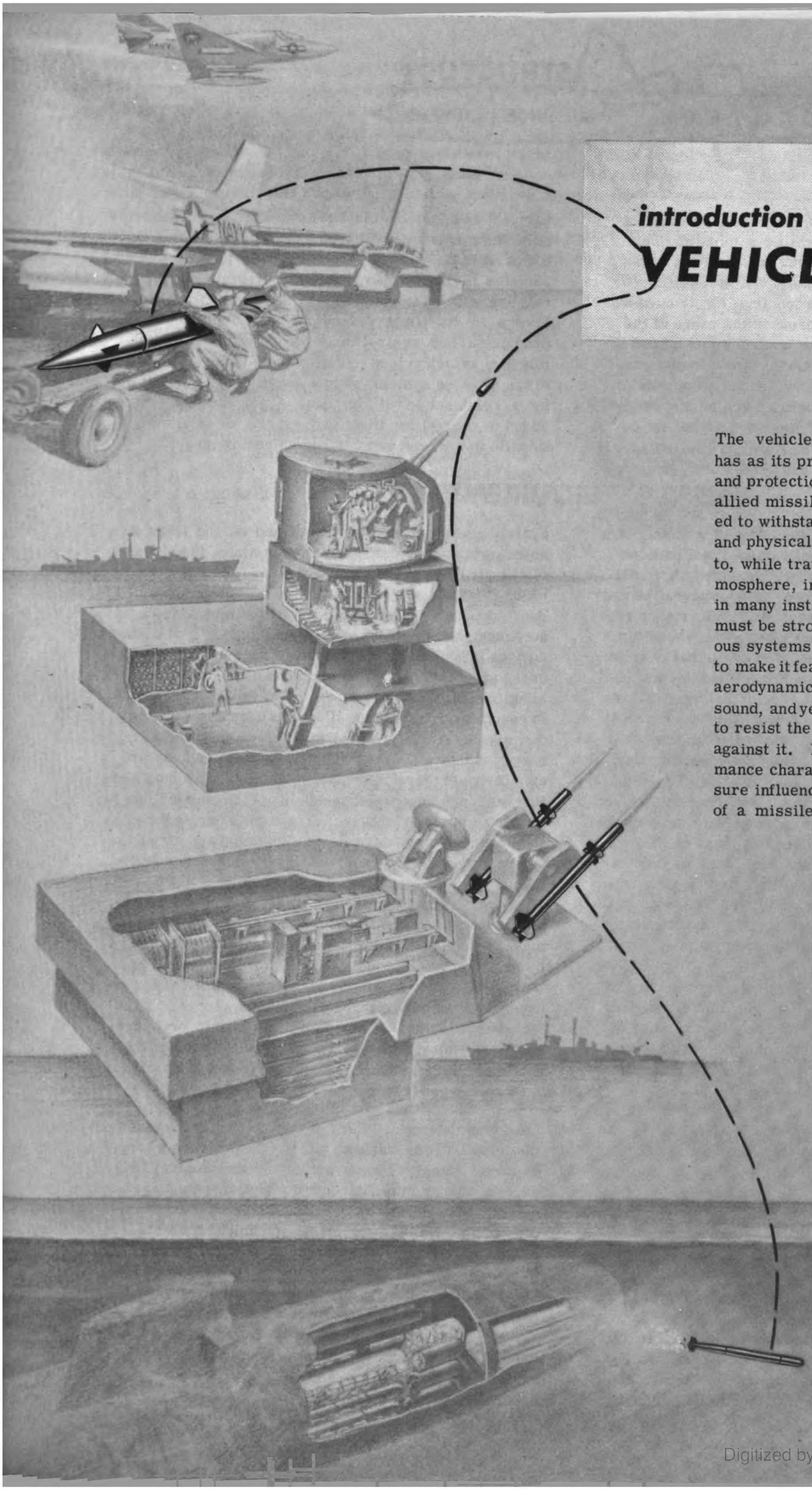
The line-of-sight beam rider course is defined as that described by a missile which remains in the line of sight from a guiding point to the target. This course requires a missile acceleration less than that required for a pursuit course, but greater than the target acceleration. There are also programed beam rider courses, not analyzed here, which differ from the line-of-sight type in that the beam is directed at the target during the terminal phase only. The major advantages of the beam rider course are its flexibility and the minimal complexity of the equipment which must be carried in the missile, since the major burden of guidance is assumed at the source of the beam.

CONSTANT BEARING COURSE

The constant bearing course (frequently called the collision course) is defined as that described by a missile which moves in a manner which holds the heading of the missile-to-target line of sight constant. This course can also be defined as the straight line course to a predicted impact point. For a constant velocity target, the flight path is a straight line.
The outstanding feature of this course is that for a maneuvering, constant speed target, the missile acceleration never exceeds the target acceleration. The major drawback lies in the fact that the weapon control system requires sufficient data gathering and data processing equipment to predict the future position of the target.

PROPORTIONAL NAVIGATION COURSE

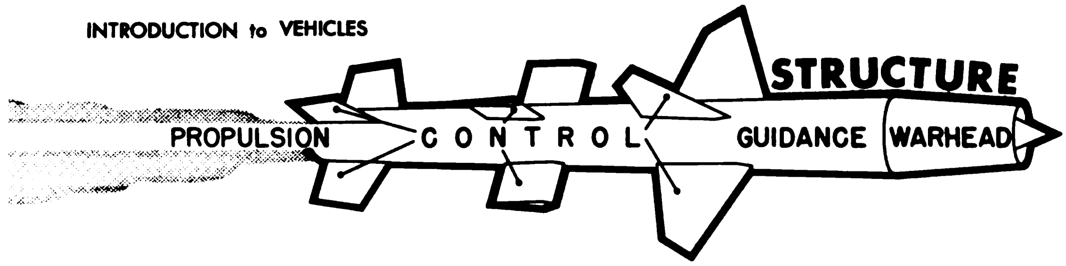
The proportional navigation course is defined as that described when the missile turning rate is equal to the turning rate of the missile-to-target line of sight multiplied by the navigational ratio (N). This course can also be defined as one in which the heading of the missile is equal to N times the heading at the line of sight plus a constant. When account is taken of the finite sensitivity of the control system, the two definitions lead to different paths. Values of N in the range from two to four lead to flight paths with the most desirable characteristics. Lead angles and navigation ratios can usually be chosen so that the missile acceleration will exceed the target acceleration only slightly.



CHAPTER 5

introduction to VEHICLES

The vehicle or structure of a missile has as its primary purpose the support and protection of the warhead and other allied missile components. It is designed to withstand the environmental forces and physical pressures it will be exposed to, while traveling either through the atmosphere, in space or underwater, and in many instances through all three. It must be strong enough to carry the various systems of the missile, light enough to make it feasible and simple to propel, aerodynamically or hydrodynamically sound, and yet resilient and strong enough to resist the forces that will be exerted against it. Its capabilities and performance characteristics to a large measure influence the success or failure of a missile's mission.



Any object that can be hurled, projected, launched or propelled into flight may be called a missile. This definition includes stones, arrows, projectiles, bombs, torpedoes, rockets, and airplanes. The scientific progress made in rocketry, jet propulsion, electronics and aerodynamics not only has increased the range of the missile, but now allows it to be controlled or guided in its flight. A guided missile carries an explosive warhead, and contains within its structure some means of controlling its own trajectory or flight path. As the end result (target kill) is accomplished directly by its destructive energy output, the warhead is the primary component of the missile. The structure supports, carries, and protects the warhead and allied systems from the forces or pressures that will be exerted on them. It is the body or fuselage of the missile and contains the aerodynamic or hydrodynamic surfaces that are utilized to stabilize the missile or to correct errors in its flight trajectory. The functions of propulsion and control may be performed by the missile, as in a guided missile; or by launch components of the weapons system, as in a gun-fired projectile; or by a combination of the two. A guidance system is added to correct for flight path errors, or to allow the missile to seek out and destroy an enemy target.

MISSILE REQUIREMENTS

The required operating characteristics of a missile are determined by a combination of varied specifications. Physical requirements of size, weight, strength, resistance to environmental conditions, and configuration are paramount parameters. In addition, performance requirements of range, speed, altitude, maneuverability, flight path, flight accuracy, stability and lethality must be considered. Cost per target kill is also important. It is desirable to use the most economical weapon possible to obtain the specific end result desired. Physical and performance requirements are not independent but are interrelated. Because missiles are normally designed for use against specific types of targets, target performance and target characteristics will determine the required missile performance, which in turn determines the physical and performance requirements of the configuration. For example, an antisubmarine missile is designed to encounter different flight and target characteristics than an antiaircraft missile. Therefore, it has different physical and performance requirements to meet which will lead to vital differences in specifications and configurations.

In impulse propelled missiles (the large majority of which are unguided) the size, weight, and strength of the missile are determined by the required potency of the warhead, and the type of propulsion and launching system required to deliver it to the target. The forces exerted on the missile at launch and during flight are an important consideration in determining the strength of the structure needed to withstand the pressures to which it will be subjected. As the missile is unguided, the flight path is determined before flight, and flight accuracy and stability are dependent on the stabilizing devices employed by the missile. Range and speed, however, are functions of the impulse force and the missile configuration.

In reaction-propelled missiles, the size and weight of the missile are determined by the required lethality, range, and speed. When the missile is guided, added circuitry and therefore extra weight become a consideration, although new developments and technological advances in miniaturization of components are reducing weight as a factor. Missile configuration and strength are dependent on the required missile maneuverability, stability, speed, and operating altitude.

Missiles which utilize both impulse- and reaction-propulsion techniques in flight, for example, the Polaris, must be designed to fulfill the requirements imposed by both forms of propulsion. In addition, missiles must be designed to withstand the environmental forces present in the media through which they travel. This includes withstanding not only atmospheric or hydrostatic pressure, but also deteriorating action due to salt spray, rain, solar radiation, and unwanted vibrations.

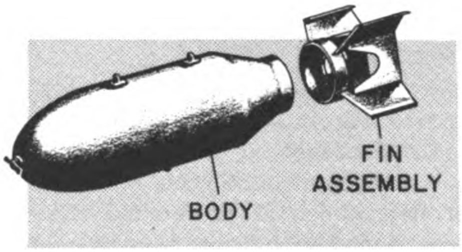
MISSILE COMPONENTS

warheads

In the design of a missile weapon system one of the primary requisites is to estimate the type and size of warhead that will be required to fulfill the mission. The defense posture of the target and the accuracy of the guidance system employed are both critical in the selection of the payload utilized. The problem is twofold: 1) If the accuracy of the guidance system and its probability of performance are of a high order, a warhead with a smaller lethal radius can be effective.

2) If a warhead with larger damage volume (nuclear explosive) is utilized, the same degree of hit probability is not required, and it is necessary only to detonate the warhead in the vicinity of the target, to obtain the same destructive result. Therefore, the decision on the warhead type determines the limits of accuracy required of the missile. The warhead effectively destroys a target or target complex by converting stored chemical or nuclear energy into destructive force; by producing high velocity fragments; by creating blast effects of tremendous potential; by releasing chemical agents which may be catalytic to living organisms; or by releasing biological or radiological agents into the atmosphere, which have destructive effects on plant and/or animal life. The warhead incorporates a fuze system that recognizes or senses the optimum time for detonation, and a safety and arming system (S & A) to prevent early or premature detonation, and to combat the effects of enemy countermeasures.

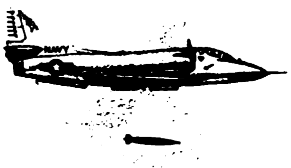
structure



The structure contains, supports and protects the warhead and associated components during launch and travel to the target. Examples of simple structures are those used in bombs, depth charges and projectiles. These structures must enclose and protect only the warhead. More complex structures, such as those used on torpedoes and guided missiles, must additionally support and protect propulsion, guidance and control systems; must be aerodynamically and/or hydrodynamically sound while providing external control surfaces that permit guidance signals to initiate flight path control action; and must be designed for a high strength-to-weight ratio.

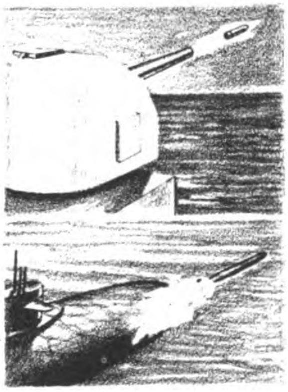
propulsion

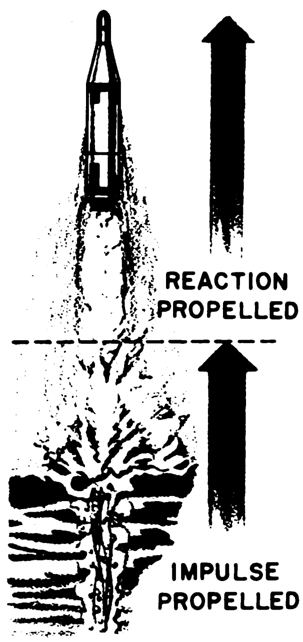
The propulsion system provides the power or energy necessary to propel or move the missile to the target. This propelling force may be generated either by an impulse propulsion system or by a reaction propulsion system. In addition, the propelling force may be obtained solely from the action of gravity on the missile. For example, an aerial bomb requires no component in the missile or in the delivery vehicle to propel it to the target - it depends only on the velocity vector of the carrier at the instant of launch, and the force of gravity.



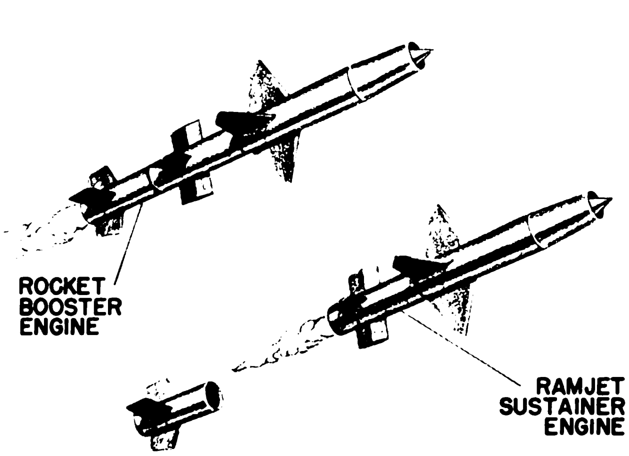
In particular instances, launching may require augmenting the initial impetus force to the missile, to insure safe clearing of the delivery vehicle. However, once launched the missile is gravity propelled.

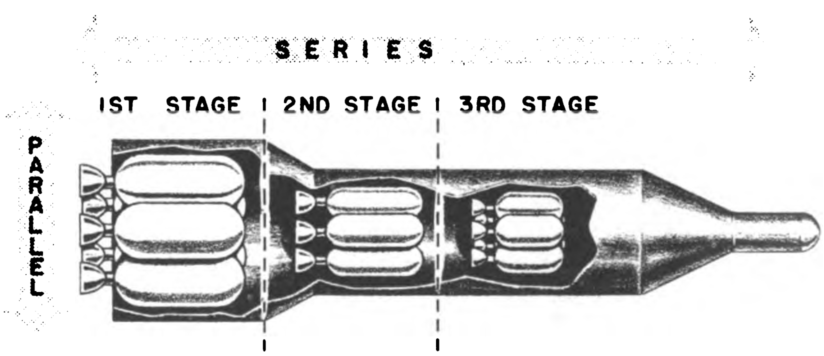
Additional propulsion may be achieved by the use of more than one propulsion unit. For example, a booster rocket may be employed to accelerate a missile to a desired flight speed prior to the operation of a sustainer engine. Once it has expended its propelling force, the booster is usually designed to separate from the missile. Beam-riding missiles often use booster rockets to rapidly accelerate the vehicle to desired flight velocity. The sustainer engine then takes over and provides the propelling force necessary to maintain the design flight speed of the missile. In a reaction-propulsion system, an engine generates a force, whose reaction moves the missile. The propulsion engine may be a jet engine such as used in some antiaircraft missiles, or a rocket engine as used in long-range ballistic missiles.





An impulse-propulsion system employs expanding high-pressure gases to generate the propelling force necessary to give impetus to the missile. A common example of this system, the gun-fired projectile, has both its propelling charge and primer located in the gun tube. The range and speed of the projectile are directly related to the energy output of the charge. Torpedoes and several airborne missile types use impulse-launching techniques to enable them to safely clear the delivery vehicle before their reaction-propulsion system takes effect. For example, the Polaris missile is impulse propelled to clear both the delivery vehicle and the water before its rocket-propulsion system takes over.





Boosters may be connected in parallel to generate high thrust or in series to generate high speed. Multistage engines, in which each stage operates for a specific time interval, are used to provide the extremely high velocities required in long-range ballistic missiles.

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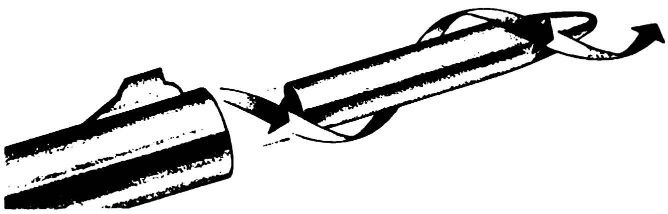
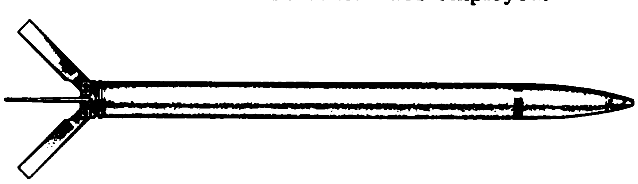
controls

UNGUIDED MISSILES. A control system maintains an unguided missile in its desired flight path by the use of reflexive stabilizing devices. The primary setting of control devices takes place before launch where initial missile atitude and interior ballistic considerations determine the required launch characteristics. Once the missile leaves the launcher, however, stabilization is the only control that keeps the missile on the desired flight trajectory. Generally, unguided missiles are maintained in stable flight by either spin stabilization or fin stabilization.

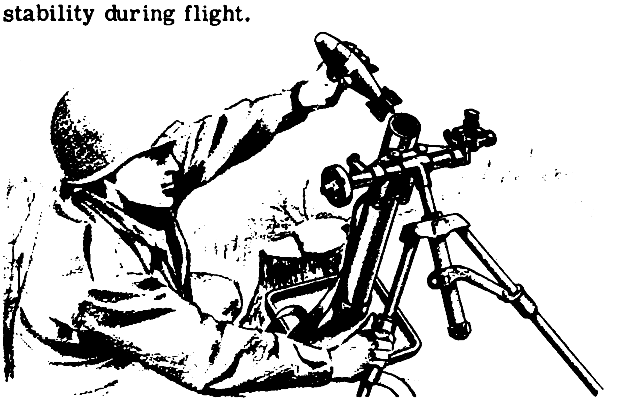
SPIN STABILIZATION. Spin stabilization is a practical and widely used method of stabilizing impulse-propelled missiles such as projectiles. Spin is imparted during launch, and the resultant "boring" motion of the projectile effectively aids the vehicle to maintain its stability about its vertical, longitudinal and lateral axes.



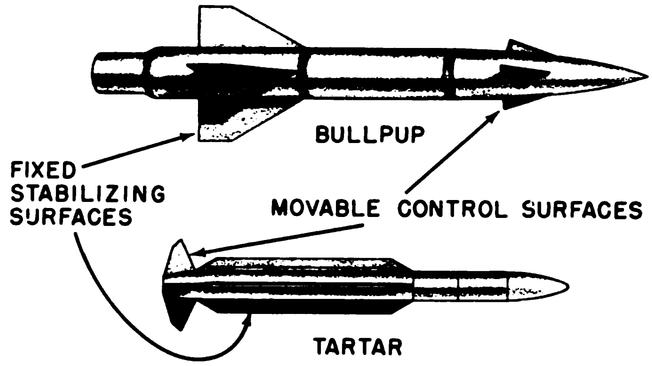
FIN STABILIZATION. Fins may be used to stabilize a missile in flight by reacting against the medium the missile is passing through. They are obviously not useful for missiles which travel in vacuum, and usefull in air only at high speed. Impulse-propelled missiles attain high speed rapidly and aircraft-launched missiles start with the imparted carrier velocity. Shipboard-launched reaction-propelled missiles, however, may require special features if fin stabilized because they move slowly at first, and the air flow over the fins is too slow for adequate stabilization. High thrust rocket boosters with canted nozzles to impart spin torque as well as forward thrust are sometimes employed.



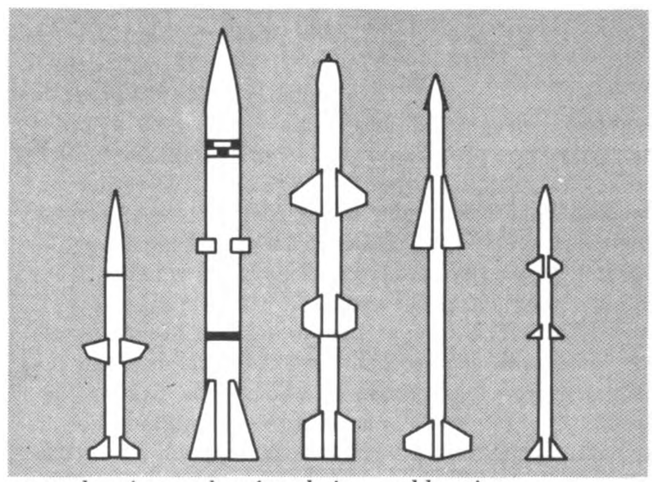
FIN AND SPIN STABILIZATION. Mortars are a specialized branch of impulse propelled missiles that employ both fin and spin stabilization methods to control their stability during flight.



GUIDED MISSILES. Guided missiles are made to follow desired flight paths by both reflexive stabilizing methods and active error sensing methods. Control surfaces which react with the air or water are generally used if possible. Ballistic missiles which travel in vacuum, however, must use reaction systems.

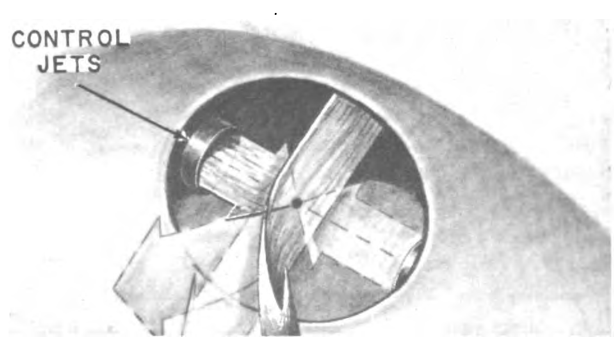


CONTROL SURFACES. Moveable control surfaces react with air or water according to the laws of aerodynamics and hydrodenamics to control a missile in flight. The control surfaces may be located in any position, depending on the dynamic and structural requirements.

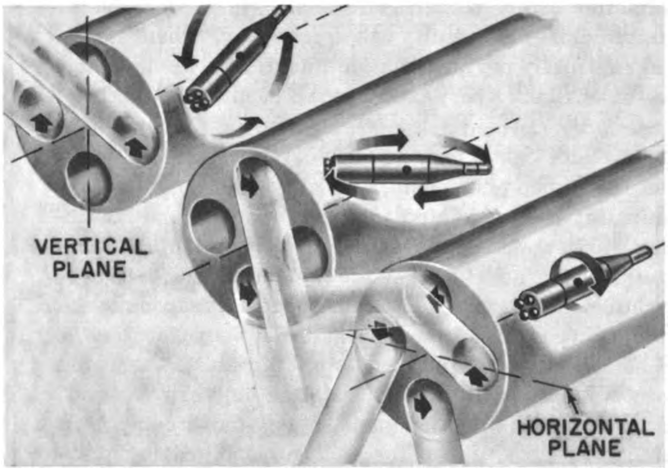


examples of control surface designs and locations

BALLISTIC MISSILES. Ballistic missiles often use thrust as a control force to stabilize the missile or change its direction of flight. Resultant thrust forces are initiated by pointing the nozzle of a gimballed engine off to one side, by the use of deflecting vanes within the jet, or by separate control jets whose thrust potential is often used to correct error deviations in flight path.



Extreme accuracy can be obtained by having four gimballed or rotating nozzles working in conjunction, and by utilizing the corresponding nozzle selected by the guidance system to correct the trajectory deviation. The two general methods of control can be combined as in a missile which uses surface control in the atmosphere and controlled thrust for stabilization and control during vacuum flight.

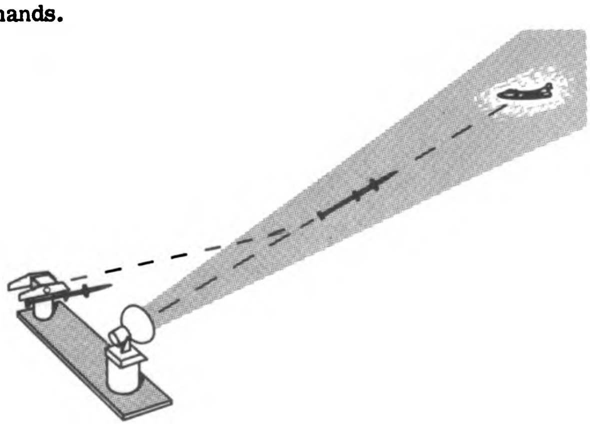


guidance

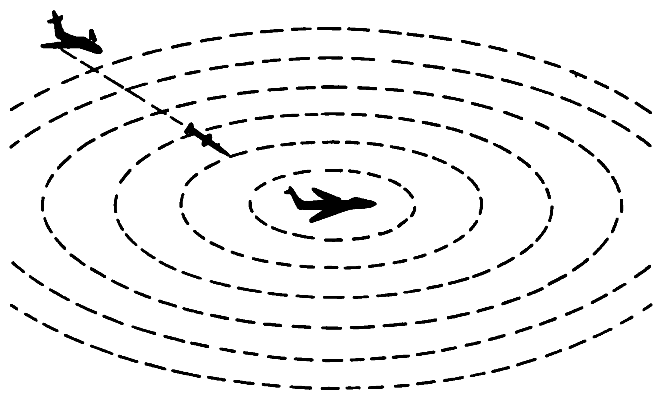
A guidance system senses the missile position and direction, compares it with the desired flight path, and initiates action to correct any deviation. A guidance system may be preset, so that the complete course is programmed into the missile before launching, or it may alter the course during flight in response to target data. The latter method permits use of more recent target data, while the former offers the advantage of being virtually immune to countermeasures. There are many types of guidance systems in use, classified by means of the method used to sense position,

Types of missile guidance systems include:

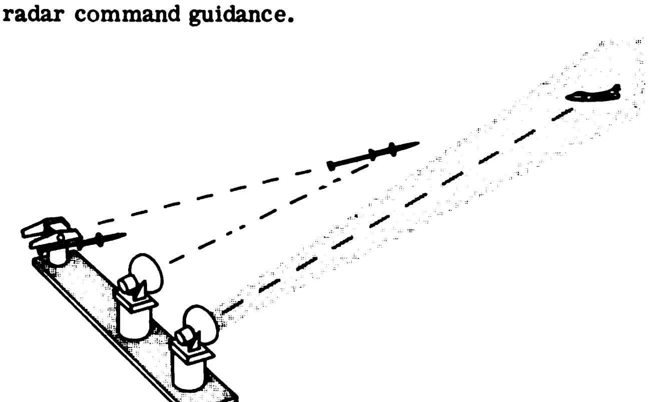
BEAM RIDER. The missile is directed along an energy line between the beam energy source and the target. Electronic sensing equipment on the ground senses a target and sends the information to the missile. The guidance and control system within the missile then interprets this data and formulates its own steering commands.



HOMING. A self-contained system guides the missile toward the target. Guidance is accomplished by receiving information directly from the target itself, rather than from an outside source. Data is gathered, using one of several types of sensors (radar, heat, and light). The target configuration and enemy countermeasure techniques are instrumental in the choice of a sensor. The Sidewinder air-to-air missile and ASROC surface-to-underwater missile use homing guidance.



COMMAND. The missile is directed along a desired flight path by intelligence transmitted to it from an external location. The Nike surface-to-air missile uses radar command guidance.



CELESTIAL. A missile guidance system which measures the existing position of the missile by taking fixes on celestial reference points (fixed stars) much in the manner of a ship's navigator. When information received reveals a deviation from planned position, control action is initiated.

TERRESTIAL. A system which makes use of the fact that the characteristics of the earth's magnetic field - lines of equal magnetic deviation, lines of equal magnetic inclination, and lines of equal intensity - can be measured and used to determine the missile position. Comparisons made with a preset position indicator show error deviation from the desired flight path.

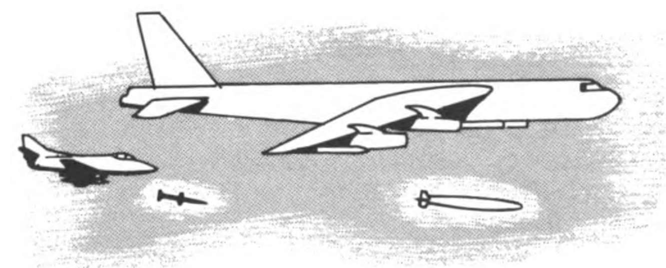
INERTIAL. A system which is designed to detect errors in the trajectory by measuring the lateral and longitudinal accelerations during missile flight, and determining the missile position from this information.

requirements STRUCTURE

The size and type of a missile warhead selected for a particular function is based on an estimate of its kill probability against a known target configuration. In determining the destructive effects of a payload, allowable miss distance, salvo size, and economics must be considered. Basically, however, the missile is designed to carry the warhead to the target. Usually the selection of the warhead determines the size and weight requirements of the missile. Guidance and accuracy requirements also are instrumental in the final determination of missile configuration.

size and weight

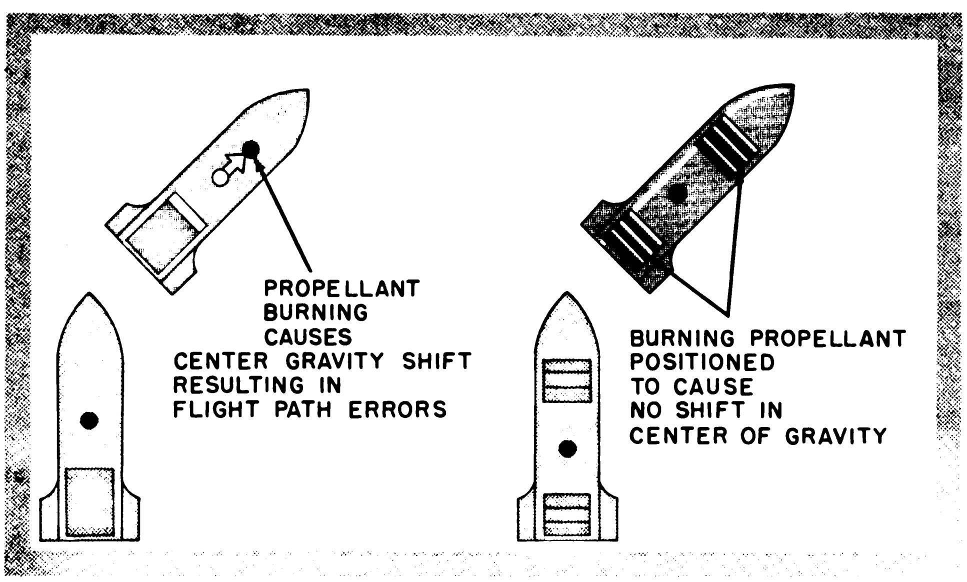
Missiles are designed to be as compact and as light as possible for many reasons. The smaller and lighter the missile, the easier it is to move, and the greater the number that can be stored within an allotted area. This means reduced weight and space requirements are imposed on the delivery vehicle. Weight and space savings are realized not only because the missiles themselves are smaller and lighter, but also because of the resulting reduced need for large and complex missile transfer, loading, and launching equipment. Usually, the larger and heavier the missile, the larger and heavier the launcher must be to support the weight of the missile, and to supply the necessary impetus to the missile, or to withstand the reaction of the missile. Some of the larger long-range missiles use liquid fuels which present severe handling and storage restrictions. Smaller missiles can be stored more easily, in greater quantity, and in closer proximity to the launcher. These advantages usually permit smaller missiles to achieve higher rates of fire. Every pound saved in missile weight means less propulsion energy required, allowing use of a smaller, lighter engine or booster.



As mentioned previously, the size and weight of the missile; its associated transfer, loading, and launching equipment; and the extent of storage facilities required have an effect on the weight of the delivery vehicle. For every additional pound of weapons system weight, vehicle structure weight may increase tenfold. Size and weight requirements often prohibit the adaption of a weapons system to an existing delivery vehicle, and necessitate the design of larger, more complex, and more expensive apparatus. Larger and heavier guided missiles are not as maneuverable as their smaller and lighter counterparts. Therefore, they require more accurate launching orientation than more compact missiles do, when operating over similar short ranges, such as encountered in antiaircraft fire.

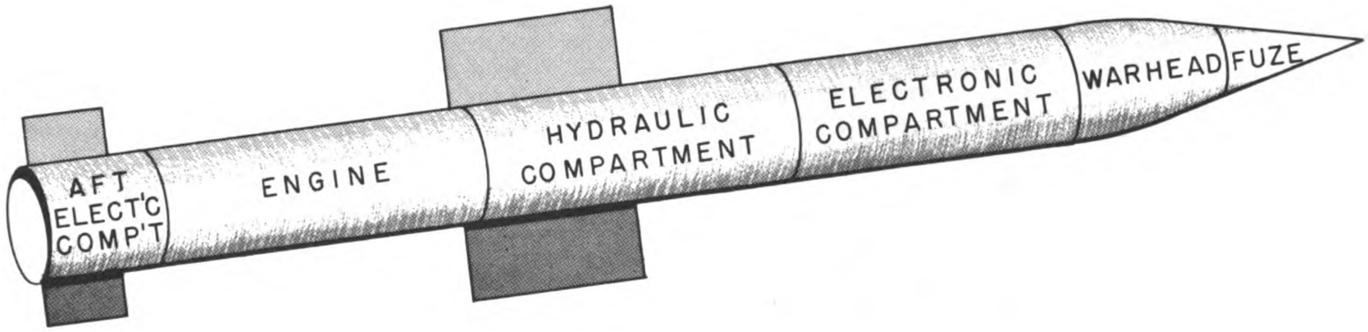
WEIGHT AND BALANCE RELATIONSHIPS

In designing a missile, weight and balance relationships must be given careful consideration. The initial location of the center of gravity is of extreme importance. A missile in flight can be considered to move about three axes (yawing, rolling, and pitching). Whenever there is a displacement or change about any of the three axes, the missile might tend to oscillate or in extreme situations go out of control, unless provisions are made to restore a balanced flight mode. Because of the consumption of propellant fuel and the possible jettisoning of a booster or stage, the missile center of gravity may change during flight. This change must be considered in the overall design. For symmetrical missiles, or for those in which the thrust axis-missile center of gravity relationship is important, the radial center of gravity location should be controlled by component location rather than by the addition of non-functional deadweight, such as ballast.



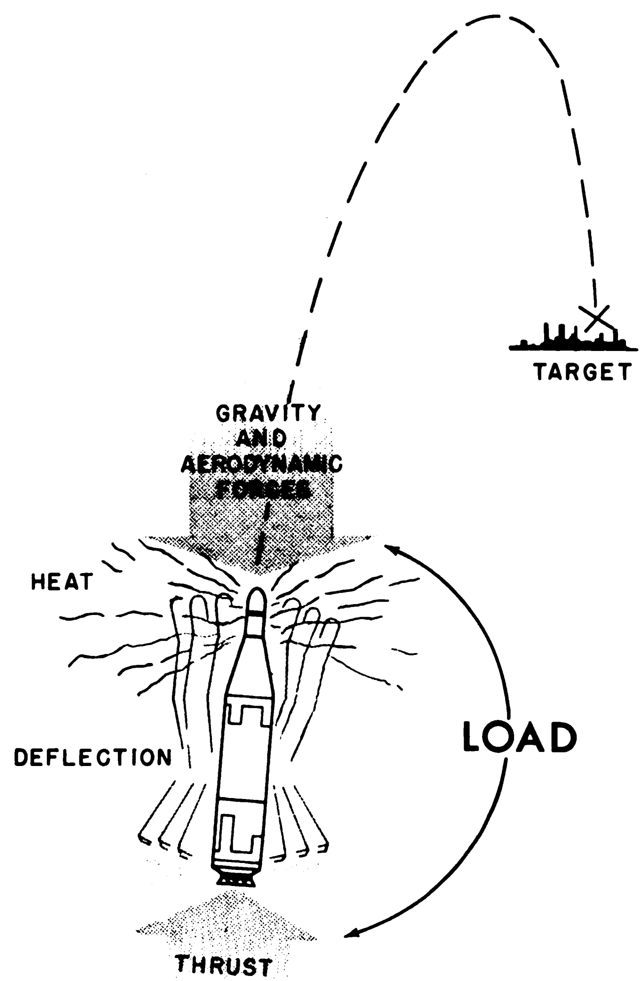
SECTIONALIZATION. Sectionalization is an important aspect of missile design. The separation and division of systems are influenced by packaging requirements, maintenance ease, weight and balance features, and a desire to have as little interaction between subsystems as possible. Central supply sources within the missile (electrical power, hydraulic power, etc.) should be located as close as possible to needed points, to minimize interaction between various components of missile systems. This is accomplished by organizing components into subsystems so that most interactions occur within subsystems, while individual subsystems interact as little as possible with each other. Basic compartmentalizing of systems is usually performed to locate systems of a similar type as close together as mechanically or operationally feasible: hydraulic compartment, electronic compartment, etc.

Minimum size and weight do not necessarily constitute the most efficient structure. There is no simple way to predict the lightest structure for a missile. It is often sound design to employ various types of structures for different sections of the missile to obtain certain design or maintenance advantages. Structural design layouts influence not only initial costs, but also affect recurring maintenance costs. For example, joints required to hold sections of the missile together are comparatively heavy components, and add additional weight to the missile. However, for simplification of production, ease of maintenance and replacement, simplification of shipping and field assembly, and for functional sectionalizing of subassemblies, it is desirable to accept the higher weight and performance penalty rather than incur the costs necessary to engineer them out of the system.



strength

The missile structure supports the warhead and other missile components prior to and during flight, and shields or protects them from the effects of static and dynamic loads. Static load conditions which influence the choice of structural materials, are due to the weight of missile components, and atmospheric or hydrostatic pressure encountered during missile flight. Dynamic load conditions reflect the forces that will be imposed on the structure during launch, while maneuvering, upon deceleration, by effects of gusts, environmental pressure loads, and by heating stresses. The design of a structure involves the mutually dependent parameters of load, deflection, temperature, and time. In the selection of materials for use in missiles, safety of personnel does not constitute the problem presented the aircraft or submarine designer. However, structural failures at, or prior to, launch may endanger friendly personnel, and safety precautions must be taken to insure safety of operation during this period.



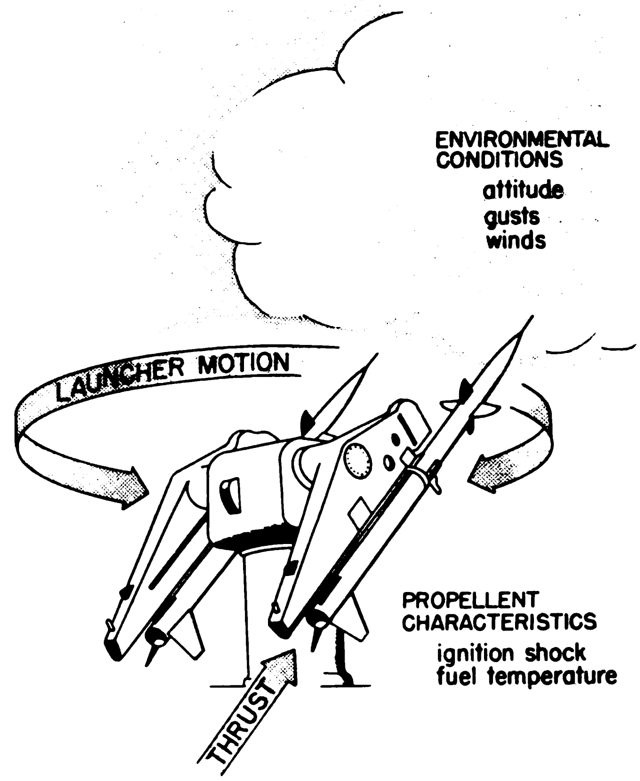
OPERATING TIME. Operating time is relatively of short duration for high-speed missiles. Since almost all missiles are one-shot affairs, the structures must withstand maximum operating loads for only a short duration of time. Fatigue factors under these conditions are not usually of paramount importance; however, materials must have a high probability of successful use after a long standby period during which time they may have been exposed to adverse environmental conditions. Structural integrity must be maintained, and deficiencies must not appreciably degrade overall missile reliability or performance.

static and dynamic loads

LAUNCHING LOADS. Launching loads and their effects are primarily the result of thrust, propellant characteristics, environmental conditions, and launcher motion prior to launcher-missile separation. Launching load factors are particularly significant in reaction-propelled missile types, such as long-range ballistic missiles using short-duration high thrust for their boost phase; and for impulse-propelled missiles which receive all their thrust at launch.

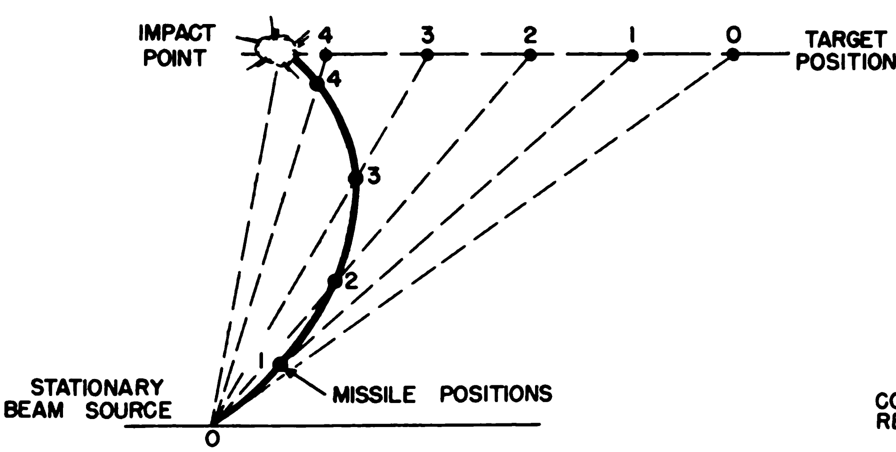
Acceleration constants and their resultant forces vary with the type and the burning characteristic of the propellant, and are further affected by the type of stabilization used, drag coefficients, and weight of the missile. In reaction-propelled missiles, in which the sustaining engine is initiated after the separation of the booster, a deceleration load factor is experienced at the instant of decoupling. It is desirable to keep this factor at a minimum because the speed lost by the missile due to this effect must be compensated for by the sustainer (or succeeding stage), or the booster must be more powerful than otherwise needed. Such deceleration load factors may vary from three to ten gravity units.

Hinge moments on movable control surfaces in the period following launch vary over wide limits, because of the extensive shift of the center of pressure experienced on surfaces designed for supersonic use when operating in the subsonic and transonic regions. (A hinge moment is the torque required to deflect the control surface to a desired angle of incidence.)

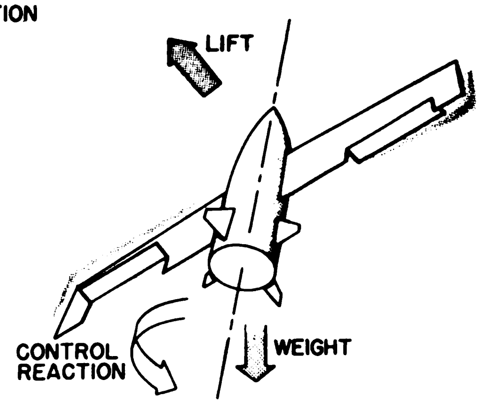


primary dynamic load conditions at launch

MANEUVER LOADS. Maneuver loads are particularly significant on short-range antiaircraft and antimissile guided missiles, which require a high degree of maneuverability to accomplish their missions. Maneuver load effects depend on speed (Mach number), altitude or depth trim conditions, and deflection of the control surfaces. (When a missile, in free linear flight, maintains a constant angle of attack, with no resultant moment in a vertical plane, it is said to be in trim condition.) Load factors must be determined for critical combinations of weight, speed, angle of attack, altitude or depth, and control system characteristics. For example, beam-riding missiles require a high turning rate in order to close with a target at high relative velocities. A high turning rate is approximately proportional to control surface deflection, and inflicts heavy maneuver loads on the missile. The missile structure must be designed to withstand the maneuver load produced by the maximum turning rate required of the missile, and then a system of automatic limiting can be applied to the control surfaces or other missile components to limit the turning rate to an optimum degree. If the missile turning rate exceeds the design maximum, structural failure may occur.



typical path of beam riding missile



forces acting on a missile during a banked turn

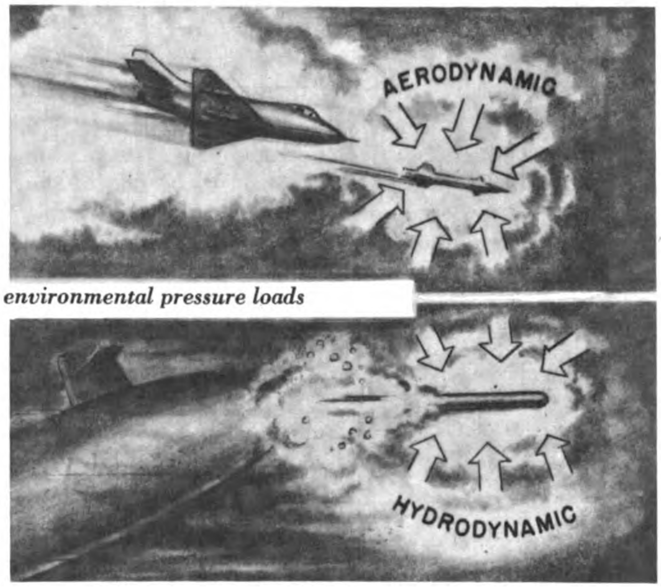
DECELERATION LOADS. Deceleration loads are the result of a missile encountering resistance during flight and decelerating. Deceleration loads are opposite in direction to launching or thrust loads. The mutual force exerted is greatest when ballistic missiles re-enter the atmosphere or when antisubmarine missiles (such as ASROC) enter the water.

GUST LOADS. Gust loads are frequently superimposed on other load conditions. These are important considerations, during launching and terminal phases of missile flight, but are not always significant at supersonic speeds. Because of the vector properties of winds, their variability with height, and their changeability with time and temperatures, wind conditions along the flight path are difficult to predict and to correct for, at time of launch.

HEATING STRESSES. Use of materials with dissimilar thermal expansion characteristics results in induced stresses caused by non-uniform expansion. These stresses may be minimized by the optimum use of insulating materials, and by consideration in design layout of expansion or contracting coefficients. Also materials closer to the heat source may be chosen to have lower rates of heat transfer.

HANDLING AND STORAGE LOADS Missiles are subjected to dynamic loads in storage, transfer, checkout, servicing, and loading.

ENVIRONMENTAL PRESSURE LOADS. Environmental pressure loads are the distributed loads on the surface of the missile due to its motion in a medium. This environmental pressure varies directly with the speed of the missile and its depth in water, but inversely with its altitude in the atmosphere. For torpedoes in submarine tubes and missiles mounted externally on aircraft, this is a pre-launch loading condition.



environmental pressure loads

resistance to effects of environment

Environmental conditions which may require consideration in missile structural design include the following: . . . .

temperature	ice
humidity	sand
aridity	dirt
salt spray	dust
pressure	rain
vibration	snow
mechanical shock	hail
thermal shock	sleet
solar radiation	abrasives
grease	fungus
oil	lightning
noise	

The above factors affect different missile designs in different ways, and many of these factors may not be appreciable for a particular missile. The determination of which factors to consider is established by tactical usage. Shipping, handling, and pre-flight testing set up additional environmental conditions that influence the design of all missile components. Environmental conditions may be subdivided into 3 phases:

(1) PRE-FLIGHT
Transportation, handling, storage, ready service storage

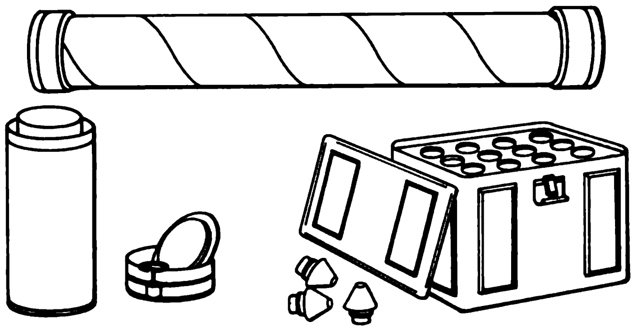
(2) TACTICAL HANDLING AND LAUNCHING

(3) IN FLIGHT
Temperature, pressure, vibration, shock, other environmental conditions

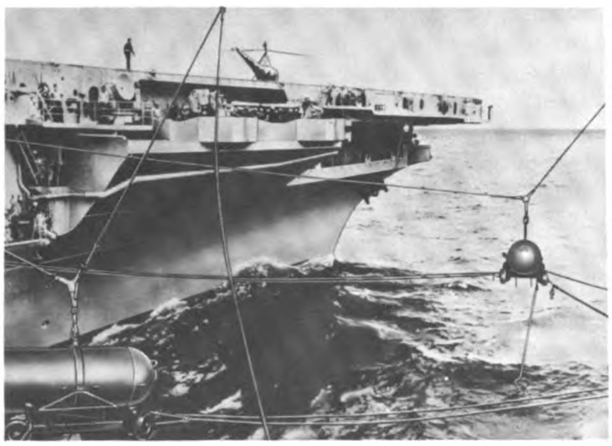
pre-flight

TRANSPORTATION. The transportation environment is often very severe, consisting, in part, of random vibrations, bumps, bounces, shocks, temperature changes, and the presence of moisture and abrasive particles. Surface damage is prevalent, and many small failures may occur. Transportation by aircraft appears to be the most desirable form of transportation for missiles from the point of view of minimum imposed forces at low frequencies, as opposed to transportation by ship, rail, or truck. In addition, the short time required for shipment by air is favorable from the fatigue point of view.

HANDLING. Because the human element is so predominant, handling is usually considered to be one of the most difficult environments to design against. Therefore, special containers, rather than the missile itself, are designed to withstand handling conditions. These containers are frequently heavy, bulky, and expensive, and therefore are normally of a reusable type.



Transfer of weapons at sea involves a severe handling environment. Missiles must be able to withstand the associated shocks and stresses and often special protective containers must be developed for this purpose.



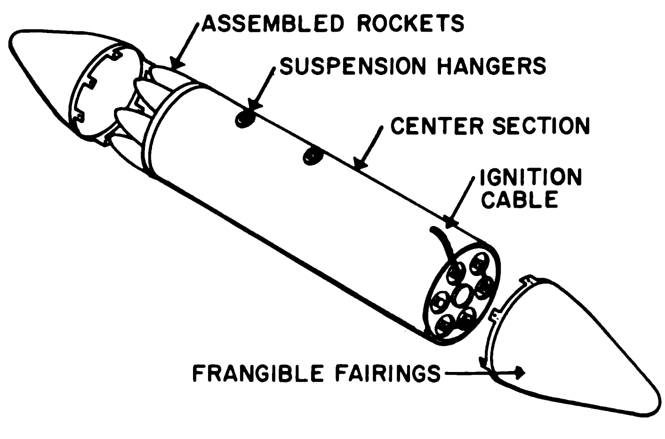
STORAGE. Environmental conditions in the storage phase depend on location and exposure of the missile. The missile may require removal from its container at regular intervals for routine maintenance such as cleaning, flushing the hydraulic systems, testing of various components to determine efficiency, or replacement of components that might deteriorate with time. Missile storage systems usually provide a controlled environment to prolong missile storage life.

READY SERVICE STORAGE. If it is not controlled, the ready service storage environment can impose various destructive forces on uncrated missiles. Parts susceptible to deterioration from temperature, humidity, dust, and moisture usually require appropriate protection. In some weapons systems, it may be better to seal all damageable elements than to provide a controlled environment, such as dust- and moisture-free air conditioning. In other systems, however, a controlled environment maintained in storage may be necessary. The missile must also be isolated from vibration, shocks, and structural flexures of the delivery vehicle.

tactical handling and launching

The tactical handling phase includes the environment encountered by a completely assembled missile from unpackaging to commencement of launching, or, in the case of air-launched vehicles, up to the time of attachment to the delivery vehicle. Handling conditions are influenced primarily by the type of missile, condition of use, size, and logistics.

During the short period in which the missile is being transferred to, and while it is on, the launcher, it can be subjected to the severest environmental conditions. However, this exposure time is brief, and therefore, the main design problems relating to short-period protection from the elements are: launcher vibration, blast from adjacent missiles, and enemy action. Rain- and splash-proofing may be considered as adequate protection, except when the missile is intended to remain on the launcher for an extended period of time. Under these conditions, consideration must be given to sealing compartments containing electronic or other operating gear unless the packages are individually sealed. In some instances, drain holes are provided in the missile to eliminate trapped water. For missiles which are loaded on a launcher for prolonged periods, special design precautions are necessary to protect against water, dust, dirt, static electrical charges, ice, and solar radiation. For air-launched missiles (such as rockets), the pre-launch environment is extremely harsh. These environmental effects may be reduced by the use of protective covers or pads. Extreme care must be exercised in the design to be certain that no internal malfunctioning can cause premature launching or failure at take-off of the delivery vehicle.



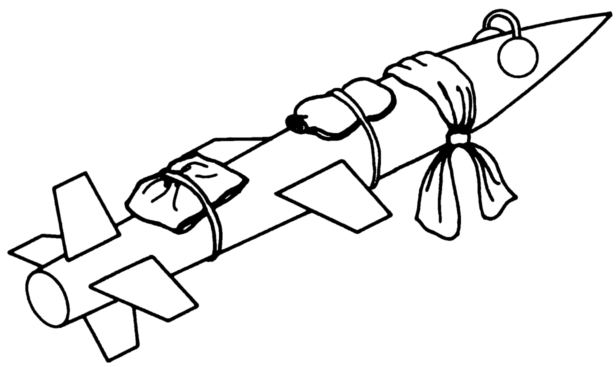
in-flight

TEMPERATURE. Ambient temperature and change in temperature due to aerodynamic heating are both factors in the in-flight phase. Temperature change due to hydrodynamic action is not a primary factor in underwater operation of a missile. The rate of rise of temperature in a structure is dependent on boundary layer temperature, time of exposure, material thickness, proximity of thermal sinks or sources, quality of insulation, and surface emissivity. Boundary layer temperature constants are a function of both Mach number and time. For example, antiaircraft missiles have a high temperature rise because of their high rate of climb and attained velocities. Because the missile will be exposed to extremes of temperature for relatively short periods of time, the time-temperature relationship is very significant. The relatively short time of flight permits using more liberal design allowances than if the materials were thoroughly exposed to maximum operating temperatures for long periods of time.

The problems of thermal expansion and thermal shock are especially important in fabricated structures, and particularly in those involving different materials. As mentioned previously, thermal expansion results in induced stresses in materials with dissimilar thermal expansion characteristics. Thermal shock would occur when a missile that has been subjected to aerodynamic heating enters a region of different temperature, such as occurs when an ASROC or SUBROC missile enters the water at the end of its flight path in the air.

PRESSURE. Atmospheric and hydrostatic pressures vary with changes in altitude and depth of the missile path. Therefore, the calculation or estimation of these pressures is an important design consideration for long-range missiles.

VIBRATION. Vibration is the severest and the most difficult environment to design against. Maximum vibration amplitudes are caused as a result of engine operation in reaction-propelled missiles and by the reactive resultants of impulse forces in impulse-propelled missiles. Air and water flow are also paramount causes of missile vibration. Stress and strain analysis can determine at which points vibration effects will produce maximum torsions. Data obtained by tests usually determine the most important points are found where critical elements of the missile are located: stable platforms, control devices, sensing equipment, and electronic computing systems. Frequency of vibrations encountered in missiles varies over a random spectrum but the apparently significant region is from 10 to 2,000 cps. In a propeller-driven missile, resonances at the propeller-shaft and the propeller blade frequencies also cause missile vibrations. Sharp changes in flight path and flameout can also lead to unwanted oscillations of the missile airframe. By correctly selecting the characteristics of the structure, the material, and the configuration, the airframe can be designed so that structural damping greatly attenuates the vibration.



SHOCK. Shock effects may be subdivided into 2 basic phases: 1) transverse or lateral; and 2) longitudinal Transverse or lateral shocks to a missile are initited by such factors as handling, launching clearances, launching platform motion, wind, and engine operation. Longitudinal shocks result primarily from booster-missile separation, ignition shocks, and aerodynamic and hydrodynamic drag.

For many designs, vibrations are more damaging than shock or impact loads, even when vibrational loads are substantially less than those due to impact. This can be attributed to the effects of repetition having a cumulative destructive buildup when vibration is continuous.

OTHER ENVIRONMENTAL CONDITIONS. When a container may be exposed to rain, hail, snow, or sleet, these factors play an important part in the design of the container. They are also of significance to the missile itself if it is exposed to the elements for long periods of time. The effect of rain as a cooling agent on a missile in supersonic flight is almost nil. However, the effects of erosion on plastic and rubber components due to hail or rain can be serious. To avoid the entry of moisture into the missile, it is feasible to hermetically seal sensitive components or even to seal the entire structure. Snow loads on most missiles are small and therefore usually considered only when the missile is exposed, for example, on a launcher. Hail and sleet are not justtified design considerations because of their infrequent occurrence. To avoid the effects of erosion and the icing of air-launched missiles, the use of pads for the launcher housing may be required. This means is often used to protect aircraft launching systems.

Dust, dirt, and sand can cause movable elements to stick, and disrupt or damage rotatable equipment by scoring or abrading precision parts. Dirt and sand are hygroscopic, and, if moisture is retained, become a source of corrosion and rust.

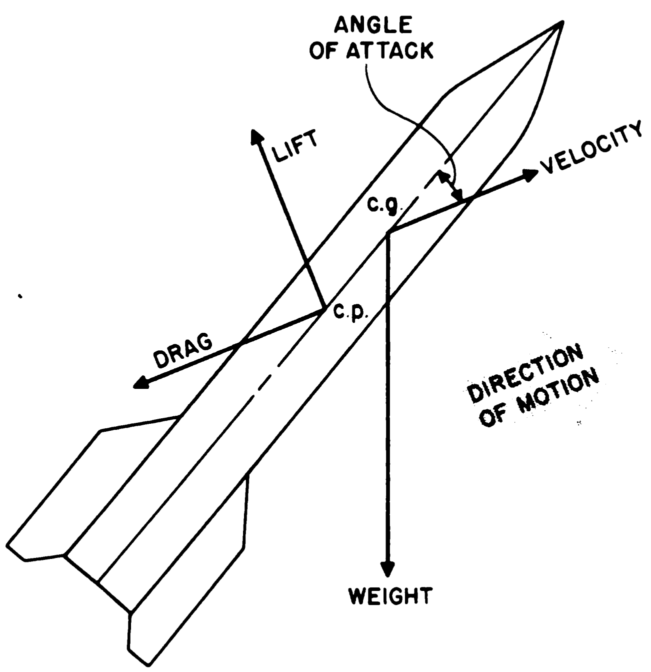
A marine atmosphere is extremely corrosive because deposits of salt are present and almost immediately initiate corrosion. Machine handling of finely finished surfaces is preferable to manual handling since deposit acids and moisture inherent in the human appendages can lead to rapid corrosion when inadvertently applied to a polished surface.

CONFIGURATION

forces on a missile in motion

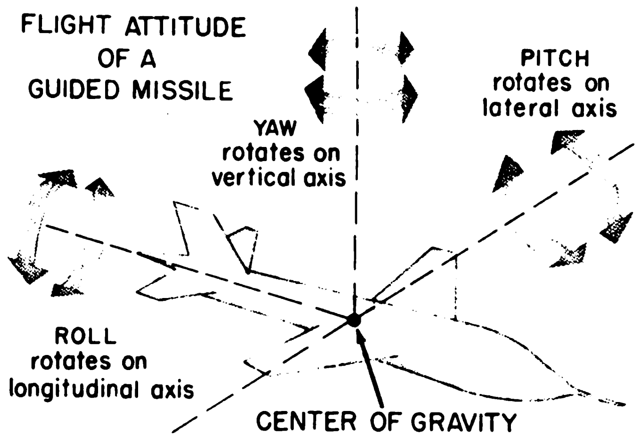
When a missile is in motion in the atmosphere or in the sea, it is subject to a number of static forces which originate from the resultant air or water pressure acting externally on the exposed parts of the missile. These forces are added to the functional resistance of the medium, the thrust required to propel it, and the gravitational force that constantly attracts the missile towards the earth. Just as the weight of the missile is designed considering the gravitational force, the configuration of the missile is designed considering the resulting aerodynamic or hydrodynamic forces.

DRAG AND LIFT. These aerodynamic or hydrodynamic forces are handled analytically by resolving them into two planar components - parallel to and at right angles to the direction of flow of the medium being traversed. The parallel (horizontal) component is designated as drag (D), and it operates in a direction opposite to the motion of the surface; the vertical force operating upwards is called the lift (L), and must be of sufficient amplitude to overcome all forces acting in the horizontal plane. Since missile thrust is required to overcome the effect of drag, we generally desire a configuration with minimum drag characteristics, to keep power requirements low. In addition, for the same expenditure of power, the lower the drag the greater the missile speed and range.



FORCES ACTING ON A MISSILE IN MOTION

CENTER OF GRAVITY - CENTER OF PRESSURE. While the weight of the missile is considered to act at the center of gravity, the lift and drag forces are considered to act at a point called the center of pressure. Lift and drag forces vectors intersect at this point, and if the center of gravity and the center of pressure are not located at the same point, as is often the case, the resulting moment of the forces acting at these points may cause unstable flight conditions. Therefore, the location of the center of pressure, determined by the configuration of the missile's structure, is an important factor in missile stability.

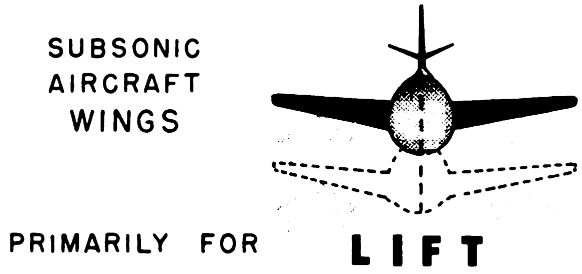


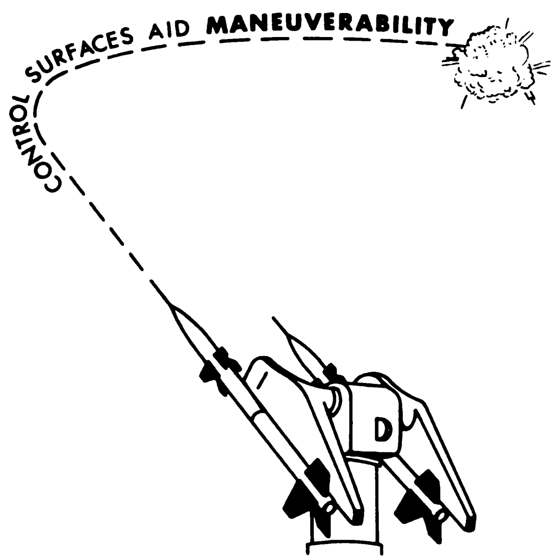
In addition to the moments tending to produce rotation about the center of gravity (roll, pitch, and yaw), there are moments applied about the hinges or axes of the missile control surfaces. When force vectors representing these pressures have their origin at the center of gravity, it is then possible to describe the rotation of the missile in terms of angular displacements about these axes. When all forces acting on the missile are resolved into component forces along the three axes, we can completely specify the resultant motion of the missile with respect to its center of gravity. The angle of attack of the missile is identified as the angle between the longitudinal axis and the direction of flow of the undisturbed medium; it is generally taken to be positive when the direction of rotation is as shown in the illustration. A missile in flight encounters various forces that may be characterized as disturbing forces or moments, inasmuch as they tend to cause undesired deviations from the operational flight path. They may be random (wind, gusts, etc.) or systematic (misalignment of thrust components). When a missile is design stable, it will return to its former position after being disturbed by an outside force. Guided missiles have the great advantage over unguided missiles in being able to correct, to some degree, both random and systematic disturbances that may occur during flight.

basic design considerations

All missiles must develop a lifting force to maintain altitude or depth and to enable maneuvering. On the other hand, drag is a retarding force which acts to reduce missile performance. Hence, it would seem that missile design should strive to maximize lift while minimizing drag. However, both of these forces are directly proportional to the square of missile speed. Thus the missile configuration chosen is that which will provide the required lift with the minimum drag over the range of speeds the missile will undergo. The necessary lift force is provided by the motion of the missile body in the medium. If greater lifting force is required to overcome gravity or to execute the required maneuvers, control surfaces may be added to the missile body to supply and control the necessary lift. The adding of external surfaces not only increases the overall missile drag but adds additional weight to the missile, and is done only when the missile body cannot develop sufficient lift forces of its own. In low-speed conventional aircraft, required lift is provided by wing configuration, and drag effects are not of paramount importance. However, at high speeds, increased turbulence effects and the increase of drag with the square of the velocity necessitate specialized configuration design. While body lift is usually considered insignificant in subsonic missiles, it may be used as the sole lifting force in some supersonic missiles. In such high-speed applications, control surfaces can be made smaller or eliminated entirely, thus counteracting the increased drag force at the higher speed.

Configuration of low-speed underwater missiles with no great maneuverability requirements, such as early depth-charges and torpedoes, were often determined by consideration of handling, launching, and water entry, rather than by hydrodynamic considerations. These configurations were sufficient when employed against targets of that day. To effectively function against faster, more evasive targets, torpedo speed and maneuverability were increased. As a result of these ever-increasing requirements, hydrodynamic considerations become more and more important in the design of underwater missiles. Water entry is an important design factor in the configuration of high-speed surface-to-underwater and air-to-underwater missiles. Richocheting or skipping on the surface of the water must be prevented, but the configuration must not, as a result, penalize the underwater performance of the missile. The lifting force necessary to maintain depth for underwater missiles is not as dependent as airborne missiles on missile surface area and velocity, due to the greater buoyancy of the denser medium of travel. However, the drag force is greater, requiring a large propulsive force to achieve relatively high speeds. Separation and cavitation, which occur at high speeds underwater (just as air turbulence occurs at high speed in air), increase the probability of detection by enemy sonars.



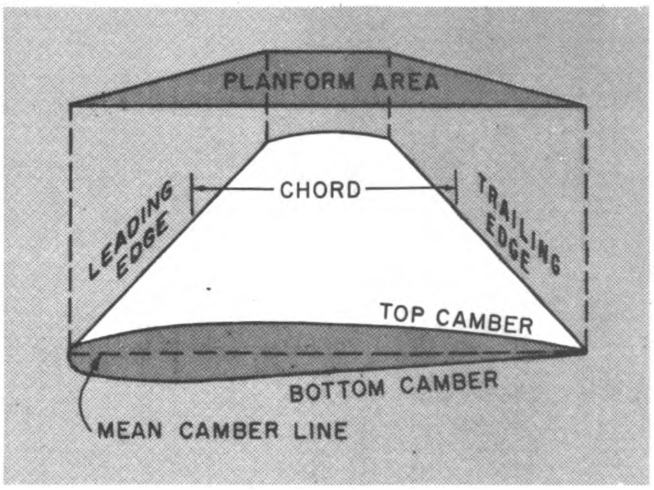


speed classification

Missile speeds are usually classified in four categories: subsonic, transonic, supersonic, and hypersonic. Speeds are called subsonic when the relative velocity of the media on all points on the missile surface is less than the speed of sound. When a point is reached at which the speed of the body is the same as that of the pressure wave, waves build up into a single pulse of greater magnitude. This condition exists when the body moves at the speed of sound. It is at this speed where transonic airflow also occurs. When a body is moving at sonic speed, there are points at which air velocities about the body are above the speed of sound, and there are points at which air velocities are below the speed of sound. Considerable turbulence and buffeting of missiles take place in this region; therefore, it is desirable to pass through the transonic region rapidly, to prevent unwanted disturbances. A missile is moving at supersonic speed when the airflow at all points about the body is greater than the speed of sound. Little turbulence is present in supersonic flow. The fourth classification of speed is hypersonic, sometimes called ultrasonic flow. As a body moves through the air at high speed, a short amount of time is necessary for the molecules of air to adjust themselves to the presence of the body, and to readjust themselves after the body has passed. This period of adjustment and readjustment is termed the relaxation time. If a body is moving at a speed greater than the relaxation time, it is in a new velocity range which is called hypersonic.

lift and drag coefficients

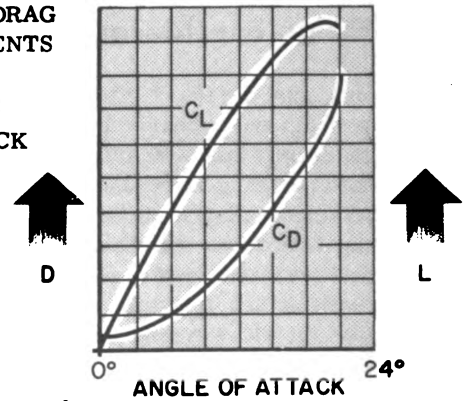
The principal forces and moments that act on a missile in motion can be computed for design purposes from experimental data taken from scale models under simulated conditions. Results are expressed in terms of model dimensions. The force acting on a body is a function of dynamic pressure (q) and characteristic area (A). We may obtain a dimensionless coefficient by dividing the force or moment in question by the product of these functional parameters. Hence, the defining equation for the lift coefficient is $C_L = \frac{L}{qA}$, and the drag coefficient is $C_D = \frac{D}{qA}$. A torque coefficient is likewise found from $C_M = \frac{T}{qcA}$.



When the body or fuselage of a missile is under analysis, the characteristic area is considered to be the cross sectional area of the component, while the planform area is utilized for a wing or control surface. The planform area is the plan area as seen from above; that is, the area projected on a horizontal plane directly above the surface. The characteristic length (c) may be any convenient compatible linear dimension; for example, the diameter of a missile, or the chord in the case of a control surface. The chord of a control surface is the length of an arbitrary straight datum line drawn between the leading and trailing edges. The mean chord length is the quotient of the control surface area, divided by the span, when the span is taken to be the overall length of a pair of surfaces from tip to tip.

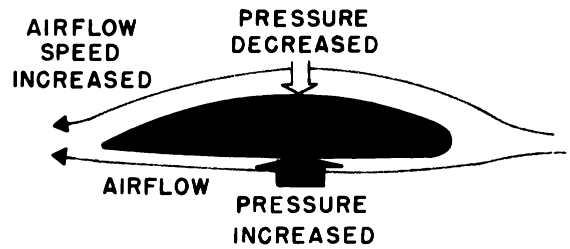
The dynamic pressure (q) is the principal variable in the force acting on a missile. Its value is determined for low-speed missiles in a fluid medium from: $q = 1/2ev^2$, where e = medium density and v = velocity. For supersonic missiles, the equation is: $q = 0.7pM^2$, where p = ambient pressure and M = Mach number.

As can be seen from the above discussion, the forces created by the motion of the missile through the atmosphere are functions of ambient pressure, Mach number, and missile size. The dimensionless coefficients, C_L for lift and C_D for drag, are functions of missile configuration and angle of attack of the airfoil, and are independent of missile size.

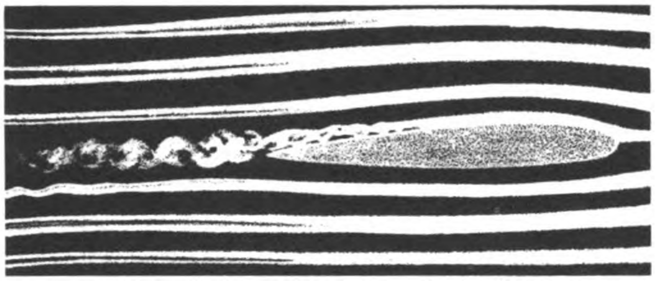


lift aerodynamics

In subsonic airflow over a conventional wing section, the air that passes over the wing must travel a greater distance than the air passing under it. Since the two components of the air stream rach the trailing edge of the wing at the same time, the air that flows over the wing must travel faster than the air flowing under it. This results in a lower pressure on the top than on the bottom of the wing, thereby increasing the lift force applied to the wing.



The differential pressure causing the lift force resulted from the configuration of the wing. Differential pressure can also be achieved by changing the angle of attack of the wing, regardless of its configuration. Increasing the angle of attack not only increases the lift force (until it reaches the burble point, at which time the lift coefficient is a maximum), but also increases the drag force resulting in a shift in the location of the center of pressure. The burble point is the point at which the airflow over the upper surface becomes rough, causing an uneven distribution of air on the upper surface of the wing. This occurs in subsonic flight when the angle of attack is increased to about 20 degrees. Subsonic missiles are designed to obtain their lift force from both airfoil configuration (camber) and angle of attack. Camber refers to the use of a curve of an airfoil section. However, because of the type of flow and pressure distribution characteristics at supersonic speeds, the advantages of camber no longer exist; the wing receives its lift as a result of the angle of attack rather than of any increase in velocity of the air stream over the upper surface as compared with the lower surface. At supersonic speed, the air is traveling faster than the speed of sound, and hence faster than the speed of pressure impulses which could warn the air of the presence of the wing before its arrival.

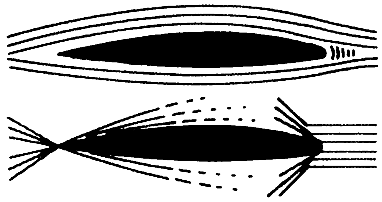


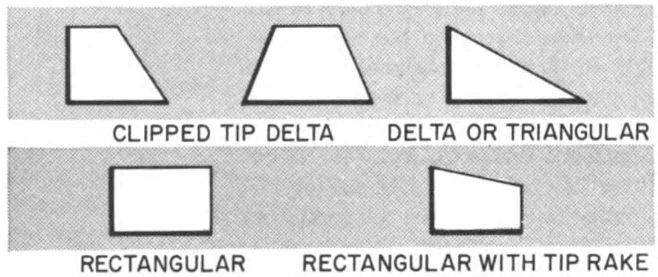
AIRFLOW AT LOW ANGLE OF ATTACK

AIRFLOW BEYOND BURBLE POINT

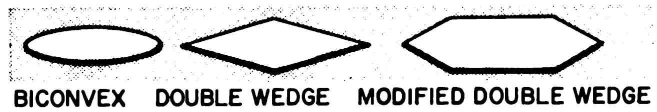
shock waves and expansion waves

A change in direction of flow of a supersonic stream of air will generate either shock or expansion waves, and the pressure on the surface of the body will be affected. A shock wave will be generated at the leading edge of exposed bodies when placed in a supersonic stream.
If the airflow is diverted in such a manner that it turns and interferes with the flow of air in adjacent stream layers, an oblique shock wave will be created. In the illustrated case of a two-dimensional wedge, the flow direction will be altered to be parallel to the surfaces. In the case of a cone, the flow direction will be diverted to a small degree and will vary thereafter as indicated in the illustration. Blunt-nosed bodies will have normal shock waves in the region immediately ahead of the nose. In the case of oblique flow under shock wave stimulation, the static pressure, density, and temperature parameters of the flow are always increased, while the Mach number is decreased. The effect of these factors is to increase the drag and to decrease the lift coefficients applied to the body. In the case of a normal shock wave, the flow direction is unchanged, and the flow becomes subsonic behind the wave. In the case of shock waves ahead of blunt bodies, wherein a normal shock blends into an oblique shock pattern, there will be a sonic line dividing the flow behind the shock wave into regions of subsonic and supersonic flow.
An expansion wave will occur whenever airflow is such that it tends to turn away from the air in the adjacent stream layer. As the name implies, expansion waves occur wherever the air is allowed to expand; the static pressure, density, and temperatures are decreased through these waves, but the Mach number and speed of the airflow are increased. An expansion wave created by a change in flow direction is fan-shaped, and the change in conditions is more gradual in the region away from the surface of the body.

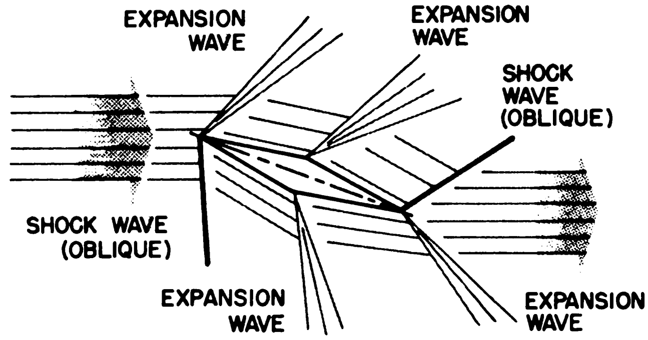




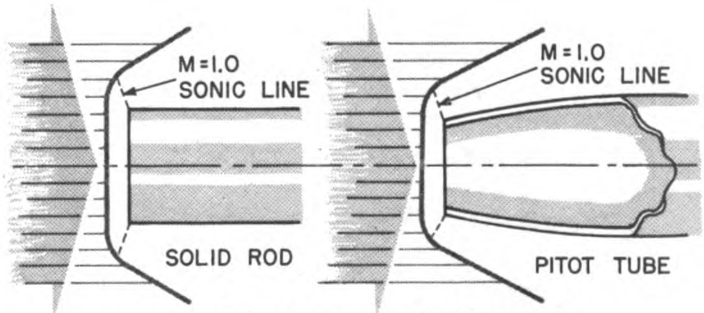
typical supersonic wing planforms



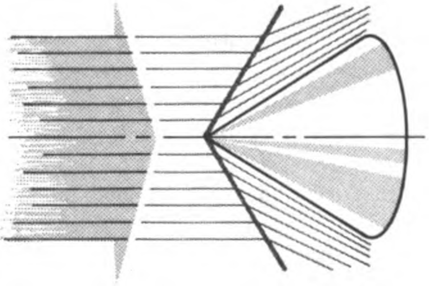
typical supersonic wing cross-sectional shapes



shock and expansion waves around a double edge airfoil at an angle of attack in a supersonic stream



creation of shock waves ahead of blunt bodies

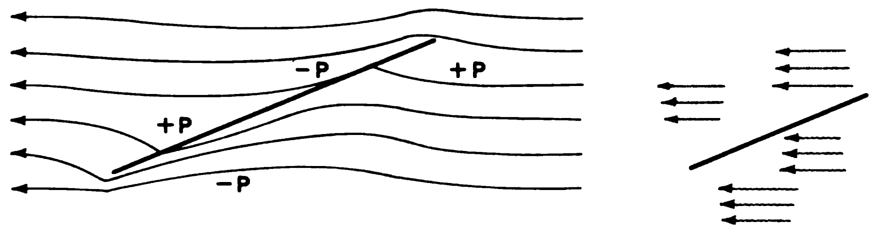


airflow over a cone at supersonic speeds

lift hydrodynamics

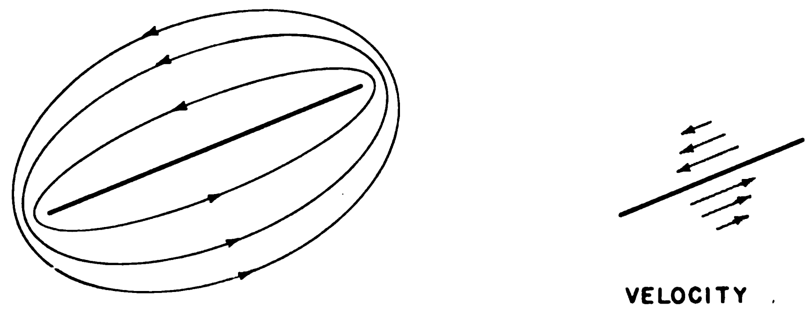
Dynamic lift can be applied to an underwater vehicle by using a control surface (hydrofoil) to deflect a mass of the liquid. The development of lift on a surface moving in water differs substantially from the same phenomenon in air because water is much close to an ideal (incompressable) fluid. The motion of an inclined plane in air causes a change in the density of the air at various points around the plane. Since the density of water is essentially invariant, however, the same motion produces a different effect. The flow of water past an inclined plane can be treated as composed of two distinct motions. The first is the streamline flow just as is encountered in air, the second is a circulation. The circulation component arises directly from the incompressable nature of the water.

DEVELOPMENT OF LIFT ON A HYDROFOIL. The flow of an ideal liquid past an inclined flat plate is illustrated. The velocity is separated into two components (a and b). The first component is the streamline flow, and is very much like the flow pattern arising from the same situation in air. The second component is the circulation, a strong contributor to the total flow in water. The pattern is elliptical at the plate, but becomes circular as the distance from the plate increases. The net flow (c) is the sum of these two components. The circulation velocity is downstream at the top of the plate, adding to the streamline velocity, and is upstream at the bottom of the plate, reducing the total velocity. The resultant velocity, therefore, is greater above the plate than below the plate.



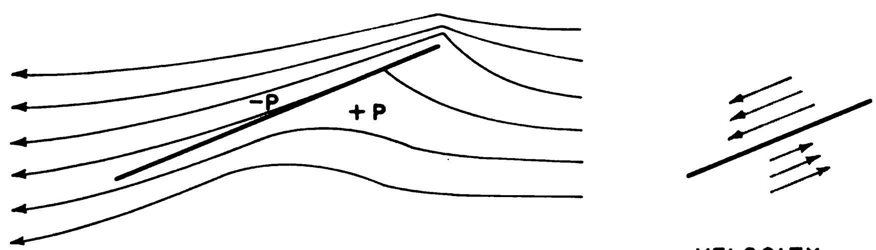
(a) STREAMLINE FLOW

VELOCITY



(b) CIRCULAR FLOW

VELOCITY



(c) RESULTANT FLOW

VELOCITY

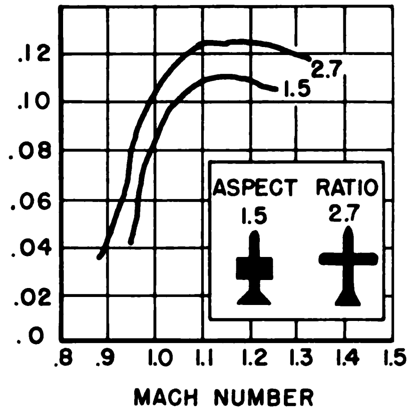
The Bernoulli energy relationship holds, and there is a region of pronounced +p on the face, and one of pronounced -p on the back, for about the same proportion of the length (+p and -p are considered relative to normal or static pressure (p)). Whereas there was at (a) only a counterclockwise moment and no net force in any direction, there is, in addition at (c), a powerful upward lift force, due to the +p's generated below the plate, and the -p's existing above it. The summation of all pressures on the top and bottom, acting over the total area of the plate, produces a resultant force on it. When resolved perpendicular and parallel to the direction of the uniform stream flow, the force components are the lift (L) and the drag (D), respectively. Considering the liquid stationary and the plate moving, the lift force (L) is always normal to the direction of motion, and the drag force (D) always parallel to it.

HYDROFOIL ANGLE OF ATTACK. The greater the lift required of a hydrofoil, the greater must be the effective angle of attach at which it moves. Increasing the angle of attack increases the hydrofoil circulation and the pressure differentials between face and back, the strength of the tip and trailing vortexes are augmented, and the induced flow velocities become greater. Within a normal range of attack angles, a hydrofoil will produce a lift foce (L) that varies almost linearly with the effective angle of attack, reckoned from the angle of zero lift position. In common with any body placed in a moving stream of liquid, a hydrofoil is subject to separation and cavitation, when certain ranges of speed or angles of attack are exceeded, or if certain unsuitable combinations of fluid velocities and attack angles are employed. Cavitation is the formation of a vacuum around a hydrofoil when it displaces water around it at a faster rate than the water can flow toward it. When these discontinuities of flow occur, lift decreases, drag increases, water trail may be formed, and buffeting and other

undesirable phenomena may occur. The real effect of these discontinuities is to change the hydrodynamic shape of the hydrofoil, and to reduce its effectiveness as a control device. In the regions of separation and cavitation there are zones of stagnation or cavities which, in effect, change the normal hydrofoil flow patterns that existed when the flow was continuous. From the zero lift angle through a maximum operable attack angle, the lift (L) and the lift coefficient (C_L) vary in a controlled manner. When we exceed the maximum attack angle, separation or cavitation occurs, the lift decreases, and the drag builds up rapidly. The region where this occurs is known as the "stalling point" (similar to the burble point in aerodynamics) and the corresponding geometric angle of attack is called the "stalling angle".

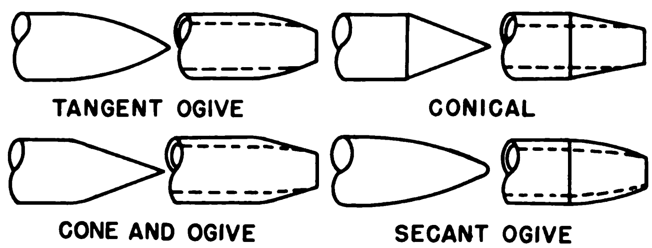
DRAG. Drag is due in part to the piling up of air or water in front of the missile and in part to the adhering of air or water to the missile surface (boundary layer). The boundary layer results from friction between the foil surface and the medium being traversed. All exposed missile surfaces resist the motion and contribute to drag. The resistance of the parts of a missile which did not contribute to lift is called "parasitic drag". Missiles are designed for minimum drag. Since drag is directly proportional to the velocity square, minimum drag design becomes of paramount importance as missile velocity increases. Boundary layer thickness distributions, surface imperfections, and aspect ratio sections of the foil are among the most important drag considerations. The more air or water adhering to the surface, the thicker the boundary layer and the greater the resistance to missile movements. By controlling the thickness and flow of the boundary layer, it is possible to substantially reduce the drag component acting on a missile. For example, boundary layer control by "area suction" has been successfully applied in aerodynamic flow. In this principle, the plate is made porous and the fluid streaming past is drawn through the plate. This action in effect "thins out" the boundary layer and effectively reduces the drag. However, if the boundary layer is made too thin, the roughness of the surface elements will be exposed, and the resulting turbulent flow will increase the drag. Surface imperfections tend to counteract the effects of streamlining, and only by developing a laminar flow can we effect a reduction in separation effects. Laminar flow refers to a condition where the layers of the fluid close to the surface are smooth although of different velocities with respect to each other.

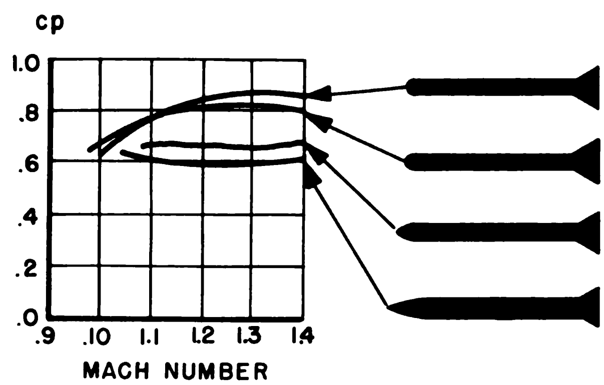
ASPECT RATIO. It was noted previously that foil section configuration influences drag; as when the use of a cambered surface at supersonic speeds was discussed. The aspect ratio is the relationship between the span and the area of a foil, and can be expressed as follows: aspect ratio $= \frac{b^2}{A}$, where b is airfoil span and A is airfoil area. The lower the aspect ratio, the lower the magnitude of the drag coefficient. Hence, supersonic missiles are designed with what is referred to as "clipped wings" as opposed to the conventional wing shape on subsonic missiles. At supersonic velocities, mutual interference between the drag of individual components results in an overall drag coefficient of total magnitude.

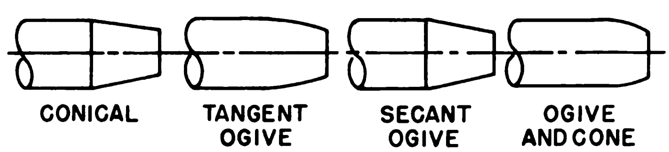


EFFECTS OF CONFIGURATION ON DRAG

The forebody or nose of the missile is designed for minimum drag compatible with its mission. In instances where search equipment must be contained in the nose of the missile, the shape of the vehicle must be designed for effective energy transmission and reception, with a minimum drag coefficient factor. The Sidewinder missile is an example of missile configuration containing infrared detection equipment in its round nose, and still having a low drag coefficient. The effects of different shapes of missile forebodies on aerodynamic drag, as it varies with Mach number, are illustrated. A boat-tail configuration as illustrated reduces the aerodynamic or hydrodynamic drag of a missile. It is a cylindrical section of the missile body where the diameter is continually decreasing toward the rear. A small boat-tail angle creates a lower negative pressure area than a standard section would, and results in lower drag.







stability and maneuverability

The primary objective in control surface design is to achieve the greatest possible resistance to unwanted motion (stability) while maintaining the necessary capability for deliberate motion (maneuverability).

compatibility with missile components, launcher and delivery vehicle

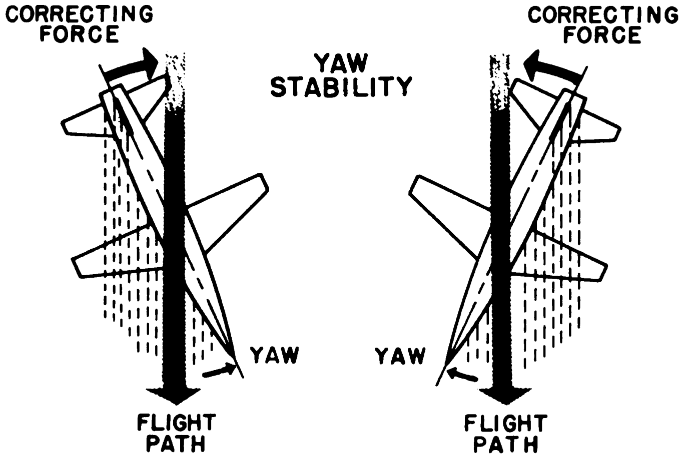
Some aspects of the configuration are often dictated by requirements other than control. The launching system and delivery vehicle may effect the configuration markedly. Ordinarily gravity-propelled general purpose type bombs require only a configuration that will maintain in-flight stability to a target. However, if these bombs require mounting on the exterior surfaces of high-performance aircraft, ordinary design would result in excessive drag. Hence, streamlined low-drag bombs have been developed. The sinking time for such missiles as depth charges must be kept as short as possible to provide the enemy with minimum time for evasive action. This is increasingly important as faster deeper-traveling submarines are constantly being designed. To effectively combat this trend, depth charge bombs have been streamlined, resulting in better ballistics and reduced missile dispersion. The Zuni aircraft rocket is fired from a launcher which does not permit the rocket to have fixed fins. Since the unguided rocket requires fin stabilization, folding fins have been designed for it. When the rocket leaves the launcher, the fins automatically open into flight position. Missile configuration is also dependent on the specific use of the missile. For example, aircraft-launched depth bombs, and ship-launched antisubmarine rockets such as the Alfa, often have flat noses to reduce the possibility of richochets when striking the water at small entrance angles. Surface-fired torpedoes often have a flat section on the nose for the same reason.

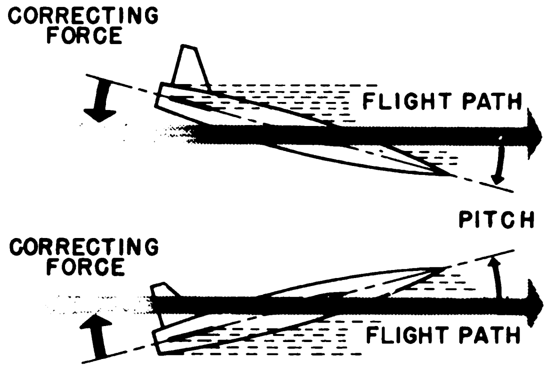
MISSILE STABILITY AND CONTROL

stability

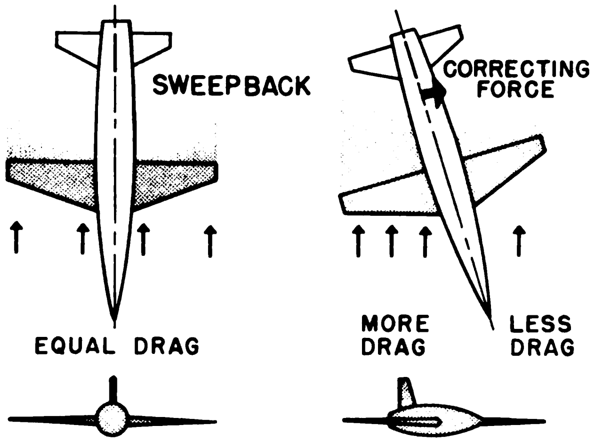
To optimize the hit probability and to minimize its dispersion factor, a missile must be properly stabilized in flight. The addition of in-flight guidance systems in the missile further increases the probability of hit, and decreases the probability of dispersion. Aerodynamic or hydrodynamic control devices are used to produce the necessary changes in flight trajectories. At the same time, the missile must have the best design configuration for the intended speed. In guided missiles, the operating mechanism (or servo system) must have the necessary response characteristics to prevent instability and to provide the required maneuverability on the missile. The use of fixed fins and spinning of the missile are the simplest means of stabilizing a missile in flight. In spin-stabilized missiles, the center of pressure is usually forward of the center of gravity. The problem of stability in design is one of developing a configuration in which the center of pressure stays close to the trajectory which is traced by the missile's center of gravity. As the launched missile moves along its trajectory, the curvature of its path becomes greater until shortly after the maximum ordinate is reached. After this, the curvature diminishes again. The effect of the initial increase in curvature is to increase the air pressure under the nose of the missile, causing the missile to precess to the right. This shift of the axis to the right results in an increase in air pressure on the left side of the missile nose which, in turn, causes a precession downward. This train of events continues, causing the axis of the missile to oscillate about a tangent to the trajectory. A missile can be made too stable. For example, if a projectile has developed a too high rate of spin, it may fail to nose over at the desired moment on the descending portion of the trajectory. This occurs when the trajectory drops at a rate faster than the precessing rate of the projectile permits it to follow. In such a case, the projectile has become so stable, and is precessing so slowly that it cannot dip far enough for its axis to remain on the rapidly dropping trajectory. Thus, overspinning projectiles carrying nose fuzes may increase the probability of duds or of erratic fuze operation. In fin stabilization, stability is provided about the three axes of the missile by stabilizing airfoils.

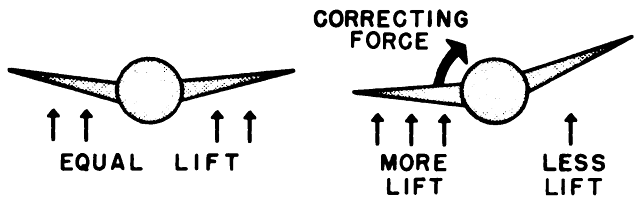
STABILITY ABOUT VERTICAL AXIS. If a missile tends to turn to the right (yaw) during its trajectory, then the pressure on the left side of the missile tends to increase. This increased pressure resists spinforce components, and actuates tail movement in the opposite direction. The addition of a vertical fin for directional control may partly or fully overcome this unwanted movement. Along with the fin, the sides of the missile body act as a stabilizing area. Vertical stability is also obtained by sweepback of foil surfaces. The sweepback angle is the angle that the leading edge of the foil makes with the longitudinal axis of the missile. When a missile yaws to the right, the front edge of the left sweepback foil becomes more perpendicular to the relative fluid force than the right sweepback foil, placing more drag on the left foil. This unbalanced drag effect tends to equalize the yawing moment, and to re-establish the original attitude.



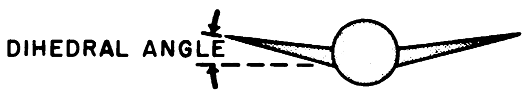


PITCH STABILITY





ROLL STABILITY



DIHEDRAL

STABILITY ABOUT LONGITUDINAL AXIS may be provided for in a missile by a dihedral - an upward angle of the wing - and the position of the foils. A dihedral angle is the angle formed by the lateral axis of the missile and a reference line through the foil surface. This angle lies in a plane perpendicular to the longitudinal axis. (Cathedral or negative dihedral is the angle that the foils make in a downward direction toward the foil tips from the body.) Dihedral produces stability by causing a change of lift on the foil surfaces as shown in the illustration. As the missile starts to roll, lift on the right wing increases, while lift on the left wing decreases. This unbalanced lift tends to roll the missile back to its original attitude. The positioning of the foils is another means of obtaining stability about the longitudinal axis. A missile has greater stability if the foils are placed about the center of gravity, than if the foils are placed below the center of gravity.

STABILITY ABOUT LATERAL AXIS, called "pitch stability", is accomplished by horizontal surfaces at the tail of the missile. These surfaces are known as stabilizers and consist of two sections, the stationary part as the stabilizer, and the movable component as the elevator. Pitch stability is accomplished by the changing forces present on the surface, when the missile changes its angle of attack. For example, with the stabilizer at the tail, if a missile tends to nose downward, the force exerted on the upper surface of the stabilizer increases, forcing the tail down and bringing the missile back to its original attitude.

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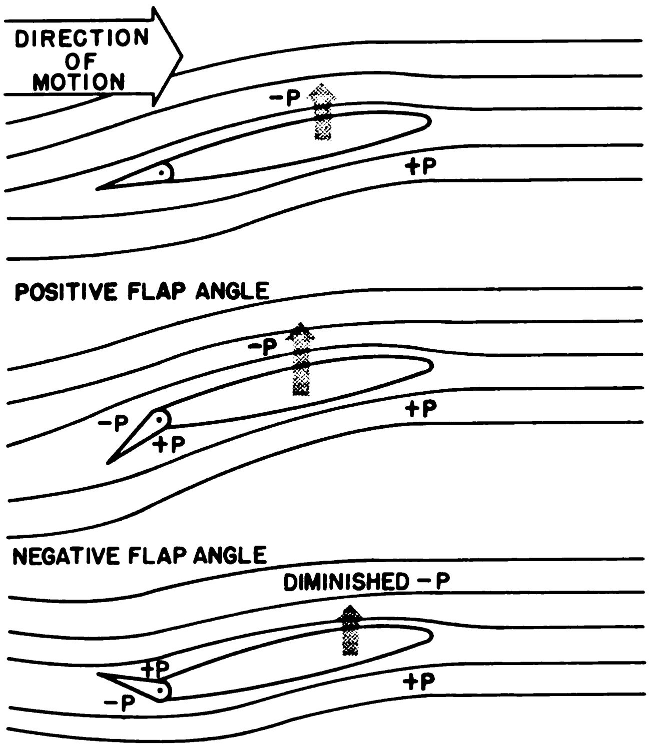
movable control surfaces

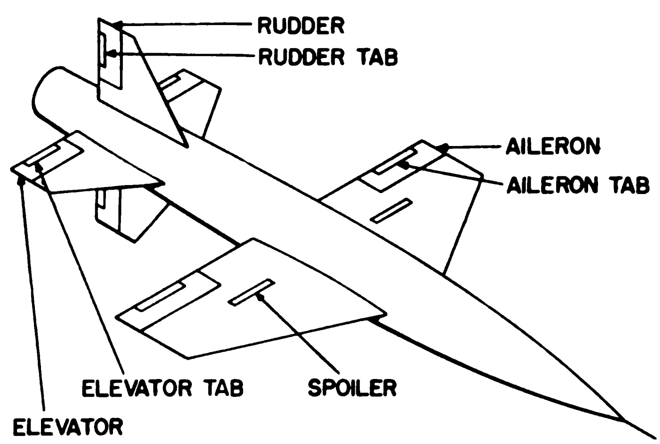
It is frequently necessary, particularly in guided missiles, to have an airfoil or hydrofoil produce variable lift in either direction, while the fluid velocity past the foil remains essentially the same, and without changes occurring in nominal angle. The more stringent the requirements for in-flight control of the missile along a flight path, the more important it is to produce this force, variable in magnitude and direction, as an addition to the force obtained from the fixed guide fins. Projectiles that do not require a high degree of accuracy (mortars) usually have fixed fins and sometimes combine this with spin stabilization for excellent results.



An antiaircraft missile (such as the Terrier) requires precise control and rapid response to in-flight guidance, and must use some means to achieve this operational requirement. One obvious solution is the use of a foil symmetrical about the base chord, that can have its attack angle varied in either direction as is done with a ship's rudder. However, a symmetrical section is not the most efficient lift-producing device. A "compound foil" with a fixed forward section and a movable "flap" or rear section hinged to the trailing edge of the fixed portion of the foil is more efficient and provides more precise control over the missile's trajectory. When both portions of the foil are in line, the foil is of the orthodox symmetrical type. When the rear section is bent about the flap hinge, a cambered section is produced which augments the flow circulation, and with it the lift force. This lift force is developed when the flow past the trailing edge of the flap is deflected to a greater angle with respect to the direction of motion, developing a +p region on the upper side of the hinge and an augmented -p region on the back or lower side of the hinge. The action of the flap placed at an angle is to develop circulation of flow and lift potential, even though the forward portion of the foil is at zero angle of attack. The +p and -p regions extend to the fixed portion of the foil as well as to the flap, and in many instances the greater portion of the lift may be exerted on the fixed or non-movable portion of the foil. A practical advantage of this arrangement is that the torque required to swing the flap is relatively small, while the fixed portion of the foil, carrying a large portion (and possibly most) of the lift, can be constructed to carry or bear optimum lift potentials. There may be occasions when hydromatic lift is a drawback, and where a reduction in lift must be made quickly, without additional complication. When a reduction in lift is required, the process is performed in reverse order. The use of the positive flap angle to increase lift has its counterpart in the use of negative flap angles to decrease the lift. The trailing edge is swung to the opposite direction so as to stop a large part of the circulation, resulting in an accompanying shift in signs of the p's over the flap area and on the main hydrofoil ahead of it.

CONTROL DEVICES. It is possible to mount a foil carrying a small flap somewhere near the overall aerodynamic or hydrodynamic center of a missile, and, by controlling its swing angle about this axis, to exert alternating and varying lifts by applying flap angles in either direction. This type of control device is called a "tab". With the use of a tab for angling or changing the direction of motion of the missile, heavy steering gears may be dispensed with, if it is not necessary to move the rudder when the missile is not in motion.

Flaps and tabs are classified as primary control and secondary control devices, respectively. Primary control devices are responsible for maintaining missiles along desired trajectory paths, and alone could, under certain circumstances, give satisfactory results. That is, if there were no unstabilizing conditions present, primary control could function satisfactorily, unaided. However, a missile can be controlled more accurately and more efficiently by the use of secondary controls, in various combinations, working in conjunction with the primary control system. Ailerons, elevators, and rudders are considered primary controls. Tabs, spoilers, and slots are secondary controls.

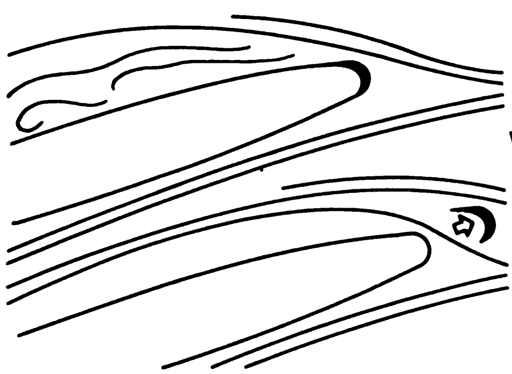


Tabs are hinged to primary control surfaces, and the force exerted by these devices is directed to act against a primary control and not the missile itself. Suppose it is desired to trim in pitch by raising the nose of the missile. A small movement of the tab on the trailing edge of the elevator causes a small force to be exerted on the primary control surface, resulting in the elevator moving in the opposite direction; thus when the tab is moved downward, the elevator moves up. Since the missile responds only to the primary control action, the tail is lowered, thus raising the nose of the missile. Tabs are classified in 3 catagories: fixed, trim, and booster. A fixed tab is preset for a given condition of stability. A trim tab is controllable, and its setting can be varied over a wide range of conditions. A booster tab is used to assist in applying force necessary to move control surfaces of large areas.

Spoilers are devices used to overcome undesired wing motion. They are recessed into the upper camber of the wings. When the spoiler is not used, the flow of air over the wing is smooth and uninterrupted. However, if a gust of air causes the left wing to drop, the control system calls for the spoiler on the right wing to extend. As the spoiler extends, the lift pattern on the right wing is reduced considerably by the turbulence created by the spoilers. The wings then tend to return to the original position.

A slot is basically a lift device, and is located along the leading edge of the foil. When the slope on the -p side of the flap exceeds actual value, separation occurs. In other words, there is a breakdown such as takes place on any foil when the angle of attack exceeds the stalling angle or burble point. If a jet of the medium is introduced into the leading end of the separation zone, the lateral force exerted by the assembly would be augmented. This may be accomplished by hinging the foil forward of its leading edge, so that at large flap angles a wide slot opens between the trailing edge of the foil and the leading edge of the flap.

The controls that have been discussed are of the conventional type. For high-speed missiles, new control surface designs have been developed. Examples of such surfaces used are elevons, rollerons, ruddervators, and aileevators. As the names indicate, these are multi-purpose control devices. For example, an elevon replaces an elevator and an aileron, allowing control of pitch and yaw by a single control mechanism. An aileva-tor is the same as an elevon. The purpose of these multi-faceted devices is to insure instantaneous responses to guidance system directions.

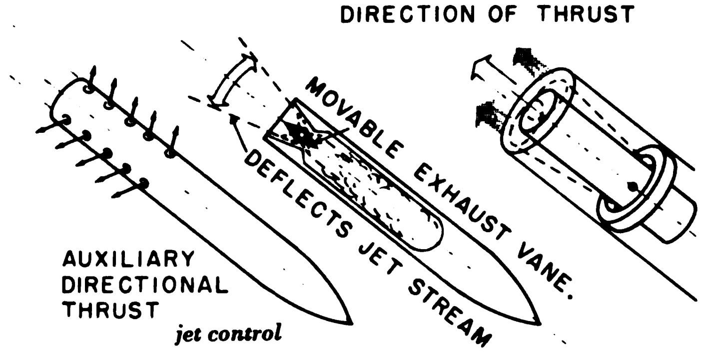


control at low starting speeds and in a vacuum

Ordinary control surfaces are not capable of control stability until a missile attains a speed at which the foil sections will have an aerodynamic or hydrodynamic effect on its motion. Also, since the control surfaces require air or water flow, or pressures of sufficient magnitudes to become efficiently operative, another means of control, other than by external control surfaces, must be provided for travel in a vacuum. This means may be supplied by the use of exhaust vanes or jet control.

EXHAUST VANES. A technique for stabilizing a missile, both for low starting speeds and for travel outside the atmosphere, is that of deflecting the exhaust gases of the jet engine by the use of exhaust vanes installed directly in the exhaust path. Changing the position of the vane deflects the exhaust, resulting in a change in thrust direction, as thrust and exhaust are always exactly opposite in polarity. Because of the tremendous heat developed, the life of an exhaust vane is generally short.

JET CONTROL. One means of jet controlling the flight attitude of a missile is the gimballed engine. In this design, the exhaust stream can be directed at any desired angle to give the desired direction of motion. Basically, this method is not greatly different from the variation of exhaust direction to obtain control stability. Two objections to this method are that the fuel lines must be flexible, and the control system that actuates the jet must be extremely strong. A typical system for controlling the stability of a missile in flight, employing a gimballed engine, is to have the control system direct the thrust chamber of the engine to correct for pitch and yaw errors. Vernier jets or engines mounted alongside the jet engine may be directed by the control system to correct or regulate the roll.



jet control

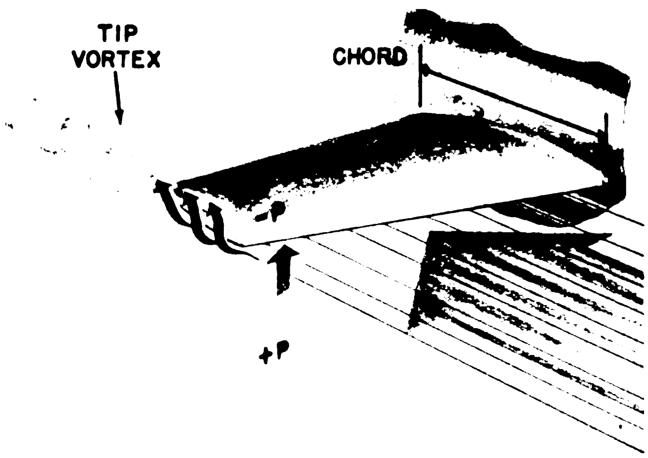
Another method of jet control is accomplished by mounting jets at strategically designed points along the missile configuration. These small thrust-directors are usually mounted on fins or control surfaces so that they can develop higher moment arms for control. Control is accomplished by activating the desired jet, thus developing thrust in the required direction. Heat shields are necessary to protect the main body of the missile from exhaust heat generated by the jets. This method of control may eliminate the need for outside control surfaces, affording a cleaner aerodynamic or hydrodynamic design.

FOIL CONFIGURATION

Operational speed is the major factor in the design of foil configuration. The main difference between the supersonic and the subsonic profile is that the leading edge is sharper in the supersonic, and the shape of the foil about the chord line can be symmetrical or unsymmetrical at subsonic speeds, while "compound foils" are utilized at supersonic speed.

Because of the type of flow and pressure distribution encountered at supersonic speeds, the advantage of camber as in the subsonic case no longer exists. Lift forces are derived as a result of the average relative angle to the oncoming stream rather than of an increase in velocity over the upper surface as compared with the lower. Likewise the sharp leading edge is required to minimize drag coefficients under supersonic flow conditions. Typical supersonic foil configurations were illustrated previously with the discussion of lift and drag forces. The dynamics of airfoils at subsonic speeds are similar to those of hydrofoils. This is due to the basic similarity in the reactions of air and water to low-speed motion of a foil. Low-speed foils are symmetrical or unsymmetrical, depending upon their intended use. Foils designed to develop lift potential principally in one direction are usually unsymmetrical about the base chord. Propeller blades that are required to produce forward thrust are an example of designed asymmetry. Foils that alternately generate lift in both directions, such as a rudder, are symmetrical.

Of considerable practical importance in ship design is the location of the line of action of lift, drag and resultant forces on a foil. In determining torque requirements necessary to activate a control surface, designers utilize these lines to determine basic reference frames to be used in final solutions. Some airfoil sections used for aircraft wings are designed especially to produce a small shift on the line of action of the lift with changes in angle of attack. With asymmetrical foils of orthodox sections, the center of pressure (CP) usually is located a quarter-chord length from the leading edge of the foil. The aerodynamic or hydrodynamic center of the section is by definition located on the base chord, and is usually used as a reference point of origin for force and moment determinations. Camber (rise of the main line above the base chord) is a factor in practically all determinations involving unsymmetrical foils. It is an element because of the utilization of camber to develop an upward or lift component of the flow at the foil nose, and a downward or reverse component of the flow leaving the tail. In hydrofoils, large cambers develop high circulation strength, causing the flow to leave smoothly from the tail. In practice, strength and rigidity govern foil thickness, rather than required dynamic performance. This is particularly true in the case of hydrofoils. Because of the extremely wide range of attack angles to which a hydrofoil may be exposed, the utilization of thick sections, or at least a relatively thick leading edge, is necessary to permit the incident flow to approach from a considerable range of angles without risk of separation or cavitation at the leading edge. A theoretical foil possessing unlimited span perpendicular to the direction of motion or of stream flow, would develop a two-dimensional flow along its length, if the flow is uniform.



When the foil is of finite length, and has free tips or ends, or when the flow is not uniform, three-dimensional flow takes place, and a certain degree of spanwise flow is developed. The illustration indicates that the liquid flows outward from the underside, upward around the tip, and inward above the top, forming the trailing vortex and setting up the cross-flow. The result of this action is to reduce the lift potential by domination of +p on the underside of the foil, and of -p on the topside of the section. All other factors being equal, the foil having the longest tip in proportion to its total area will be the least effective in lift potential. The one having the shortest tip and the greatest span per total area is the most effective. The loss of lift at the tip appears to be a function of the rate of change of circulation with spanwise distance, as well as the fore-and-aft length of the tip. It is wise, therefore, to taper a foil so that the ratio of mean chord to tip chord is large, or that the tip chord is small in comparison with the mean chord.

If area cannot be sacrificed for this purpose, and if the foil is designed primarily for lift in one direction, as in a screw-propeller blade, other procedures may be followed. The camber of the foil tip sections may be reduced, or the sections deliberately twisted to diminish their angle of attack relative to other sections toward the mean or root chord. In either case, tip circulation is deliberately reduced, to compensate for pressure leakage around the tip.

basic types of missile configuration

Missile configuration depends upon various factors: purpose, maximum operational speed, type of control and guidance system to be used in conjunction with it, maximum turning rate, whether the missile will remain in the atmosphere, in space, in the sea or in a combination of environments, and the maximum range of the missile. All of the above listed factors impose demands upon the structural design of the missile, and often meeting the criteria of one requirement reduces the effectiveness of another. Missile configuration may be divided into four general types, depending on the location of the primary control and/or lifting surfaces.

These may be classified as

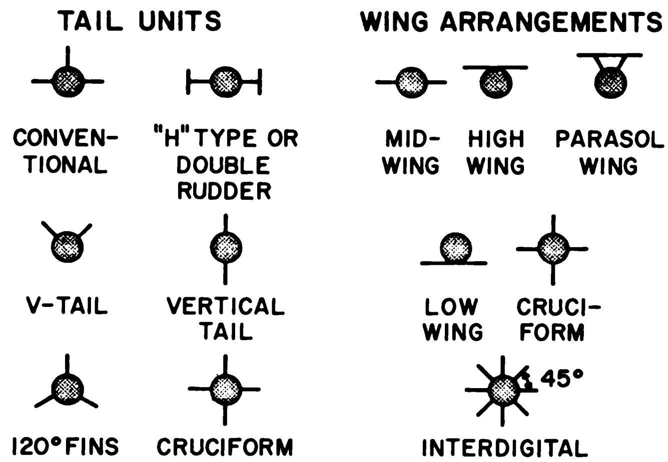
1 CANARD 2 WING CONTROL 3 TAIL CONTROL and 4 TAILLESS

CANARD TYPE. The canard type is characterized by small control surfaces, well forward on the body, while the main lifting surface is rigidly attached to the afterbody. Lift is obtained by altering the angle of attack of the forward control surfaces, which effectively increases the angle of attack of the missile.

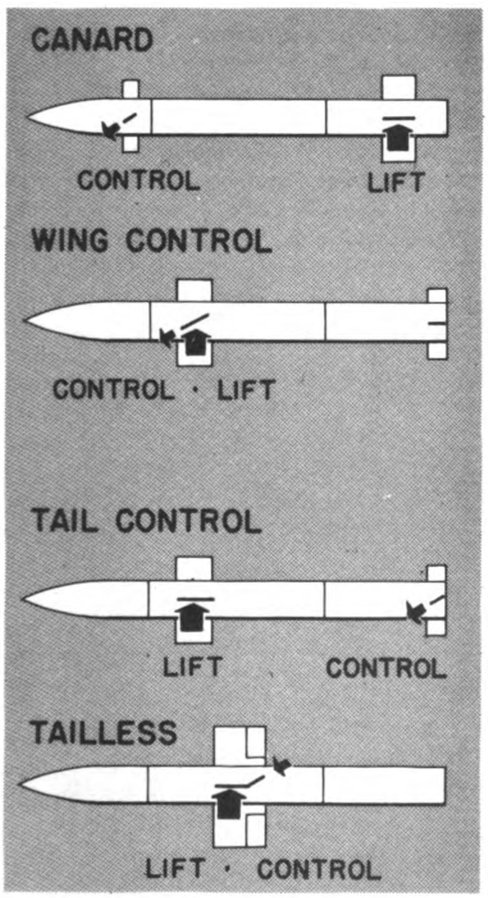
WING CONTROL TYPE. In the wing control type, the control surfaces are located near the center of the configuration, and are also the main lift surfaces. The entire lift surface is controllable, increasing or decreasing the lift in response to control or guidance information, thus changing the missile velocity factor without affecting the missile attitude.

TAIL CONTROL TYPE. The control surfaces of the tail control type are at the rear of the body. Lift supplied by fixed rigid surfaces near the midsection, and deflection of the tail control surfaces are used to alter the missile angle of attack. A wingless tail control design is obtained by omitting the main lift surfaces and achieving the desired lift from the body alone.

TAILLESS TYPE. The tailless type generally involves a set of lift surfaces with control flaps on their trailing edges, the fixed surface providing lift, and the flaps providing lateral and longitudinal control. Each type of missile configuration may have the various combinations of surfaces as illustrated.



common arrangement of foils



Small control surfaces operate at large angles of attack.
High Power required to produce sufficient control surface rates to produce desired manueverability.

Control surfaces located near center of missile.
Does not require large angles of attack relative to free stream.
Low control surface speed and large hinge moments utilized for easier manueverability.

Control surfaces located at rear of body operates efficiently at small angles of attack relative to free stream.
Control surface rate must be high to rapidly change body angle of attack and to control overshoots.
High power required to produce high control surface rates for manuevering.

Lift surfaces and control flaps on their trailing edges.
Compact and limited number of control surfaces.
Control surface location is critical.

missile maneuverability

Missile maneuverability is determined by the speed with which the control surface varies in lift, and the magnitude of the variation. The degree of maneuverability is dependent on missile weight and lift force. Therefore, control surface area and position, missile speed and attack angle, and control system response time are factors affecting maneuverability. The performance characteristics of the basic configuration types are discussed in more detail below.

CANARD CONFIGURATION. In the canard missile configuration, control surfaces are deflected in a positive manner; that is, the leading edge is raised to provide for a positive attack angle. To obtain sufficient lift potential, the control surface must be positioned (with reference to the free stream) at large attack angles that are characterized by increased loads and hinge moments on body surfaces. Time duration of surface movements in response to control signals is of paramount importance in the determination of configuration efficiency. High control surface rates (and hence high power requirements) are necessary to minimize the time duration needed to attain the required attack angle, and thus the desired maneuverability.

WING CONTROL CONFIGURATION. The wing control design has the advantage of being able to produce rapid maneuvers without requiring large inclinations of the missile to the oncoming stream. This is desirable when employed in conjunction with a propulsion system such as a ramjet, or in the use of homing equipment, where a change in missile attitude would adversely affect the efficiency of the system. The magnitude of a dynamic overshoot of the attack angle during a maneuver will likewise be less, thereby increasing the dynamic stability of the missile. Maneuvers are initiated at the instant the wings are deflected, and are not dependent on an increase in body angle to provide the necessary lift. Control surface hinge moments may be higher than for the canard or tail control types; however, the power required may be no greater because of the lower control surface speeds that can be employed. Tests have indicated that changes in the location of the center of pressure, and the resulting hinge moments on control surfaces may be kept reasonably low when employed with all movable control surfaces. Since the wing is mounted near the center of the missile longitudinally, the wing control system has the disadvantage that a rocket sustainer unit must be located somewhat aft of the overall center of gravity, causing a sizeable shift in the center of gravity during flight as the propellant burns. This tends to equate the maneuverability per unit wing deflection as a function of propellant usage. Increasing the available wing deflection can compensate for the lower body trim angle of attack caused by the shift in the center of gravity, although at the expense of increasing tail loads and body-bending moments for a given maneuverability. The concentration of all control equipment at one location tends to stabilize the missile performance characteristics. Wing surface deflections for the control of longitudinal and roll deviations can be actuated from a central control unit, the location of which is aerodynamically sound. Any increase in the body angle of attack in the same direction as the wing incidence increases the angle of attack of the main control surface.

TAIL CONTROL CONFIGURATION. With the tail control design, control surface deflections are in the direction opposite to those of the angle of attack, thereby keeping at a minimum the relative angle between the control surface and the free stream direction. This results in low control surface and body bending loads as well as low hinge moments. Since the forward main lifting surfaces are not deflected, wing-tail interference effects are reduced. Further, the control surface is small, thereby again producing low hinge moments and minimum drag. However, as in the case of the canard, the control surface rate must be kept high to minimize the time required to increase the body angle of attack and to minimize control of overshoots. Power requirements will be affected accordingly. Surface deflection for longitudinal and roll control can be designed for centralized interaction.

TAILLESS CONFIGURATION. The tailless type has the advantage of compactness and small number of control surfaces. The most obvious disadvantage of this type results from the small distance between the center of gravity and the control surface due to the necessity of attaching the wings (which are the stabilizing surfaces) at a central location for required stability. Locating the surfaces too far aft would result in excessive stabilizing moments, and would necessitate extremely large control surface sizes with consequent large downward forces being required to provide positive angles of attack of the body-wing combination. Conversely, moving the surface forward and reducing the stability characteristic also reduces the moment arm and resulting available control moment. In order to overcome the longitudinal moment of inertia of the missile as well as the stabilizing moments of the surface (in order to increase the angle of attack), the control moment must be of sufficient magnitude not only to negate opposing moments but also to develop required lift for necessary maneuvers.

missile launcher fittings and adapters

A missile must contact its launcher at one or more points. Missiles which require some holding force to be applied by the launcher often make use of fittings to convey this force. This holding force may be required to perform one, or a combination, of the following functions: to suspend a missile, to restrain its travel along rails in a tube, or to prevent missile motion until a required thrust is built up.

These structural fittings are in the form of hooks, buttons, or slippers and are required to resist tensile, compression, or bending loads. For some missile dedigns, the hooks must be retractable because the drag penalty due to a fixed extended hook would be inordinately high; however, short protruding hooks which are not near the nose will be immersed in the boundary layer and the drag will be small. At supersonic speeds, any protrusion will have an affect on missile performance, so that retractable hooks, when employed, must be designed to extend or rotate in launch position, with provisions to spring back into a defined orifice upon

launch. The launcher design should provide for the front and rear hooks to leave the launch rails simultaneously, unless it is decided that tip-off (an alteration of the trajecting path due to imperfections in launch components) is acceptable in the design at hand. Providing adequate tolerances for hook location is a critical problem that can be design-simplified by having all hooks attached to the same structural section of the missile, e.g., to a mid-section casting.

These hooks must be designed to withstand the loads and stresses imposed in missile launch. In the case of naval aircraft, some of the most severe loads occur during catapulted take-off and arrested landings. The rolling pullout of fighter aircraft induces high tensile stresses in the missile-carrying hooks. In addition, the hooks may be used to support the missile while in storage, such as in the Terrier weapon system where the hooks are utilized to aid in missile movement along the rails in and out of the magazine area.

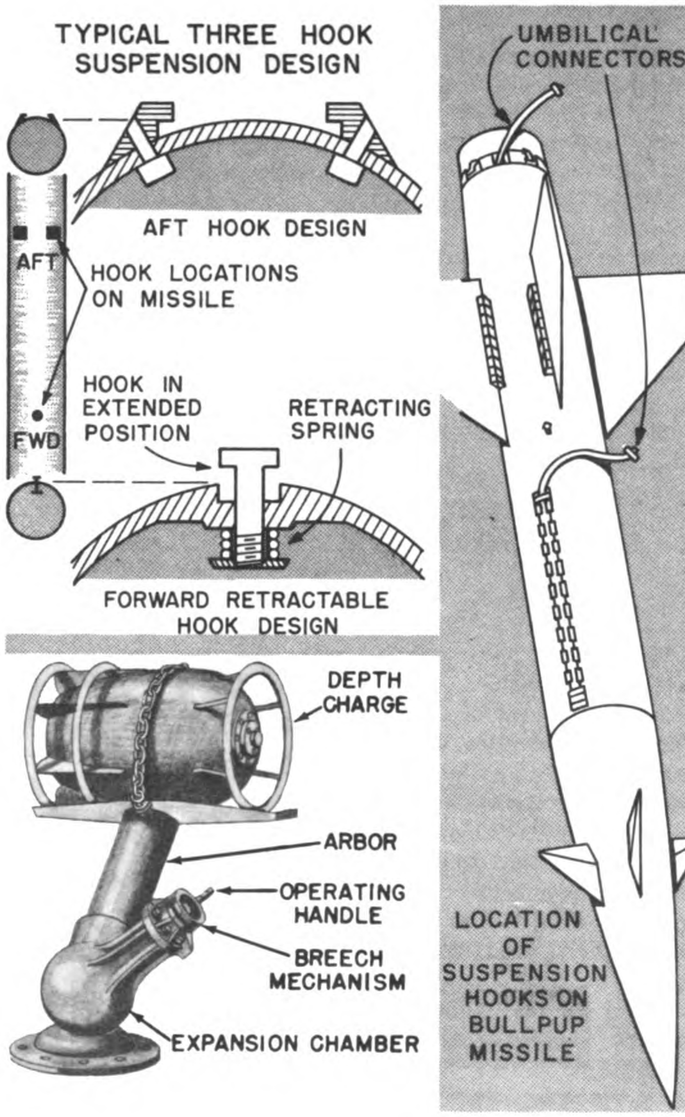
Impulse-propelled missiles require a means of sealing high-pressure gas in the bore and positioning the missile in the launcher, and a method of directional control during missile flight. The common means of accomplishing this in projectiles is in the design of a rotating band and a bourrelet on the projectile body. The rotating band is usually located near the after end of the cylindrical part of the projectile, and the bourrelet at the forward end. These two surfaces, slightly raised above the body, provide the means of positioning and imparting spin to the projectile in its passage through the launcher.

Adapters such as sabots may be used to seal the pressure gases in the gun barrel, and then drop away after the projectile leaves the muzzle. This device provides higher muzzle velocity to the projectile, but has the disadvantage of being dangerous to nearby friendly personnel. On the other hand, "skirted" projectiles have many of the advantages of the sabot without its disadvantages. These projectiles are equipped with flanges which furnish a gas seal, being squeezed inwardly by a tapered reduction in the gun bore. The resulting slight loss of accuracy is not too great a deterrent for the employment of these so-called "squeeze-bore" projectiles.

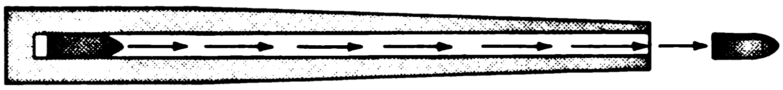
Because of their streamlined shapes, missiles may be difficult to handle, and in some cases difficult to launch. Some missiles compensate for this defect by adding to their configuration. The teardrop-shaped depth charges, Mark 9 and Mark 14, for example, have added circular rings for handling and storage which do not appreciably affect the hydrodynamic properties of the depth charges.

When preparing a guided missile for launch, checkout procedures may require external sources of power, with connections to the missile that must be broken at the instant of firing. Various types of electrical devices are employed to effect the separation smoothly and safely. For example, in a shipboard missile system, the electrical connector on the missile is circular and has a rubber cover for moisture proofing. The mating connector in the launcher arm contains points which pierce the rubber coating to make contact. In airborne guided missiles, missile-to-launcher electrical connection is usually provided by an umbilical-type connection as illustrated previously in the Bullpup missile. When a booster is used in conjunction with a missile, a clamping arrangement is used to mate them. Often these fittings are external and must be considered in the missile configuration. When the booster propellant is consumed, a release mechanism may be activated to release the booster, which falls free of the missile.

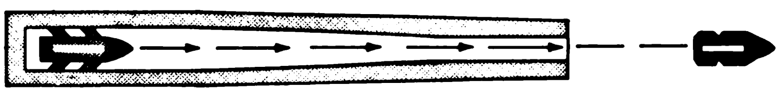
IMPULSE-PROPELLED MISSILES. Impulse-propelled missiles include gun-fired projectiles, depth charges fired from K-guns, and torpedo and ballistic missiles ejected from tubes. The most common type of impulse-propelled missile is the gun-fired projectile.



conventional gun firing "sabot" projectiles



high-strength long gun firing conventional projectiles

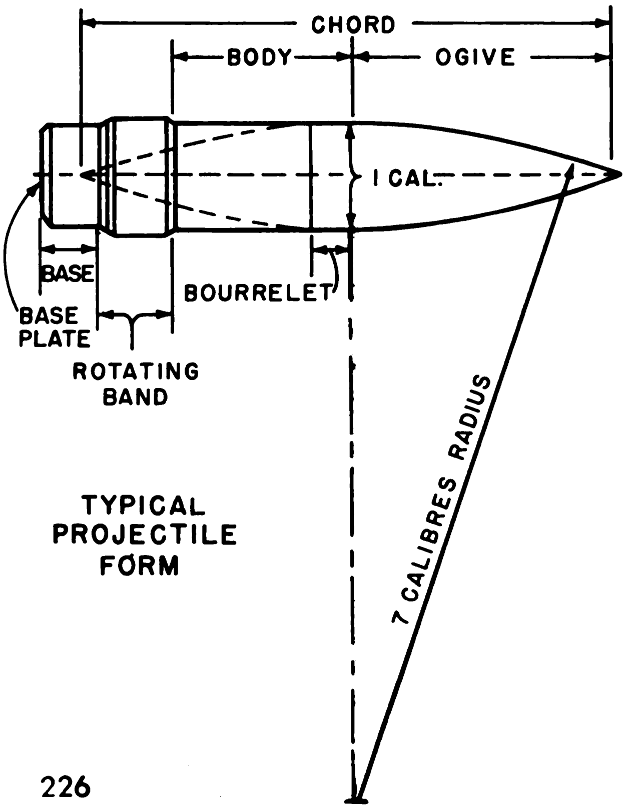


tapered-body gun firing "skirted" projectiles

projectiles

Projectiles, like all missiles, are designed to fulfill a specific mission or missions. The projectile may damage by impact alone, or it may be designed to carry a warhead and inflict damage either by blast or fragmentation effects. The external shape of the projectile is chosen to obtain desired flight characteristics of stability and minimum air resistance. The projectile is designed for optimum warhead weight to strength ratio. In addition, the distribution of weight is a matter of considerable importance. The center of gravity should be in the longitudinal axis and close to or abaft the center of pressure to insure the required missile stability. Extremely high standards of precision are demanded in production to ensure uniform characteristics, and therefore predictable "hit" probability constants.

CLASSIFICATION OF PROJECTILES. Since targets differ in character, projectiles differ in design. The primary classification is into three general types: penetrating, fragmentating, and special purpose. Penetrating projectiles include armor-piercing (AP) and common (COM). They are designed to penetrate, respectively, heavy and light armor. The usual bursting charge for these types is explosive D, which is insensitive enough to permit penetration without premature detonation. Fragmenting projectiles are designed to inflict damage primarily by emission of small, high-velocity fragments. Some damage may also be incurred by blast effect, but this is a secondary design consideration.
Special purpose projectiles are not designed to inflict damage by explosion or fragmentation. Their construction incorporates no special configuration other than that required to withstand discharge from the launcher without damage to the contents. Some of the common varities are illuminating (ILLUM), smoke or white phosphorus (WP), window (W), and target or blindloaded (BL) projectiles.



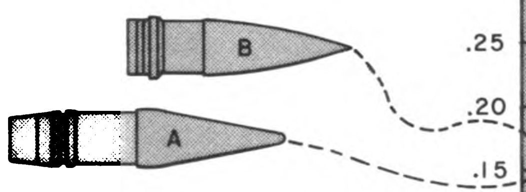
TYPICAL PROJECTILE FORM

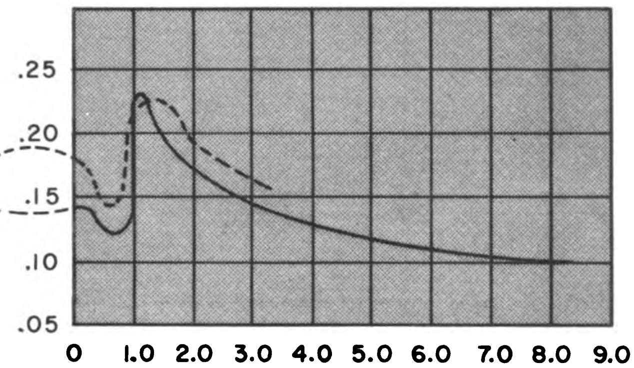
PROJECTILE FORM. The projectile configuration is designed for stability and minimum drag. The shape of the forward end which was found to have the least resistance to airflow was the ogive. An ogive may be described as the arc of a circle whose center is on a line perpendicular to the axis, and whose radius is expressed in calibers. In a projectile, the chord (as shown in the illustration) is the axis of the projectile, and the radius used in about seven to nine times the diameter (caliber) of the projectile. In smaller caliber projectiles, a cone is sometimes used instead of an ogive. For effective armor penetration a blunter head may be necessary.
The term,"projectile body", applies specifically to the cylindrical portion between the bourrelet and the rotating band. Its diameter is less than either of the adjoining sections to prevent contact with the rifling. The base of the projectile is that part to the rear of the rotating band. When the cylindrical shape of the body is continued to the base area, the projectile is said to have a square base. When the after portion is slightly tapered or conical, the projectiles are described as boattailed. The length and degree of the boat-tailing is a function of velocity. At velocities under Mach 1, the boat-tail reduces retardation due to air resistance, increasing the range. At velocities above the speed of sound, dispersion increases and performance suffers.
The bourrelet and rotating bands are slightly raised above the body, to provide the support and bearing which steady the projectile in its passage through the gun.
On large caliber projectiles additional bourrelets, abaft and forward of the rotating band, are added to provide better support, especially during ejection from the muzzle. The rotating bands are composed of relatively soft metal securely sealed in the body of the projectile. As the projectile travels down the bore, the soft band is engraved by the bands of the rifling, and the rotation necessary for in-flight stability imparted. The three primary functions of the rotating band are to seal the bore, to position and center the rear end of the projectile, and to impart rotation to the projectile. The rotating band has considerable effect on muzzle velocity, range accuracy, and the life of the gun. The forward edge of the band is slightly conical, to facilitate engagement with the origin of rifling in the gun barrel. Except for the bourrelet, the projectile does not require a fine machined finish.

DRAG COEFFICIENT. The configuration of a moving projectile determines the manner in which air will behave as it flows over the projectile surface. A tapered or pointed projectile encounters less resistance in its movement through air than a blunt-nosed projectile, and a projectile with a tapered base allows air to flow by it more readily than one with a square base. The plot of the drag coefficient against the Mach number for any type or shape of projectile will indicate an increase in the value of the drag coefficient as the projectile approaches the speed of sound. The sudden increase in drag in this region is due to the fact that as a projectile moves through the air, the air tends to compress and pile up in front of the projectile, and can flow smoothly past it only at velocities less than Mach 1. As the projectile velocity approaches the speed of sound, the air

can no longer flow past the surface of the projectile, forcing the projectile to split the air stream, producing shock waves and increasing drag coefficients. At speeds greater than that of sound, the entire character of the airflow is changed. At lower velocities, a projectile is retarded primarily by the skin friction between the air stream and the projectile surface. As the velocity of the projectile is increased, the air stream is unable to close in behind the base, and a decided turbulence appears behind the projectile (wake). The projectile must then overcome drag coefficients from both skin friction and wake. A further increase in velocity beyond the speed of sound will introduce resistance in the form of shock waves. Therefore, a projectile traveling at supersonic speed encounters retardation, insofar as velocity is concerned, which is the combined effect of skin friction, wake, and shock waves.

DRAG COEFFICIENT vs MACH RATIO FOR DIFFERENT PROJECTILE SHAPES





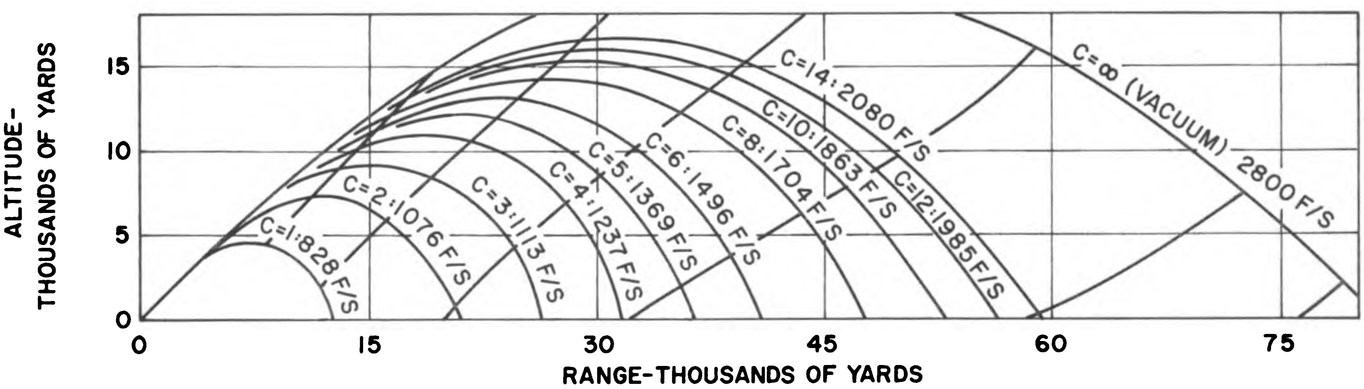
BALLISTIC COEFFICIENT. The ability of various shaped or dissimilar weighted projectiles to overcome air resistance may be calculated, and the results expressed in the form of graphs or ballistic tables. The ballistic coefficient (C) indicates the manner in which a projectile overcomes the resistance of the medium being traversed. The larger the value of coefficient (C), the lower the retardation effects of the medium. The formula for the determination of the ballistic coefficient is:

$$C = \frac{W}{id^2}$$

where

- W is the weight of the projectile in pounds
- d is the diameter of the projectile in inches
- i is the form factor which relates the drag coefficient of the projectile under consideration at a given velocity, with a standard projectile at the same velocity.

In the trajectory plots illustrated, the effect of the ballistic coefficient of the projectile, fired with the same or different mutual velocities and at the same angle of elevation, is indicated. At relatively low initial velocities, the loss of velocity due to air resistance is small as compared with the loss of velocity when the initial velocity is high. When it becomes necessary to fire projectiles with high initial velocities, they should be constructed as heavy and as narrow as aerodynamic demands permit. The ballistic coefficient also indicates a reduction in projectile range for small values of C as compared with that obtained when C = (in vacuum).



PROJECTILE STABILIZATION. Two methods are employed to stabilize projectiles in flight: fin stabilization and spin stabilization. Most projectiles are stabilized by a spin imparted by the rifling (helical or spiral) in the bore of the weapon. The twist of the rifling determines the rate of spin of the projectile and is most important. The twist may be "uniform" (generally about 1 in 15 or 20 times the bore diameter) or "increasing" (as in the 40mm gun) so that the twist becomes sharper as it nears the muzzle. Projectiles launched by guns without rifling may utilize fins to control flight.

reaction propelled missiles

In the design of reaction-propelled missiles, there are special considerations that depend on the particular purposes for which, and the tactical conditions under which, the missile is to be employed. Reliability, complexity, maintainability, tactical usage, packaging, and productivity must all be taken into account and perhaps compromised with the usual consideration of desirable aerodynamic characteristics.

categories

Reaction-propelled missiles may be divided into different categories depending on the principle purpose for which a particular missile is intended:

AIR-TO-AIR · SURFACE-TO-AIR
AIR-TO-SURFACE · SURFACE-TO-SURFACE
SURFACE-TO-UNDERWATER

AIR-TO-AIR. The air-to-air missile is one that is launched from one aircraft against another, both of which may be expected to be flying at extremely high speeds. As long as it is intended to launch these missiles straight ahead from the delivery vehicle, the aerodynamic problems are rather straightforward. The principal difficulty under such circumstances is possible interference with the launching aircraft due to characteristics of airflow between the missile and the delivery vehicle. Safety devices must be designed to prevent the missile from turning into the launch vehicle, or to be seriously deflected during launch, thus affecting its resultant flight path. Air-to-air missiles that are launched in a direction other than that of the flight direction of the delivery vehicle present a very serious problem of dispersion due to crosswind forces. Air-to-air missiles generally have a relatively short range and are of the "launch and coast" type. The propulsion systems utilized are usually of the single-stage type, that is, no special launching booster is used, the missile using its own power plant for attaining maximum velocity.

SURFACE-TO-AIR. The surface-to-air missile is generally used for antiaircraft defense and may vary from a short- to a relatively long-range missile. The desired maximum range will affect the aerodynamic design. For example, in a short-range missile, solid- or liquid-fuel rockets can generally be used; for long ranges, the ramjet type proves advantageous from economy and weight standpoints. For short range, the beam-riding type of guidance will provide sufficient accuracy for reasonable probabilities of kill, whereas with the long-range types some form of homing system becomes necessary. Each type of guidance usually affects the aerodynamic design features of the missile.

AIR-TO-SURFACE. The air-to-surface type of missile presents problems in relation to the delivery vehicle similar to those of the air-to-air missile, except on a larger scale. In general, a smaller number of such missiles would be carried in or on a delivery vehicle (bomber-type plane). The design requirements of this missile, however, are markedly different from those of the air-to-air type because of the comparatively low maneuverability requirements of the air-to-surface missile. The employment of a rocket or ramjet type of configuration in this design category is again dependent on the desired range and location of the target. Ramjets must generally attain supersonic velocities to insure proper ignition and acceleration and are primarily designed for the moderately high supersonic range. The use of a ramjet would require utilization of a boosting rocket which the designer would probably prefer to avoid, in the interest of size and simplicity, when mounting the missile on or in the delivery vehicle. Again the type of guidance and control characteristics would influence the particular aerodynamic design.

SURFACE-TO-SURFACE. Missiles used for surface-to-surface operations are of two varied types depending on the desired maximum range (short or long). The short-range type is of considerably smaller size and its maneuverability requirements are simple. The long-range missile generally uses a booster to accelerate the missile to flight velocity, and requires a complex control and guidance system to effectively deliver it to a target area.

SURFACE-TO-UNDERWATER. The air- and surface-to-underwater missiles are generally used in antisubmarine warfare and are usually of a short-range type. The missile design is determined both by its aerodynamic characteristics and its water-entry problem as well as atmosphere re-entry, if ballistic. Solid-fuel rocket engines are normally used. The missiles, which are guided underwater, generally use homing guidance for the submarine kill.

effect of propulsion system choice on aerodynamic configuration

Basically the simplest ramjet consists of a cylindrical tube open at both ends wherein the air flows through the center of the missile, and the associated equipment must be packaged in the annular space between the inner and outer skins. Such a design is rather inefficient from the packaging standpoint, and would necessitate a somewhat larger missile diameter than for a corresponding rocket missile.

A ramjet engine designed for flight at supersonic speeds must be launched and boosted to a supersonic speed, approximately equal to its operating speed, before ignition. Only at these speeds can a satisfactory margin of thrust over drag be achieved. With such an engine, the thrust varies with the atmospheric pressure and Mach number. A rocket, unlike a ramjet engine, carries within its own structure all the energy necessary for its operation and is independent of the surrounding medium. The configuration used in either case would be dependent on the operating characteristics and the associated equipment needed to operate either the ramjet or the rocket-type engine efficiently.

To improve the performance of the engine, and to isolate it from the main airframe, designers often utilize a pad arrangement wherein the engines are supported at a small distance from the airframe by means of fixed wings or short arms. This involves an acceptance of certain structural and aerodynamic disadvantages that are compensated for by an improvement in missile performance. Liquid-fuel rockets and ramjet engines present a certain amount of flexibility insofar as location of the propellant and the propellant fuel system is concerned. The basic requirement is that the main center

of gravity of the fuel be located at, or close to, the overall center of gravity of the missile, so as to minimize center-of-gravity shift during flight. Solid-fuel rockets are more compact in design than liquid-fuel rockets, and,because of the need for a container of great strength to act as a storage chamber as well as a combustion chamber, this section of the missile body is inherently quite rigid. This rigidity minimizes aero-elastic body-bending effects through this section.

LAUNCHING AND BOOST PHASE. As discussed previously, it is often necessary to use boosters to attain sufficient velocity for the missile control surface to perform properly. The total weight required to transport a payload a given distance can be minimized by the use of a booster which, when its purpose is accomplished, is detached from the missile, and thus does not have to be carried beyond the boost stage. However, the portion of the missile which accommodated the booster must be carried along for the entire flight, and constitutes an undesirable weight penalty. In the case of ramjet propulsion, some manner of boost is essential to bring the missile up to the flight speed required to permit automatic action to begin. In general, ramjets will not operate below approximately transonic speeds, and systems designed for operation in a given Mach number range will generally not be able to provide an excess of thrust over drag at speeds much below this range. It is necessary, therefore, that the boosters be able to accelerate such missiles to some minimum flight speed greater than transonic.

TYPES OF BOOSTERS. Boosters may be employed either in series to meet velocity requirements or in parallel (clustering) to achieve thrust requirements. Staging of boosters in series has the advantage of each booster stage being discarded upon completion of its function, thus minimizing structural weight considerations. Total configuration length is increased, however, as the stages are usually located in series behind the missile. Cluster arrangements of boosters are usually placed well forward on the airframe to allow the missile fins to stabilize the entire configuration. As all the engines in a cluster provide their full thrust from launch, they are separated from the missile at the same instant, and special design provisions must be incorporated to insure that the boosters clear the missile without interference in missile performance. An advantage of parallel mounting of boosters is that, comparatively, the missile length is shorter by 10 to 15 percent, reducing storage and handling requirements. Clustered boosters (with canted nozzles) pointing to a common center of gravity minimize small errors in thrust vector alignment, or negate moments due to thrust or mechanical asymmetries.

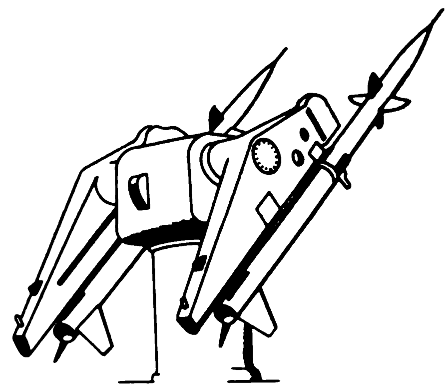
dispersion and loads

The chief purpose of a boosting network is to accelerate the missile to near normal flight speed, and, within the limits imposed by its control and guidance systems, place the missile in a position where it may easily attain its normal flight path. The latter objective is not quite as critical for homing missiles as in beam rider types. Dispersion or deviation from normal flight paths may be caused by misalignments, wind velocities, and lack of stability of the missile configuration. Trajectory deviations are usually the result of thrust misalignments at initial launch velocities, caused by unstable aerodynamic features of the missile or variations in propulsive forces initiating the thrust. Unwanted moments or torques exert the greatest effects when the missile velocity is low, and the angular alignment of the aerodynamic surfaces is such as to provide these unwanted forces a surface to act against. Aerodynamic misalignments are more common during the final stages of boost, even though stabilizing devices utilized by the missile tend to keep the effects of misalignment small. To achieve a high degree of aerodynamic stability when booster configurations are added, the effects of known wind gusts must be programmed into pre-launched flight path determination. Once a missile is launched, especially during boost phase, it is extremely difficult to negate the effect of sidewinds. During the boosting operation, maximum stress or loading of the airplane occurs, and control devices are not nearly as effective as they are under normal flight conditions. High stability characteristics are desirable to minimize the moment due to disturbances that result in flight path deviation. However, the higher the degree of stability the poorer the resultant maneuverability of the missile, and a compromise setting of stability devices and control mechanism usually results. The heading of the missile at separation is of extreme importance from a control viewpoint. Most control devices are intended to correct for slight deviations, and a complete re-orientation of the missile's position would be extremely difficult.

SEPARATION. The separation action of booster from missile is extremely crucial. Although the act of disengagement takes only an exceedingly short time (approximately 0.2 second), various phenomena that occur can seriously and adversely affect the flight of the missile. No matter how smoothly and efficiently the separation is effected, the original body with its combined aerodynamic and structural characteristics must undergo severe stress when the booster system is separated from it. The new trim conditions of the missile must be aerodynamically sound to insure a stable and easily maneuverable final missile configuration. It is imperative that the missile-booster combination achieves a prescribed heading at the instant of separation to prevent any re-engagement or impact forces resulting from a bumping action of booster and missile. Extreme damage to the missiles can result from a collision of the two bodies. In the base of a cluster arrangement, all rockets must be disengaged from the missile at the same instant. Various devices are employed to effect the separation. The general custom is to have the rockets rotate backward about a fitting attached to the missile, falling free and outward without striking any external aerodynamic surfaces. Clustered rockets may be separated in much the same manner as a stage is separated in a multistage missile. In the case of a ramjet missile the pressure on the inside of the ramjet can be utilized as a repellent force on the booster, causing the separation to occur at a predetermined velocity. Alternate means of providing missile stability during boost include the use of jet vanes in the booster nozzle to produce motion-controlling moments, or canted nozzles to perform the same function.

launcher adapter fittings

Launcher length is a function of 1) time necessary for booster configuration to attain launch velocities, and 2) length of missile. New developments in rocketry and launch techniques have considerably shortened launch rail lengths, allowing launchings to be effected from limited operating areas (shipboard, etc.). Utilizing shorter rail lengths, especially if the forward and aft booster fittings (shoes) which support the configuration travel on the same rails, causes the missile nose to tip downward slightly when the forward shoes leave the rails with the aft shoes still bearing. This unwanted effect is called "tip-off" and has led to launcher rail design wherein the forward and aft shoes ride on separate rails of the same length, allowing both sets of shoes to leave the rails at the same instant. This launch technique prevents the missile from initiating a pitching or vibrating motion, at the instant of separation between missile and launcher.



TACTICAL AND DESIGN CONSIDERATIONS

OBJECTIVE OF MISSION. Tactical considerations influence the choice of a weapon required to accomplish a mission (objective of an attack). Weapons are often designated by their tactical usage: antitank, anti-aircraft, antimissile, antipersonnel, and antisubmarine. Missiles designed for a specific tactical problem may be ineffective or even uneconomical when utilized in a situation other than their original design required. Therefore, we design missile systems to meet as many different tactical situations as practical, and adapt these existing systems to meet less common target configurations. The most important single component of a missile system is the payload that is being delivered to a target area. Every payload is designed to accomplish a particular military objective. The wide range of payloads in a modern arsenal includes fragmentation-controlled, high-blast, chemical, biological, and nuclear types. The choice of a particular payload depends on the target's characteristics and the degree of kill probability that is militarily required. The damage volume, or the effective radius of damage of a payload depends not only on the strength of the explosive agent, the form of energy released, and the attenuation due to the medium, but also on the accuracy and dependability of the control and guidance system.

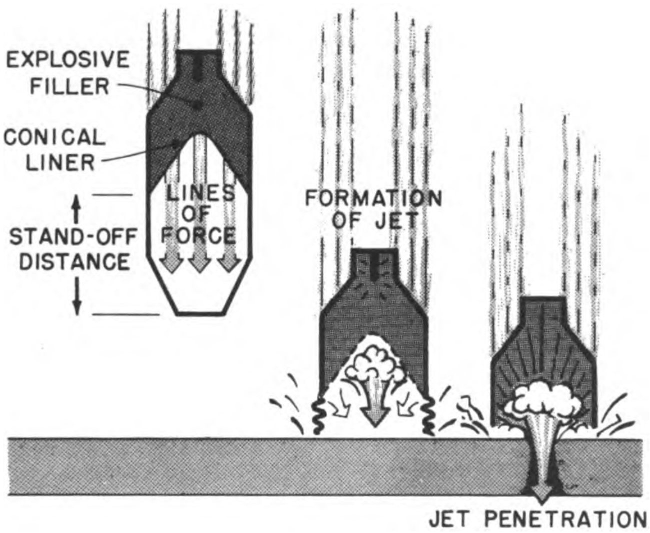
SELECTION OF A GUIDANCE AND CONTROL SYSTEM Dispersion or deviation of a missile from a flight path can be caused by aerodynamic or hydrodynamic instability, environmental conditions, or by artificial stimuli (countermeasures). Guided missiles have built-in control systems able to respond to guidance commands and to negate any deviation from true flight paths. However, because of the speed of travel, and the time delays encountered between converting electrical impulses into mechanical movements, or other unforeseen conditions, errors in guidance are caused, and missile dispersion results. New homing devices have decreased the tolerance of the miss, but it still exists. If the dispersion angle of the carrier is great, it becomes necessary to increase the payload capacity to insure a "kill". However, increasing warhead potential increases armament weight, thus increasing vehicle strength requirements. The selection of the guidance system is therefore of paramount importance in missile design.

POWER REQUIREMENTS. To determine the power requirements of a missile, a designer must consider the desired missile velocity, the medium traversed, the distance to the target area, the configuration of the missile, and the total weight that must be propelled by the launch mechanism. The target location, maneuverability, speed, and performance will determine the capability of the power plant required for the missile to complete its mission. The environmental conditions which the missile will encounter (temperature, humidity, pressure, etc.) demand of the designer a vehicle capable not only of carrying the various components of the system, but also of shielding them from environmental forces. The combined design, then, requires a vehicle which can accurately control its motion, has the ability to maneuver in flight, remains resistant to environmental forces, and is strong enough to withstand the strain and stresses to which it will be subjected while delivering a payload to a designated area. A complete design package must produce the optimum missile configuration to perform a particular mission.

target characteristics effect missile configuration

Many enemy targets, for example, have protective armor. To destroy these targets, armor-piercing missiles such as AP projectiles and AP bombs are used. For the heavier armor plating defenses, shaped charged warheads are used to direct their explosive force into a small and concentrated jet of destructive energy. To pierce heavy armor, shaped charges are detonated at relatively short distances from the armor, allowing jet action to occur. The warhead containing the shaped charge, for example, an antitank rocket head, is therefore designed to provide this stand-off distance, yet have the nose conform to sound aerodynamic design principles. Fragmentation projectiles are designed for use against aircraft. Upon initiation, a high explosive filler breaks the casing into particles with sufficient velocity to damage or kill aircraft targets.
Clustered warheads are used to increase the damage volume of a missile. Instead of delivering one warhead, the missile delivers several warheads, which are detonated in such a manner as to increase their potential

destructive power many times. Missile configuration must allow for storage of multiple warheads and still remain aerodynamically stable. When the tactical situation requires action against underwater targets, the missile must be designed for travel underwater as well as in the environment in which it is launched. Missiles which travel through the air and then under the water usually have a configuration designed to prevent ricochets and also be aerodynamically and hydrodynamically clean. For short ranges, ASW missiles launched underwater may travel the entire distance submerged to the target. However, at longer ranges, it may be advantageous, from a propulsion standpoint, to design the missile to leave the water, fly a ballistic trajectory, and re-enter the water for the kill. The missile must then be designed for re-entry into the atmosphere as well as into the water. The airframe must be able to withstand the temperature variations as well as the tremendous stresses to which it will be subjected. An underwater-launched ballistic missile fired against surface targets involves design problems similar to the underwater-to-air-to-underwater missile mentioned previously.



RE-ENTRY BODY HEATING. The re-entry problem involves the protection of the warhead from very high deceleration and heating loads, and the minimization of re-entry dispersion. The re-entry body approaching the atmosphere possesses a large amount of potential energy because of its altitude, and a very high kinetic energy due to its supersonic speed. During re-entry, it loses most of this energy as it falls toward Earth, and decelerates to a fraction of its initial re-entry velocity. A supersonic-entry body traveling through the atmosphere carries with it a flow field of air contained within a shock envelope. The gas molecules in the field tend to fill up against the surface of the nose and create a subsonic flow region. The areas covered by this subsonic flow are referred to as stagnation points. Separating the supersonic and subsonic regions is a shock front. The shock process entails a reduction of the relative velocity of the gas molecules from supersonic to subsonic flow - which results in very high temperatures.

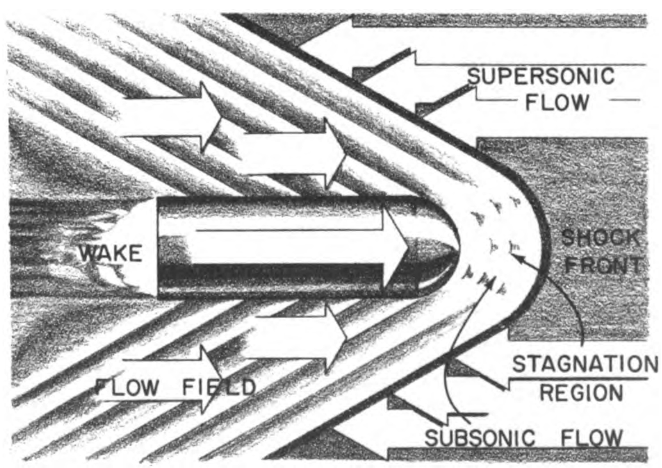
When the re-entry body has a pointed nose, the created shock wave causes a flow of air at extremely high temperature to pass over the body configuration, inducing excessive heat-transfer rates to its surface. The heating effect is most pronounced at the front of the re-entry body, and the very sharp point will tend to melt rapidly, or even vaporize; a blunt shape, however, will not. With a blunt-shaped nose, the shock wave becomes detached, although air of the same high temperature is still in contact with it. The blunt shape provides more effective heat absorption capacity per unit area of surface than the pointed shape, thus reducing the tendency to overheat. Blunt-shaped re-entry bodies manage to make the air absorb up to 99 percent of the heat generated by the dissipation of their kinetic energy. An expression for the time rate of heat transfer during re-entry, evaluated at the stagnation point, may be written as:

$$g_s = \frac{K\sqrt{\rho}\,V^3}{\sqrt{R_n}}$$

where g_s = time rate of heat transfer
K = constant of proportionality
ρ = density of undisturbed air ahead of shock wave
V = speed of re-entry body
R_n = radius of re-entry body at stagnation point.

Examination of this expression shows that to minimize g_s, it is necessary to reduce K, ρ or V, or to increase R_n. Increasing R_n means increasing the front area, making the shape of the nose more blunt.

It is also evident that re-entry bodies with high initial velocities have a greater kinetic energy (due to greater velocity) to dissipate. From the total heat standpoint, then, it is desirable to have the re-entry body commence re-entry at a velocity as low as possible and a steep angle of descent. This can be realized by selecting a ballistic missile trajectory as close as practical to a minimum energy potential, on the high or "lofted" side. Even with the use of very blunt-shaped re-entry bodies in long-range missiles, there remains a considerable amount of heat to be absorbed by the re-entry body itself. Therefore, it is important for the re-entry body to have some additional form of heat protection.



heat protection methods

Several methods of heat protection are available for absorbing or rejecting re-entry heating. Some of the most proficient devices are listed below.

1
SOLID HEAT SINKS

Employ the thermal capacity of a metal to absorb convective and radiative heat from the air around the missile.

2
ABLATION COOLING

Involves transferring heat energy away from the re-entry body by vaporizing the surface material of the re-entry body shell.

3
MECHANICAL COOLING

Uses circulating coolant fluid to absorb heat from the inner surface of the re-entry body.

4
TRANSPIRATION COOLING

Injecting a gas or liquid through the skin to carry away heat.

5
INSULATION

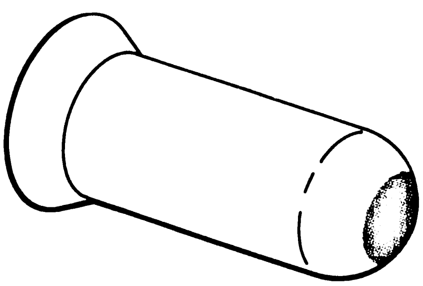
Used to reduce heating effects and to shield the structure and payload.

6
RADIATIVE SYSTEM

Dissipates convective heat input by radiation from the surface of the re-entry body.

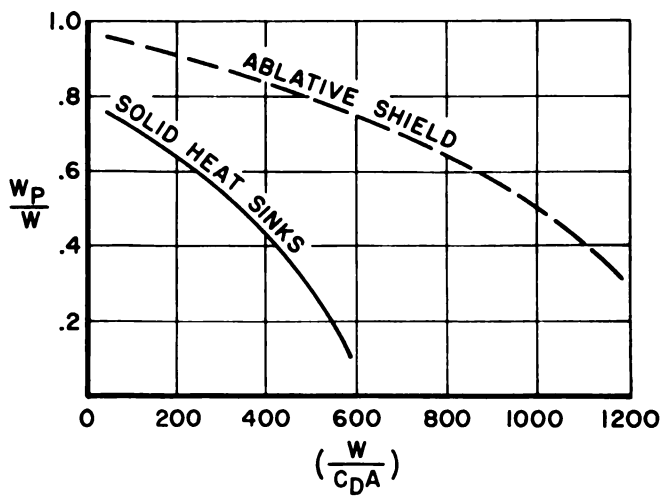
7
MAGNETIC FIELDS

Repel the hot, ionized gases which surround the re-entry body.



HEAT SINK
MOULDED ON REENTRY BODY

The requirement for simple reliable devices to control the re-entry body against excessive heat generally rules out mechanically complicated systems such as mechanical cooling, magnetic fields, and transpiration.
Radiative cooling is not practical due to excessive insulation requirements. The solid heat sink is "heat rate limited" by the heat capacity and the conductivity of the metal employed and its melting point. As the peak heat rate increases, the metal has more and more difficulty conducting the heat from the outer surface of the sink and distributing it throughout the metal sink. At a maximum heat potential, the heat sink melts because it cannot conduct the heat away fast enough. Heat sinks work well with intermediate range missiles, but as the missile range increases, the application of heat sinks becomes limited.
An ablative heat sink or shield not only absorbs heat due to its own solid state heat capacity, but may also melt and vaporize, and this change in shield material to the gaseous state is in effect a transfer of mass. The material vaporized leaves the re-entry body, bearing heat potential away with it. An added advantage of this mass-transfer heat-protection device is that the vaporized material escapes into the boundary layer and acts as an insulation shield for the re-entry body.
In contrast to solid heat sinks, most ablative shield materials have a low thermal conductivity, and are good insulators. Ceramics, refractory oxides, and plastics have been used successfully, and can handle large quantities of heat. Beryllia B_eO is a refractory oxide which will absorb 10,600 BTU/lb when it sublimes. As a result, only thin lightweight layers of beryllia are required on the frontal area to provide heat protection. This lightweight heat-protection system permits a much greater percentage of re-entry body weight to be devoted to payload, as well as protecting the payload from the effects of heating.



W_P = useful payload weight
W = total re-entry body weight
C_D = drag coefficient of re-entry body
A = effective frontal area of re-entry body

design criteria and considerations

OPERATIONAL REQUIREMENTS

A guided missile weapons system is designed to meet a specific operational requirement; a tactical military need derived from known and estimated offensive and defensive capacities of a target. The operational requirement is then translated into performance specifications which establish the necessary capabilities of the weapons system (range, altitude, speed, maneuverability, etc.) Target performance, location, size, number, and armor dictate the type of warhead to be utilized, and the desired kill probability affects the selection of a guidance control system to perform the required function. The type of launching vehicle to be employed will establish limits in the system, size and weight. Structural stresses incident to launching, and stabilization requirements will have to be taken into account. System complexity must be intelligently related to personnel capability in maintenance and testing.
The performance specifications establish the required capabilities of the guided missile system, which is composed of integrated smaller systems operating in unison to form a composite and complete missile system.Close coordination in research, testing, and layout procedures is essential to the development of the various components so that they are compatible and operate as a fully integrated system.

DESIGN FOR ENVIRONMENTAL CONDITIONS

The optimum criterion for environmental design is to meet conditions established by tactical usage. Environmental extremes to which a missile system may be exposed (temperature, pressure, vibration, shock, corrosion, spray, solar radiation, etc.) lead to intensive research and development to negate their effects. Often the shipping, handling, storage, and assembly requirements introduce environmental factors of great importance to the design engineer.

ENVIRONMENTAL TESTING

Environmental tests are designed to check the reliability of components and systems, and to determine their effectiveness in withstanding the extremes they will be subjected to during their life expectancy. Temperature tests require a simulation of the temperature and humidity with which missile structure and components will be confronted under all conditions. Vibration, shock, acceleration, and centrifuge tests similarly explore every facet of missile performance in laboratories and under actual flight conditions.

Comprehensive environmental testing of missile components, subassemblies, and assemblies is an important consideration in determining the reliability of a missile system. Included in any environmental test program is a measure of life expectancy of industrial or assembled parts. Life-appraisal tests apply particularly to components normally subjected to deterioration in their performance capabilities (batteries, rotating equipment, gears, relays, etc.). Generally it is cheaper to test individual components in destructive environmental tests than it is during missile performance runs.

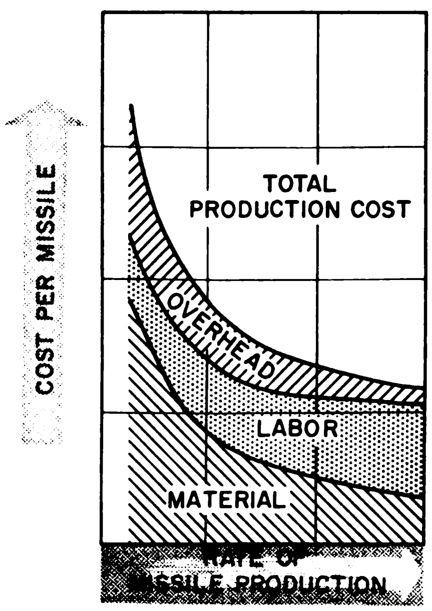
COMPONENT PACKAGING

Component packaging is important in obtaining optimum size and weight relationships for the missile. Provisions must be made for maintaining and servicing the missile and its components, with quick accessibility provided to components requiring frequent maintenance and service.

COST EFFECTIVENESS

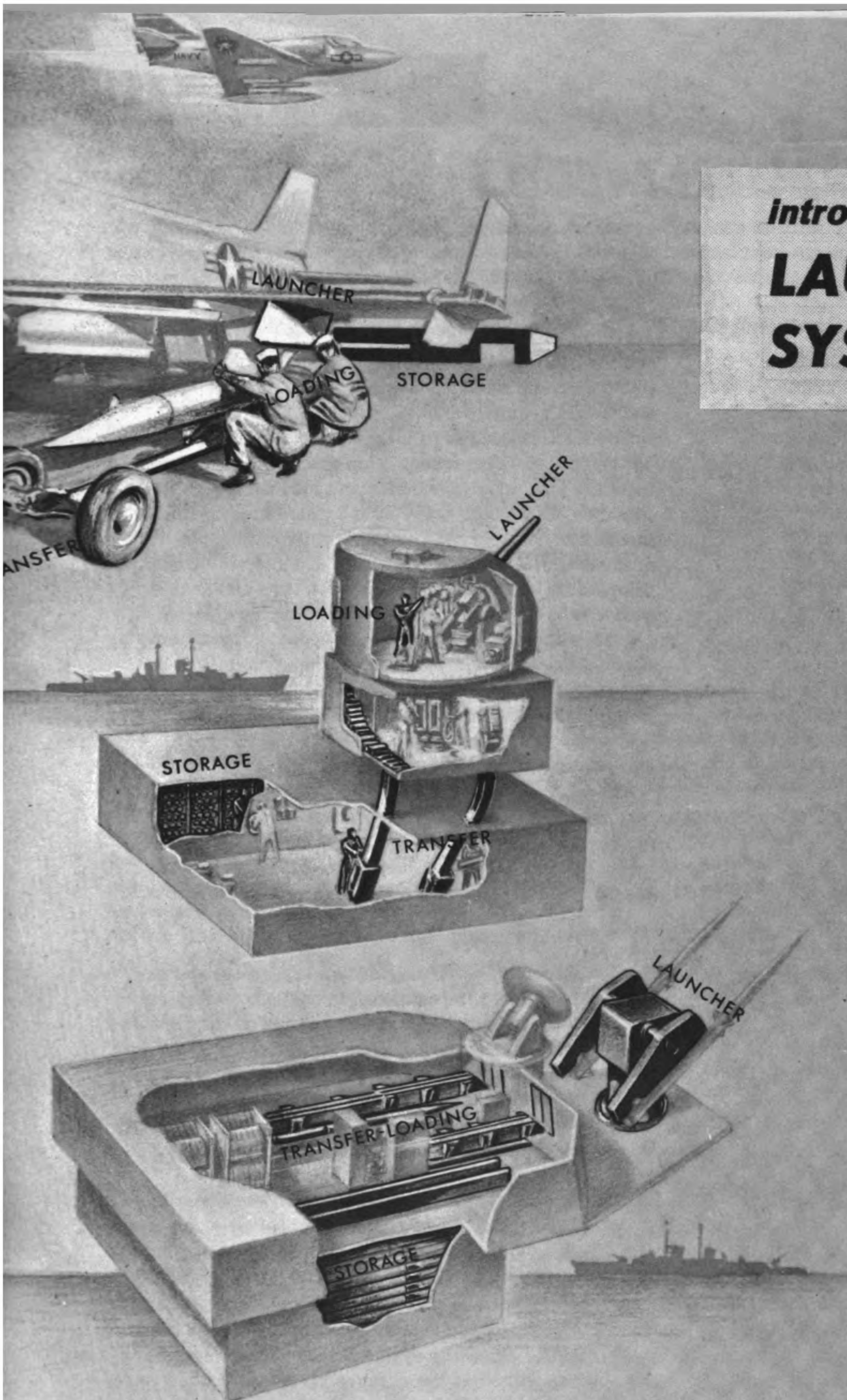
Another factor of major importance in determining the performance of an overall weapons system or the feasibility of a proposed system, is the cost effectiveness or cost-per-kill of the system. This factor can be applied to existing systems which accomplish the same specific mission, to determine which system operates more efficiently on a cost-per-kill basis. Applied to proposed systems,this factor,if accurately predicted,can be used as a primary deciding point in comparing different system approaches to the same objective. Costs to be considered in missile system design include costs incurred before deployment, such as research, development, production, etc. Improvements or modifications of subsystem components are frequently the source of an unanticipated developmental costs and time delays. Therefore,improvments and changes are often omitted, unless absolutely necessary, in the final stages of design and development.

general relationship between total cost per missile and rate of missile production

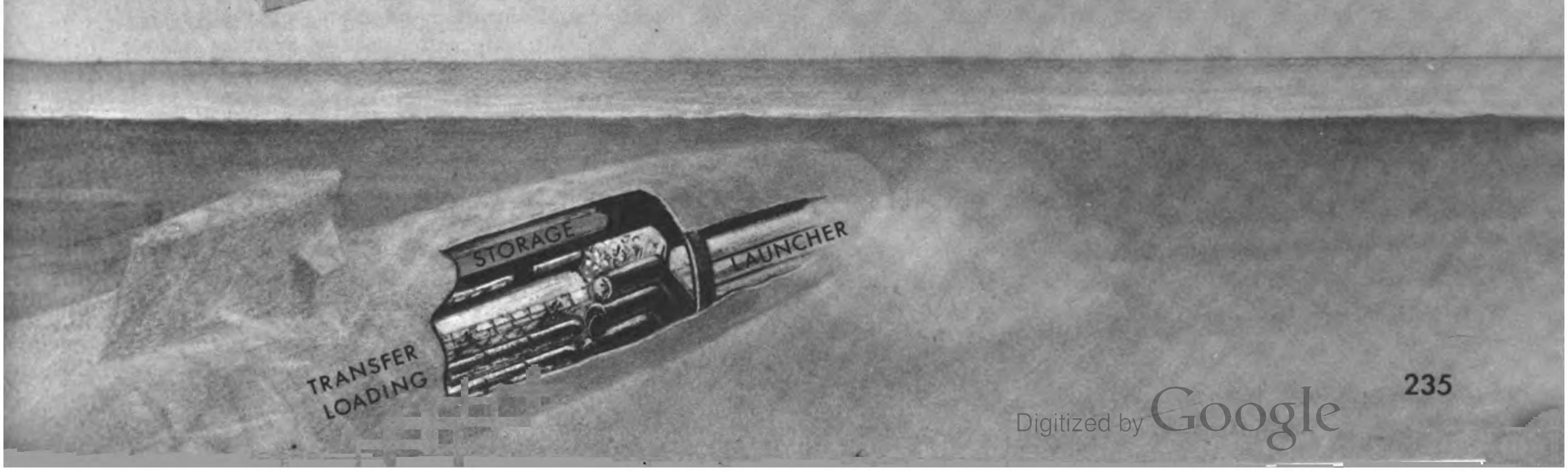


CHAPTER 6

introduction to LAUNCHING SYSTEMS



The purpose of a launching system is to place a missile into a desired flight path in a safe and efficient manner and as rapidly and frequently as the situation demands. The weapon system selected to meet the tactical military requirement will determine the launching technique that will be employed (gravity, impulse, or reaction). The state of readiness of a launch system is of vital importance, as it relates to the time of receipt of alert information to launch. It is highly important that launching occur at the optimum moment so that the system may function successfully in performing its specified task in a given operating time. Finally, a launching system must safely withstand the stresses the propulsive forces will subject it to. The reliability of the launch system is of extreme importance in achieving the high degree of kill probability that is a requirement of a weapon system.



LAUNCHING REQUIREMENTS

The most significant general requirements of launching systems can be summarized in the following words: SPEED, RELIABILITY, SAFETY, COMPATIBILITY.

SPEED

One of the necessary requirements of a launching system is that it be capable of rapid initial employment and a high rate of fire. These characteristics are essential because of the short period of time available for weapon employment against high-performance targets, the likelihood of simultaneous attacks by multiple targets, and the low probability of kill of many types of missiles. It is possible to achieve a satisfactory kill probability with only one launching tube when missile control techniques and warhead potentials are maximized. Often, however, this is not feasible. One alternative is to use several launcher tubes whose combined rates of fire raise the kill probability to an acceptable value. This technique is often used with low kill probability missiles - such as projectiles and rockets - and with large, bulky missiles that are inherently difficult and slow to load - such as torpedoes. The use of multiple launching tubes, however, generates large space and weight requirements.

Another alternative is to increase the rate of fire of individual launching tubes. This results in a requirement for fast missile transfer and loading.

In either case - with many simple, slow rate of fire launchers - delivery vehicle capacities limit the number of launchers that can be carried and operated efficiently. Thus, there will be a definite limit in the number of targets a vehicle can engage effectively during a given period of time. If enough targets appear simultaneously, all the launchers will be engaged, the system will be saturated, and some targets will get through. The problem of multiple targets is a basic consideration in the design of a weapon system.

RELIABILITY

Reliability in a launching system involves simplicity of operation and simplicity of design. It means that a system should be simple to operate and simple to maintain and repair. Therefore, it is simple to train men to operate, maintain, and repair the launching system. Simplicity in design infers that a system is inexpensive and easy to manufacture, thus readily available in large numbers at comparatively low cost.

SAFETY

Safety of vehicle and personnel is of vital importance in the operation of launching systems. Protection must be provided to the vehicle's structure and to operating personnel from the normal effects of launching. For example, protection from rocket motor blast and from the possibility of shooting into the structure of the launching vehicle must be provided. Prevention of injury to personnel by moving machinery, fires, and explosive is of great importance. In addition, the safety of the whole delivery vehicle may depend on the capability of the launching system to localize and contain explosions and fires. Lastly, safety involves the protection of the launching system and operating personnel from the effects of enemy attack. This is accomplished by the use of armor and compartmentation.

COMPATIBILITY

In addition to the requirements for speed, simplicity, and safety, there is another general launching system requirement - compatibility. A launching system must be compatible with the rest of the weapon system, and with the delivery vehicle in which it is installed. The launching system must be compatible with many characteristics of the weapon system. For example, the design of the launching system is dependent upon the type of missile propulsion employed, and the size, shape, and weight of the missile it must launch. In general, a launching system is designed to handle a specific type and size of missile, and will accommodate no others. A launching system is usually designed for general and even specific types of delivery vehicles. Therefore, the characteristics of the delivery vehicle have a strong influence on the launching system design. For instance, the size and weight of the launching system must be compatible with the limitations imposed by the space and weight available in the vehicle. Some other major considerations are the environment in which the vehicle operates (air, surface water, underwater) and the dynamic characteristics of the vehicle (speed, maneuverability).

For example, launching systems aboard ship must be designed to withstand the ravages of salt, air, and sea spray, the effects of vibration and shock, and the working of the ship's hull. Aircraft launching systems must be light and simple, and produce minimum effect on the airflow surrounding the aircraft.

Launching systems, in turn, affect the design of delivery vehicles. Small, simple launching systems are easily attached and removed and have little effect on delivery vehicle design. However, large, complex systems usually influence the basic design of the vehicle. That is, the delivery vehicle must be designed to carry a particular weapon system.

FUNCTIONS

The operation of a launching system can be divided into two distinct phases: prelaunch and launch. In the prelaunch phase, the launching system provides for missile storage, missile transfer, and missile loading. During this phase, the launcher also provides physical support for the missile. Orientation and control of the missile and communication between the missile and the control center are functions that are common and necessary to both phases. In the launch phase, which covers the period of time between actual firing of the missile and total separation of the missile from the launching system, the effectiveness of the propulsion, the probability of optimum flight path selection, and the degree of control and guidance of the missile are determined. In order to accomplish its purpose, the launching system must successfully perform all of these distinct functions. Often, many of the functions are combined into single actions performed by multipurpose devices.

LAUNCHING

Force must be provided to separate the missile from the launching device. The propelling force may be in the form of gravity, in the form of an impulse from the launching system, or in the form of reactive thrust from the propulsion system of the missile. Flight initiation occurs when the forces of separation actively operate on the missile and cause motion of the missile relative to the launching system. Prior to and during missile launching, the launching device provides physical support and orientation to the missile.

STORAGE

To put missiles into flight as rapidly and frequently as the situation demands, a safe and readily accessible stowage must be provided for missiles until they are needed. Storage capacity should be sufficient to provide for the sustained firing operations necessary to meet expected threats. For the missile to be put into flight as rapidly as possible, some missiles must be stored in a location and condition in which they are ready for immediate service.

TRANSFER

In a launching system, there must be physical transfer of missiles from storage to the vicinity of the loading device at a rate commensurate with the required rate of fire. In addition to transferring missiles from storage, a launching system must provide for the transfer of missiles to storage, such as the return of unused missiles to storage and the striking down of newly received missiles.

LOADING

Before a missile can be put into flight, it must be placed or loaded onto the launching device in a ready-to-fire position. The loading operation must be accomplished quickly and continuously. In general, it is desirable to have the rate of loading as high as possible. However, the loading rate is inversely proportional to the size and weight of the missile. The missile loading function also involves the unloading of unfired missiles in good condition and the jettisoning of unusable missiles.

CONTROL

To put a missile into a desired flight path, the launching system must provide the necessary orientation in space so that the missile will, when launched, follow the desired flight path to the target. Thus, the launching system must have some means to control the missile orientation. In general, this may be accomplished by proper orientation of the delivery vehicle velocity vector or by proper directional adjustment of the launching device mounted on the delivery vehicle.

COMMUNICATIONS

Before a missile is launched, it is often necessary to have communication with it, i. e., missile flight instructions, launching orders, and warhead firing instructions. Thus, the launching system must provide a communications path to the missile, from internal and external stations, at some time while the missile is in the system. In addition, many missiles require that certain other operations, such as missile assembly and checkout, be performed somewhere in the launching system before the missile is launched. Missile communication may also be effected throughout the entire launching cycle (prelaunch and launch) to ascertain correctness of operation.

PRINCIPLES OF OPERATION

LAUNCHER SYSTEMS

general

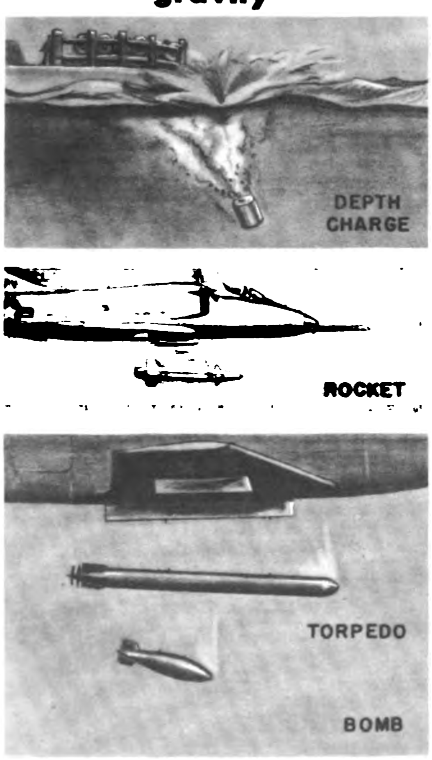
The launcher is the elemental unit of the launching system. It provides support, both static and dynamic, for the missile prior to and during launch. Initial flight orientation is provided to the missile by aiming the axis of the launcher in the necessary direction in space to achieve the desired flight path, and constraining the missile to the line of the launcher axis. The launcher facilitates flight initiation by providing a transmission path to the missile for the firing signal which may be initiated at the launcher or at a remote station. Launchers may be rigidly attached to the delivery vehicle or they may rotate with respect to the vehicle about one or two axes. In the former case, the delivery vehicle must be properly oriented in space before a missile can be launched into a desired flight path. Fixed launchers of this type are usually simple, light in weight, and reliable, but limit the tactical flexibility of the delivery vehicle, because it is necessary to orient the vehicle along the desired flight line of the missile before and during the launching operation. Rotating launchers are heavier and more complex. They are used when the delivery vehicle is too large, too slow, or is tactically restricted and cannot supply proper launcher orientation. Use of several rotating launchers on a vehicle permits greater flexibility of employment and enables a vehicle to engage several targets at once.

A launcher usually has the form of a rail or tube in order to provide support and orientation to a missile prior to and during the launching phase. A launcher must be strong enough to withstand forces attendant to launching and to support a missile under the static and dynamic loads associated with the vehicle and the medium in which it travels. The launcher rail or tube often acts as a missile stowage area. For example, torpedoes, depth charges, and POLARIS missiles are normally stored on or in the launcher until used.

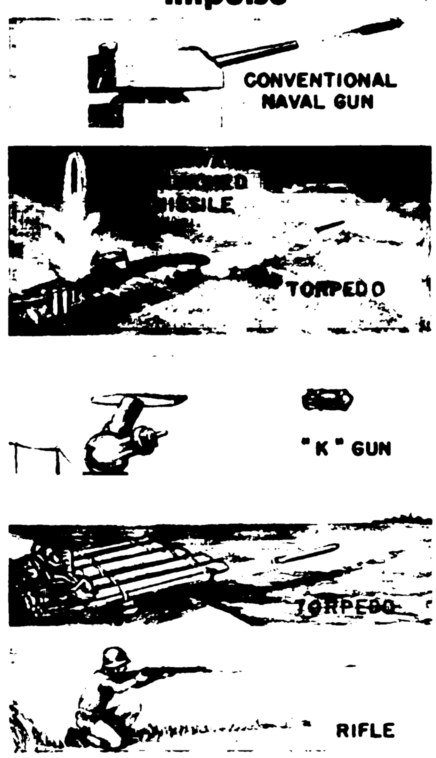
launcher types

Launchers may be categorized by the source of the force used to effect separation of the missile from the launcher. The three common designations are:

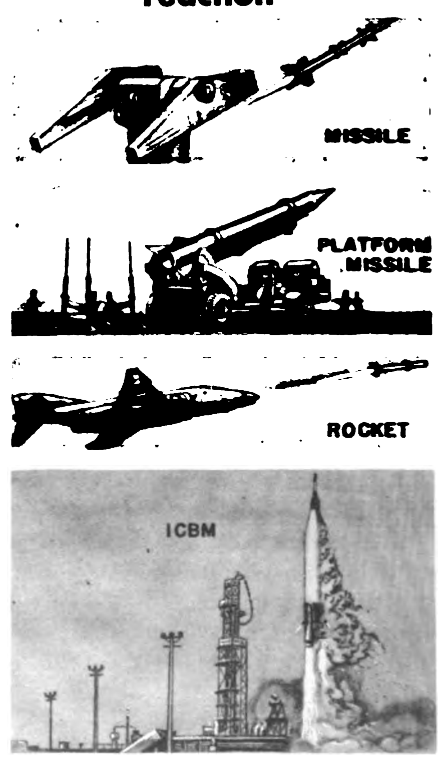
gravity



impulse

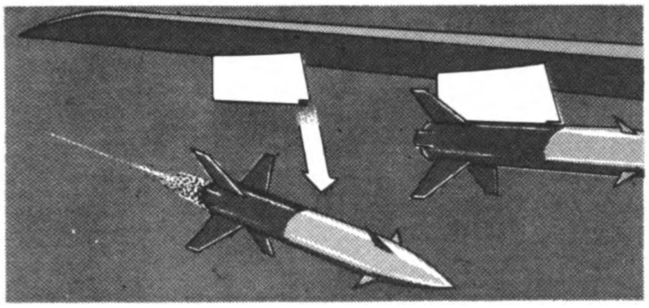


reaction



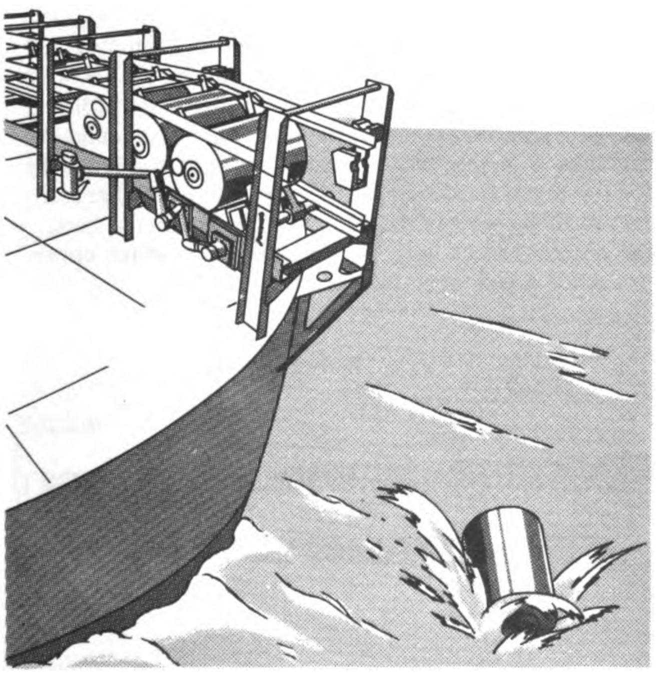
GRAVITY-TYPE LAUNCHERS

Gravity launchers are noteworthy for their extreme simplicity because they rely upon gravity to cause separation of the missile from the launcher.

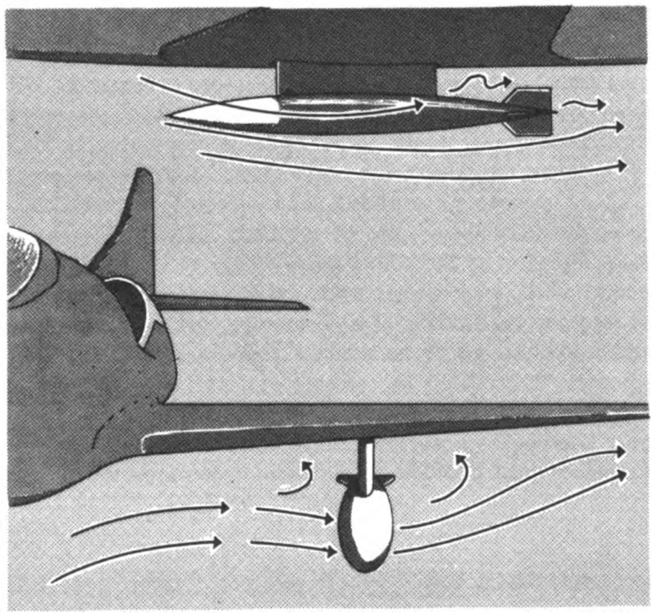


Since no additional forces are applied to the missile at launch, the launcher does not sustain any sudden shocks. The launcher provides positive support during delivery vehicle approach and releases the missile at the required instant. Therefore, a simple, lightweight structure is all that is needed. Depth charges and bombs are normally launched by gravity launchers. Other missiles, such as torpedoes and rockets, can also be launched from gravity launchers.

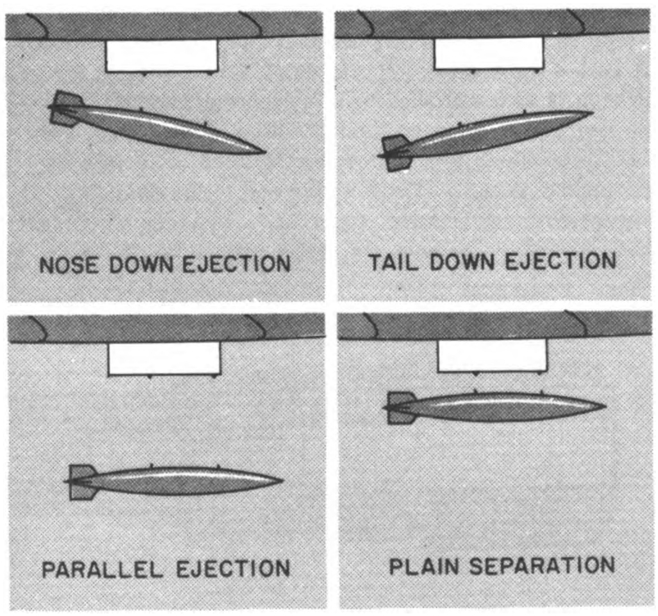
Since the initial velocity and orientation of the missile is supplied solely by the delivery vehicle, a high-speed, maneuverable vehicle, such as an aircraft, PT boat, or destroyer is most suited for this type of launcher. Aircraft, because of their high speed and maneuverability, are ideally suited to use gravity-launched missiles and to benefit from the inherent simplicity and light weight of these launchers.



Unfortunately, the high speeds of modern high-performance aircraft create an airflow field which can cause undesired motions of the missile. When conventional bomb bays are used in high-speed aircraft, the airflow field is sufficient to cause the bombs to rattle about in the bomb bay after being released. Similarly, if the missiles are dropped from launchers mounted on the wings or fuselage, sufficient motion can be generated to cause the missiles to crash into the aircraft. Even when no danger of collision exists, it is often difficult to predict the trajectory of the missile. The danger of collision and trajectory deviation can be minimized by careful design and proper weapon employment.



One solution is to forcibly eject the missile from the launcher to prevent danger to the transmission vehicle.



IMPULSE LAUNCHERS

Impulse launchers are characterized by the use of an impulse which is provided by the launcher to project the missile into flight.

The launching impulse may be of sufficient magnitude to project the missile along the entire length of the desired flight path to the target, as in the case of guns launching projectiles, or it may be of small magnitude, sufficient only to cause the missile to clear the vehicle safely, as in the impulse launching of torpedoes and rockets. Thus, there is impulse for both missile propulsion and missile ejection.

gun-type launchers

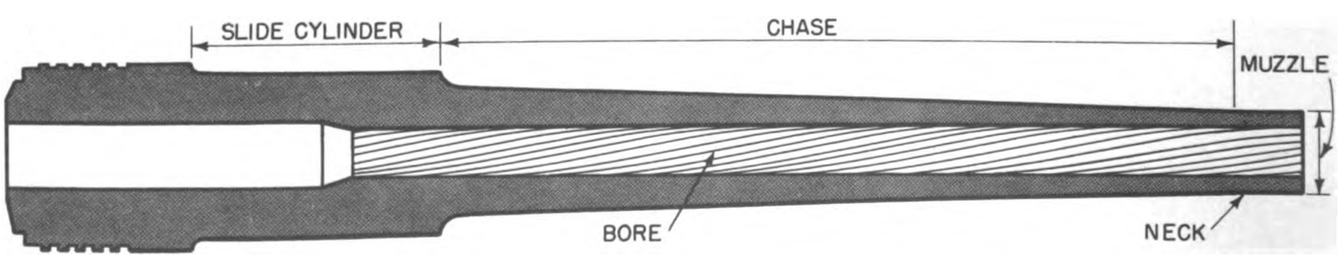
When impulse provides the total propulsion of a missile, as in guns, a large amount of momentum is imparted to the launcher. The launcher will transmit this momentum directly to the delivery vehicle unless something is done to absorb or dissipate the momentum. Naval guns use devices known as recoil and counterrecoil mechanisms to control and absorb the momentum. Often, the momentum imparted by launching a projectile is employed to do useful work, such as extraction and ejection of cartridge cases, and the loading of a succeeding round. Launchers which provide all of the missile propulsion also control missile flight. That is, the guided phase of missile flight occurs on the launcher. Since this is usually the only control or guidance the missile receives, the launcher must provide accurate missile orientation. Therefore, a long, close-fitting tube, or barrel, is needed to provide accurate directional control to the missile while it is in motion on the launcher.

This type of launcher is exemplified by naval guns and is the most familiar of all launching systems used by the Navy.

Naval guns are often referred to by the type of ammunition they fire. Case guns are those which fire metal shell encased ammunition, while bag guns are those which fire ammunition in which the propellant powder charge is packed separately. If the total propellant powder required to attain initial projectile velocity for a large gun were placed in a single rigid container, its size and weight would make loading exceedingly difficult and slow. By packing the powder grains in bags, it is possible to divide the total charge into units that can be handled expeditiously by one man.

GUN BARRELS

The launcher consists of a gun tube, or barrel, supported by a structure attached to the ship. A barrel is a simple tube which is closed at one end. The barrel provides support and orientation for the projectile during its launching phase, and by confining the propellant gases, provides for proper propulsion. A typical gun barrel assembly is shown below. The end of the gun which can be opened (for loading) and closed (for firing) is called the breech. This opening and sealing of the breech is accomplished by a breech mechanism, which is basically a movable block or plug. Just forward of the breechblock is an enlarged chamber to contain the propelling charge. The forward end of the chamber tapers down to the bore, which has a constant diameter to the front, or muzzle end, of the gun. In all but a few special-purpose guns, the bore is rifled - that is, a set of spiral grooves is engraved into its surface. In larger guns, which tend to wear more rapidly per round fired than smaller weapons, the rifling is cut in a liner which can be replaced when worn. The cylindrical after part of the barrel is called the slide cylinder. It moves in bearings in a structure called the slide during recoil. Forward of the slide cylinder the barrel tapers (this tapering part is the chase) then (in many gun designs) thickens to form a bell which discourages any tendency for the metal to split. The narrow part of the barrel just abaft the bell is the neck. In newer gun designs, the muzzle has no bell, but may have lugs which serve to anchor a jack used in replacing the liner.



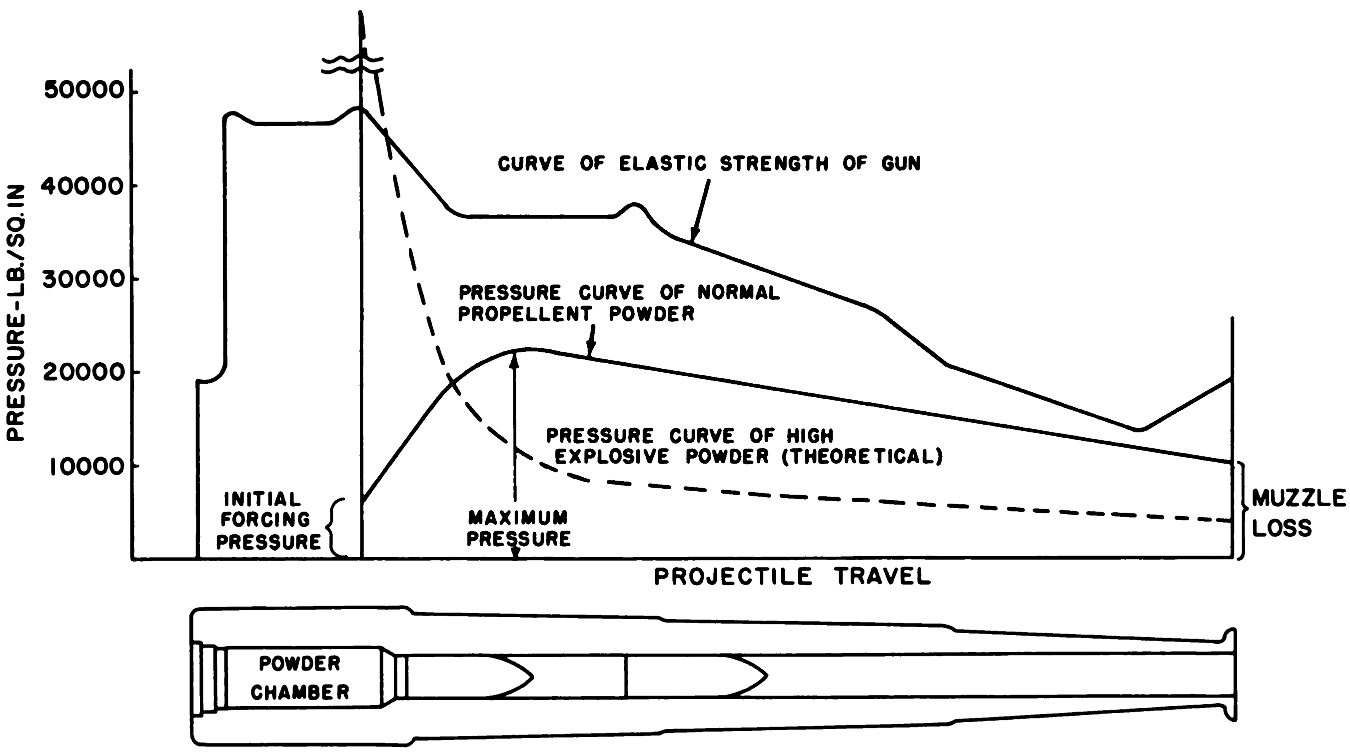
Gun barrels must be constructed with sufficient strength to withstand the tremendous internal pressures developed by the propellant gases. The barrel is thickest at the chamber because this is where the greatest pressure occurs. As illustrated, the barrel thickness and strength diminishes toward the muzzle in general conformance with the pressure travel curve. All modern gun barrels are made of steel, and they are generally prestressed to make them more resistant to internal (bursting) pressures. The object of prestressing is to make the outer layers of metal in the barrel bear a greater proportion of the bursting load.

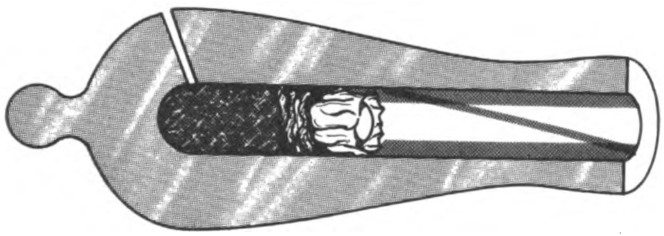
The built-up method of prestressing is to heat steel ring-shaped jackets, or hoops, to high temperatures, then slip them over the gun tube and allow them to cool. As the hoops cool, they contract, until at the end of the process they squeeze the tube with a pressure of thousands of pounds per square inch.

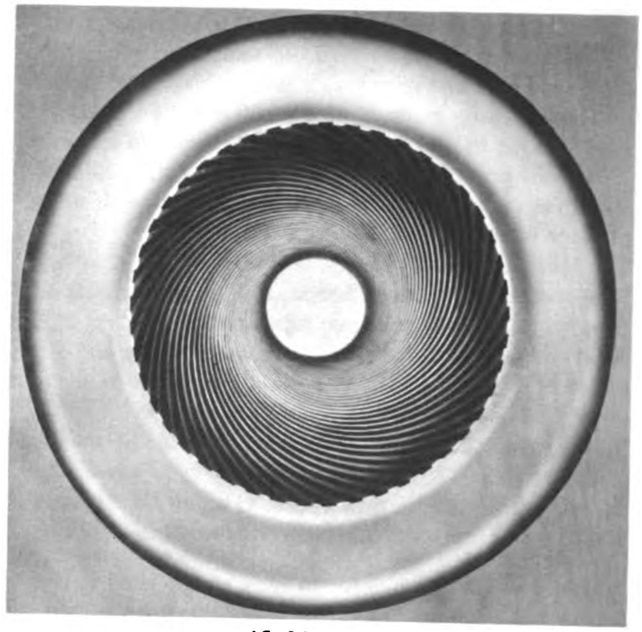
Most modern gun barrels are of one-piece or monobloc construction. They are prestressed by a process of radial expansion. In this process, a gun tube with bore slightly smaller than the caliber desired is expanded by hydraulic pressure. When the pressure is released, the outer layers of the tube tend to return to their original dimensions, while the enlarged inner layers tend to maintain their enlargement. Thus, the inner layers of metal are severely compressed by the contraction of the outer layers, as if a hoop had been shrunk on. The built-up and radially expanded methods may be incorporated in a single gun. The 8"/55 caliber gun, for example, has a jacket shrunk on a radially expanded tube.

Smaller guns are made from a single steel forging with neither radial expansion nor hoops. The pressures in small guns may be higher than in large guns, but the forging, which is not excessively large in any event, can be made bigger. This type of construction is limited presently to guns of 3-inch caliber and smaller.

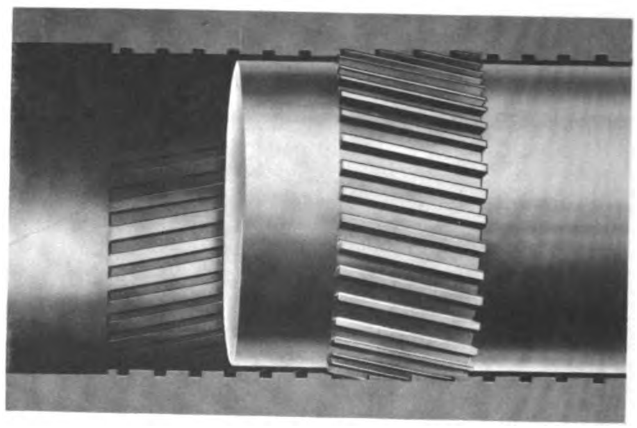
A basic problem of early gunnery was the inaccuracy of the fire and the unstable and erratic behavior of the projectile in flight. Aerodynamically, a projectile or missile can be spin stabilized by being made to spin on its long axis. This problem is solved by modern naval guns which are constructed with spiral grooves cut into the gun tube liner.



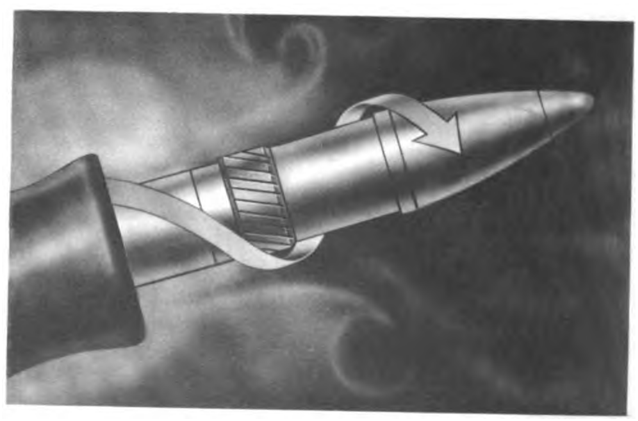




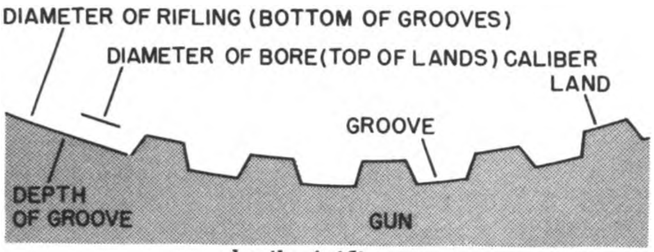
rifled bore



rifling engraves the rotating band



projectile spin in flight.



details of rifling

The lower surfaces of the rifled barrel are called grooves. The raised surfaces between the grooves are known as lands. The caliber of a rifled gun is measured from the top of one land to the land on the opposite side of the bore. Since the projectile's rotating band is slightly larger than the nominal gun bore diameter, the rifling cuts into or engraves the softer metal of the rotating band. As the projectile moves down the bore, the rifling causes the projectile to spin at an increasing rate as the propellant gas accelerates it.

In all naval guns and small arms, except the .45 caliber pistol, the rifling has a right-hand twist. With a right-hand twist, the spin of the projectile is clockwise when viewed from the breech. The twist may be uniform (generally one turn in 15 to 20 calibers) or increasing, where the twist becomes sharper as it nears the muzzle. The rotational speed imparted to the projectile depends upon the magnitude of twist and muzzle velocity. It varies considerably among guns. For example, a 16"/50 projectile will leave the muzzle at about 4000 RPM, while a 40-mm projectile may spin at about 40,000 RPM.

Naval gun launching systems are usually designated in accordance with certain characteristics of the gun barrel or bore. The caliber of a naval gun is the diameter of its bore, usually measured in inches or millimeters. Gun launchers which have a caliber of 3 inches or larger are usually described by stating the caliber and length of the bore (plus the powder chamber) in calibers. For example, a 3"/50 caliber gun barrel has a caliber of 3 inches and is 50 calibers, or 150 inches, long.

A complete designation of a gun launching system is usually made in one of two ways. The larger caliber guns are designated by the caliber in inches, followed by the length of the gun in calibers and by Mark and Mod numbers. A Mark designation denotes a unique equipment design. A Modification number indicates a minor change to the basic design. For instance, a 5"/54 Gun Mk 42 Mod 8. Smaller caliber guns are designated by the diameter of the bore in millimeters, followed by the Mark and Mod numbers. For example, a 40-millimeter Gun Mk 3 Mod 4.

In addition to the specific designations just noted, gun systems are classified into general groupings according to bore diameter:

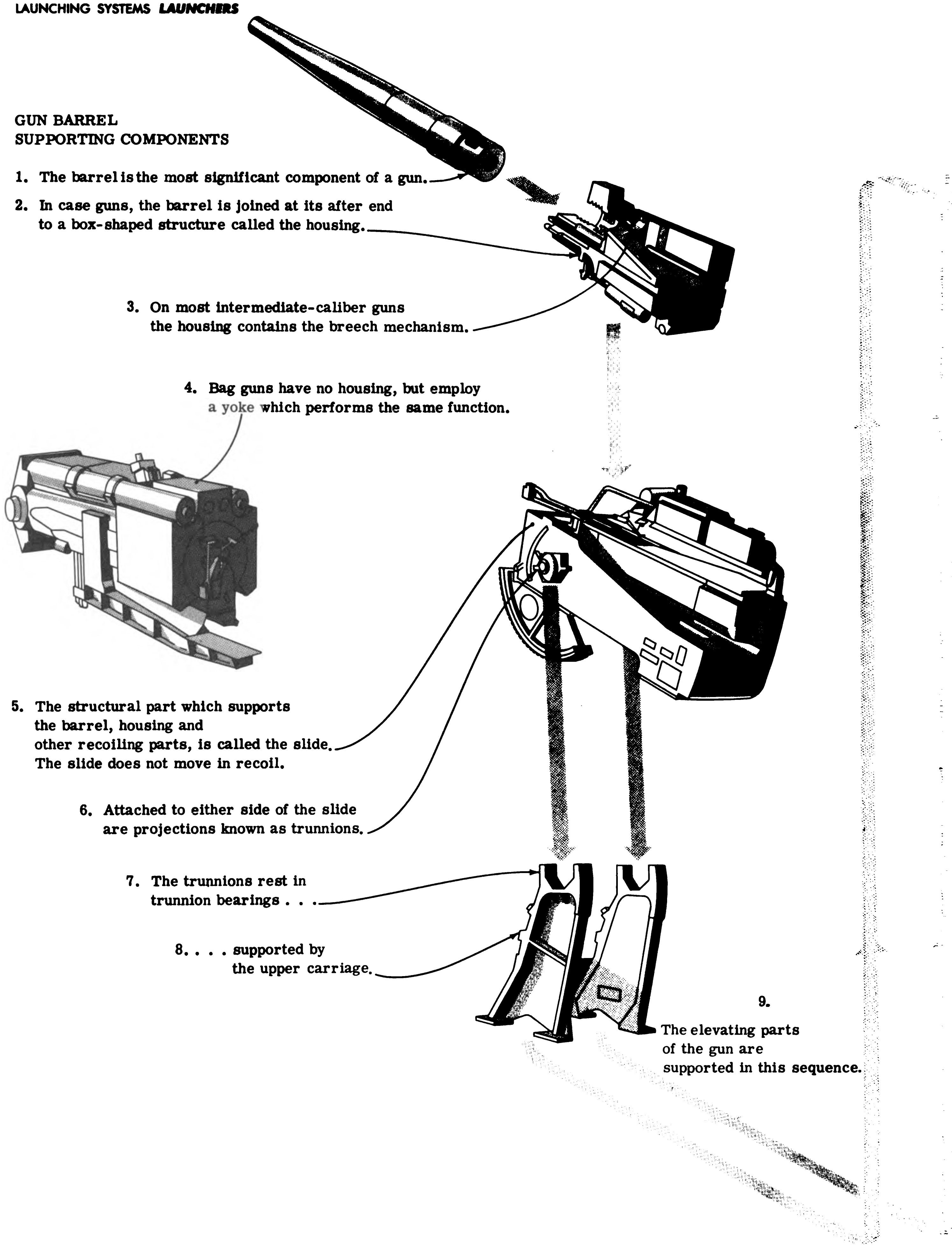
Major caliber -	8 inches or larger
Intermediate caliber -	greater than 4 and less then 8 inches
Minor caliber -	greater than 0.60 inch but not more than 4 inches
Small arms -	0.60 inch or smaller

All major caliber and some intermediate caliber guns are supported in heavy, armored, rotatable structures called turrets. A turret is an elaborate gun mount. Guns of smaller caliber are supported in various types of mounts. Some of these are enclosed and outwardly look like turrets. However, the enclosure, or shield, is light in weight and offers only weather (and in some cases, splinter) protection for the gun and crew. Other mounts are open and provide no shelter. Mounts may have more than one gun. Those with two guns are twin mounts and those with four are quad mounts.

The types of ships on which the various naval gun systems are installed are listed below. The gun systems marked by an asterisk are currently found only on ships in the reserve fleet.

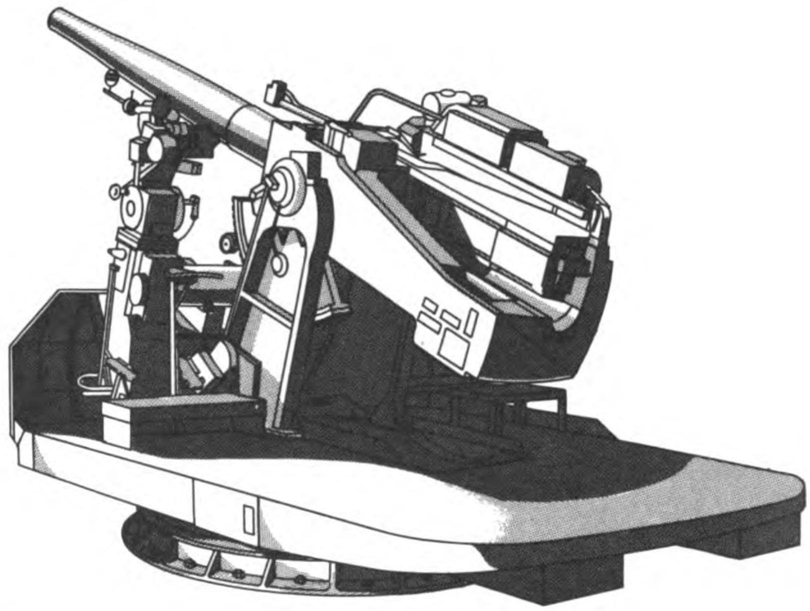
GUN	CARRIED ON
16"/50 cal*	Battleships
16"/45 cal*	Battleships
12"/50 cal*	Large cruisers
8"/55 cal	Heavy cruisers and guided missile cruisers
6"/47 cal	Light cruisers
5"/54 cal	Large carriers, destroyers, and frigates
5"/38 cal	Battleships, cruisers, destroyers, carriers, and auxiliaries
5"/25 cal*	Submarines
3"/70 cal	Frigates and tactical command ships
3"/50 cal 40-mm 20-mm	Any ship from patrol craft to battleship

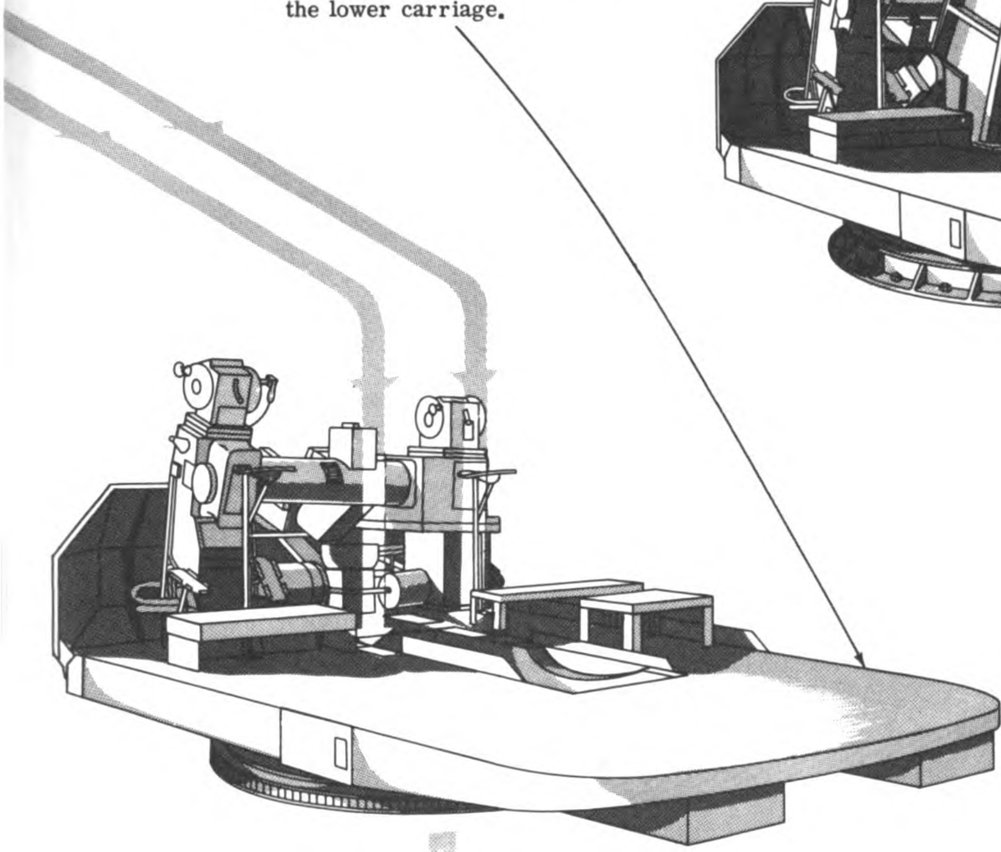
GUN BARREL SUPPORTING COMPONENTS



1. The barrel is the most significant component of a gun.
2. In case guns, the barrel is joined at its after end to a box-shaped structure called the housing.
3. On most intermediate-caliber guns the housing contains the breech mechanism.
4. Bag guns have no housing, but employ a yoke which performs the same function.
5. The structural part which supports the barrel, housing and other recoiling parts, is called the slide. The slide does not move in recoil.
6. Attached to either side of the slide are projections known as trunnions.
7. The trunnions rest in trunnion bearings . . .
8. . . . supported by the upper carriage.
9. The elevating parts of the gun are supported in this sequence.

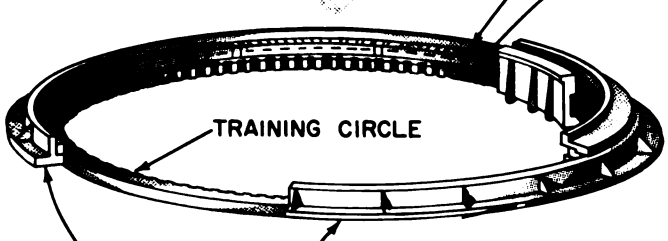
10. The upper carriage is supported by
a platform known as
the lower carriage.

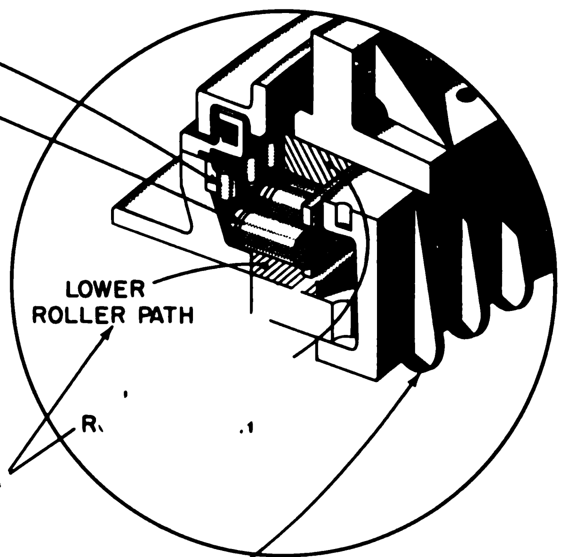




Note
these bearings take up
the horizontal and vertical forces
exerted between ship and mount

11. The lower carriage
rests upon
vertical
and
horizontal
roller
bearings . . .

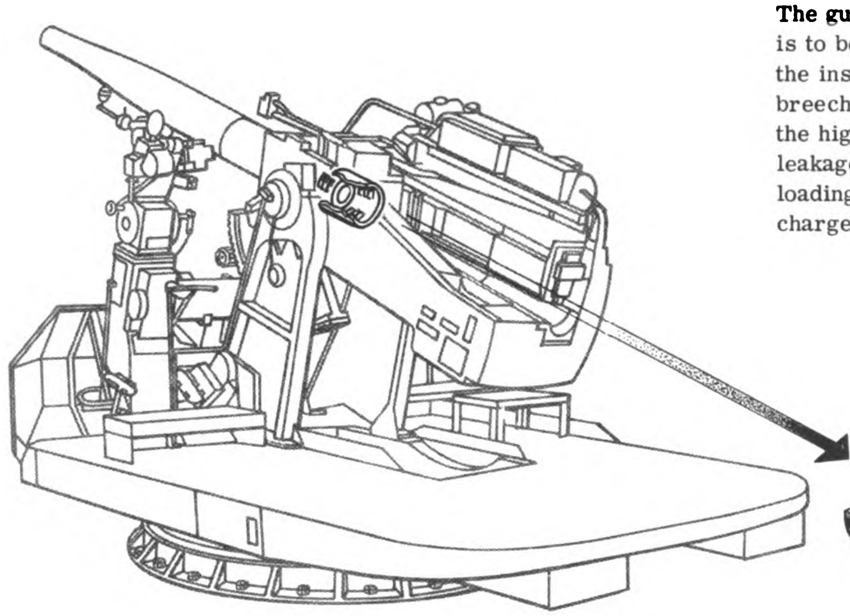




12. . . . which in turn
rest upon the roller path
of the stand.

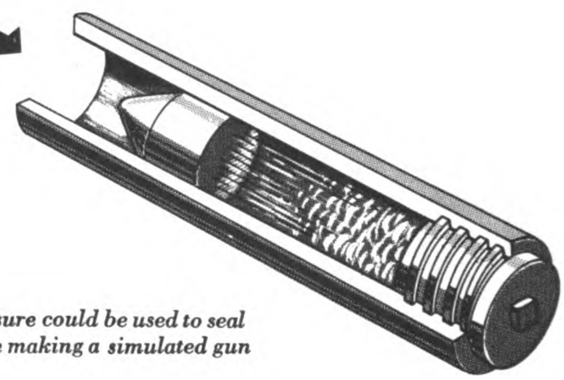
13. The stand
is stationary
and is bolted
to the ship structure.
It supports the combined
carrier, slide and gun.

14. Holding down clips
hold the carriage and stand together
and prevent the carriage
from jumping off the stand during the firing
of the gun or when the ship pitches and rolls.



BREECH MECHANISMS

The gun barrel is a tube open at both ends. If the tube is to be used as a gun, the after end must be closed at the instant of firing. The device which does this is the breech mechanism. It must be capable of withstanding the high pressures developed in the chamber without leakage of gas. It must also provide a means for rapid loading of the gun and a dependable means for firing the charge. Above all, it must be safe in operation.

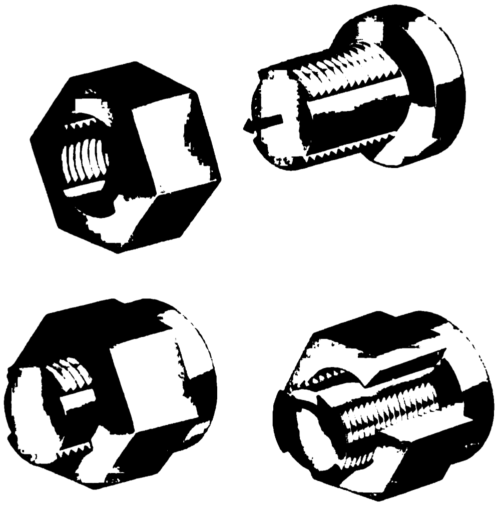


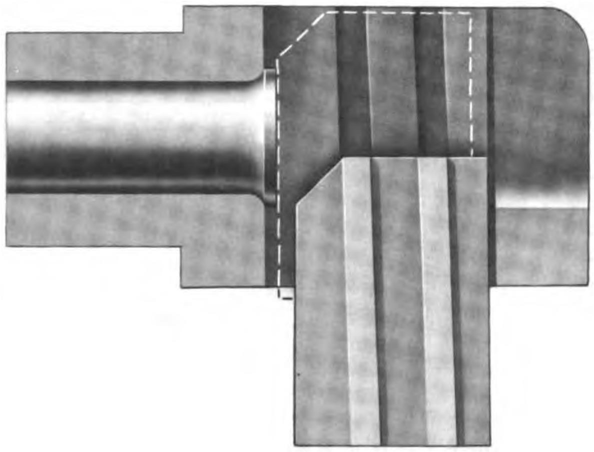
a continuous screw closure could be used to seal the breech end of a tube making a simulated gun

The gun barrel and its supporting components have been discussed in the preceding pages and are shown assembled above. Up to this point, the modern naval gun is not essentially different from the early muzzle-loading cannon. Modern gun fire imposes two additional requirements, however. The long length, the need for rapid firing, and the nature of ammunition used in modern guns rules out muzzle loading. A mechanism to seal the barrel while permitting loading from the rear is necessary. The large amount of energy released in a modern gun makes a system to absorb the energy imparted to the gun necessary. These two additional components: breech mechanisms and recoil and counterrecoil systems are discussed in the following pages.

Two types are used in naval guns larger than 20-mm: the interrupted screw and the sliding wedge. The former is characterized by a cylindrically shaped, partially threaded plug, which screws into the breech of the gun and plugs it. The distinctive part of the latter is a grooved block that slides across the face of and blocks the breech of the gun. In either case, the breech mechanism comprises the system of operating parts which enable the closing device to fulfill its functions of securing the after end of the gun tube and firing the propelling charge.

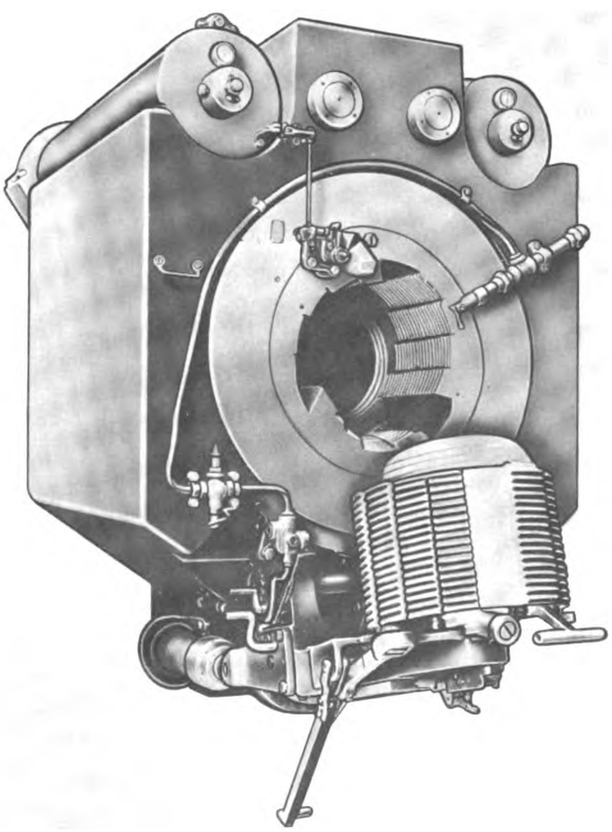
The screw is a widely used device for securing against a heavy thrust. A continuous screw closure is an effective device for sealing the breech end of a gun barrel. Such a breech closure or plug would, of course, require unscrewing to open the breech after every firing. The mass of such a device, designed to withstand the 40,000 psi gas pressure developed in a typical large-caliber gun, would inevitably be considerable. Turning such a screw through several revolutions would be difficult and time consuming. Application of the principle of the interrupted screw reduces the number of turns required to a fraction of one revolution.



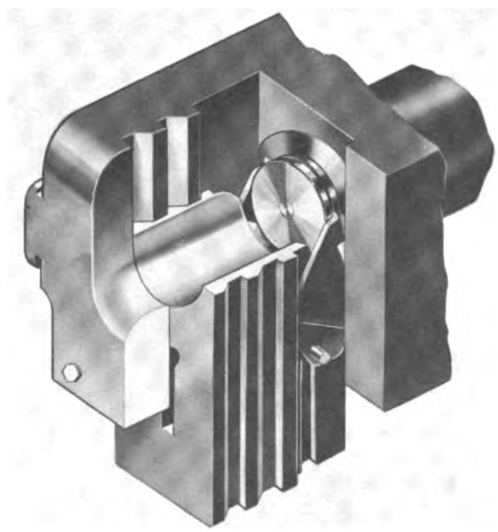


sliding wedge breechblock

The principle is shown in the illustrations. If half the threaded area is removed from a bolt (representing the breech plug) and the nut (representing the breech of the gun tube), then it is possible to insert the bolt and engage the two by turning the bolt 90 degrees. The actual breech plugs in use today are modifications of the simple interrupted screw just described. They provide even greater holding power and require less than 30 degrees of rotation for full engagement. Today, only bag-type guns use the interrupted screw breech mechanism.

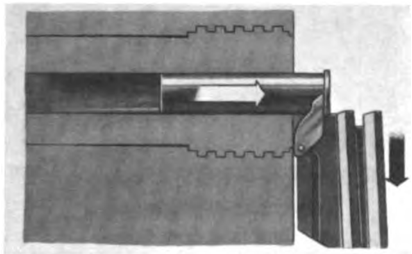


The sliding wedge breechblock is used on rapid-fire, automatic and semiautomatic case guns larger than 20-mm. It consists of a sliding block that automatically seals the breech and seats the cartridge in the chamber. The block rides vertically on grooves cut in the housing. The grooves enable the block to withstand the pressure developed in the chamber. The grooves in which the block moves slant forward slightly so that the sloping base of the block wedges the case into the chamber as it rises. This is sometimes referred to as a vertical sliding wedge to distinguish it from the horizontal type used in some guns.

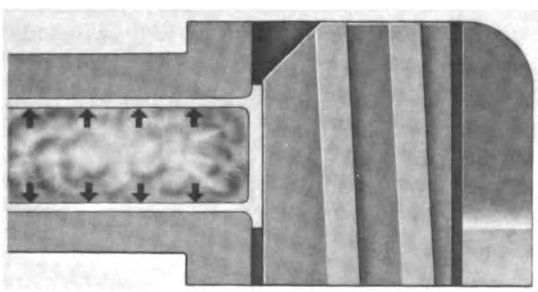


vertical sliding wedge breechblock

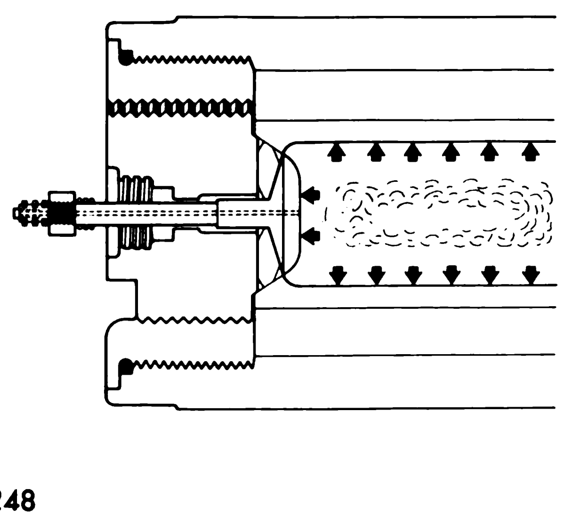
Extractors, which contact the rim of the spent cartridge case and pull it out of the chamber, are activated when the sliding wedge falls during the unloading portion of the operating cycle.



No breechblock or plug can be so carefully fitted to the breech that it will completely prevent the escape of gas from the chamber to the rear. Gas escaping in this manner is known as blowback and, if abnormal, may cause death or serious injury to gun crews. It also erodes the accurately machined surfaces of the breechblock, rendering them less gastight. A breech mechanism must therefore be supplemented by a gas check. Two kinds of automatic checks are in use, both of which employ the pressure of explosion to seal the breech: sealing by expansion and sealing by compression.



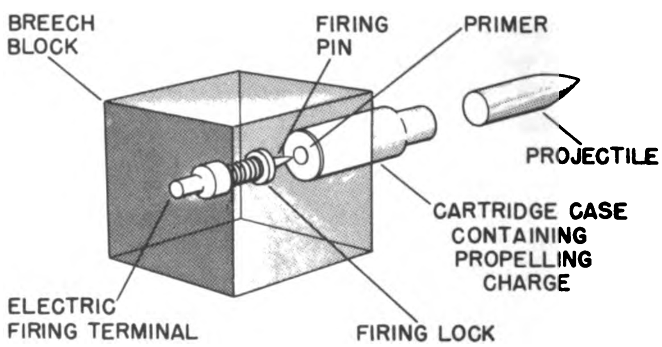
The cartridge case provides the gas check in case guns. The pressure develops within the case and expands it outward against the surface of the chamber. Theoretically, the surface of the case and chamber must be perfectly smooth to provide a tight seal. Actually, there are some irregularities, but the total area of surface in contact is great enough to make the seal effective. In bag guns, a separate gas check is provided to form the gas seal; there is no metal shell case to perform that function.



As illustrated, a gas-check pad is firmly pressed against the gas-check seat as the breech is closed. When the gun is fired, the pressure of explosion pushes against a mushroom-shaped valve, forcing it slightly to the rear. The curvature on the after side of the mushroom valve compresses the pad, causing it to spread radially against the gas-check seat. The greater the chamber pressure, the harder the valve pushes against the pad. The seal is automatic in the sense that it adjusts itself to the pressure within the gun. The gas-check seat and the surfaces between the plug and valve are sealed by the pad. The primer vent is sealed by the expansion of the primer case against the primer chamber walls in the outer end of the valve stem. To prevent accidental opening of the breech in the event of a misfire, a salvo latch is attached. It is a device that locks the breech so that it can be opened only by a deliberate effort. The salvo latch is a positive lock which in present designs is cammed to open automatically during recoil of the gun. It will not open automatically if the gun does not recoil.

FIRING MECHANISMS

The firing mechanism is the device which actually explodes the primer and initiates the firing of the gun. Guns are fired by percussion or by electricity. Some guns have provision for either firing mode.



typical firing mechanism

In the screw-type breech plug mechanism, the firing mechanism is fastened to the rear end of the mushroom valve stem and is called a firing lock. In the sliding wedge type of case gun, it is called simply the firing mechanism and is located on the inner side of the breechblock, where it is accessible for removal and servicing.

In combination and electric firing systems, the firing pin is electrically insulated and is part of the electric firing circuit. The pin makes contact with the primer of the loaded propelling charge case when the gun is loaded and the breech is fully closed. The pin is withdrawn into the breechblock when the breech is not fully closed. In combination and percussion firing systems, the firing pin strikes the primer when actuated by a mechanical firing linkage, provided the gun is in battery and the breech is fully closed.

In electrical firing systems, the firing mechanism proper consists of electrical elements only - an insulated firing pin and a quick-disconnect terminal to which a firing lead is attached.

RECOIL AND
COUNTERRECOIL SYSTEMS

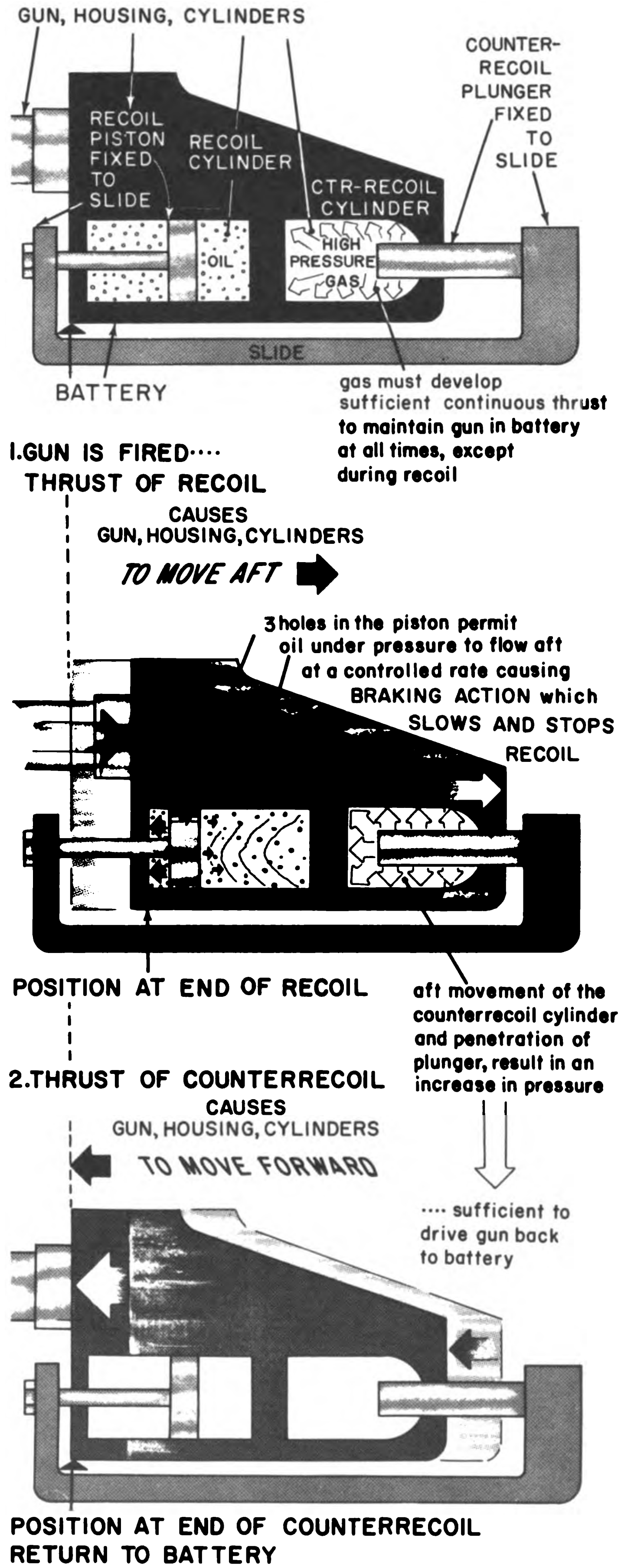
One of the basic functions which an impulse launcher must perform is to control the momentum imparted to it as a result of launching a missile.

The movement of the gun to the rear as a result of the work done upon the gun by the powder gases is known as recoil, and the length of this movement is known as the length of recoil.

All modern naval guns recoil in their mounts when fired. The recoil movement is introduced to control the momentum imparted to the gun. In the case of a gun having no recoil, the force acting on the mount because of the firing of the gun is the product of the area of the bore and the effective powder chamber pressure. This product amounts to several million pounds in the case of major caliber guns and is of considerable magnitude for all calibers of modern naval guns. From this it is obvious that, for a gun having no recoil, the proportions of the mount would be unreasonably large, making it cumbersome to handle and, on account of weight, unsuitable for use aboard ship. By permitting the gun to recoil a limited distance, the forces which would otherwise act on the ship are greatly reduced and can be regulated to suit the character of the vessel for which the mount is designed, thus making possible the use of larger caliber guns aboard ship.

The return movement of gun to battery after firing is known as the counterrecoil, and is equal to the recoil. Recoil generally takes place in the direction of the axis of the gun barrel. In special cases, where it is necessary that the breech clear the platform on which the gun is mounted, as in field artillery, the recoil may take place parellel or slightly inclined to the platform. Mounts on small, light vessels have a longer recoil than mounts for similar guns on battleships, where the deck structure is more substantial and capable of withstanding greater forces. When the gun recoils in the mount, the forces acting on the mount depend upon the resistance offered by the mount to the recoil of the gun, rather than on the chamber pressure and the diameter of the bore. For the same gun and the same pressure curve, the forces acting on the mount vary inversely as the length of the recoil. For major caliber guns, the length of recoil is limited to about three calibers because of the physical restrictions of the turret and barbette. For light vessels, where the deck structure is incapable of sustaining large forces, the length of recoil is increased considerably and is usually six calibers or more. Usually, however, there are limitations on the length of recoil imposed by the ship's structure, and the determination of the proper length of recoil is compromised by several conditions. For example, in turret guns, an increase in length of recoil results in a larger barbette diameter and greatly increased weight. In minor caliber guns, longer recoil results in increased trunnion heights in order that the breech may clear the deck at extreme angles of elevation.

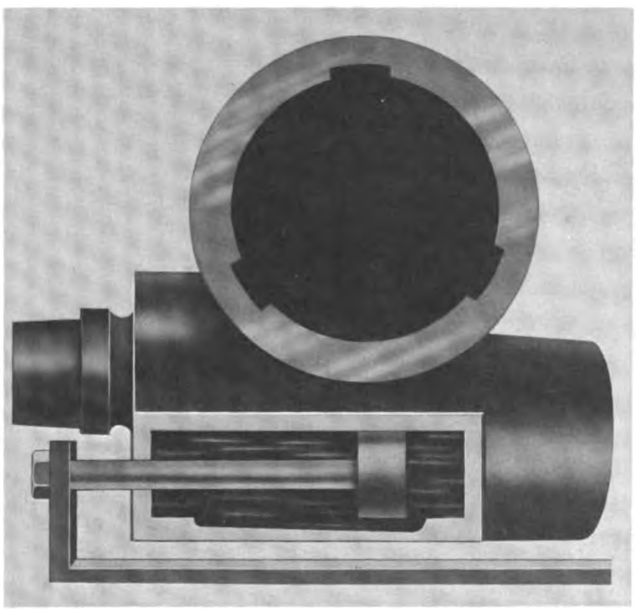
simplified example of the general principles of recoil and counterrecoil of a 5 inch mount

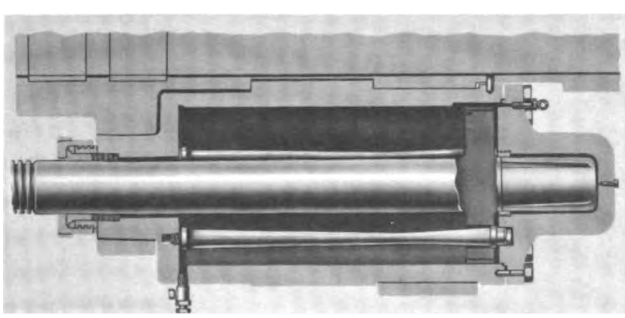


recoil mechanisms

The mechanism incorporated in the gun for the purpose of checking the recoil and bringing the gun to rest within a limited distance is called a recoil brake.

In most naval guns, the major portion of the energy of recoil is absorbed by hydraulic recoil brake which comprises the principal part of the recoil system. The counterrecoil system and the friction of the gun in the slide contribute a small part of the resistance to recoil, whereas the gravity component of the recoiling weight exerts a varying affect depending on the angle of elevation.





Most hydraulic brakes consist of four simple elements: a cylinder, a piston, a liquid, such as glycerin and water, and some form of orifice connecting the ends of the cylinder on either side of the piston. Because the cylinder is filled with liquid, the motion of the piston within the cylinder forces the liquid through the orifice from one side of the piston to the other. The work required to force the liquid through any given orifice can be definitely determined from the laws of hydraulics and depends on the area of the orifice, the area of the piston, the velocity of the piston, and the weight of the liquid.

The work done on the piston is equivalent to the work done on the liquid. The work done on the piston is utilized to overcome the movement of the gun during recoil, whereas the work done on the liquid during the same time is indicated by a rise of temperature of the liquid. It can be shown that the work absorbed by the hydraulic brake can be fully accounted for in the rise of temperature of the liquid. Under rapid fire conditions, the temperature rise is accumulative from shot to shot and results in a considerable rise of temperature which must be taken into account in designing a recoil system.

The advantages of the hydraulic brake can be attributed to the large amount of energy that can be absorbed in an unreturnable form,to its simplicity and reliability, and to the facility with which the resistance offered to the movement of the gun can be regulated. The energy absorbed is converted into heat and dissipated by the mount into the atmosphere. Springs or compressed air are not suitable for checking the recoil of guns because of the limited amount of energy that can be absorbed and also because the energy absorbed during recoil is returned again to the mount during counterrecoil.

The hydraulic brake is so proportioned that the total resistance offered by the recoil system, counterrecoil system, and friction, with a proper allowance being made for gravity forces, is constant and of sufficient magnitude to bring the gun to rest in the prescribed distance. With the resistance constant, the velocity of the gun during recoil will vary from zero at the beginning to a maximum and back again to zero at the termination of recoil. The resistance offered to recoil by the liquid at any point of the recoil cycle would be proportional to the velocity of the piston at that point if the area of the throttling orifice were constant. To obtain a constant resistance, it is necessary to vary the size of the orifice so that it is proportional at any point to the velocity of recoil at that point. For example, the area of the orifice must be greatest at that point where the velocity of recoil is greatest.

In the application of hydraulic brakes to guns, the recoil cylinder may be attached to the slide and the piston to the gun (throttling-groove type). However, in many cases the reverse arrangement is used (throttling-rod type).

A recoil mechanism moderates the firing loads on the supporting structure of a weapon by prolonging the time of resistance to the propellent gas forces. The source of this force is the pressure generated by the hot, expanding gases (force = pressure x area). From basic physics, it is known that the integral of the forces acting on a mass is equal to the change in momentum of that mass, or symbolically

$$\int_{t_1}^{t_2} F\, dt = m_2 v_2 - m_1 v_1$$

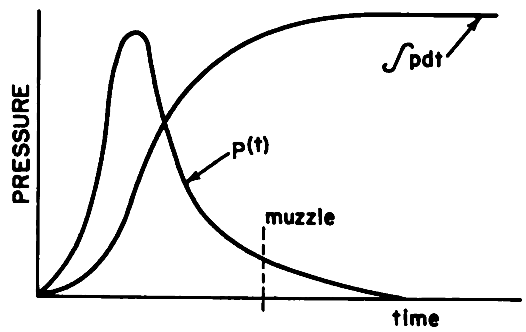
If it is assumed that the expanding gases exert the only force acting on the recoiling parts in the condition known as free recoil, then

$$A \int p dt = m_r v_f$$

where

A = area of bore
p = chamber pressure
m_r = mass of recoiling parts
v_f = velocity of free recoil of recoiling parts.

A typical p(t) curve and a typical $\int$pdt curve are shown.



The pressure-time curve, then, can be used to obtain the curve of v_f versus time by rewriting the equation:

$$v_f = \frac{A}{m_r} \int pdt$$

If the assumption is made that under conditions of free recoil the velocity of the center of mass of the products of combustion is half the velocity of the projectile, then while the projectile is in the bore, and only while it is in the bore, the following momentum balance exists:

$$m_r v_f = m_p v_p + m_c (\frac{1}{2} v_p)$$

or

$$v = \left[\frac{w_p + \frac{1}{2} w_c}{w_r}\right] = v_p$$

where

w_p = $m_p g$ = weight of the projectile
w_c = $m_c g$ = weight of the propellant
w_r = $m_r g$ = weight of the recoiling parts
v_p = $v_p(t)$ = velocity of the projectile

Note that the quantity $\dfrac{w_p + \frac{1}{2} w_c}{w_r}$ is a constant for a given system; hence, while the projectile is in the bore,

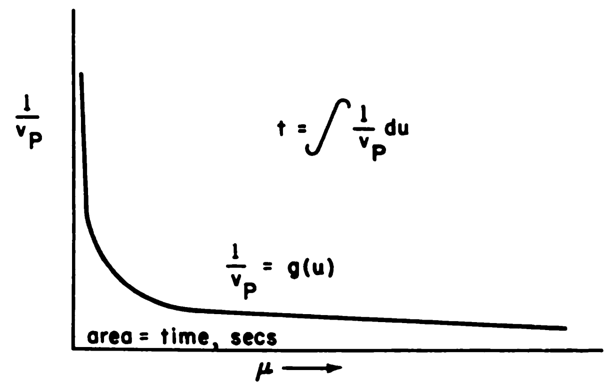
$$v_f \propto v_p.$$

Now that it has been demonstrated that v_f is proportional to v_p, it is necessary to write an equation of v_p versus time. Since values of v_p for each value of u are known, a plot of $\dfrac{1}{v_p}$ can be made.

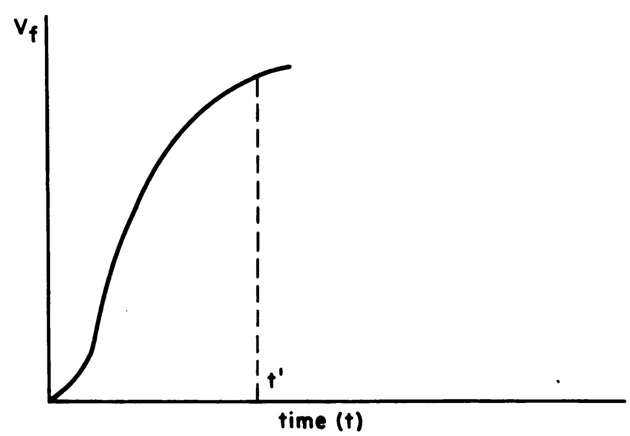
Since

$$v_p = \frac{du}{dt}, \text{ then}$$

$$t = \int \frac{1}{v_p} du$$



Thus, values of t corresponding to each value of v_p can be determined by graphical integration and a plot can be made of v_p versus t. Then, by a simple change of scale, a plot of v_f versus time can be made for the time that the projectile is in the tube. Such a plot is shown, wherein t' is the time at which the projectile leaves the muzzle.



Since the propellent gases continue to act on the gun for a brief time after the projectile leaves the muzzle, the maximum velocity of free recoil will be greater than the maximum value given by the previous equation. Based on experiments, the following empirical equation has been developed:

$$V_f = \frac{W_p V_p + 4700 w_c}{w_r}$$

where

V_f = maximum velocity of free recoil
V_p = muzzle velocity (instrumental)
4700 = empirical constant representing velocity of propellent gases after projectile leaves the muzzle

velocity of free recoil vs time

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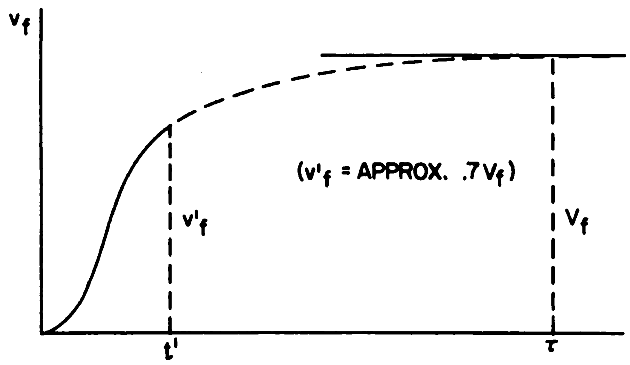
Once the maximum velocity of free recoil is attained, the gun would continue to recoil at the velocity indefinitely, since in this hypothetical condition absolutely no resistance is offered.
Now the remainder of the v_f curve can be constructed from v_f (t′) to V_f. This is done by continuing the curve as a smooth one from v_f (t′) to the point where it becomes tangent to the horizontal line at the height V_f. There is no formula, empirical or otherwise, which will yield a plot of this portion of the curve. It is done by eye.



velocity of free recoil vs time

Note that the time at which v_f becomes V_f is denoted by the symbol τ. Time τ is the total time the propellent gases act on the recoiling parts.
In the discussion so far, all resistances to recoil have been neglected. When the gun is mounted on a carriage, the recoil brake and other forces begin to act as soon as recoil begins. The velocity of retarded recoil is consequently less at each instant than the velocity of free recoil. Assume for the purpose of this treatment that a constant force R, representing the sum of recoil brake force, counterrecoil system force, and friction force, acts to retard the recoiling weapon. Then, starting with the v_f curve, the designer can establish the kinetics of the recoil system as shown. First, $A\int pdt$, which is the momentum of the recoiling parts in free recoil, is plotted. Then the integral of R with respect to time is plotted, giving the momentum transmitted through the recoil system.

The difference between these two momentum curves is the momentum of the recoiling weapon.

$$m_r v_r = A\int_0^t pdt - \int_0^t Rdt$$

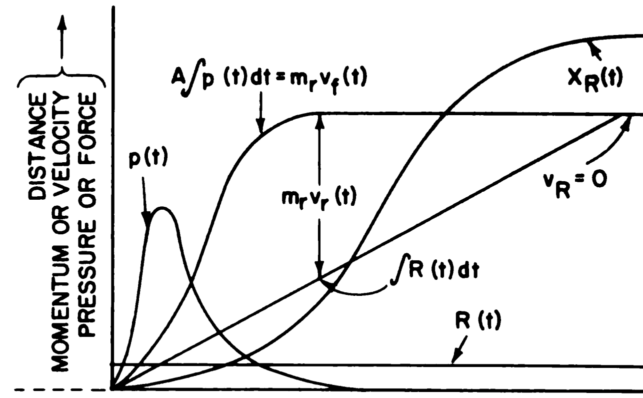
Since R is assumed constant

$$m_r v_r = m_r v_f - Rt$$

Then, dividing by m_r

$$v_r = v_f - \frac{R}{m_r} t$$

This is a velocity equation, since R/m_r is a force divided by a mass, yielding acceleration, which when multiplied by time, t, becomes a velocity term. It can be seen that Rt/m_r is the velocity that would be attained in time t if a force, R, were acting along on a mass, m_r.



recoil system dynamics

Integration of the v_r formula will produce a curve of recoil distance versus time, or

$$\int_0^t v_r\, dt = X_r = \int_0^t v_f\, dt - \int_0^t \frac{R}{m_r}\, dt$$

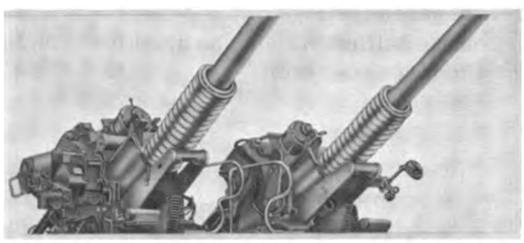
where X_R = recoil distance at time t. If the maximum recoil distance is acceptable, this phase of the problem is completed. If the distance is too great or too small, it can be changed by increasing or decreasing R until the desired maximum value of X_R is obtained. Any variation of R with time can be used, provided, of course, that this variation can be designed into or produced by the recoil braking system. It should be noted that the use of a constant force imparted instantly will produce overshoot (dynamic effects in the immediate supporting structure). For example, a suddenly applied constant force will produce a peak force of 2R in a simple undamped spring-mass system. To reduce this effect, it is desirable to increase the force gradually, if practical.
It should also be noted that firings at various temperatures will change the pressure-time curve, so that some safety factor should be used.
After the kinetics of the system have been established, calculations of the hydraulic recoil break orifice sizes can be started. This can be done in the usual way, selecting an orifice size and coefficient (a function of Reynold's Number) and forcing flow conditions into a turbulent region. Note that all hydraulic devices are means for dissipating, not storing, energy.

counterrecoil mechanisms

Counterrecoil mechanisms are incorporated in a gun mount to return the gun to the in-battery position after the recoil mechanism has brought it to rest at the end of recoil. Energy for this purpose is obtained from the momentum of the gun itself during recoil and is stored in a suitable medium, usually helical springs or compressed air.
The functions of any counterrecoil system are: 1) to return the recoiling parts of the gun to battery after recoil; and 2) to hold the recoiling parts in battery. Thus, a counterrecoil system must not only provide thrust during counterrecoil, but must also develop enough continuous thrust at all times to hold them there except while the projectile is actually being propelled through the bore. (Recoil brakes develop their reverse

thrust for braking only while the gun is moving in recoil, and at other times exert no forces on the gun parts.) Because its thrust follows through to the end of the counterrecoil stroke, any counterrecoil system tends to drive the recoiling parts into battery with considerable shock. Hence, all counterrecoil systems for guns 40 mm and up have a counterrecoil buffer to take up this terminal shock.

Counterrecoil buffers are dashpot devices used to control the velocity and cushion the impact of counterrecoiling parts at the end of counterrecoil. In present designs, they are physically located in the recoil mechanisms and incorporate needle valves which can be adjusted (within limits) to modify the speed of the counterrecoil stroke.

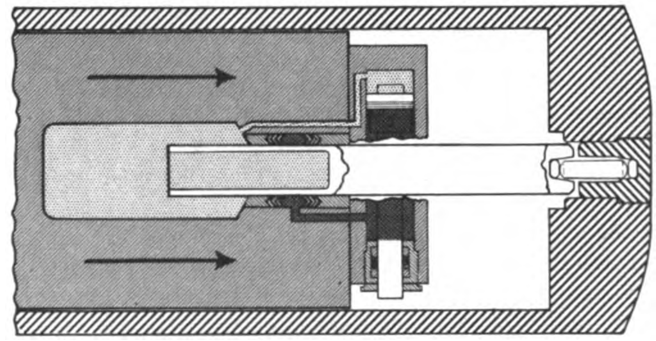


In all naval guns smaller than 5-inch, coil springs provide counterrecoil thrust, In 3"/50 mounts and in most 40-mm mounts, the springs surround the exterior of the barrel. During recoil, the gun moves to the rear, compressing the spring. Upon termination of recoil, the energy stored in the compressed spring is used to return the gun to battery. At full recoil, the springs may be compressed as much as 50 percent of the length they have when the gun is in battery.

Formerly, all counterrecoil systems were of the spring type. However, the introduction of antiaircraft guns of high elevation and the increasing of the elevation of heavy turret guns made such increased demands on the counterrecoil system that it was found necessary in many cases to replace the spring system with a pneumatic system. Satisfactory counterrecoil springs for heavy guns are very difficult to obtain and if they are broken while the gun is in use, the gun is likely to be put out of commission. Also, it requires considerable time and work to replace the springs. Counterrecoil springs in antiaircraft guns are likely to become permanently set or to have their free length shortened if the guns are normally secured or stowed in an elevated position.

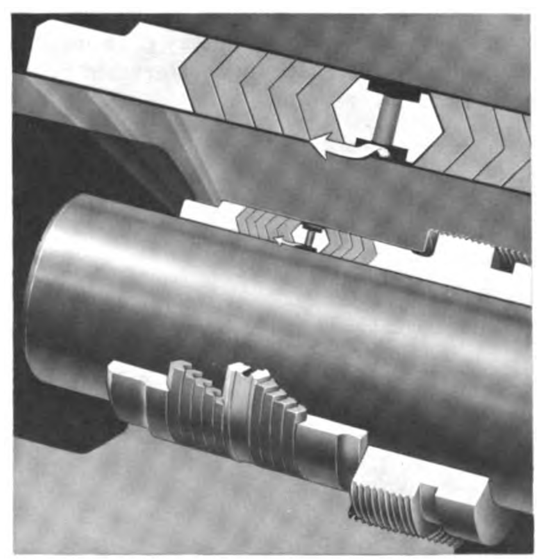
For these reasons, it will be found that the spring system is used on minor caliber mounts and on intermediate and major caliber mounts of moderate elevation, whereas on antiaircraft mounts and on intermediate and major caliber mounts of comparatively large elevation, the pneumatic system is used. A few turret mounts use the spring-pneumatic system.

Today, 5-inch and larger guns use pneumatic recuperators which depend on compressed gas (generally air or nitrogen) to provide counterrecoil thrust. Since the very high-pressure gas used in such systems is sealed by use of packings under hydraulic pressure, such systems are often called hydropneumatic counterrecoil systems.



To counteract the tendency of the high-pressure air to escape around the plunger, a liquid-filled chevron packing system is used. Liquid under higher pressure than the air is led to the packing, causing it to press tightly against the plunger and prevent the escape of air. The pressure is maintained by the use of a differential cylinder.

A simplified hydropneumatic counterrecoil system consists of a counterrecoil cylinder charged with an inert gas such as air or nitrogen to a pressure of about 1500 to 1800 psi. The cylinder may be mounted in the housing or on the slide. A counterrecoil plunger fits into the cylinder. When the gun moves to the rear in recoil, the plunger enters farther into the cylinder, compressing the gas and effectively storing some of the energy of recoil. At the end of the recoil, the compressed gas returns the stored energy by pushing the plunger out of the cylinder, thereby returning the gun to battery.



Since the weight of the gun is held in battery solely by compressed air, it is essential that some positive means be provided to prevent the gun from sliding out of battery when it is elevated and not in actual operation. This is accomplished by using a locking device. The housing is connected to a bracket on the slide by a safety link and pin. The link is of sufficient strength to support the weight of the gun in battery, but would be parted without damage to the gun if it were in place when the gun was fired. Before firing, a check is made of the recuperator air pressure. If it is satisfactory, the safety link is removed and the mount prepared for firing. The safety link is kept in place at all other times.

other mechanisms

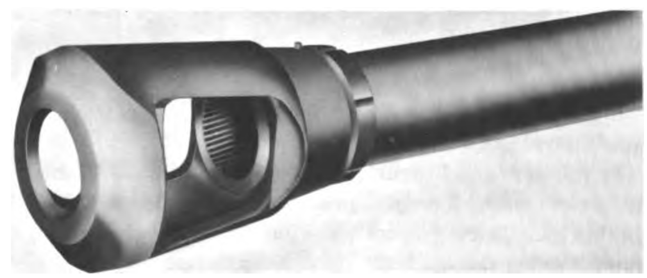
The preceding discussion of recoil and counterrecoil systems has been limited to the most common applications found in naval guns. It is of interest at this point to examine a few other solutions to the recoil problem not found in naval guns.

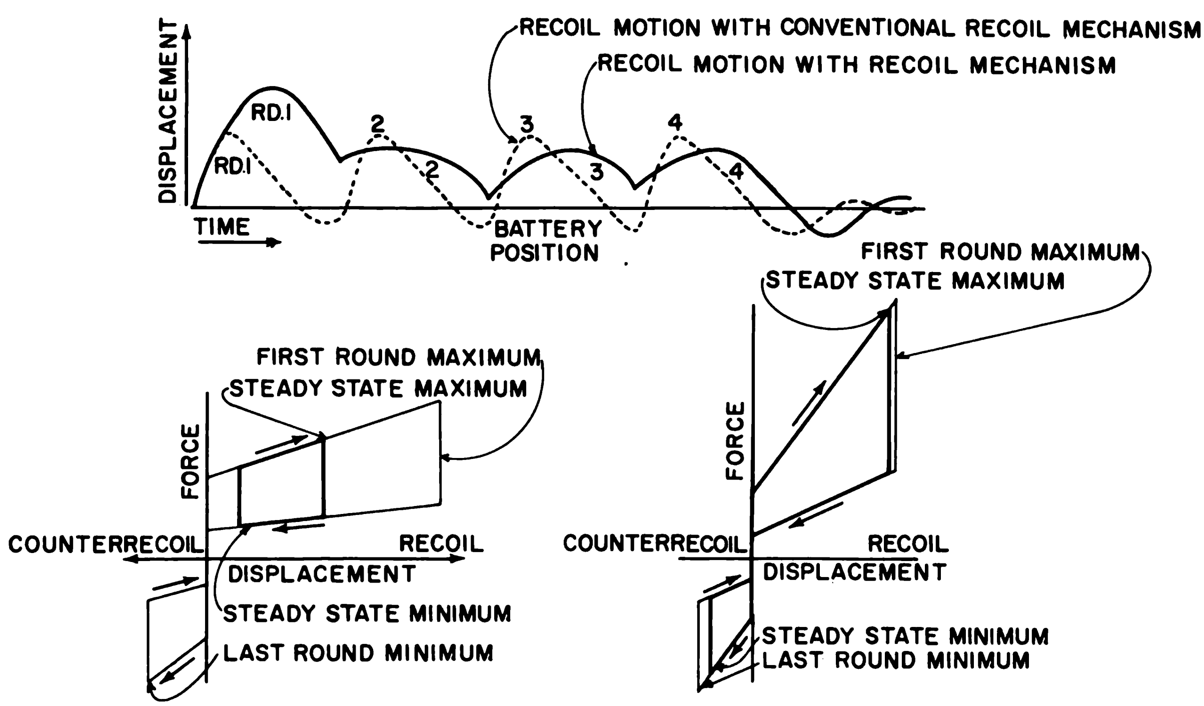
In the development of medium caliber automatic weapons for aircraft, it was found that the high velocity projectile used caused excessive reaction forces at the weapon trunnions. These forces were transmitted to the aircraft structure and required that additional structural strength be built into the airframe in order to absorb the stresses. In an effort to reduce these reaction forces to a minimum, the principle of soft recoil was applied to these weapons. Using a hydrospring recoil mechanism, it was found that the steady-state variation of the effective horizontal forces at the trunnions was approximately 25 percent of that obtained when the standard recoil mechanism was used. It was also possible to increase the rate of fire of the weapon, since only a portion of the total basic recoil cycle was being used. For single barrel weapons, rates of fire in excess of 1300 rounds per minute have been obtained.

The maximum trunnion forces can be reduced by lowering the deflection rate of the recoil spring. When the spring rate is reduced sufficiently, a recoil condition is reached in which the period of the recoil cycle exceeds the time of the gun's firing cycle. During this soft recoil condition, each round (after the first) is fired before the gun returns to battery position and the momentum of recoil is partially counteracted by the momentum in counterrecoil.

A stable round-to-round pattern of recoil motion is obtained by introducing some form of damping to absorb a portion of the recoil energy. One form, the hysteresis of a ring spring, is illustrated by the calibrations of the figure. When gun operation is maintained wholly on the recoil side of battery position, the horizontal forces at the trunnions also are maintained in that direction. Consequently, a soft recoil mechanism frequently reduces the total variation in force during the steady state by as much as 50 to 70 percent. Because of the softening of the recoil spring, the maximum recoil force of the first round and the maximum counterrecoil force at the end of the burst are also lower than those obtained with a conventional recoil mechanism.

A muzzle brake is a very simple device, consisting of one or more baffles placed ahead of the gun muzzle. Propellent gases issuing from the muzzle at high speed impact on the baffles, causing an axial force on the barrel which resists recoil.





SOFT RECOIL MECHANISM

CONVENTIONAL RECOIL MECHANISM

Another solution to the recoil problem is the recoilless rifle. This operates on the basis that if the momentum of the propellent gases discharged to the rear is equal to the momentum of the projectile and the gases expelled forward, then the rifle itself will have no momentum imparted to it. Recoilless rifles are generally recoilless in the mean. That is, the total momentum applied to the gun over the entire firing cycle is zero. They may sustain large, unneutralized forces during parts of the firing period and other oppositely directed forces during other portions of the period. The total momentum imparted to the recoilless rifle, however, is essentially zero.

PROJECTOR-TYPE LAUNCHERS

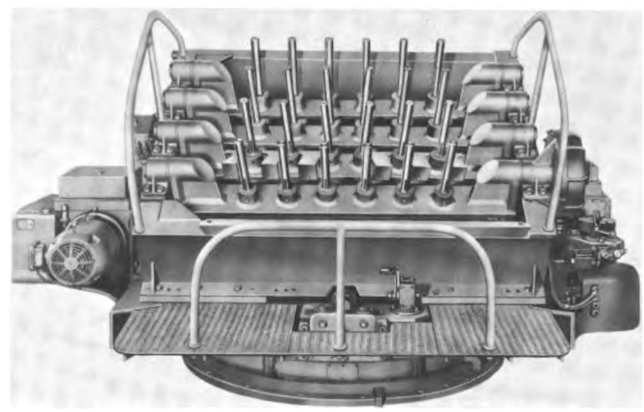
Projectors are another type of impulse launcher similar to guns, wherein the launcher provides all missile propusion and control of guided flight.

The projector shown, commonly called a K-gun, is used to launch 300-pound cylindrical or the 200-pound depth charge. It consists of a smooth launching tube into which the arbor fits. A black powder charge provides the necessary impulse to project the depth charge and arbor into flight.

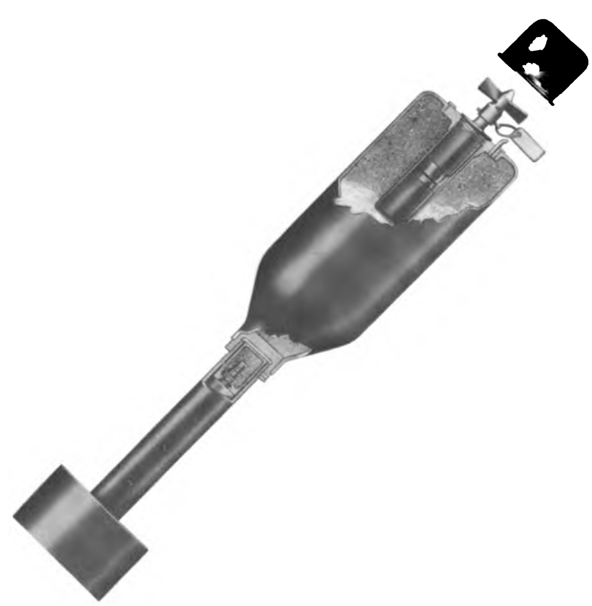
projector Mk 6

The projectors have no means of controlling the momentum imparted by launch and simply transmit it to the vehicle. Therefore, the total momentum acceptable to launcher and delivery vehicle is limited by structural strength. Thus, the momentum imparted to the missile is limited, and the missiles are of short range.

The Mk 11 Projector shown is interesting in that the launching tube is on the missile and fits over a long rod or spigot on the launcher, which provides directional control. This is feasible because the propellant charge is fairly small, resulting in low gas pressures and a fairly light launching tube. In addition, this method adapts itself to the configuration of fin-stabilized missiles.



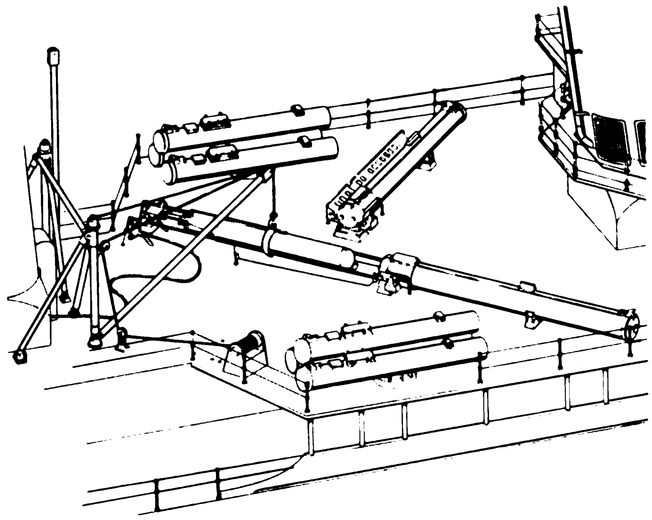
projector Mk 11



7.2 inch projector charge

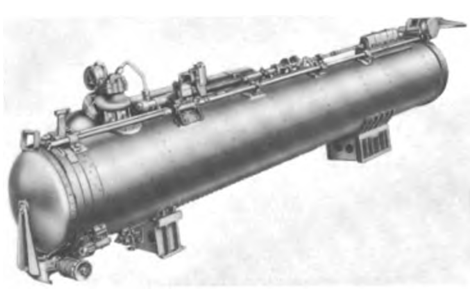
EJECTOR-TYPE LAUNCHERS

Impulse launchers for missile ejection are employed with both free fall and self-propelled missiles. Their main purpose is to insure that the missile safely clears the delivery vehicle. Ejection is usually accomplished by the expansion of high-pressure gases from a compressed air supply or from ignition of a propellant charge. Because it is used for ejection purposes only, the impulse is small, and the launcher can be built to withstand the shock of launching without the need for excessive structural strength or special devices. Thus launchers of this type are fairly light and simple in design. Impulse launchers for missile ejection are most often used with self-propelled missiles.



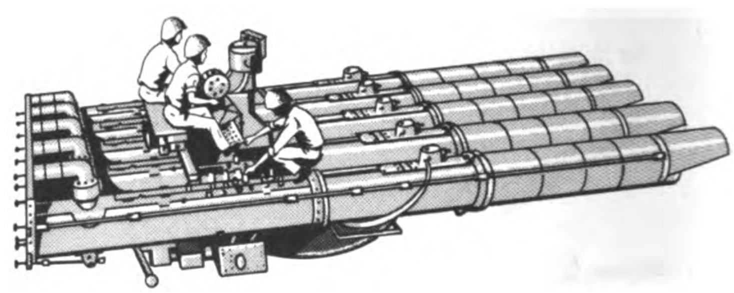
Frequently, however, these missiles are large and heavy, making transfer and loading a slow, laborious process. A launcher of this type usually has several launching tubes, and the missiles are stored within the tubes until needed. The most common launcher of this type is the torpedo tube. Torpedo tubes are installed in ships and submarines and may be of fixed or trainable design. Torpedo ejection is caused by expanding gas from a black powder propellant charge, or from a compressed air supply.

Fixed torpedo tubes are installed in the bow and stern of most U.S. Navy submarines. Fixed tubes are also found in surface craft such as PT boats and destroyers. Aboard ship, fixed tubes are usually located internally with only the muzzle protruding from the side of the deck house.

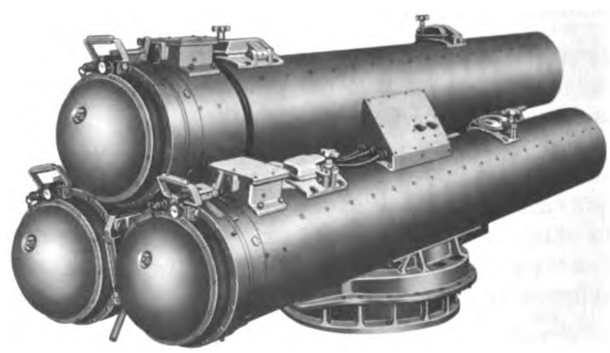


The fixed torpedo tube pictured uses compressed air for the ejection of standard torpedoes and homing torpedoes.

Trainable torpedo tubes, on the other hand, are installed in a clear deck area so that they can be oriented as required. Two types of trainable torpedo tubes are shown. The first illustrates a quintuple-tube launcher which is normally mounted on the centerline of certain destroyers, permitting torpedoes to be launched from either side. It is designed to eject the torpedo with a propellant charge of black powder.

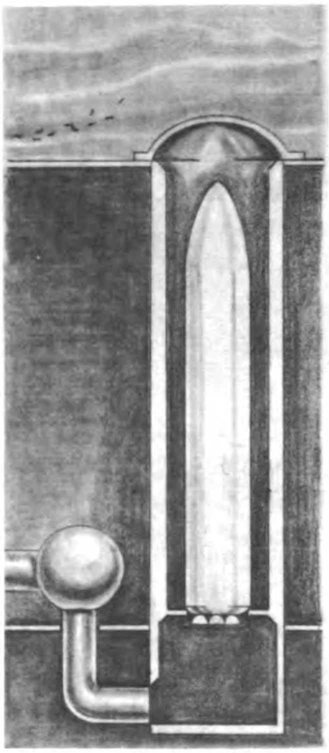


The second type is the triple-tube launcher, an excellent example of modern methods in reducing space and weight requirements. To reduce space, the tubes are stacked one above the other. By fabricating the tubes of reinforced fiberglas, weight is reduced to a minimum.

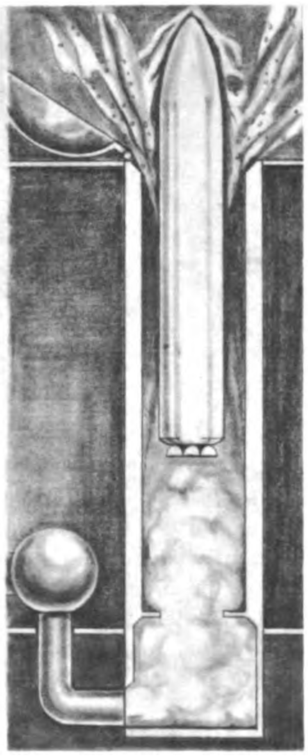


This torpedo launcher is designed to eject the small ASW torpedoes by compressed air. It is normally mounted near the side of a ship and can be fired over the side only.

The launching system installed in submarines for Polaris ballistic missiles consists of a series of vertical tubes which hold the missiles in storage until launch. Ejection is provided by compressed air.



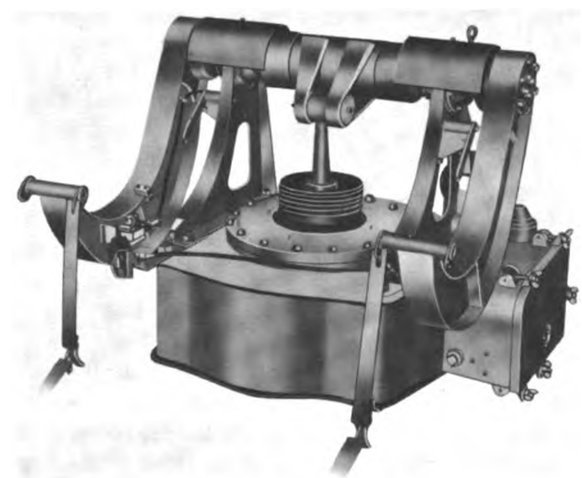
storage



launching

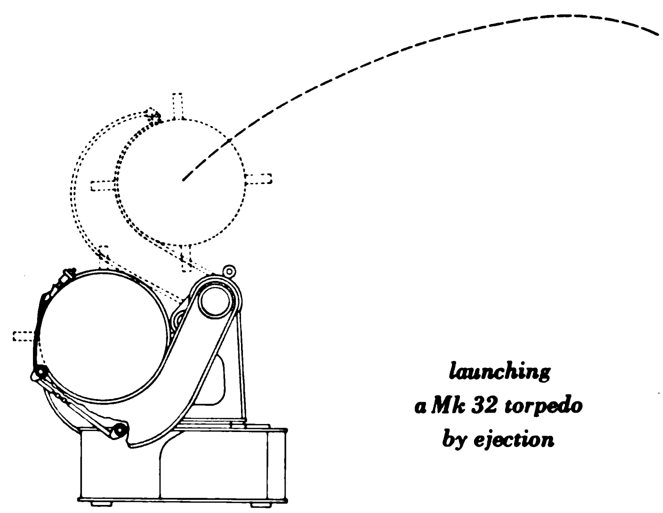
An interesting application of ejection launchers is in the F4H Phantom II, which carries Sparrow missiles. By partially submerging some of the missiles in the fuselage, more missiles can be carried and airflow problems which affect the performance of the aircraft and missile launching safety are minimized. The partially submerged missiles must be ejected clear of the aircraft before they can operate safely. Ejection is accomplished by firing a cartridge producing high-pressure gases which force the missile from the aircraft.

Another example of an impulse launching by ejection is the torpedo launching rack.

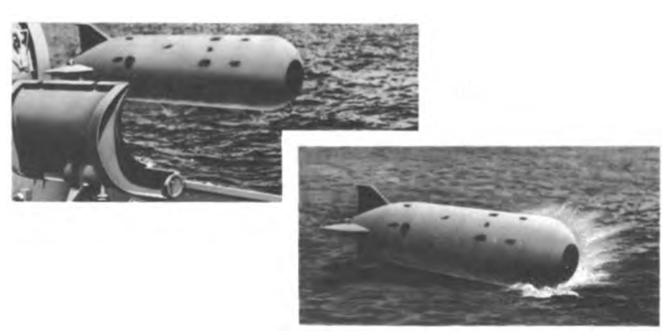


torpedo launching rack Mk 4

The torpedo is supported by a launcher which mechanically tosses it clear of the launching ship. The ejection impulse is provided by a pneumatically operated piston.



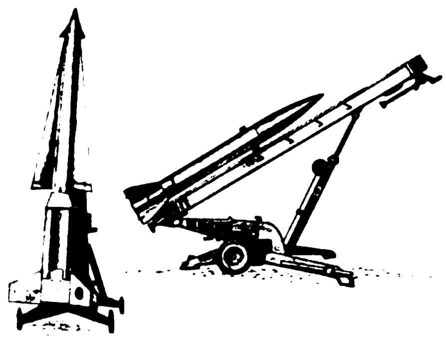
launching a Mk 32 torpedo by ejection

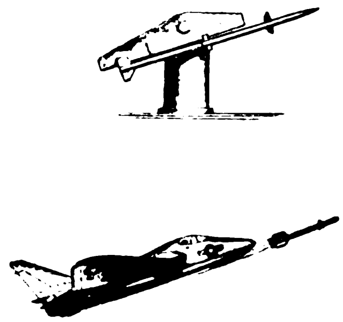


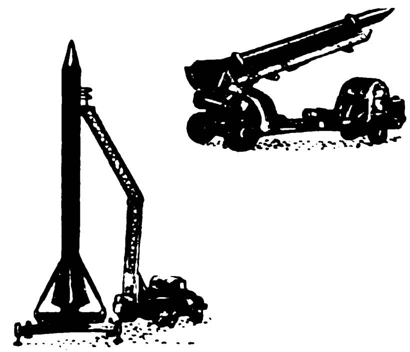
REACTION LAUNCHERS

are divided into three classes

rail **zero length** **platform**





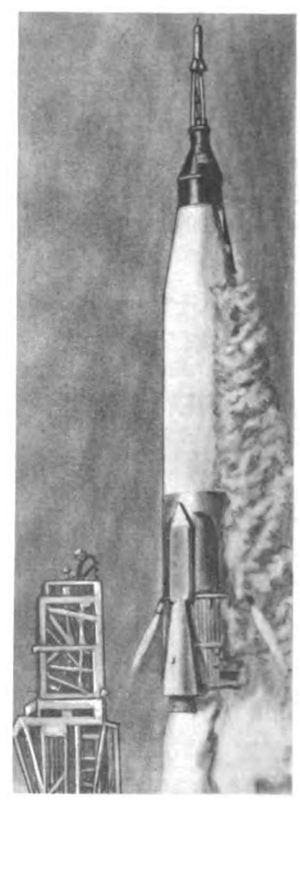


A reaction launcher is one in which the force separating the missile from the launcher is provided within the missile. The propulsion system of the missile itself may be used to provide the separating force, or an auxiliary propulsion unit may be attached to the missile to provide the necessary force. Thus, most self-propelled missiles, if not launched by ejection, are put into flight by reaction launchers.

Although reaction launchers are of many types and sizes, they all have certain function in common: 1) to provide static support for the missile and 2) to provide initial flight orientation. Support is provided by mechanical devices which hold the missile securely to the launcher until the desired time of release. Orientation is provided by attaching the launcher to the vehicle and orienting the vehicle or by rotating the launcher until its axis coincides with the desired direction of launch.

Reaction launchers are usually small and light because they do not have to sustain large forces during the launching phase. The launcher must be strong enough, however, to hold the missile securely in place in spite of large external forces, such as those occuring during radical maneuvering of an aircraft or during the arrested landing of an aircraft. They must also be strong enough to restrain the forward motion of the missile until sufficient thrust has been developed to sustain missile flight properly. Reaction-propelled missiles often depend upon wings or fins to provide lift, and must use rocket thrust to overcome gravity temporarily and to propel the missile to desired flight speed. If, during the development of thrust, a missile is free to move along the launcher, it might not have sufficient thrust or lift to overcome gravity at the time it leaves the launcher. Thus, the missile could fall to the deck of a launching ship or become completely disoriented before sufficient thrust or lift had been developed to sustain missile flight. To prevent this from happening, the missile is restrained on the launcher until sufficient thrust is generated. The restraining device or hold-lock device may be simply a pin which is sheared when the missile develops the required thrust or it may be a more complicated reusable device containing springs which release the missile when the required thrust is exerted.

Vertically launched missiles, such as ICBM's have a similar problem. Although the missile will not lift off until sufficient thrust is developed to counteract gravity, these missiles usually have several engines and all must operate properly for a safe flight. Therefore, a remote-control "tail grabber" is generally used. This device is fastened to the base of the missile and holds the missile on the launcher until released. The release mechanism is connected to the engine monitoring system. The "tail grabber" is released only when the monitoring equipment verifies the operation of all first-stage engines.



Due consideration must be given to protecting the launcher itself from the blast and erosive effects of the rocket motor exhaust stream. This is usually achieved by appropriate protective covering of the launcher and by blast deflection plates or shields. The launcher structure must be able to resist the deteriorating effects of the rocket gases, and the structural design of the launcher should be such as to minimize loading or stresses due to rocket blast or recoil momentum. As with any launcher, the reaction launcher must be compatible not only with the space and weight limitations imposed by the delivery vehicle but with the environment in which the vehicle operates as well.

For example, airborne launchers must be small and light and must produce minimum aerodynamic drag and must provide adequate storage for the missiles until they are fired.

Reaction launchers must also be compatible with the missile to be launched. Therefore, the missile size and configuration must be considered, as well as the missile propulsion system and method of flight control. For example, the launcher must be designed so that wings and fins on the missile do not strike the launcher or delivery vehicle during launch. The missile propulsion system may be entirely enclosed within the missile or attached to it in clustered or tandem units.

The launcher must be constructed to accommodate and often support these external propulsion units. Launcher design is influenced by the degree of control and guidance required of the weapon system. An uncontrolled rocket may require a considerable amount of flight control by the launcher before it is released into free flight, while a guided missile does not require this initial control to the same degree.

Rail-type launchers are composed of a set of rails which support the missile and a structure to support and orient the rails in the desired direction. The rails provide static support for the missile at rest and orientation control to the missile as it moves along the launcher during launchings.
The term rail launcher may be applied to launchers making use of rails, tubes, long ramps, and even tall vertical towers. All provide, to a varying degree, constraint to the missile while it is moving on the launcher and thus provide a considerable amount of flight control. The launcher rails are of varying length, some being shorter than the missile being launched, others considerably longer. For uncontrolled missiles, such as rockets, the rails must be fairly long so that the rocket is constrained for a longer portion of the rocket motor burning time, thus providing necessary initial velocity vector control. If the missile is guided sufficiently during flight to correct for launcher dispersion, the length of the launcher may be reduced.

Since the missile flight path is dependent on the velocity vector of the missile, the desired flight orientation can be obtained by constraining missile flight for a period of time by means of rails or tubes. Thrust developed parallel to the axis of the rails or tube will propel the missile along the rails and into a proper trajectory. Vertical components of the thrust will force the missile against the rails or tube and thus against the launcher. The launcher must, therefore, be capable of withstanding a portion of the missile thrust as well as of supporting the missile. Lateral constraint of the missile is necessary in order to prevent the missile from lifting from the rails or wandering during its motion along the launcher rail. This is usually achieved by lugs on the missile which ride in slots on the launcher rail. In general, the longer the launcher rails, the better the initial flight control and the less the launching dispersion. However, long rails, like long beams, can sag and bend. The longer the beam, the greater the possible deflection, unless the rail is well supported along its entire length. This kind of support is not practical in tactical launchers, so a certain amount of beam deflection or droop will occur at the end of long rail launchers.

This droop and the effects of vibrations set up as the missile moves along the rail cause the rail to whip and produce unwanted deviations in the missile orientation. These effects can be minimized by reducing the length of the launcher rails. Thus, material and structural characteristics will limit the length of a rail launcher for tactical usage.

This necessary restriction in rail length is not as serious as it might seem because the effectiveness of a rail launcher in constraining missile travel to the direction of the rails varies as the ratio of time of rail travel to the total time of boosted uncontrolled flight. Since a missile starts from rest on the launcher, the time of rail travel will vary as the square root of its length.

$$\text{Since} \quad S = \frac{1}{2} at^2$$

$$\text{Then} \quad t = \sqrt{\frac{2S}{a}}$$

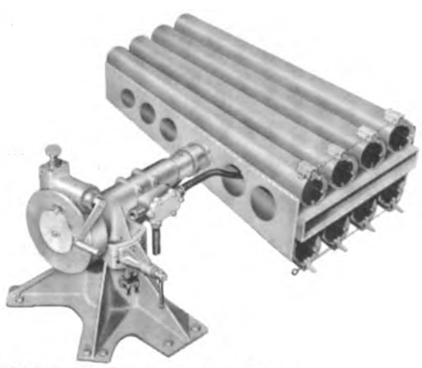
Where

S = rail length
a = missile acceleration
t = elapsed time of travel

Therefore, a relatively short rail will still provide considerable initial flight control to a missile.

From the preceding discussion, it should be apparent that high thrust, short burning time rocket motors are desirable for missiles requiring effective initial flight control. Thus, short range missiles with fast-burning motors can get sufficient initial flight control from short rails, while longer range missiles, which must achieve higher velocities, need longer burning times and require longer launcher rails.

short range missiles need only SHORT launcher rails or tubes

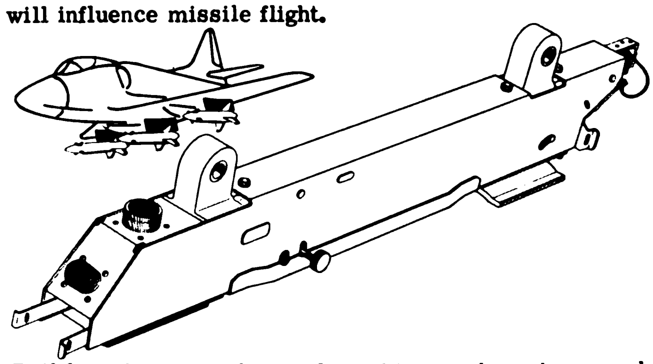


***long range missiles require LONG** launcher rails or tubes*

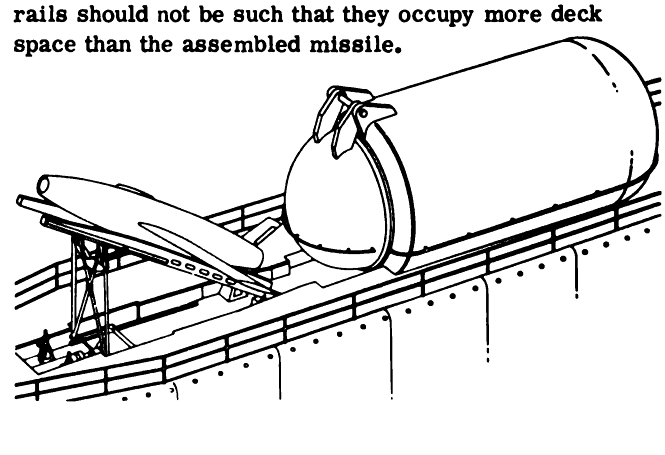
A major disadvantage associated with rail launchers is tip-off. Tip-off occurs when gravity and aerodynamic forces act on the unrestrained forward part of the missile while the after portion of the missile is still constrained by launcher vibrations and whip. Tip-off can be eliminated by providing separate rails for the forward and after supports of the missile, arranged so that the missile leaves all the launcher rails at the same time.

Rail launchers are simple, fairly light, and may be fixed or movable. They may serve as ready service storage for missiles and provide facilities for fueling and servicing. The simplicity of design of rail launchers also promotes reliability and ease of maintenance and repair.

Airborne launcher rails, because of space and flow field problems, are usually very short. As a rule, the greater the speed the missile has achieved at separation from the launcher, the less the air flow field effects will influence missile flight.

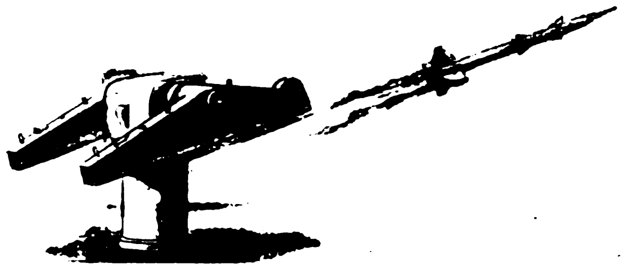


Rail launchers may be used on ships, submarines, and aircraft. For use aboard ships and submarines, a good general approximation is that the length of the launcher rails should not be such that they occupy more deck space than the assembled missile.



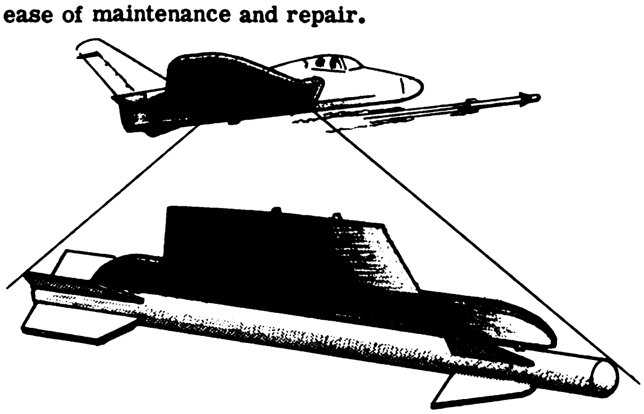
ZERO-LENGTH LAUNCHERS

Tip-off is avoided in a zero-length launcher because the first motion of the missile carries it free of the launcher.



A zero-length launcher supports the missile until it is launched and gives it initial orientation. Since the missile separates from the launcher as soon as it starts to move, no effective flight control is provided by the launcher. Therefore, the launchers are used with missiles that can immediately assume stable flight. Generally, the launching of uncontrolled missiles from zero-length launchers is restricted to airborne operations, since the missile is already moving through the air at the speed of the aircraft after it leaves the launcher. When launching controlled, i.e., guided missiles from zero-length launchers, the missile flight control system must be capable of controlling missile flight immediately after separation from the launcher. Whether the missile is controlled or uncontrolled, it is very important that the line of action of the missile thrust sector pass through the center of gravity of the missile. Otherwise, an angular motion will be imparted to the missile which will cause erratic flight, thus endangering both missile and delivery vehicle.

The chief advantages of zero-length launchers are their small size, light weight, comparative simplicity, and ease of maintenance and repair.

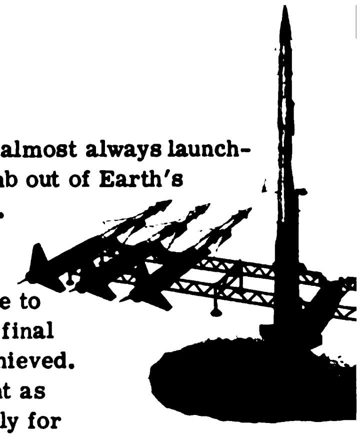


The main disadvantage is the lack of control over the missile during the initial stages of flight. Other disadvantages of zero-length launchers are the increased difficulty in jettisoning a dud from a shipboard launcher and the increased danger of an air-launched missile's striking the launching aircraft due to the action of non-uniform flow fields.

Because of their small size and weight, the zero-length launchers require a minimum of deck space and are easily moved in train and elevation. Therefore, they are used extensively in both shipboard and airborne installations for the launching of guided missiles.

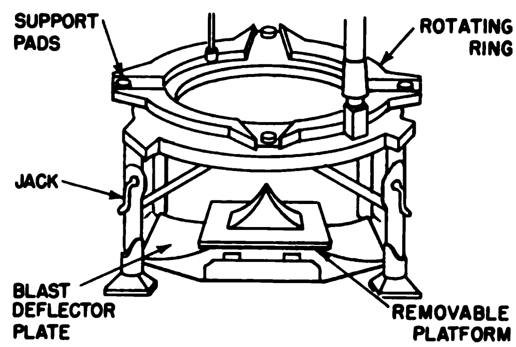
PLATFORM LAUNCHERS

Long-range ballistic missiles are almost always launched vertically so that they may climb out of Earth's atmosphere as quickly as possible.

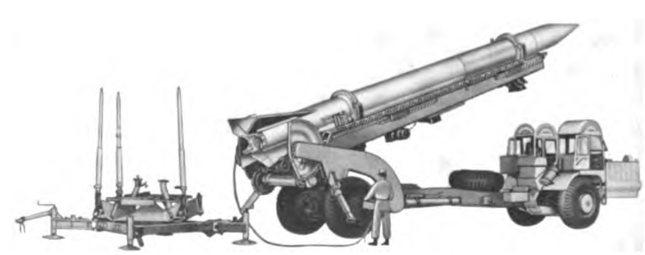


This minimizes velocity losses due to aerodynamic drag. Thus, greater final velocity and longer range can be achieved. Ballistic missiles are built as light as possible and are stressed primarily for longitudinal loads. Therefore, launching them at any angle other than vertical is uncommon.

Since these missiles are very large, heavy, and bulky, a rail-type launcher would have to be unduly heavy. Therefore, long-range ballistic missiles are usually launched from a platform launcher.



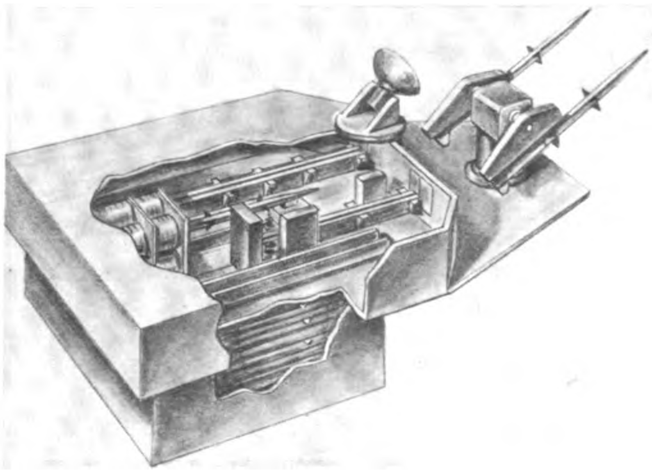
A platform launcher is simple in construction and can be adjusted through a few degrees of tilt. Built-in jacks provide a means of raising, lowering, and leveling the launcher table. A blast deflector plate deflects the rocket exhaust stream horizontally away from the launcher, thus preventing damage to the missile and violent erosion of the area beneath the launcher.



Provisions for erecting, fueling, and servicing the missile are required. Another significant requirement is that the launcher deflect the rocket motor blast so that it spreads out parallel to the ground. Otherwise, the blast would be reflected upward, damaging the missile, and would also act to erode the surface beneath the launcher.

STORAGE SYSTEMS

To achieve and sustain optimum firing, the launcher selected for a given weapon system must include a storage system capable of performing its allotted functions in a minimal period of time. The capacity of the system must be adequate to sustain the desired firing operation and the facilities must be readily accessible.



A magazine is a term used to denote a storage area needed to provide the space and the safety facilities for storage of large quantities of either gun ammunition or reaction missiles.

The ready service storage areas are located in close proximity to the launcher and are necessary to minimize the time between alert and launch. This function may be provided for by handling rooms, where missiles may be stacked or stowed in racks, or in rotating ready service rings.

To achieve maximum rapid employment, ready service missiles are sometimes stowed on the launcher. This is the case when the missiles are large, bulky, and difficult to load, or when the delivery vehicle cannot accommodate elaborate transfer and loading equipment. For example, torpedoes are stowed in the launching tubes; Polaris missiles are stowed in launching tubes; and aircraft-launched rockets and guided missiles are stowed on the launcher.

Missile storage areas must provide for the necessary protection of the missiles in storage and for the safety of personnel. Most magazines are fitted with systems designed to control the environment (temperature and humidity) to maintain favorable storage conditions. Measures to protect missiles from explosions, fire and spread of fire, and from enemy action are found in all missile storage areas. Automatic sprinkling systems, compartmentation, and flameproof missile transfer paths are methods used for fire protection. On large ships, such as cruisers and battleships, armor plate surrounds the entire launching system.

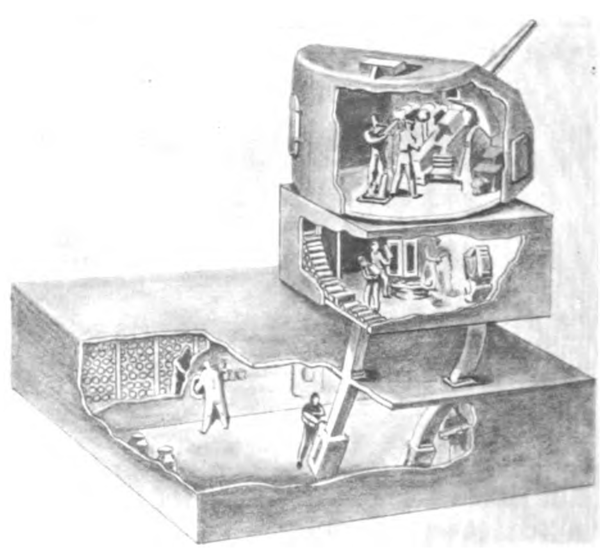
storage of gun ammunition

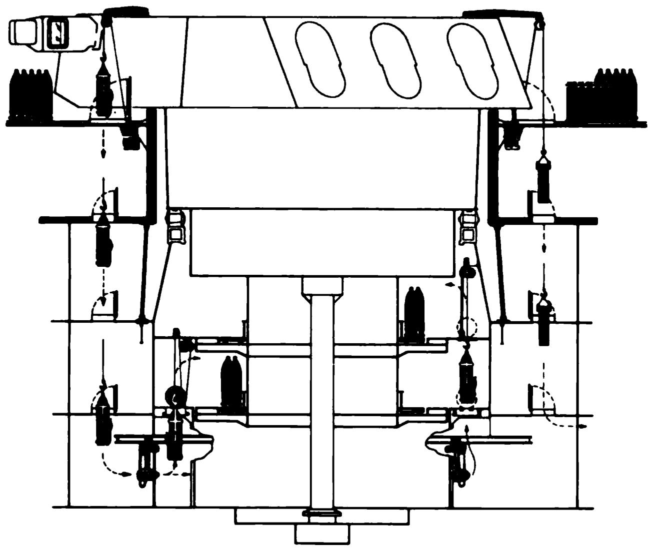
Since modern naval guns have high rates of fire and thus expend a large amount of ammunition in a relatively short period of time, provision must be made for storing large quantities of ammunition on board ship. The illustration shows the storage areas for a typical gun-type impulse launching system, a 5-inch twin mount.

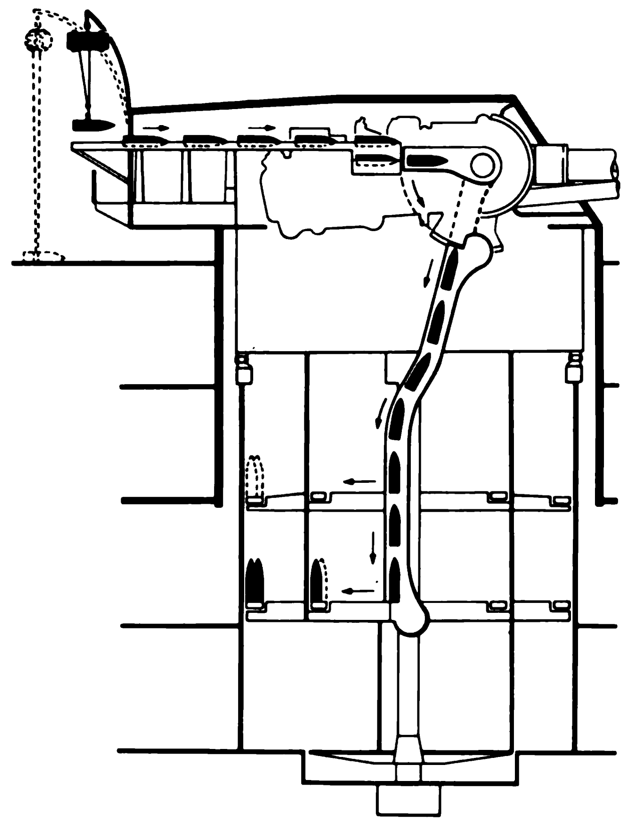
At the lowest level is the magazine, in which the propellant charges are stacked. The magazine partly surrounds the lower handling room which is separated from the magazine by a flameproof bulkhead. Powder cases which are stored in the magazine are passed by hand through scuttles in the magazine bulkhead into the lower handling room. Projectiles are stored in the lower handling room.

To fill the magazines, ammunition must be brought aboard ship. Ammunition may be carried aboard by hand, or hoisted aboard by highlines, small cranes, etc. It is usually hoisted aboard in "skip boxes", in special straps or slings, in cargo nets, on pallets, or in special containers. The ammunition is struck below by lowering it through hatches and down long vertical trunks to the magazine areas. Ammunition is also struck below by operating the transfer equipment (projectile and powder hoists) in the reverse direction.

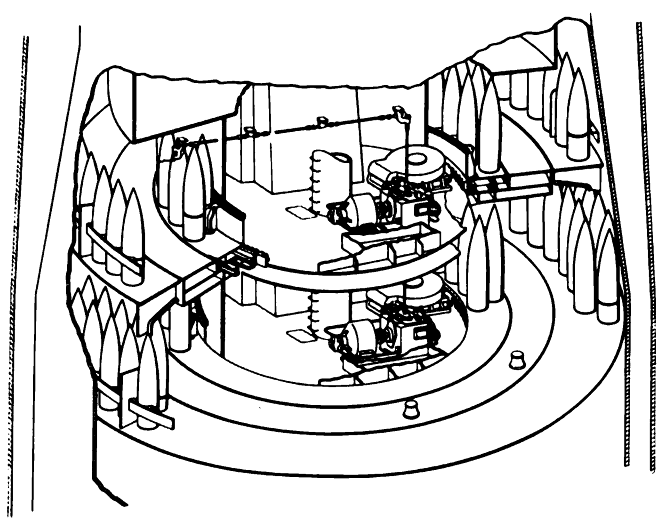
Magazines have special ventilating equipment to control the storage environment and high-capacity, automatic sprinkling equipment to protect the ammunition from temperature extremes and fires. With semifixed bag ammunition, the projectiles are stored separately from the powder in handling rooms, in magazines, and in projectile flats (turret guns).

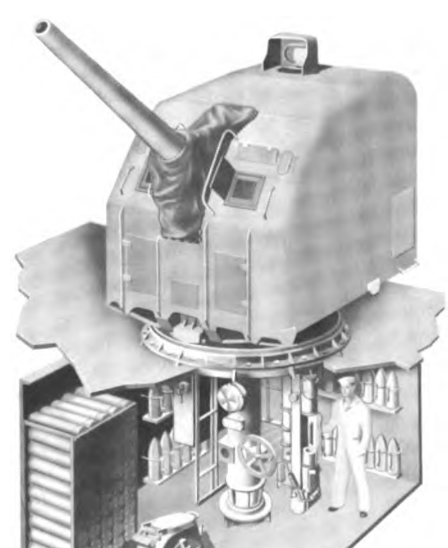






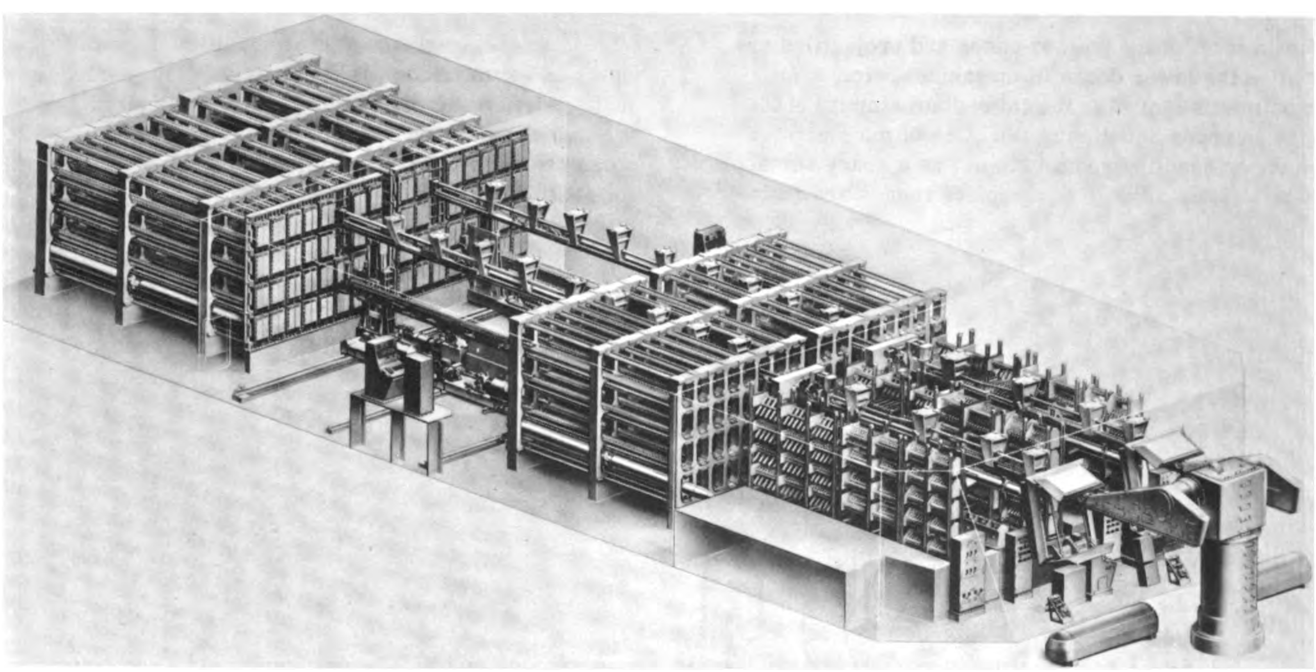
Although most of the powder cases and projectiles are stored on the lower decks in magazines, etc., some ammunition is stored in the immediate vicinity of the gun, in an upper handling room. One of the functions of an upper handling room is to act as a ready service area in which a number of complete rounds are maintained in ready racks so that ammunition service to the guns can be initiated without delay. For long periods of sustained fire, however, the entire ammunition storage and transfer system must be in operation because the ready service ammunition is sufficient for only a few minutes of firing.





storage of reaction missiles

To put missiles into flight as rapidly and frequently as the situation demands, a readily accessible supply of missiles must be available. This supply of missiles must be kept in a safe, protected, ready-to-use condition and must be available in numbers sufficient to permit the sustained firing operations necessary to meet expected threats. The problem of storing missiles and satisfying these requirements is compounded by the large size, weight, bulk, and fragile nature of these missiles. Because of these inherent characteristics, a large space is required to provide a safe, readily accessible storage for the missiles. Since space is limited even on the largest of delivery vehicles, the space available for missile storage, and thus the number of missiles actually held in storage, is limited.

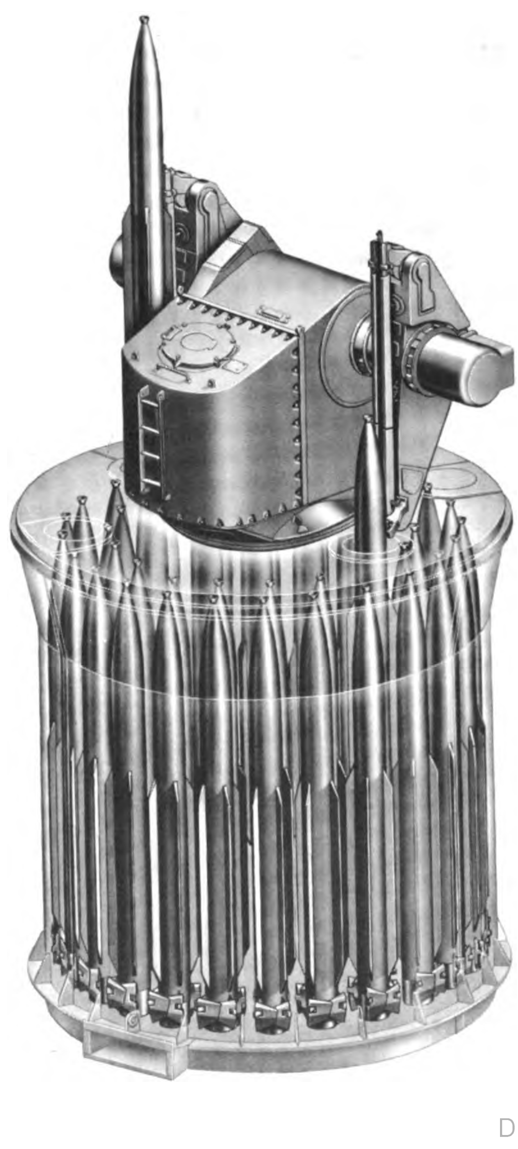


As a rule, a delivery vehicle will not be able to carry as many guided missiles as would be tactically desirable, and each missile must be employed with maximum effectiveness.

To promote rapid initial employment and a rapid rate of fire, the missiles must be readily accessible. Therefore, the missiles must be maintained in a condition and location that permits rapid transfer and employment. The surface-to-air missile launching system shown below illustrates how this may be accomplished with the smaller, shorter range missiles.

The missile must be protected against all influences that will tend to keep it from operating as designed. The storage facilities must include equipment which supports the missiles in a safe and secure fashion and isolates the missiles from the damaging effects of shocks, vibrations, and structural flexures of the delivery vehicle, such as the motions of a ship in a storm, or the maneuvers of a high-performance aircraft.

In missile storage, consideration must be given to the characteristics of the reaction motor as well as to the missile as a whole. Reaction motor propellants, particularly solid propellants, are affected by variations in temperature and humidity. Therefore, careful environmental control must be maintained if optimum performance is to be achieved. In addition, most reaction motor propellants, solid or liquid, are highly inflammable. Every precaution must be taken to avoid accidental ignition of the propellant, and to provide for the safety of missiles, men, and vehicle in the event of accidental ignition. Special storage areas are often necessary to provide for the storage of equipment components, such as spare guidance packages, and alternate components, such as special warheads.



TRANSFER SYSTEMS

Missile transfer equipment is designed to move missiles and associated components from storage areas to loading points at the launcher, or to ready service storage. In addition, transfer equipment must be available to return safe or unfired missiles or ammunition to the magazine.

To achieve rapid initial employment, the transfer system must be capable of moving the missiles at a rate commensurate with the launcher rate of fire. If a transfer line or channel has a transfer rate less than the required rate of fire, then two, or even three transfer lines may be necessary to feed the launcher.

When transferring missiles to rotating launchers, it is necessary at some point along the line to shift from nonrotating transfer equipment to equipment rotating with the launcher. This becomes a problem of considerable magnitude for a high-rate-of-fire system where the missiles must be transferred while the launcher is rotating. This can be accomplished by a manual transfer, or by complex automatic equipment, as in high-rate-of-fire naval guns. When the missiles to be transferred are large and bulky, it is often necessary to have the rotating launcher return to a specified position before a missile is transferred to it.

Missile flight preparation facilities are often found in the transfer system. En route to the launcher, operations such as missile assembly, checkout, servicing, and even communications to the missile may be performed.

Missiles in transfer can be dropped, dented, bent, twisted, ignited or exploded. Any of these accidents tends to make the missile unfit for use and, at the least, results in the loss of a useful missile. To prevent damage to ship and personnel in the event of missile explosion or ignition, the transfer system must be designed to localize and contain detonations, fires, rocket motor blast, etc. In addition to the lethal effects of the missile itself, personnel can be injured by the moving and rotating machinery of the launching system. Thus, in the transfer system, as well as in every component of a launching system, careful attention must be given in the design of equipment to minimize the danger of accidental damage.

transfer of gun ammunition

The effectiveness of a naval gun depends; other things being equal, on the number of rounds per minute it can put on the target. This, of course, depends upon the efficiency of the equipment and personnel responsible for transferring the ammunition from storage to the mount and loading it into the gun. In gun launching systems, ammunition must be transferred from stationary magazines (usually located deep inside the ship) at a high rate of speed to a rotating gun. As a result, mechanical transfer systems are necessary to move heavy ammunition continuously at rapid rates from magazine to gun. The transfer system transfers the ammunition by manual or mechanical means from a stationary installation to equipment rotating with the gun.

In a gun launching system, as shown, ammunition is transferred manually from the storage areas to the mechanical transfer equipment in the lower handling room. The ammunition is then loaded into dredger hoists (one for each gun) which haul it up to the upper handling room. Each dredger hoist can handle both projectiles and powder cases. In the upper handling room are located the upper ends of the stationary dredger hoists and the lower ends of the rotating projectile hoists and powder hoists. The handling room crew removes the projectiles and powder cases from the dredger hoists and loads them into the rotating projectile and powder hoists.

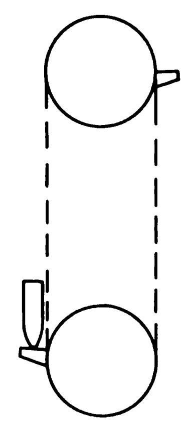
The powder and projectile hoists move the ammunition from the upper handling room to the vicinity of the gun. Here it is manually transferred to the loading equipment on the gun, which is the first step in the loading operation. Thus, the system just described, a typical 5-inch transfer system, is a combination manual and mechanical operation. Some automatic transfer systems are essentially all mechanical.

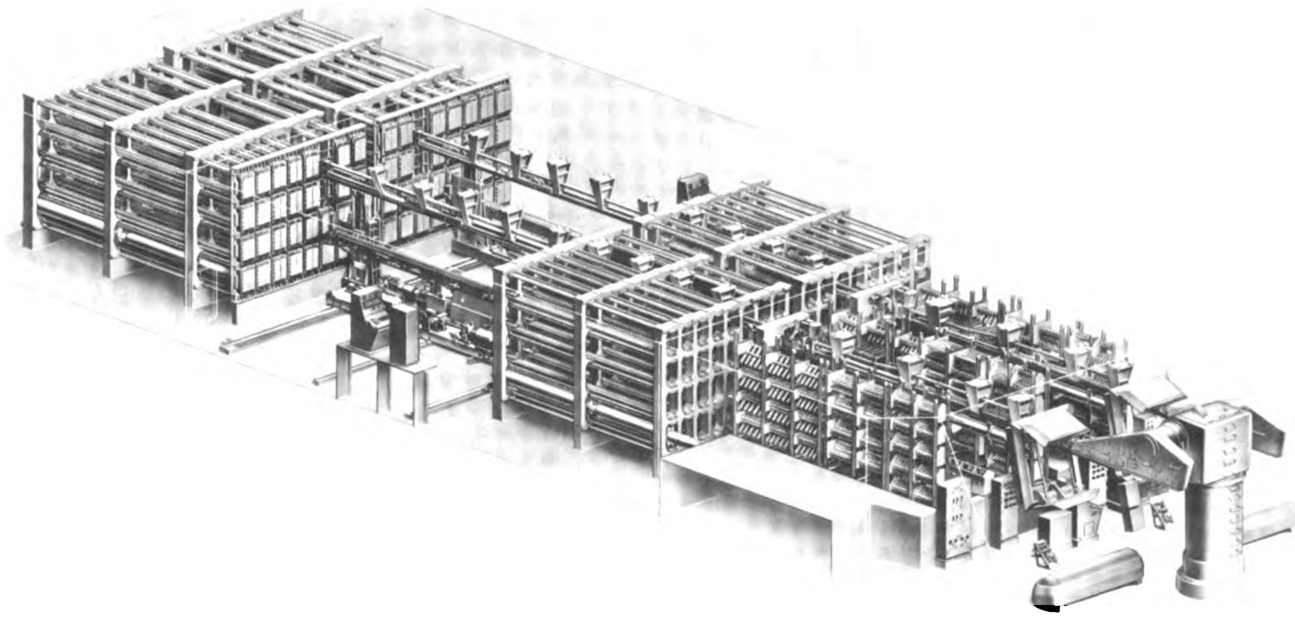
Transfer of gun ammunition from the magazines and handling rooms to the gun is normally accomplished by hoists. Hoists in use in modern U. S. Naval vessels generally fall into one of the following categories:

a. Endless chain
b. Elevator
c. Pawls

endless chain hoists

The endless-chain hoist is the most common type of hoist and includes all dredger hoists, conventional 5-inch powder hoists, and a number of others used in turrets and elsewhere. Fundamentally, it consists of articulated endless chain with supports or flights secured to it at regular intervals. Powder cases or projectiles are loaded by pushing them into the hoist in the path of the flight. When the hoist starts, the chain is driven





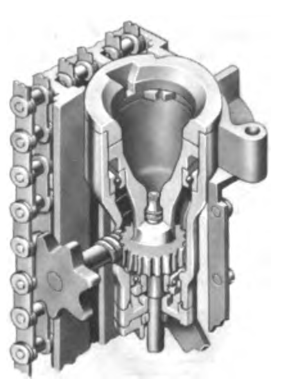
upward until the next vacant flight is in loading position. When the next unit is loaded, the hoist goes up one more flight, and so on. Endless-chain hoists are driven by rotary hydraulic motors whose functioning is controlled by valves.

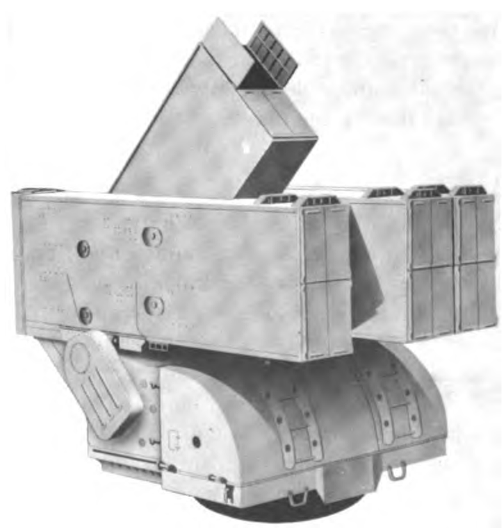
Endless-chain hoists generally can be operated in reverse to lower ammunition units, as is required when taking ammunition aboard. In either mode of operation, the hoist moves one flight at a time, intermittently, in the same direction. Only one side of the chain is used.

There are many variations in the applications of the endless-chain hoist. One type for example, employed with the 5"/38, makes use of both sides of the chain. There are two flights, arranged so that when one is at the top of the hoist on one side, the other is at the bottom of the hoist on the other. The chain runs first in one direction, then the other, and the flights always move from all the way at the top to all the way at the bottom (or vice versa). The arrangement is similar to that of the old-time well with two old oaken buckets, one of which descended while the other went up.

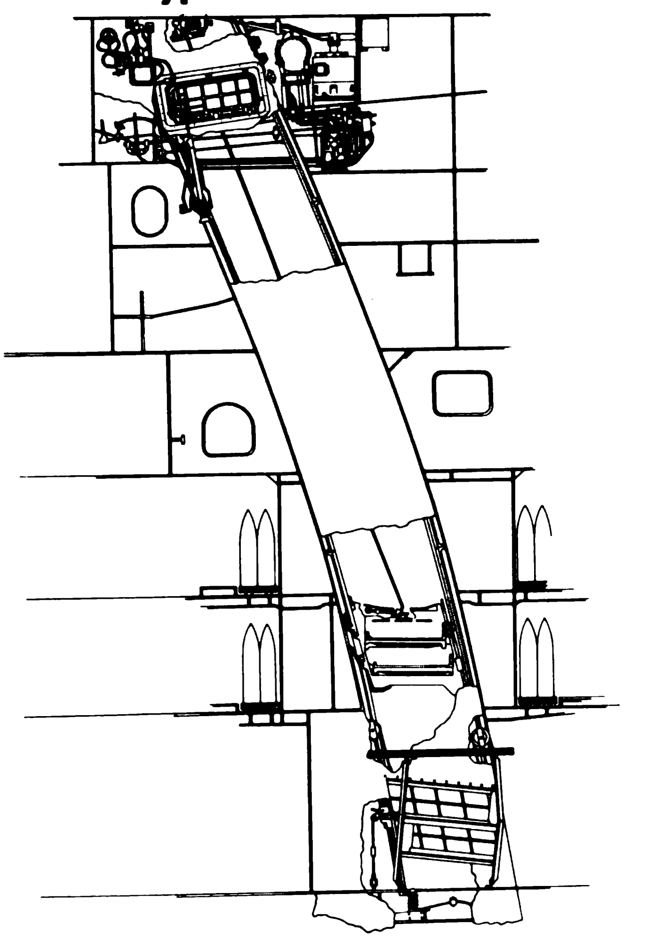
The projectile is loaded into one side, and automatically the hoist starts if the top is empty. As the loaded flight ascends, the empty comes down. The cycle reverses for the next projectile.

This type of hoist can be used for hoisting only. It is not safe to attempt to lower projectiles in it. However, it can be used to perform an additional function, the setting of projectile time fuzes. By installing a suitable mechanism in each flight, the projectile can have its fuze set as it rides up to the gun. This eliminates the necessity for performing the fuze setting operation at the gun.





elevator-type hoists



Elevator-Type Hoists. This kind of hoist is a single-stage system with a car which is moved up or down a hoistway. The car is secured to an hydraulically operated system of cables - a hoisting cable and a downhaul cable. Both are always in tension, and provide positive control of the car position. Unlike conventional elevators ashore, the car has no counterweight.

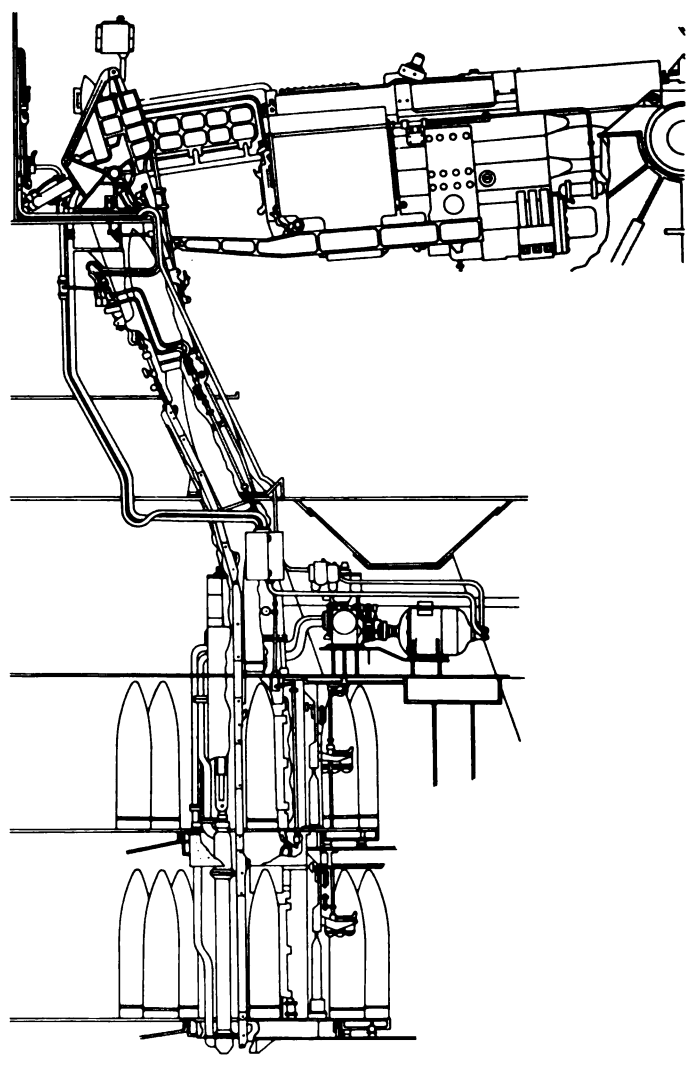
The principal application for hoists of this type is to haul powder in bag-type turrets. Although protected by interlocks, such hoists are generally manually controlled. All loading and unloading points in such installations are protected by interlocked, flametight doors. In 16-inch turrets, a single elevator-type powder hoist serves each gun. Operating procedures provide for opening the hoist upper doors only when conditions in the gun compartment are safe.

pawl-type hoists

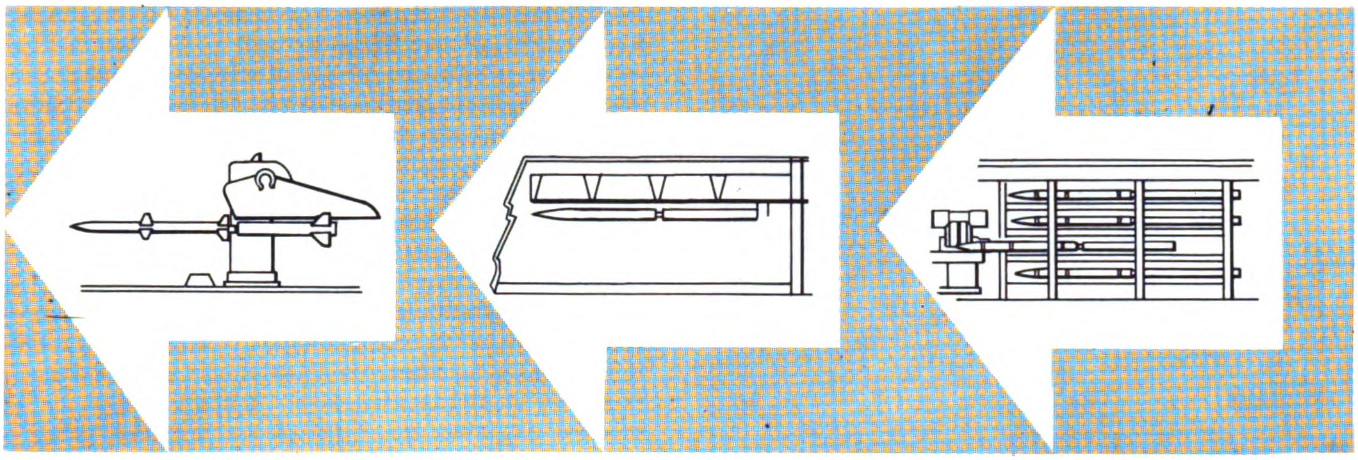
Pawl-Type Hoists. Pawl-type hoists are used for hoisting projectiles in big gun turrets. Each hoist consists of a built-up circular tube, a rack casing attached to the tube and enclosing a moving rack, a system of fixed pawls mounted in the tube, a hydraulic cylinder and piston with crosshead attached to the rack, an electric motor and pump with pipe system connected to the cylinder heads, a control mechanism, and a pivoted cradle and spanning tray mounted in a fulcrum at the top of the tube. The projectiles are raised, one stage at a time, by pawls attached to a movable lifting rack. The lifting rack is an assembly of connecting links and pawls enclosed in a casing designed to house and guide the rack in its working path. A crosshead from the rack is connected to the top of a piston rod which is driven by the hydraulic cylinder. The hydraulic cylinder is essentially an hydraulic ram. A full stroke of the piston within the cylinder is equal to one full stage of projectile lift. Five stages of lift are required to raise a projectile from the handling room to the gun compartment. A projectile serrated on a lifting pawl is raised one stage when the hydraulic piston makes a full stroke. Fixed pawls in the hoist tube at the top of each stage support the raised projectile when the piston retracts the lifting rack. These fixed pawls normally project into the tube, but retract as projectiles pass, and spring out below to support the raised projectile.

This one-stage-at-a-time hoisting process repeats until the projectile is at the top of the hoist. It must be removed before another cycle can begin.

With smaller mounts, like the 3"/50, 40-mm, and 20-mm hoists are relatively unimportant. Generally, their ammunition is stowed in ready service lockers nearby and is hard carried to the mounts, through hoists may be used (depending on the installation) to replenish supplies. In big gun turrets, the entire ammunition supply system, except the powder magazines, is inside the turret and rotates with it.

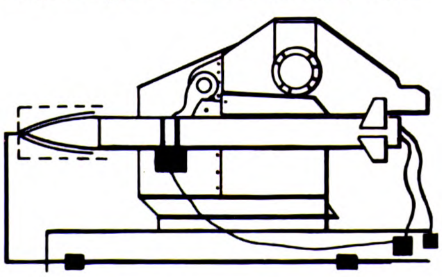


transfer of reaction missiles

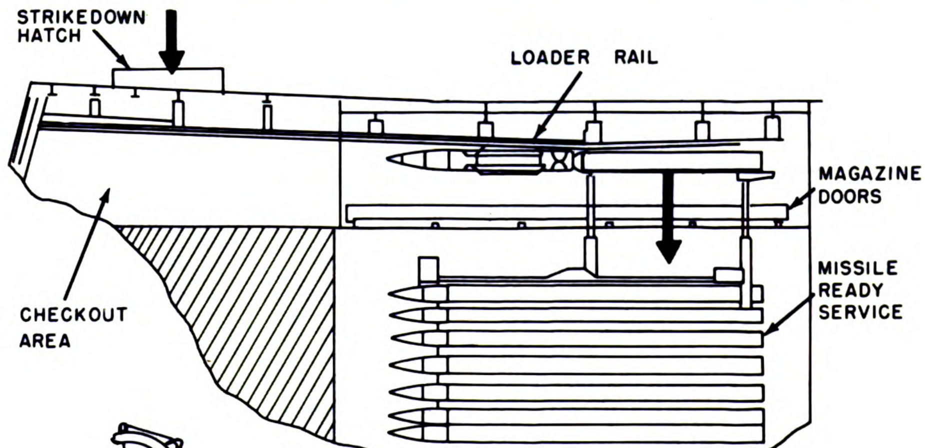


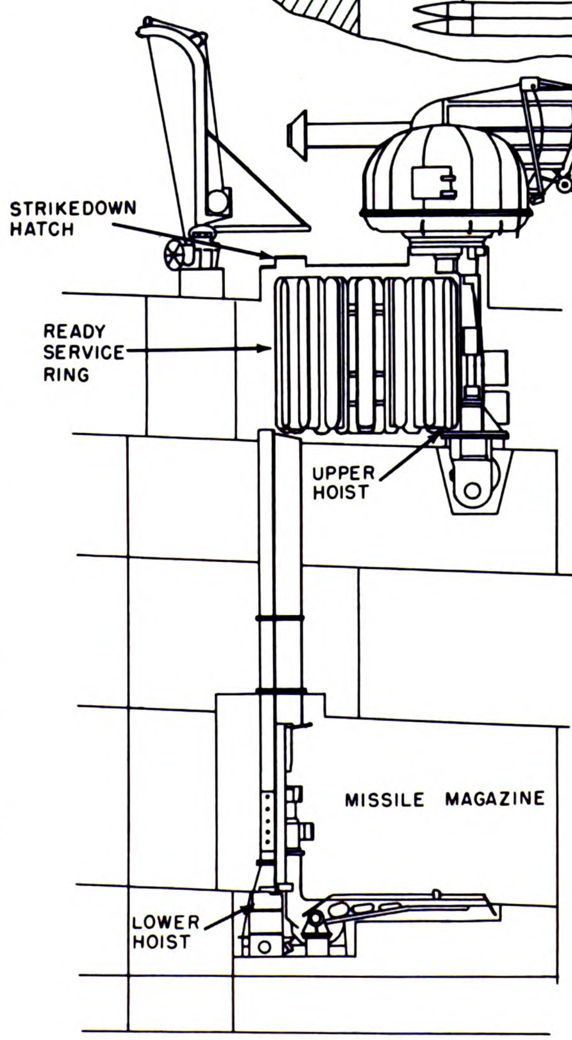
The main purpose of a missile transfer system is to move missiles from storage to the vicinity of the launcher at a speed and with a precision consistent with the rate of fire requirements of the launching system and the size, weight and slope limitations imposed by the missile itself. In addition to transferring missiles to the launcher, a transfer system is usually required to return safe, unfired missiles to storage, to strike down newly received missiles, and to provide missile flight preparation facilities.

Most reaction missiles (guided missiles, for example) are complex devices and require some degree of servicing, assembly and check before they are ready to fire. Therefore, missile checkout and assembly areas are often designated in the transfer system route between storage and launcher. They provide for wing and fin assembly, go no-go checks on various missile components and warmup power for the missile. When these services are not or cannot be performed in the transfer system they are performed on the launcher.



missile checkout on launcher





examples of missile strikedown

Most reaction missiles are large, and they take up a lot of space. Since storage space aboard ship is quite limited, it is an essential requirement that ships must be able to replenish their supply of missiles at sea. In most cases, reaction missiles can be transferred at sea with conventional rigs. When a missile has been received on deck, it is entered into the missile transfer system via a strikedown hatch and sent to a checkout area, ready service, or magazine storage.

The return of an unfired missile from the launcher to storage is performed in a similar manner, with the loading and transfer equipment simply operating in reverse.

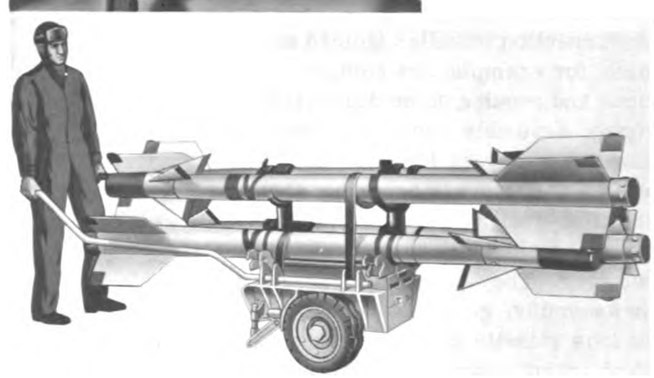
Because of the large size, weight, bulk, and fragile nature of most reaction missiles, their physical transfer requires great care and precision. A slow, positive means of transfer is necessary to prevent the missile from being bent, twisted or bumped. Even a small dent in a guided missile will render it inoperative or unsafe to use. Therefore, extreme caution must be exercised during missile transfer to avoid missile damage. As a result, rapid transfer of large reaction missiles is very difficult to achieve and transfer rates are usually slow.

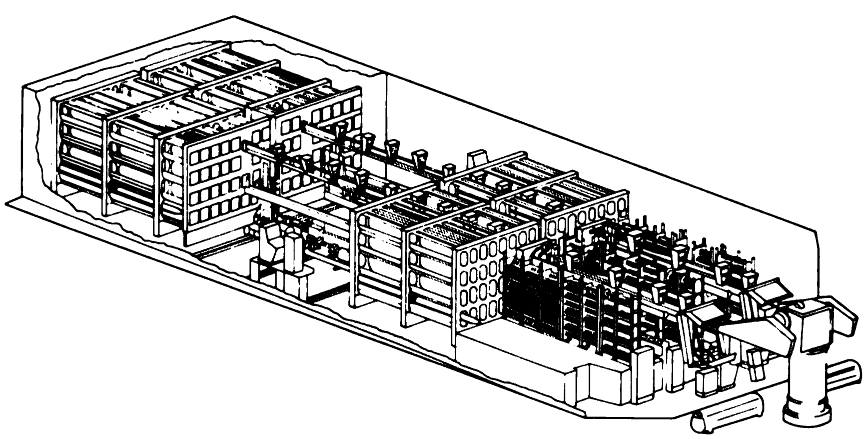
To compensate for the inherent slow rate of transfer of large missiles, many shipboard launching systems employ two missile transfer channels and a dual-rail launcher. Cumulative transfer rates are still fairly low, however, compared with those for naval guns.

To minimize transfer time, and to reduce the amount of transfer equipment required (thus saving space and weight), missiles may be stored in ready service rings directly adjacent to the launcher. Transfer then involves, essentially, the moving of missiles into the ready rings. This sort of scheme can be employed for the smaller reaction-propelled missiles when manual assembly is not required and when missile checkout can be accomplished on the launcher. Transfer equipment may be provided to replenish the ready service rings from magazines, or the ready service rings may contain the total supply of missiles, when storage space in the delivery vehicle is especially limited.

The transfer of reaction missiles from storage to the vicinity of the launcher can be as simple an operation as moving an airborne missile on a cart from the storage magazine, via elevators, to the launcher mounted on the aircraft, or it can be as complicated as a system which provides for the automatic transfer of large guided missiles to a dual-rail launcher.

In this system, a hoist elevates the missiles one at a time to the loader rail. Each missile is transported on the loader rail to the assembly area, where final missile servicing and flight preparations are accomplished. At the end of the loading rail, the missile enters the loading phase, which will be discussed in the next section.

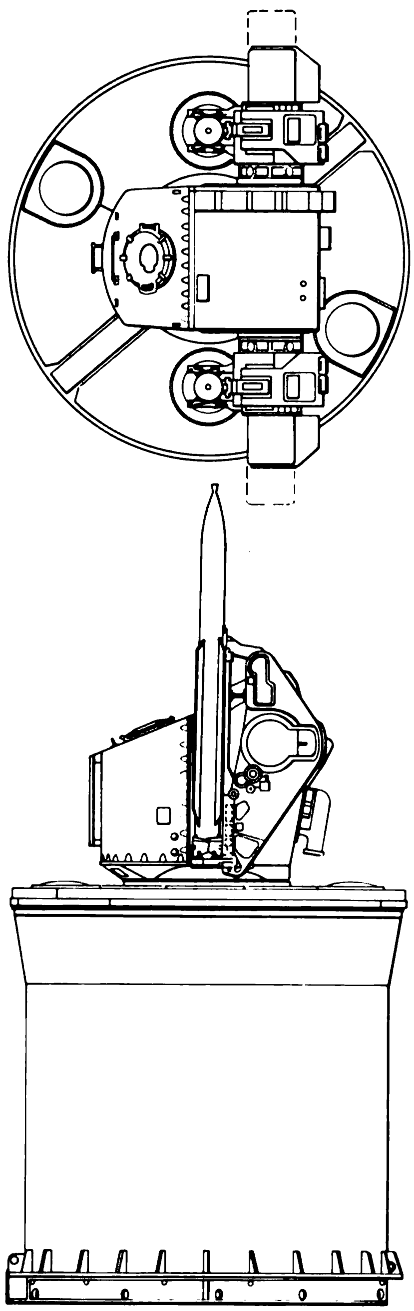


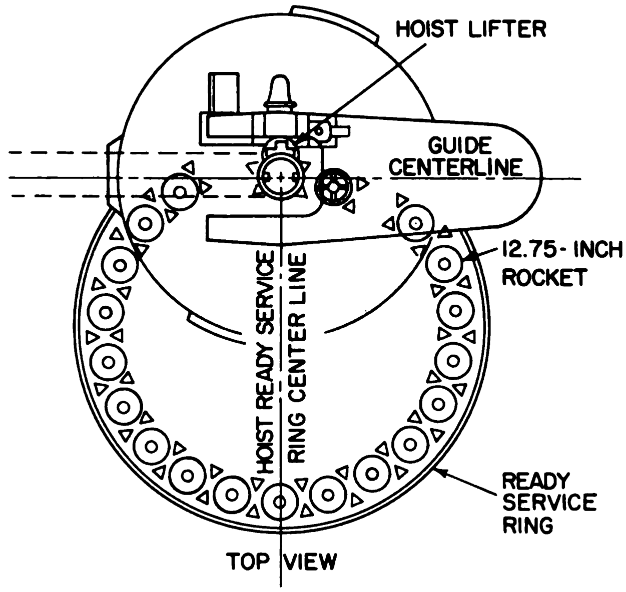


Because of the difficult handling problems inherent with larger missiles, it is usually not practical to transfer them while the launcher is rotating. Thus, the transfer and loading systems are integrated with respect to the ship and to one another. To achieve ready load conditions the launcher must return to a given position in train and elevation before receiving a missile. The resultant reduction in rate of fire must be accepted in order to insure safe and efficient delivery of the missile to the launcher.

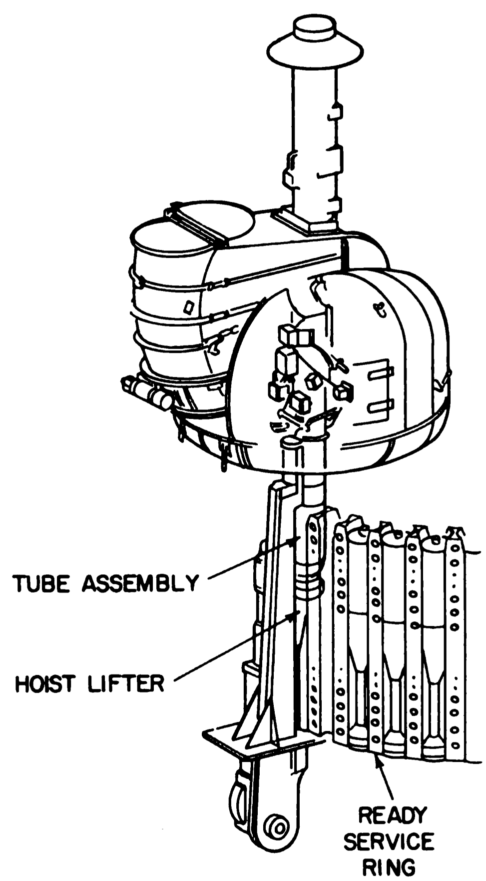
When missiles are small enough (usually those with integral propulsion systems) loading may be conducted to a continously rotating launcher by making the line of missile transfer coincident with the train axis of the launcher, and thus independent of launcher train. In systems of this type, missile transfer is completed when the missiles are deposited in the ready service ring. The next phase is to load them, by ramming, into the launcher tube or rail.

The ready service ring steps around automatically each time a missile is loaded, much like a revolver, thereby maintaining a steady flow of missiles available to the launcher until the ring is emptied. The ring must then be replenished by the transfer system from magazine storage or from some external source of supply. launching systems of this type have higher rates of fire than comparable systems where the launcher must return to a fixed position for loading.





TOP VIEW



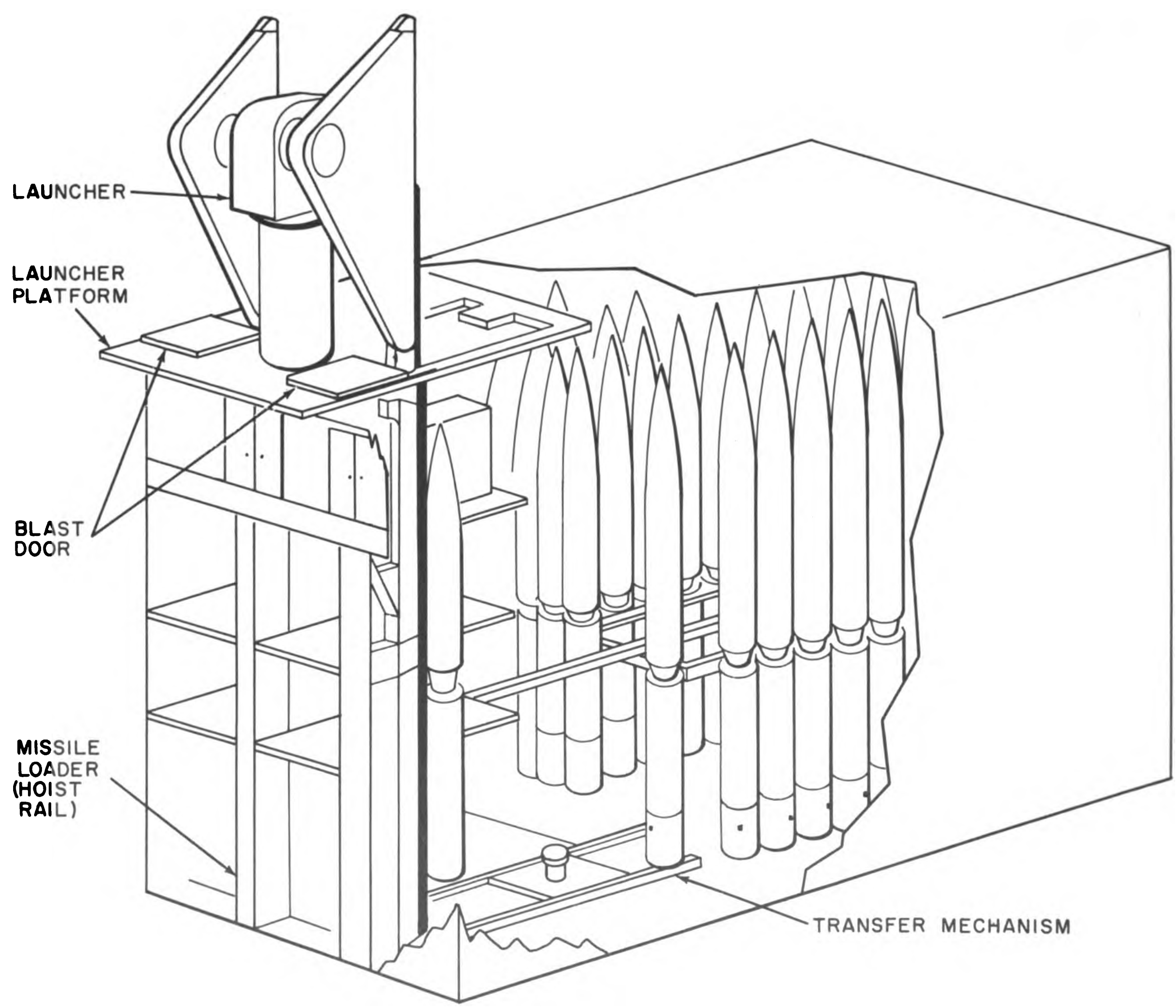
SIDE VIEW

LOADING SYSTEMS

Missile loading equipment is needed to place the missile in firing position on the launcher in a fast, reliable, safe manner. The loading operation consists of moving the missile from the transfer equipment to the launcher, and positioning it on the launcher. This operation involves a transfer to the launcher and the moving or ramming of a missile along rails or trays into firing position on the launcher. The transfer and ramming functions can be performed manually, mechanically with human operator control, or automatically with human monitoring. In some types of fixed launchers with low rates of fire, the ramming operation is not necessary. The missile is simply hoisted or lowered into position on the launcher. However, loading high-rate-of-fire launchers requires a fast, precise ramming cycle.

Some means must be provided for the unloading of unfired but usable missiles and for the jettisoning of unsafe and dud missiles. The loading equipment, in most cases, is designed to accomplish these functions when it is not desirable that they be accomplished by hand. In some cases, special equipment must be provided to perform the jettisoning function.

To achieve rapid weapon employment and to maintain a high rate of fire, the loading rate must be commensurate with the required launcher rate of fire. As with transfer equipment, this often requires the use of more than one loading unit per launching tube or rail or more than one line or channel feeding missiles to the ramming equipment.



loading of gun ammunition

Transfer equipment functions to bring the ammunition to the vicinity of the gun. The ammunition must then be placed in firing position in the gun. This operation is performed by the loading system. The loading system must provide for the following general functions:

a. Moving the ammunition from the hoist to the gun.
b. Ramming the projectile and propellant into the chamber of the gun.

Although not directly related to the basic purpose of a loading system, the following operations are normally associated with it.

a. Closing and opening the breech mechanism.
b. Removal and disposal of the empty cartridge case.

With heavy, semifixed ammunition, separate projectile and power cradles and transfer trays are often used. When loaded, the projectile and powder cradles swing into alinement with their respective trays (which remain parallel to the gun bore), and shift the ammunition units into the transfer trays. These trays then transfer the ammunition into alinement with the gun bore and remain there during ramming. The automatic loading equipment thus performs the second function of a loading system.

Automatic loading systems have been successfully designed and operated. In practice, they double and even triple the rate of fire of the gun. For example, the 5″/54 with automatic loading has a rate of fire of 40 rounds/barrel/minute as compared to 15 rounds/barrel/minute for the semiautomatic 5″/54. The penalty for the increased rate of fire is reduced reliability, increased weight, and increased maintenance requirements.

The method of loading a gun, along with breech operation and cartridge case ejection, is often used as a means of classifying guns as to degree of automatic operation. The broad classifications in general use are: 1) automatic guns, 2) semiautomatic guns, 3) nonautomatic guns, 4) rapid fire guns.

The broad classifications in general use are:

AUTOMATIC GUNS • SEMIAUTOMATIC GUNS • NONAUTOMATIC GUNS • RAPID FIRE GUNS

automatic guns

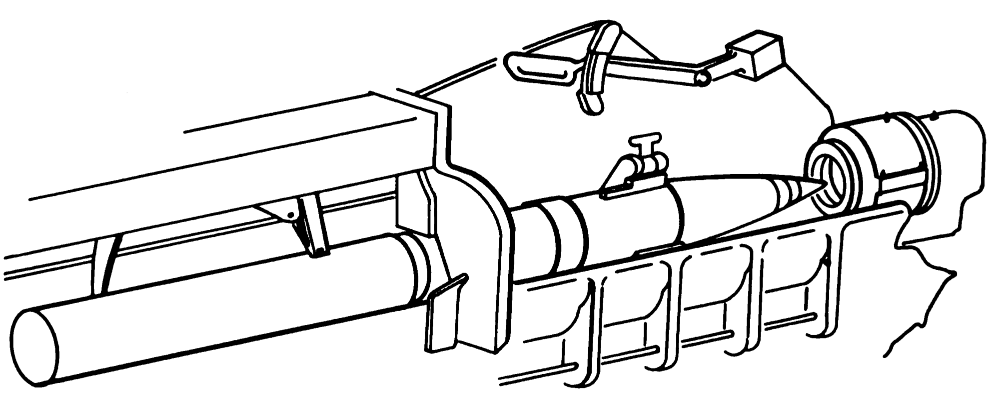
Automatic guns are those using case ammunition in which some of the energy of the propellant explosion is used to open the breech, eject the empty case and operate a device which automatically loads another round of ammunition. An automatic gun can continue to fire so long as ammunition is supplied and the trigger is operated. An example of an automatic gun is the 5″/54 Gun Mount Mk 42.

semiautomatic guns

Semiautomatic guns are case guns in which some of the energy of the propellent explosion is used to open the breech, eject the empty case, and automatically close the breech when another round is loaded. Semiautomatic guns, unlike automatic guns, must be loaded either by hand or by auxiliary equipment. An example of a semiautomatic gun is the 5″/38.

nonautomatic guns

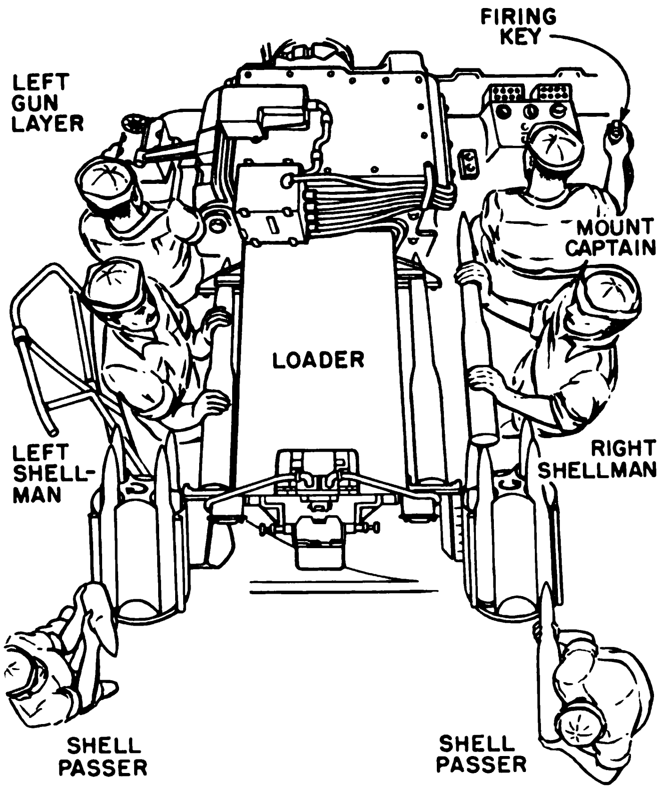
Nonautomatic guns are those in which none of the energy of the propellant is used to perform breech opening, closing, or loading functions. All guns using bag ammunition are of this type.



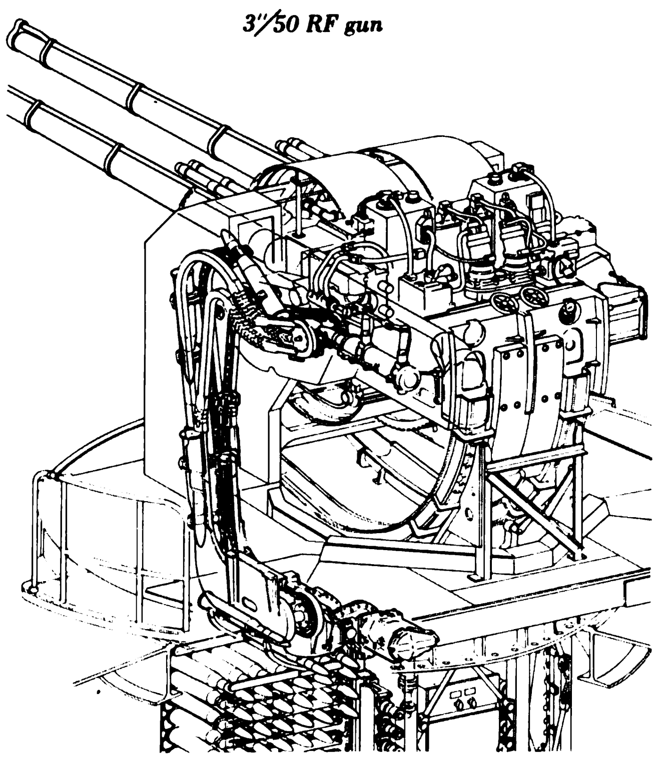
fuze setter in a 6″/47 gun

rapid fire guns

Rapid-fire (RF) guns are automatic guns in which loading, firing, empty-case ejection, and breech operation are performed automatically but are powered by a source of energy other than the propelling charge. The 3"/50 Rapid-Fire Gun is an example of this type of weapon.



3"/50 RF gun



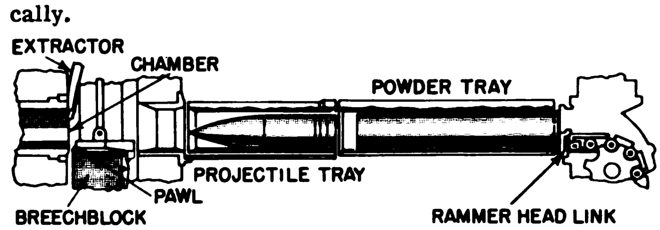
3"/70 automatic gun

New rapid-fire 3-inch and 8-inch mounts and turrets include a great deal of almost entirely automatic ammunition-handling gear. In the turrets, this equipment transfers the ammunition from the hoist to the slide, rams it into the chamber, and then disposes of the empty cartridge cases after firing. In the 3-inch gun, as in 20-mm and 40-mm machine guns, the ammunition is loaded manually into a loading device on the slide of the gun, and the ammunition is handled automatically from that point.

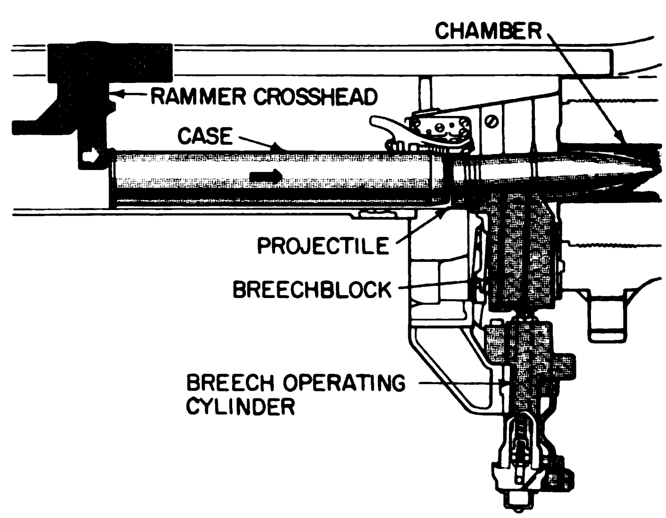
To increase the rate of fire, automatic equipment has been developed to transfer ammunition from hoist to gun for 3-inch, 5-inch, and 8-inch guns. These systems are quite complex because they must transfer ammunition at a high rate of speed regardless of the angle of elevation of the gun. Most of these systems operate in a manner similar to that of the 5"/54 Mount Mk 42. A rotating cradle first positions itself in line with the hoist and receives a round of ammunition. It then swings until it is parallel to a transfer tray on the gun and shifts the round onto the tray. From the transfer tray, the round is either rammed directly into the chamber or it is moved to a loading tray and then rammed home. This method can be used for semifixed as well as fixed ammunition.

ramming of gun ammunition

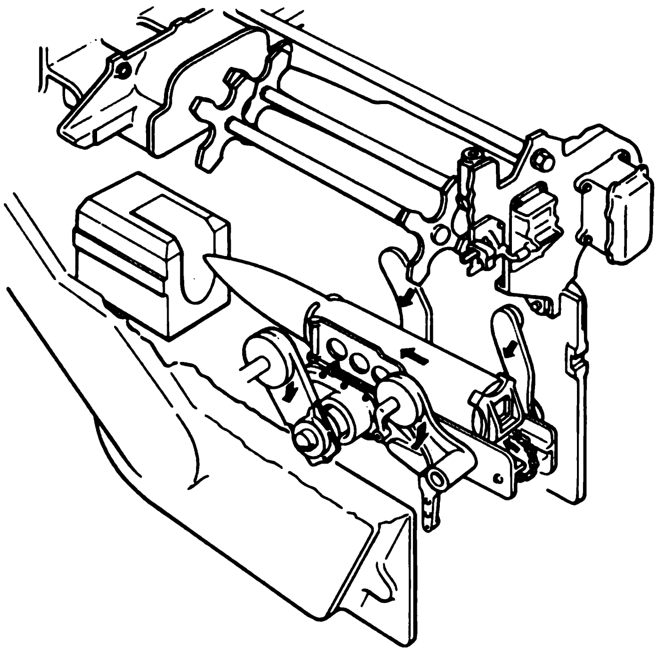
When gun ammunition is positioned in line with the gun bore, ramming takes place. The most common type of rammer consists of an hydraulically operated shoe or spade, which merely pushes the case ammunition along a long loading tray into the chamber of the gun. When the cartridge case enters the chamber, it releases the breech mechanism and the breechblock rises automatically.



8"/55 RF gun



6"/47 dual-purpose gun



3"/50 RF gun

Although the details of operation differ from one type of bag turret to another, all of them work on the same principle. A rotary hydraulic motor drives a sprocket which engages the links in a rammer chain. This is somewhat like an exaggerated bicycle chain, except that the straight chain will bend in only one direction.

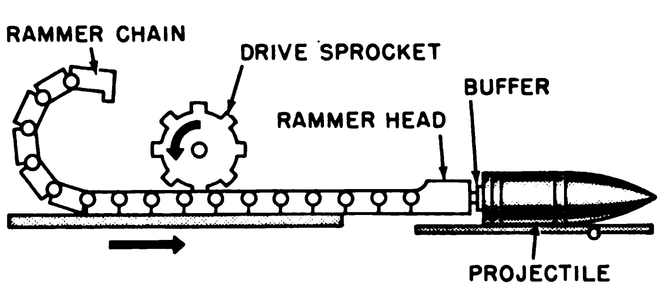
At the end of the chain is a buffer to protect the ammunition component being rammed. In bag-gun turret installations, the rammer operation is always under manual control, generally by regulating pump output to the hydraulic motor that drives the chain. The ramming operation requires two ramming strokes. In the first, the projectile is rammed home in a full maximum-thrust stroke, to insure that the rotating band engages the rifling. Then the rammer is retracted, and the powder bags are rammed much less forcibly in a second stroke. After the second retraction the breech can be closed two strokes are necessary because the maximum-thrust stroke needed for the projectile would damage the powder bags.

The tray which holds the ammunition units in line with the gun bore during ramming is collapsible and is attached to the turret, not the gun. This means that the gun must return to a given elevation angle before the spanning tray can be extended and the ammunition loaded onto it. After ramming, the breech plug is closed by manual control, not automatically.

It should be apparent that the loading of a bag gun is a relatively slow operation and limits the rate of fire to only a few rounds a minute. This, of course, is one reason why bag guns have been replaced with case guns which can be loaded more rapidly. However, space and weight restrictions and technical knowledge have prevented development of case guns larger than 8 inches.

Guns using fixed ammunitation can speed up the ramming operation by catapulting the round into the chamber. A round is received into a transfer tray which is automatically lowered into alinement with the gun bore. The tray then swings forward as the rammer chain pushes the cartridge forward. When the transfer tray reaches the end of its forward travel, it releases the cartridge, catapulting it into the chamber. The breechblock is released and rises automatically before the cartridge has a chance to rebound.

Bag-type turret guns have long chambers to accommodate (in 16-inch guns, for instance) up to six powder bags plus the projectile. This means that the rammer stroke must be very long. The length of a single hydraulic cylinder for such a rammer would be prohibitive. Such turrets are therefore equipped with chain-type rammers. Some rapid-fire guns also employ chain-type rammers when the design of the ammunition transfer equipment and loading platform preclude the use of a spade-type rammer.



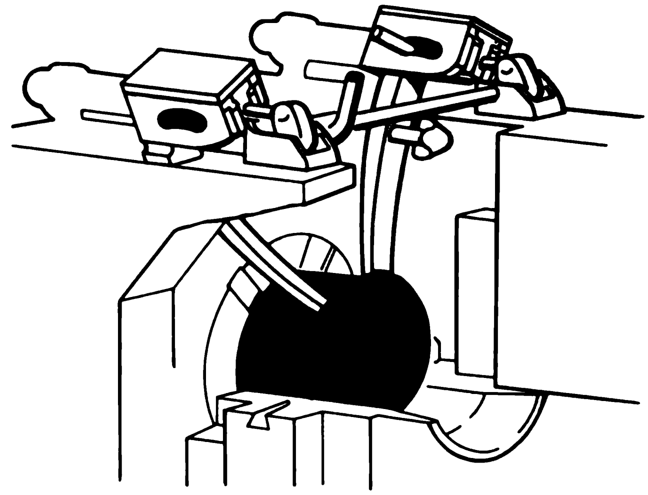
breech operation

Closing and opening of the breech can be accomplished mechanically, under manual control, and automatically. For high rates of fire, automatic operation is necessary. As previously described, most sliding-wedge-type breechblocks are designed to close automatically when the cartridge case is in the chamber. After firing, the force of counterrecoil is often used to lower the breechblock, or this can be accomplished by an external power source. Coupled with the need for automatic breechblock operation is the requirement for rapid removal and disposal of the empty cartridge case.

case removal and disposal

Cases must be removed or extracted as rapidly as possible so that the next round can be loaded into the chamber. Disposal of the case well clear of the gun is of considerable importance to guns with a high rate of fire because the cases will quickly pile up around the gun and obstruct its operation.

Empty cases are removed by extractors which operate independently or are coupled to the breechblock. Cartridge cases are usually extracted with sufficient force to eject them aft with considerable speed. For low-rate-of-fire guns, a hot shellman can catch the ejected case and toss it aside. For high-rate-of-fire guns this is not feasible, and some means of controlling the ejected cartridge is necessary. This is often accomplished by curved chutes which guide the moving case and expel it from the front of the gun mount or turret.

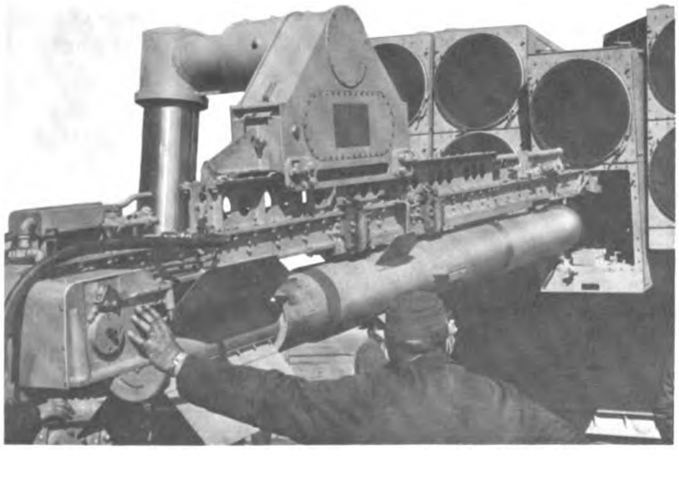


8" / 55 RF gun

loading reaction missiles

To achieve high rates of fire, it would be desirable to be able to load the launcher continuously at any angle of train or elevation. Because of the configuration and construction of large reaction missiles, however, it is very difficult to accomplish this without damage to the missile. Therefore, most loading systems for large missiles are fixed and require that the launcher return to a predetermined angle of elevation for loading. The loading angle may be vertical, horizontal, or near the horizontal. This loading at a prescribed angle of elevation is usually coupled with loading at a preset position in train, and the net effect is to restrict missile loading to fairly low rates. Those launchers that handle the smaller reaction missiles are often capable of loading at any angle of train and have, in general, much higher rates of loading and firing.

The size and weight of many reaction missiles require the use of manually controlled mechanical equipment to effect the positioning of the missile on the launcher, as in the ASROC loading equipment shown.



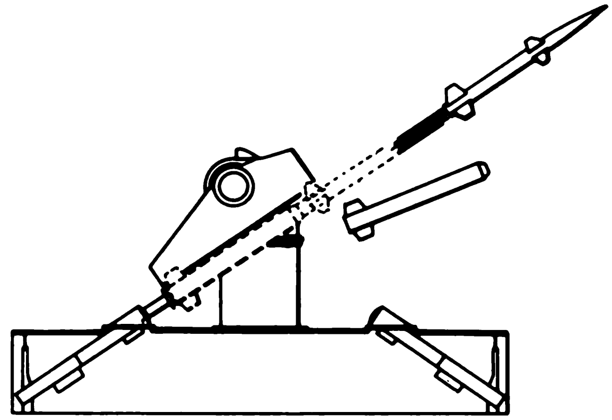
Here, the missile is supported by a loader and carefully rammed into position on the launcher rail. The loader is then positioned in line with the next launcher rail, and the loading cycle is repeated, until all launcher rails are filled. This is a slow process and is suitable only for loading launchers that are not expected to engage a large number of targets in a short period of time.

Launching systems employing automatic transfer and loading equipment are usually concerned with engaging multiple targets and are designed to maintain a sustained rate of fire. As previously indicated, it is impractical at this time to build a system capable of continuous loading of large reaction-powered missiles. Therefore, the launcher must return to a loading position (in train and elevation) before it can receive a missile.

Loading systems for smaller reaction missiles are often capable of higher loading rates. Although they service the launcher at a fixed angle of elevation, the missiles can be loaded at any angle of launcher train. This is possible when the loader axis and the launcher train axis are coincident, as illustrated and described in the preceding section on transfer systems.

A reaction missile may remain on the launcher at the conclusion of firing or as a result of a loading practice. Because these missiles are very expensive and because the supply in storage is so limited, the missile must be unloaded if it is in a safe, usable condition. Unloading the missile involves operating the transfer system in reverse and returning the missile to the transfer system or to ready service storage. Often, it is not safe to unload the missile when it is a dud or when the tactical situation does not allow for the time required for unloading.

In such cases, the missile is jettisoned clear of the delivery vehicle. Missiles can be jettisoned from aircraft by dropping them in a safe area. In jettisoning a missile from a ship, it is necessary that it be ejected well clear of the ship.
Small to medium sized missiles may be ejected by an impulse from a pneumatic rammer.
With large missiles, the thrust from an auxiliary rocket unit is often needed to propel the missile safely clear of the ship.



COMMUNICATIONS AND CONTROL

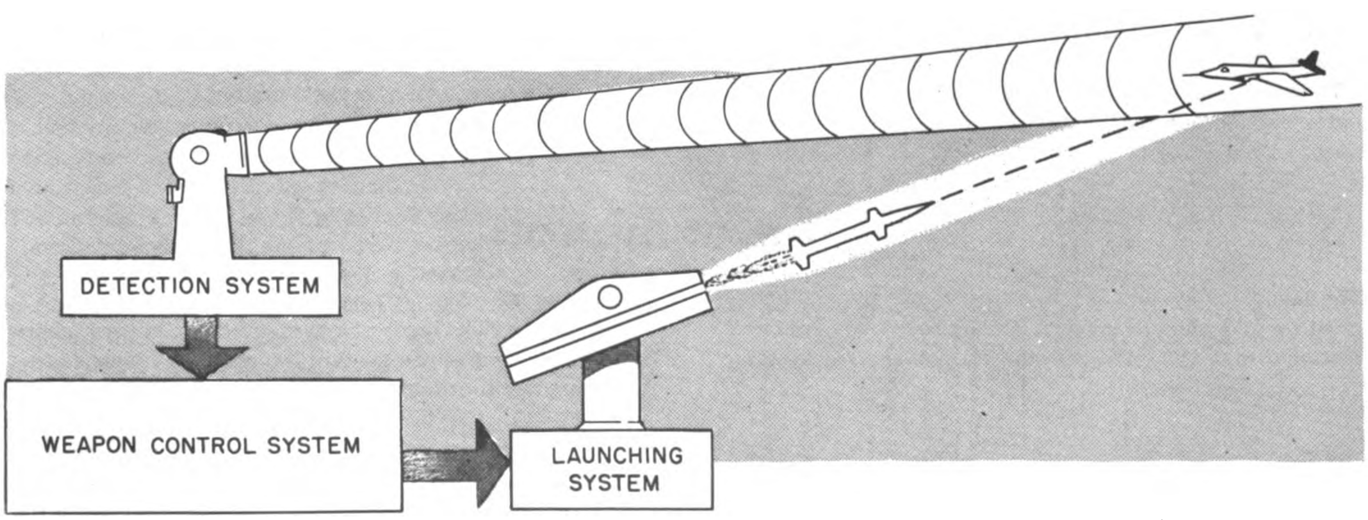
In a typical naval weapon system, a detection system determines the presence of a target, gathers information relative to its position and motion, and transmits this information to the weapon control system. In the weapon control system, this data is processed, weapon control orders are generated, and the orders are transmitted to the launching system (and to the missile). The launching system and the missile operate in response to orders from the weapon control system and/or to orders from human operators at control stations.

Thus for a weapon system to function effectively, there must be :

a. A flow of orders and information between stations within the system.

b. Equipment to receive, transmit, and respond, in a predetermined manner, to the orders and information.

In general, the launching system is subordinate in character and must respond to certain types of orders, such as what to launch, where to launch, and when to launch.



selection of a missile

A launching system will often have several types of missiles stored within its confines, so it must receive orders regarding the type or types of missiles to be launched. For example, in a shipboard gun launching system several types of projectiles are in the magazines. Control station personnel must select the type of projectile to be fired and notify the launching system of its choice. In a shipboard guided missile launching system, both homing and beam-riding missiles may be available. The launching system must receive orders regarding which type of missile to launch. In addition to the type of missile, the launching system must know how many missiles to launch. For example, a submarine launching system may fire one, two, or a whole spread of torpedoes. A guided missile launching system may fire a one-/or two-round salvo and a gun launching system may fire continuously until a target is destroyed or out of range.

flight orientation

In order to launch a missile into a desired flight path, its initial flight orientation must be controlled by the launching system. Therefore, the weapon control system generates missile orientation orders to which the launching system must respond. In cases where the launcher is rigidly fixed to the delivery vehicle, the whole vehicle must respond to the orientation orders.

The launching system must respond to orders in a rapid, accurate, predictable manner. An uncontrolled missile, such as a projectile, must have very accurate directional control exerted on it during its motion on the launcher because this is the guided phase of flight and the velocity vector of the missile at departure from the launcher determines the trajectory it will follow.

Controlled missiles must also be launched with minimum dispersion. For example, a beam-riding guided missile must be launched in such a direction that it intercepts a moving radar beam. A homing missile must be launched so that it will be in a position to acquire and lock on a target. Thus, as a general rule, control of the launcher orientation in space is required.

When the launcher is fixed to the delivery vehicle, the vehicle must maneuver, using its steering system, to provide the proper vector for the missile at launch. This method is employed mainly with fairly small, maneuverable vehicles, such as aircraft and PT boats. The slow response of the pilot or helmsman and the lack of precision of the operation make this type of delivery most suitable for missiles with loose initial orientation requirements, such as guided missiles, short range uncontrolled missiles, and torpedoes.

When a launching system is mounted in a vehicle that is not small or maneuverable, some means must be provided to orient the launcher independently of vehicle motion. A source of power must be connected to the launcher and so controlled that it will position the launcher along a line in space in response to orders form the weapons control system. In spite of vehicle motion (course, speed, yaw, roll, pitch) the launcher must be positioned in space so that the missile, when launched, will follow the desired trajectory to the target. Thus, the launcher line of orientation must be isolated from vehicle motion. A logical device to use for this task is an inertial sensor, such as a gyro. Applications of gyros in the weapon control system will be examined later.

A rotating launcher for uncontrolled missiles must orient the launcher line very accurately in response to orders from the weapon control system. This means that the dynamic response of the launcher drive to orientation signals must be fast, precise, and stable. The dynamic response characteristics of the launcher drive must be such that the launcher can overcome the motion of the delivery vehicle and can be positioned properly regardless of target motion.

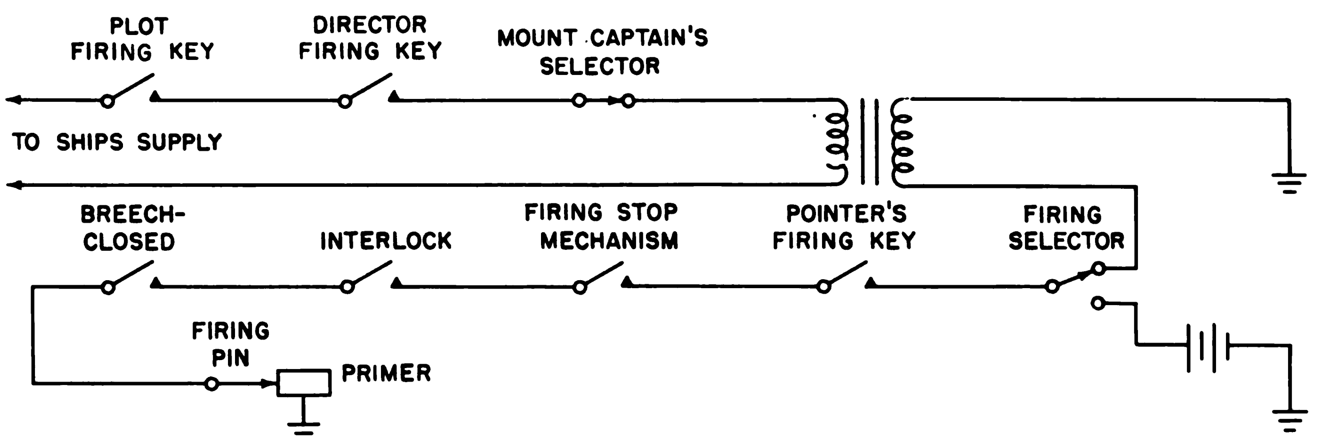
time of launch

The launching system is subordinate in character and must be told when to launch. Personnel at control stations, in conjunction with the weapon control system, determine and order when to commence and when to cease firing. The launching system must, in turn, keep the rest of the weapon system informed of response to weapon control orders and readiness to launch.

SAFETY

Launching systems must provide for the safety of operating personnel and for protection of the vehicle in which they are installed. A great number of safety features are included in the various components which make up the entire system.

gun launching systems



Modern naval guns are designed primarily for electrical firing. The illustration shows schematically the elements found in a typical electrical firing system for a gun mount or turret. The diagram does not show the physical appearance nor the locations of these elements.

Under normal service conditions, guns are fired by using the ship's 115-volt AC supply. One firing key (a spring-loaded switch which is normally open but may be latched in the closed position) is located in the plotting room. Another firing key is in the director.

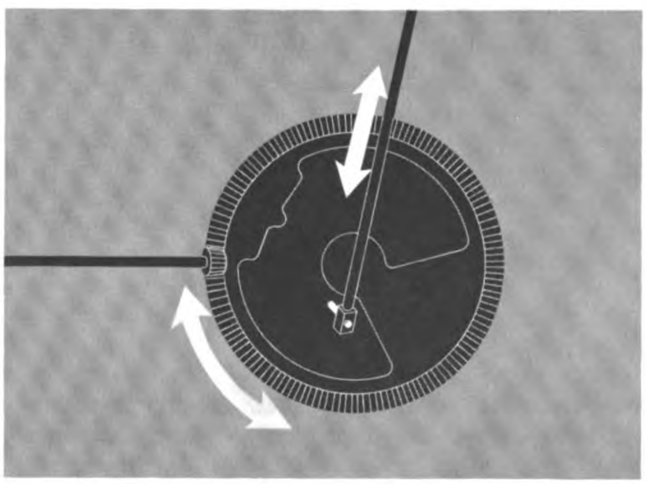
In some mounts, the mount captain has a selector which can be used to cut out one or both of the guns of a twin mount from the firing circuit.

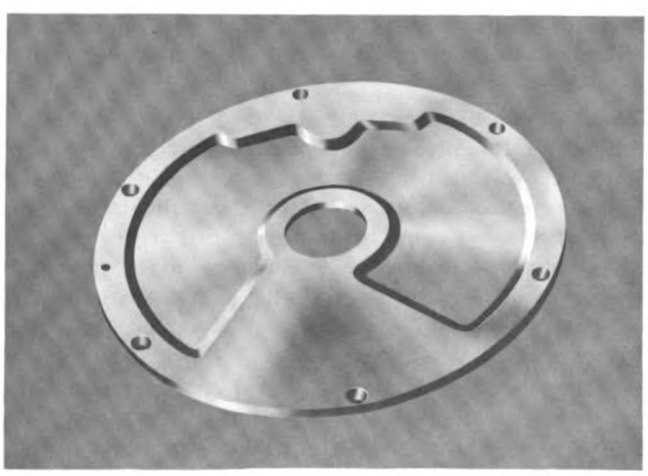
The pointer's firing key is generally on one of the elevating handwheels, and is connected to the circuit by a flexible cable. The firing stop mechanism switch is part of the firing stop mechanism (to be described later). It opens the firing circuit when the gun endangers part of the ship's structure. Some mounts have no interlock switch or relay, but such interlocks are common with automatic loading equipment or hydraulically operated breech mechanisms. The one shown in the schematic may represent up to six or more, each of which registers that a certain mechanism or part is in a position that is safe for firing. The breech-closed contact is a common variety of interlock. In addition (and not shown in the schematic) are the safety devices in breech and firing mechanisms which prevent contact between primer and firing pin when the breech is not fully closed or the gun is not in battery.

The last part of the circuit is the firing pin's contact with the electric primer. The circuit is completed through the filament in the primer of the cartridge case. Note the emphasis of safety. All the switches and keys are in series. Any link in the circuit can break the entire circuit if conditions are unsafe at that point. Yet the mount is capable of firing under local control if the remote system has failed.

firing stop mechanisms

At any considerable range, the axis of a gun's barrel deviates from the line of sight to the target. The greater the range and/or relative motion, the greater the deviation. It is possible for the pointer or trainer to have a clear field of view even though the gun barrel is directed at some part of the ship structure.
It is therefore necessary either to prevent alinement of the gun bore with the ship's structure or to prevent the gun from firing under these conditions. The latter method is more common.



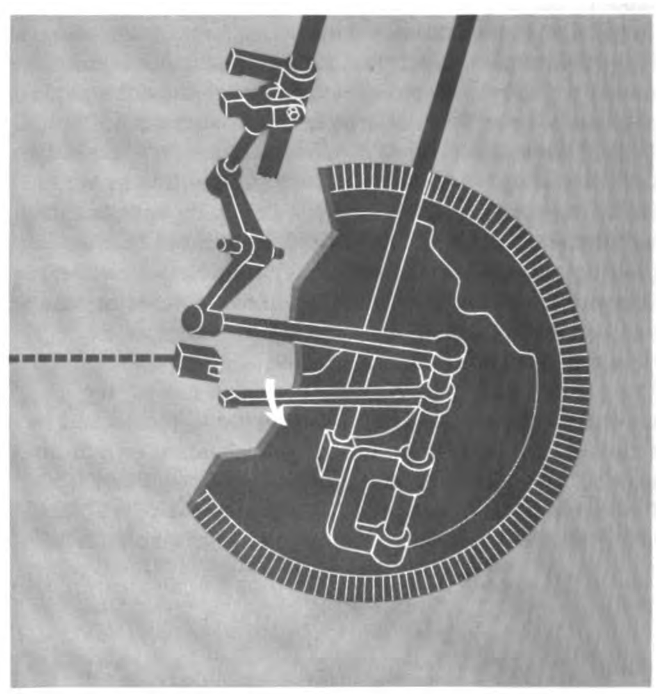


This is the function of the firing stop mechanism, which disables the firing system when the gun is so aimed as to endager the ship it is mounted on.

Firing stop mechanisms are designed to interrupt the mechanical and electrical firing circuits whenever the guns are trained or elevated to a position where firing would endanger ship's personnel or damage the ship. They should not be confused with the frameworks of steel tubing or depression-stop cams that are used to limit the movement of some light machine guns to safe zones of fire. Firing stop mechanisms do not interfere with the free movement of the gun; this is done by the train and elevation limit stops.

The importance of firing stop mechanism layout, adjustment, and maintenance cannot be overemphasized. Many casualties caused by a ship's guns firing into her own structure have been traced to neglect of firing stop mechanisms or have resulted from somebody's deliberate bypassing of these mechanisms. Every one of these casualties could have been prevented if the firing stop mechanism had functioned properly.

The basic firing stop mechanism is essentially a plate or disc type cam in which the inputs are gun train (which rotates the cam) and gun elevation (which moves the cam follower approximately radially across the cam).



The elevation input shaft moves toward the edge when it depresses. At the end of the elevation input shaft is a spring-loaded plunger which contacts the cam plate. Each point on the surface of the cam plate corresponds to a specific position (in train and elevation) of the gun barrel; the complete surface is a profile map of the gun's safe and unsafe firing areas.
When the gun mount is installed, part of the cam plate surface is cut away so that when the gun is in a position where it is safe to fire, the spring-loaded cam follower plunger will be depressed, while in unsafe positions of the gun (i.e., aimed at the ship) the plunger will be forced upward. The upward movement of the plunger interrupts both the electrical and percussion firing circuits. A switch opens the electrical firing circuit and a mechanical clutch disconnects the percussion firing linkage (if there is one).

gas ejectors

When a shot is fired from a gun, the bore is filled with residual powder gas. The gas is unsafe to breathe and is likely to be either combustible or actually burning. It is sometimes capable of spontaneous combustion when mixed with air. The gas ejector, which is installed in every 5-inch and larger enclosed mount, forces this residual gas out of the bore by air blast.
When a gas ejector fails, the gun can continue firing, but caution is necessary to ensure safety. The rate of fire may have to be reduced.
In case guns, the gas ejector is designed to open and shut off automatically during normal operation, though it can be operated manually. In bag guns, the gas ejector goes on automatically when the breech plug opens, but must be shut off manually.
In bag guns this practice is normally carried out by one of the gun crew after "Bore Clear" has been given. A member of the gun crew sings this out after he has assured himself by observation that there are no flaming gases, powder fragments, or pieces of bag remaining in the bore of the gun, or on the face of the plug.

salvo latches

A salvo latch is a device that locks the breech closed. It can be opened only by deliberate effort. Its function is to prevent accidental manual opening of the breech in the event of misfire (i.e., an apparently unsuccessful attempt to fire).
Salvo latches are part of the breech mechanisms of all guns larger than 40-mm, except for the very newest designs of automatically loaded guns like the 8-inch case gun used in Salem class turrets.
The salvo latch is a positive lock cammed to open automatically during recoil. It will not open automatically if the gun does not recoil.

safety links

The safety link is a metal strip that couples the breech yoke (in bag guns) or housing (in case guns) to the slide. It is intended to hold the gun in battery in the event of failure of the counterrecoil mechanism, or if the counterrecoil mechanism is disabled. It is used in guns equipped with hydropneumatic counterrecoil systems.
If the gun is fired with the safety link engaged, the link will part. However, it is part of the normal gun operating procedure to disconnect and stow the link before firing. The link must be replaced when the mount is secured.

ammunition hoists

Noteworthy safety features of ammunition hoists include:

1) Automatic hoists have doors or gates which will permit them to start up only after the ammunition item has been completely inserted into the hoist and the loader's hands have been withdrawn.

2) Automatic hoists will not start automatically when loaded if the top level discharge point is occupied with ammunition.

3) All hoists in which powder bags are handled are equipped with flametight doors and interlocks to prevent an open flame path between lower handling room and gun deck.

4) Most power-operated ammunition hoists are equipped for manual operation in event of power failure.

5) Ammunition hoists are equipped with hydraulically actuated brakes or hydraulic locking to prevent loaded flights or cars from falling or drifting down the hoistway.

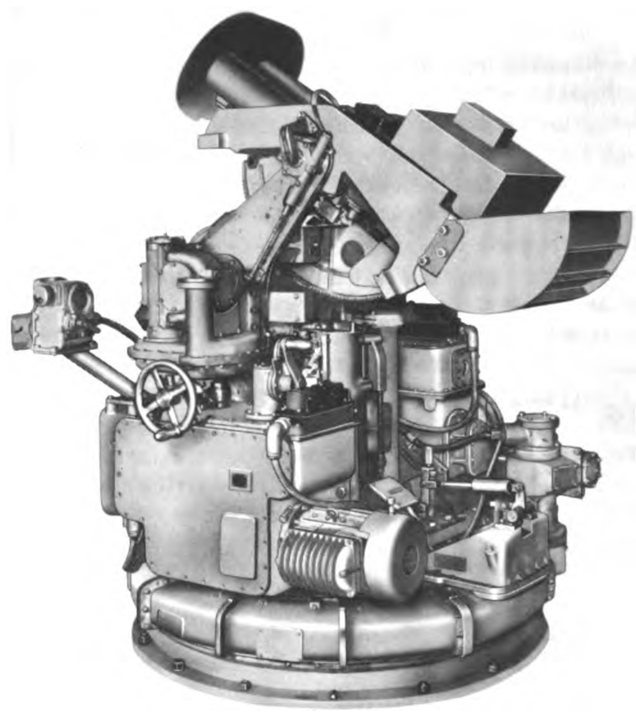
6) Most hoists are equipped with indicators to show that an ammunition item is present at the receiving end of the hoist.

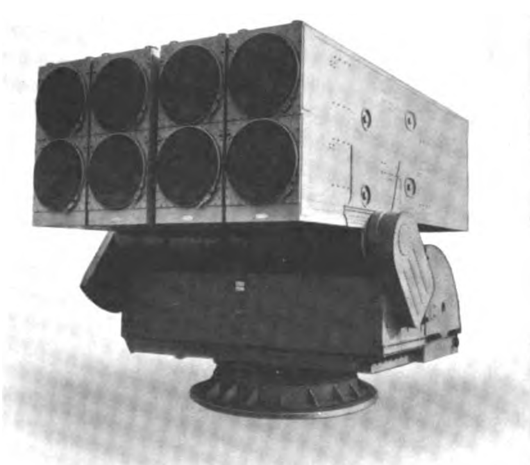
reaction missile launching systems

The exhaust stream from a reaction-propelled missile produces a considerable hazard to personnel and equipment. The exhaust from a rocket motor can cause blast damage and severely erode objects in its path. The exhaust stream consists of high-temperature, high-velocity gases accompanied by bits of solid matter, such as closure discs, igniter wires, and powder fragments. Protective measures must be taken to insure the safety of the launcher, the delivery vehicle, and operating personnel.

launcher safety

Common methods of launcher protection have been previously discussed in this chapter. Briefly, they consist of designing the launcher with materials and configurations that resist or nullify the blast and erosive effects of the rocket exhaust.





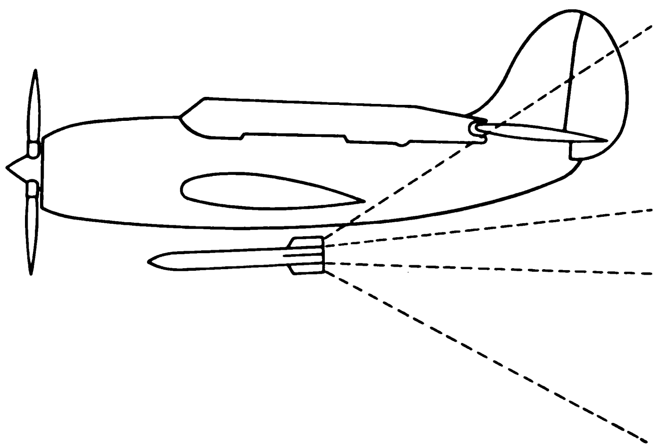
The ASROC Launcher pictured provides protective ready service storage for the missiles until they are fired by sealing both ends of each missile housing. Before the missile is fired, the muzzle doors are opened. When the missile rocket motor is ignited, the exhaust gases blast out the protective covering at the rear.

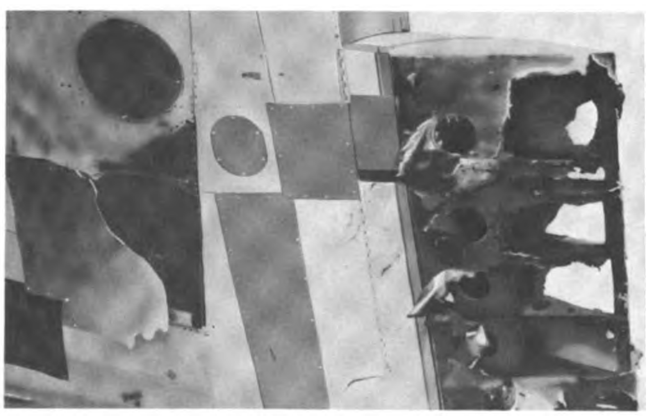
A common example is the use of blast shields on the muzzle of a launcher to deflect the exhaust gases of the departing missile. Also, if the rocket motor and exhaust gases are confined during any portion of missile travel on the launcher, some means must be provided for venting the gases before damage to missile and launcher can occur.

The method used in the Weapon "A" launcher shown is interesting because it not only provides an exit path for the exhaust gases, but also deflects them upward, away from the ship's structure. This allows the launcher to be employed in confined areas aboard ship. A large, clear deck area surrounding the launcher is not necessary, nor is the protection of adjacent parts of the ship's structure required.

vehicle safety

The protection of the delivery vehicle from rocket exhaust damage involves the design and location of the launcher on the vehicle, as well as the maintenance of an unobstructed area around the launcher and the use of protective materials. In aircraft, the blast from a missile rocket motor can cause quite serious damage.





The delivery vehicle must also be protected from the missile itself. That is, precautions must be taken to avoid launching the missile into the vehicle, just as firing cutout mechanisms prevent shooting a projectile into a vehicle. However, while automatic firing cutout protection is comparatively simple for gun mounts, it is rather complex for reaction-propelled missiles.

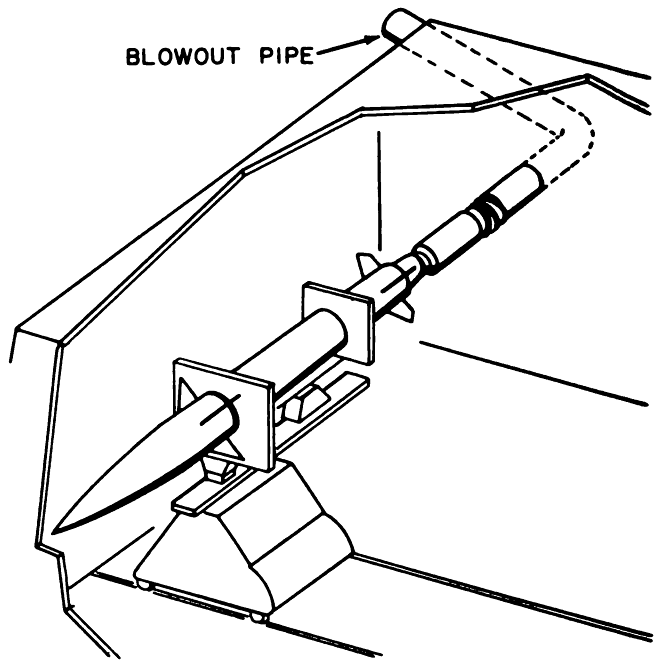
The firing cutout protection for gun mounts is intended to prevent the firing of a projectile when the gun is pointed at any part of the ship's structures and installations. Because of the high velocity at which a projectile leaves the gun, it will clear the ship in a few hundredths of a second after the time of fire, and the effects of gravity, wind, and ship motion relative to the projectile's trajectory during this time are insignificant. Therefore, the firing cutout zone for a gun mount is determined after the installation of the mount by simple gun boresighting of the ship's silhouette plus the addition of a prescribed safety clearance.

The present types of shipboard guided missiles, however, have relatively low velocities during the initial acceleration period, and the time required for the missile to clear the ship may be a second or more. During this time, the effects of gravity, aerodynamic and jet malalignments, wind, and ship motion cause appreciable relative motion between the airborne missile and the ship. Because these factors are independent and non-linear, and vary with the distance from the launcher, the simple optical plotting method and fixed safety clearance mentioned above for gun mounts do not provide adequate firing protection in connection with missile launchers. Furthermore, it is necessary to insure that guided missiles on launchers are not pointed in such directions relative to the ship that, if inadvertently launched, they could hit any part of the ship's structure or installations. The areas created by this restriction are called the pointing cutout zones. Also, supplementary firing cutout protection for additional safety is provided within similar areas called the firing cutout zones.

The pointing cutout control is performed automatically by means of cams cut in accordance with the pointing cutout zone data, switches, solenoids, valves, and the limit stop devices in the launcher's train and elevation power drives. The pointing cutout control causes the launcher to decelerate when approaching the pointing cutout zone and stops the launcher when the boundary of the cutout zone is reached. In addition, when the launcher approaches the vertical boundaries of the pointing cutout zone, it decelerates in train, simultaneously elevating to an angle sufficient to clear the zone, after which it resumes its train motion and resynchronizes with the train and elevation order signals at the other side of the cutout zone. The effect of this automatic control is that the projection of the line of sight through the center of a missile on a launcher arm will approach and follow the pointing cutout zone curve in a continuous, encircling, up-and-over manner, but will not cross the boundary. During the time the launcher is out of synchronism with the train and elevation order signals, the electric firing circuit is opened by an interlock contact.

Conventional firing cutouts, similar to those used for gun mounts, are provided also for guided missile launching systems and are intended to serve as backup protection in case of malfunction of the pointing cutout control. Separate firing cutouts normally are provided also for each arm of a dual-arm launcher. These cutout mechanisms contain switches connected in the firing circuits. These switches are activated by cams cut in accordance with the firing cutout zone data and are driven by train and elevation response shafts.

personnel safety

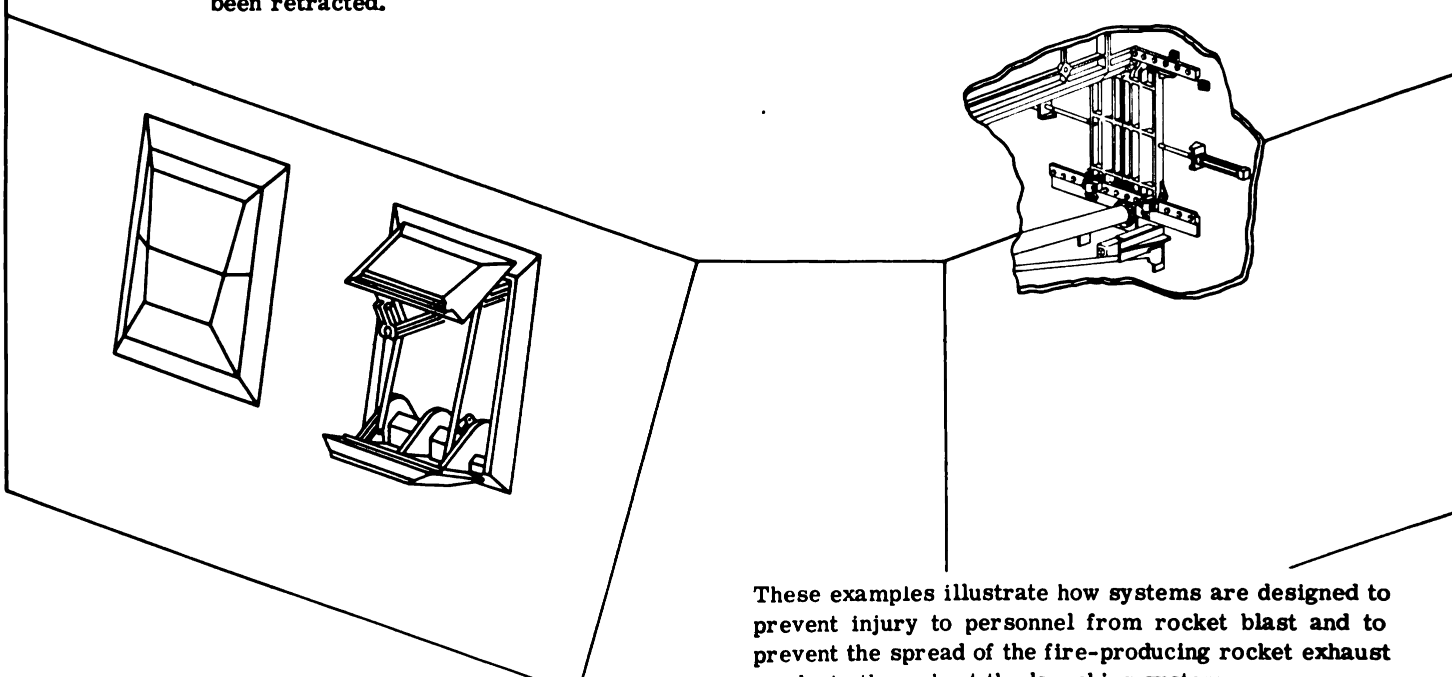


Personnel are protected from the rocket motor blast associated with normal launchings by stationing them in protected areas within the vehicle. However, inadvertent ignition of a rocket motor inside the vehicle can have very serious consequences. Not only is there danger of blast damage and fire, but when hot exhaust gases fill a compartment, they can be quite toxic to personnel.
A measure of protection can be achieved by the installation of vents in the bulkheads and the use of special blowout pipes when the missile is being serviced.

Personnel safety is highly dependent upon personnel training. By teaching the men the proper procedural steps to be followed in the operation of the launching system, injuries can be minimized. One method already employed to promote personnel safety in missile servicing is to have each man step behind a safety screen clear of the transfer system and depress a safety switch. The system will not function until all switches are depressed, insuring that all personnel are clear of moving machinery.

In reaction launching systems, numerous safety devices and interlocks are installed to guard against and to minimize the effects of accidental rocket motor ignition.

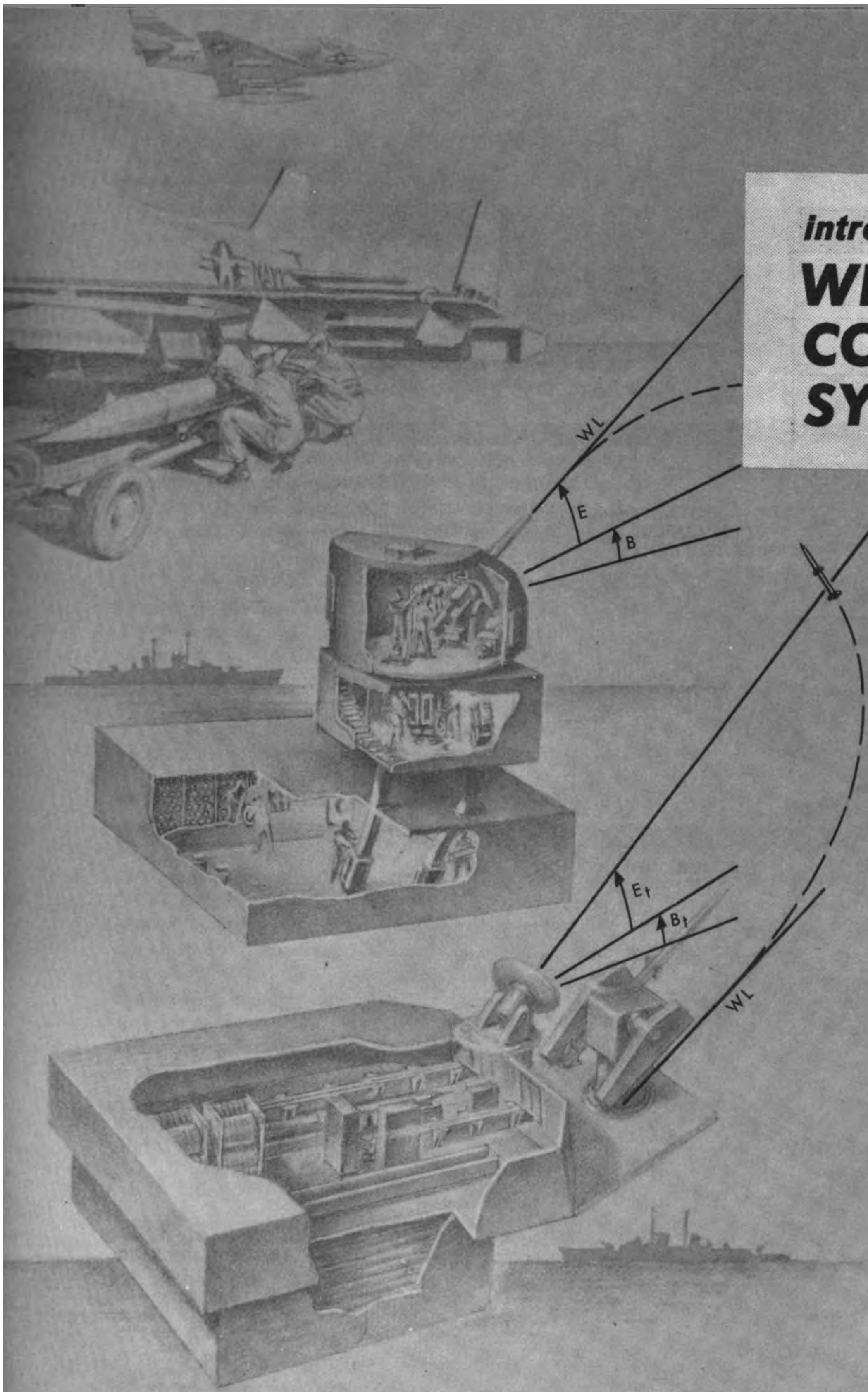
For example, in a guided missile launching system, the blast doors leading to the launcher cannot be opened when the missile magazine doors are open; the missile firing circuit to the launcher will not function when the blast doors are open; and the launcher train drive will not move the launcher until the loader spanner coil has been retracted.



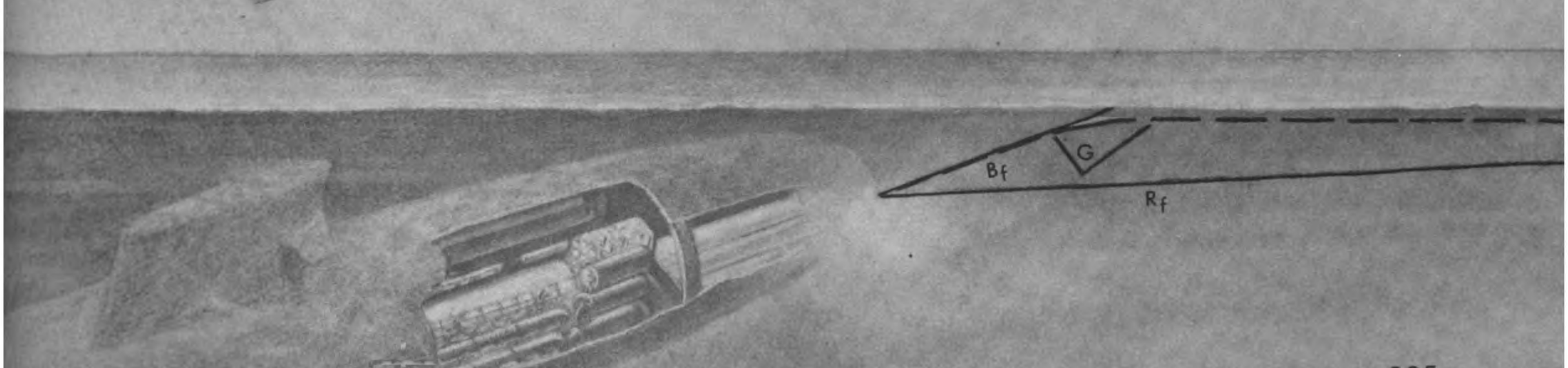
These examples illustrate how systems are designed to prevent injury to personnel from rocket blast and to prevent the spread of the fire-producing rocket exhaust products throughout the launching system.

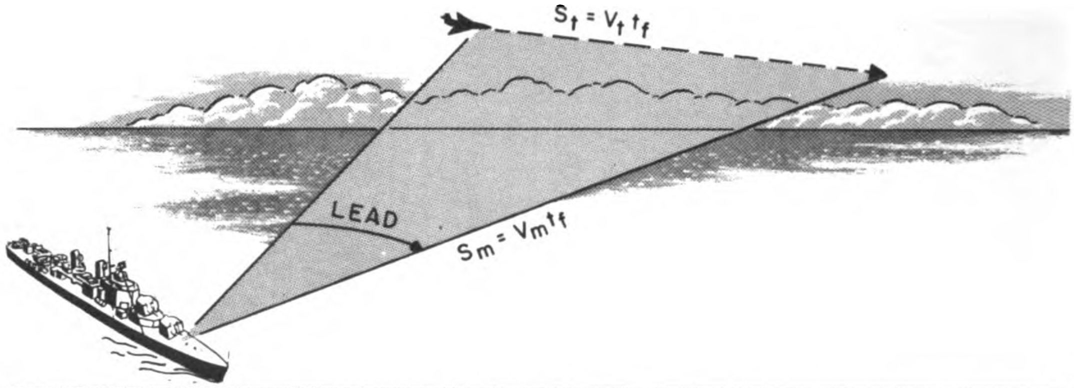
CHAPTER 7

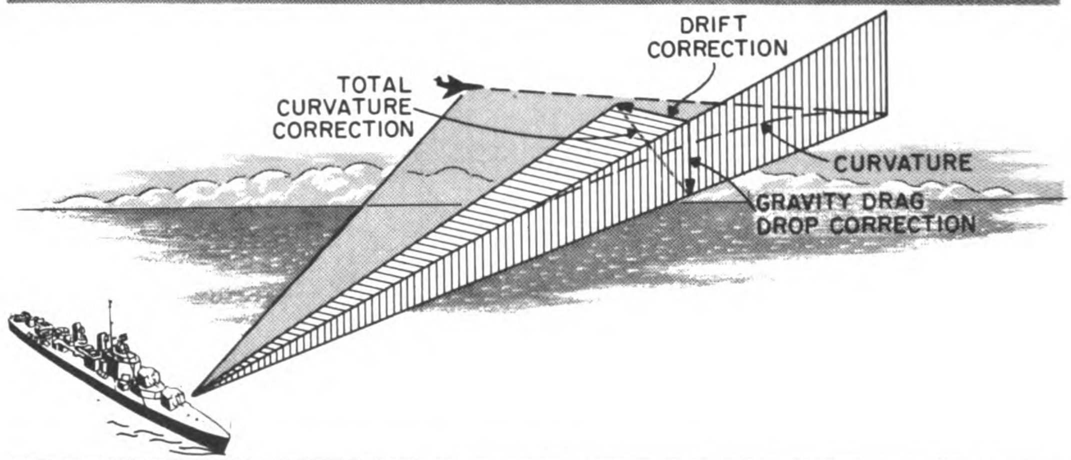
introduction to WEAPON CONTROL SYSTEMS

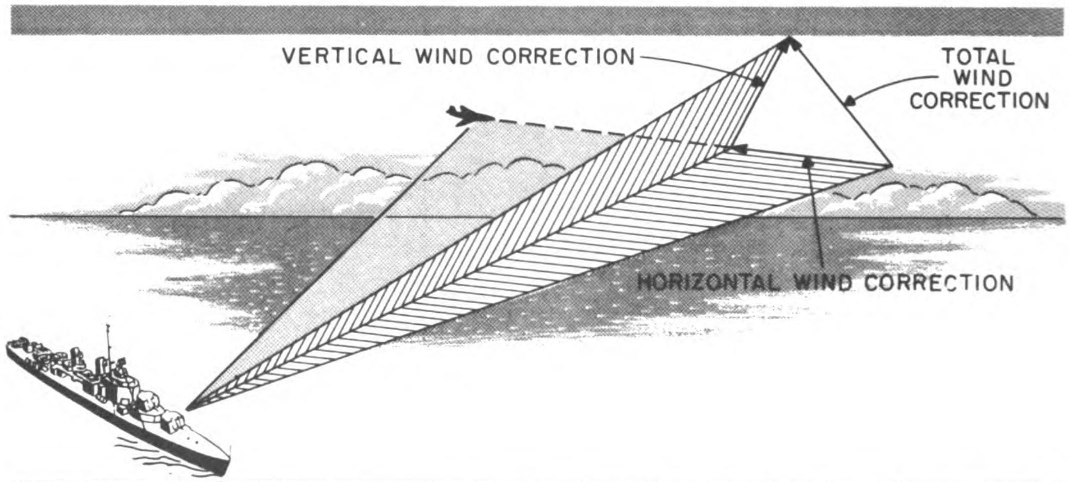


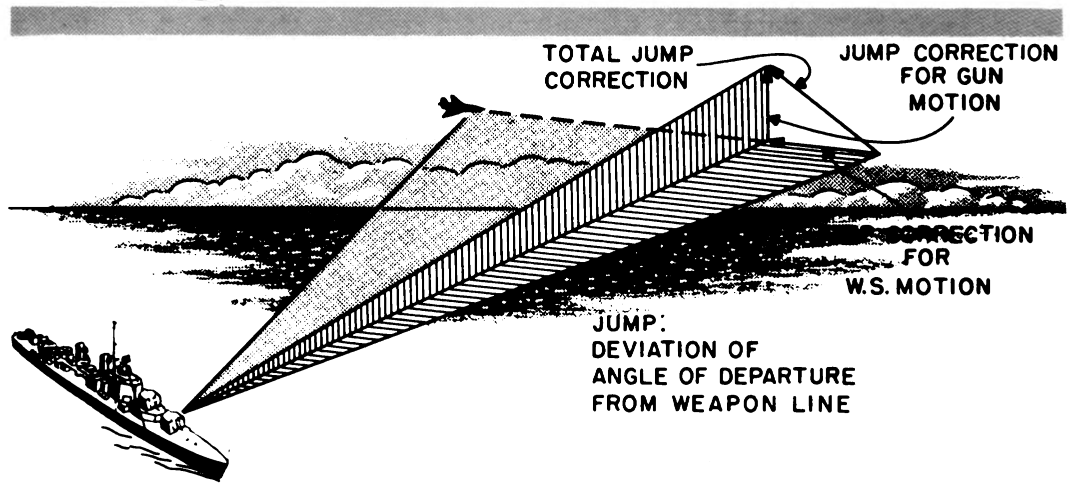
A weapon control system is the nerve center of a complete weapons system, and has as its elemental purpose the control of the interceptor device (projectile or missle) to insure the maximum kill probability of the weapons system. The various functions of the control system may be categorized : 1) to receive and assimilate tracking information from system sensory equipment regarding location, velocity, and posture of a target, or targets, 2) to calculate or compute the predicted line of fire of the weapon to insure an interception of target and warhead damage envelope, 3) to position the weapon launch device in accordance with its computed prediction data, and 4) in the case of a guided missle, to control its flight path after launch to insure target interception. Weapon control may involve surface-to-surface, surface-to-air, air-to-air, air-to-surface, surface-to-underwater, or air-to-underwater interception with projectiles, bombs, torpedoes, or missiles as the warhead carrier.




$S_t = V_t t_f$
LEAD
$S_m = V_m t_f$


DRIFT CORRECTION
TOTAL CURVATURE CORRECTION
CURVATURE
GRAVITY DRAG DROP CORRECTION


VERTICAL WIND CORRECTION
TOTAL WIND CORRECTION
HORIZONTAL WIND CORRECTION


TOTAL JUMP CORRECTION
JUMP CORRECTION FOR GUN MOTION
CORRECTION FOR W.S. MOTION
JUMP: DEVIATION OF ANGLE OF DEPARTURE FROM WEAPON LINE

BASIC CONCEPTS AND TERMS

The term weapon control system has been defined to encompass the entire series of measurements and computations used in the control of a weapon system, beginning with target detection and ending with target interception. Fire control terminology has developed its own language and nomenclature which require definition to simplify future discussions of the fire control problem.

weapon velocity vector

In many cases, common usage of the term velocity implies the speed of any object. However, in the study of science and engineering, one differentiates between the terms speed and velocity. Speed refers to a scalar quantity or magnitude which answers the question "How fast?" However, velocity refers to a vector quantity which has a magnitude (the scalar speed) and a direction (the unit velocity vector). Therefore, the velocity of a projectile or missile (weapon velocity vector) determines not only its speed, but also its direction. Fundamentally, a weapon control system entails the determination of the proper magnitude and orientation of the weapon's velocity vector to insure target interception.

weapon station

is the location of the launcher device. Weapon stations may be fixed or movable and located either on the surface, in air, in space, or underwater. Examples of surface type stations are a land based gun battery or a weapon complex located on a surface ship (cruiser or carrier). Air type weapon stations include launch devices located in or on airplanes or missiles, while space weapon stations can be guided missiles moving through space to a target area, or space platforms rotating around the earth waiting for the target to come into its range of weapon. Underwater stations can be mobile as submarines that launch torpedoes or missiles, or they can be fixed stations waiting for an identifying signal to initiate launch.

line-of-sight (los)

is an imaginary line between the weapon station and the target at the instant of target detection.

weapon line

is the direction or line along which a weapon should be launched to insure target interception after a predetermined time of flight. It is separated from the line of sight (LOS) by a prediction angle that is computed from available data.

prediction angle

is the angle between the LOS and the weapon line. It is a resultant angle of computations based on target velocities, missile capabilities, and computed corrections for the physical factors that could cause missile or projectile miss. These factors are: 1) the relative motion of target and weapon station, which require lead determinations; 2) the environmental forces that could cause variations in missile speed and trajectory, such as force of gravity, wind, etc. that require ballistic determinations and corrections, and 3) forces that cause a deviation or jump from the desired weapon line (usually at the instant of launch) normally as a result of launcher movements, etc.

lead correction

corrects for target motion.

curvature correction

corrects ballistic deviations.

jump correction

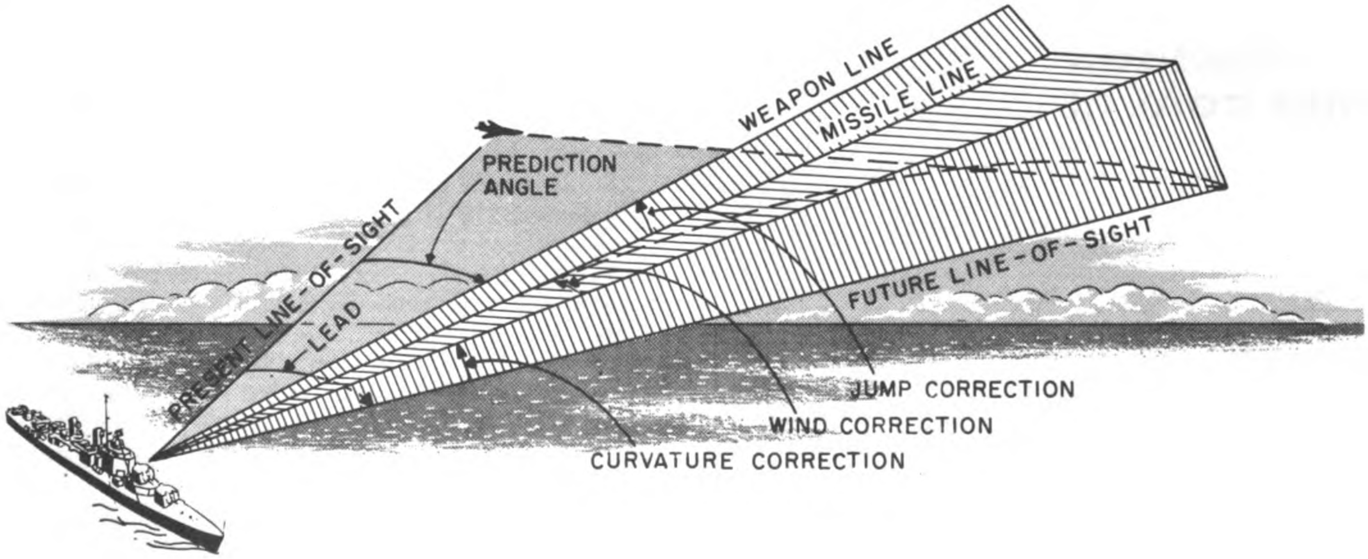
corrects for jump effects.

time-of-flight

is the length of time from launch to interception.

future line-of-sight

is an imaginary line between the weapon station and the target at the instant of target interception.



FUNCTIONS OF THE SYSTEM

data gathering

The gathering of data for use as input to the fire control prediction unit or computer is called DATA GATHERING and is one of the prime functions of a weapon control system. The data must contain sufficient information to allow the computer unit to do more than make an educated guess in its determination of a prediction angle.

data processing

Once this required data is gathered, it then becomes necessary to compute the prediction angle within a predetermined length of time so that the weapon may be launched with the proper velocity vector to insure a hit. Computation of the prediction angle is the basic requirement of DATA PROCESSING, a second prime function of a weapon control system.

weapon line control

The launching system provides the weapon line control at launch. The delivery velocity is the controlling factor if the launcher is fixed, while for movable launchers, the launcher line determines the weapon line. After the missile is launched, the weapon line is the missile line. The weapon line drive, which acts to drive the weapon line to its correct position in space, can mean several things. For movable launchers, the term weapon line drive indicates the launcher drive mechanism, while the weapon line drive for a fixed launcher is the steering control system of the delivery vehicle, submarine, or surface ship. Guided missiles have two separate weapon line drives, one for the launching phase, which is identical to that for the unguided missile, and

The principal components of a weapon control system are the data gathering elements; data processing, data conversion, and coordinate conversion devices utilized in the computer to calculate the prediction angle; and the weapon line control devices that effectively control the flight orientation of the weapon on order from the computer section of the system.

data conversion

During the operations of DATA GATHERING and DATA PROCESSING, it may be necessary to convert data from one reference frame to another or from a particular coordinate system within a reference frame to a second coordinate system. For instance, target data obtained during tracking may be measured in a stabilized deck frame, and environmental data obtained may be measured in a stable earth frame. If the computation space used by the computer is a stabilized deck reference frame, the environmental data must undergo a process called DATA CONVERSION so that these data become equivalent to data measured in the stabilized deck reference frame.

coordinate conversion

If the target data are measured in spherical coordinates, but the computation space is determined by rectangular coordinates, then the spherical coordinate data must be transformed into rectangular coordinate data within the stabilized deck reference frame so that the computer may operate properly. This process is called COORDINATE CONVERSION.

The computer produces the prediction angle as its output. It remains to position the weapon launcher so that the weapon line is offset from the tracking line by an angle equal to the prediction angle. In order for this to be accomplished, orders must be sent to the launcher directing it to slew to a position coincident to the computed weapon line position. However, it must be remembered that the reference frame of the launcher may not be the same as the one utilized at the computer. For example, let us assume the weapon is a missile launcher on a cruiser and that its reference frame is an unstabilized deck frame. Launcher orders from the computer must then take into account pitch and roll. In other words a further DATA CONVERSION process which transforms the launcher orders from a stabilized to an unstabilized deck reference frame is required.

a second weapon line drive for the missile flight control system.

The output of the computing system, which is the calculated weapon line position, must be transformed into an order for the weapon line drive equipment. For movable launchers, these orders are simply the signals driving the motors positioning the launcher. The weapon line drive orders for fixed launchers are actually delivery vehicle orders and, as such, involve the delivery vehicle flight control system dynamics. In the case of guided missiles, it is necessary to position the launcher initially so that the solution of the prelaunch fire control problem is implemented, and also to provide continuous missile orders to the missile after launch so that the solution of the post launch fire control problem is implemented. These missile orders are supplied while the missile is on the launcher and in some systems via a data link to the in-flight missile. The computer must then consider the effects of missile flight control systems dynamics in its solution.

Therefore, we can generally summarize the areas of weapon line (or missile line) control as:

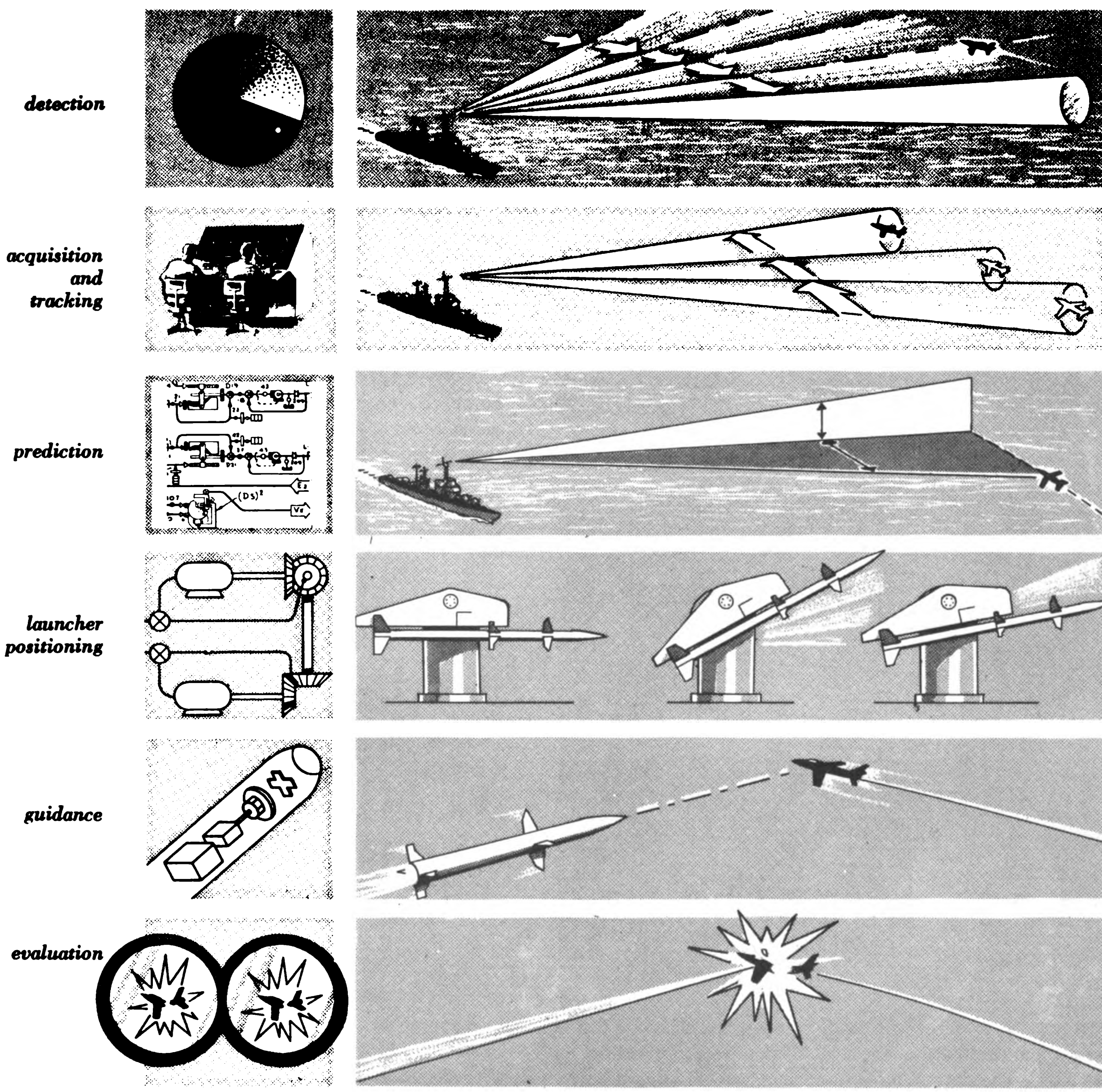
1) Control of the initial weapon line (and missile line) for all movable launchers.

2) Control of the weapon station line which in turn controls the initial weapon line for fixed launching; and

3) Control of the missile line during the guided phase of flight for guided missiles.

SECTION 1

WEAPON CONTROL PROBLEM



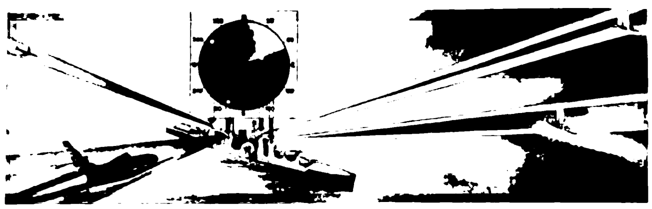
This section defines the weapon control problem and describes the information needed, and the various tasks which must be performed for its solution. The following sections describe the gathering and processing of data, and the actual missile control functions.

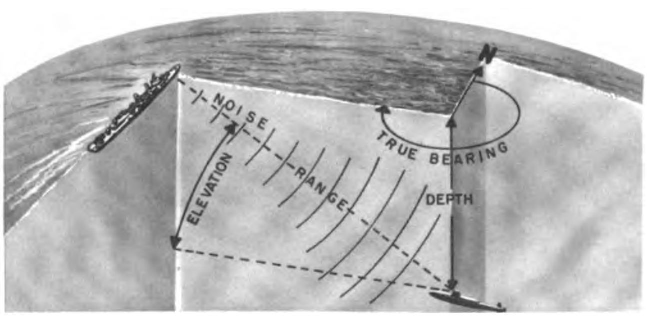
WEAPON CONTROL

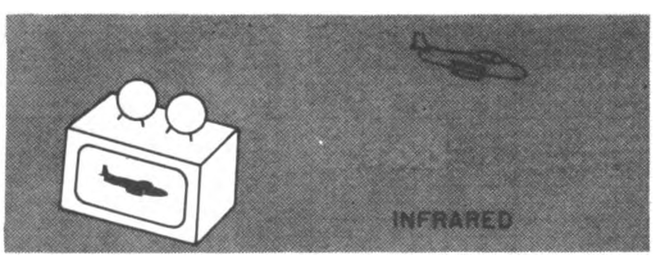
GENERAL

The fire control problem may be stated as follows: How may a missile be fired at a target (stationary or moving) from a launcher (stationary or moving) to obtain a hit on the target?

The first fundamental and necessary step in the problem's solution is the gathering of all relative information on the problem. One of the most critical pieces of information that must be determined is that a target exists. This requires the use of some form of detection system. In early naval history, the detection system was simply a man up in the mast searching the horizon for the sails of enemy ships. Today, visual detection has been virtually superceded by radar, sonar, and infrared detection systems which permit operation at extreme distances, in darkness, and during all types of weather.







The method of detection is, of course, a function of the target and the state of the art of the various detection devices. The detection system must sense some quality of the target which makes it readily identifiable from its background. This quality may be the energy it emits or reflects. This energy can be heat, sound, visible light, or radar energy. The equipment which detects the target can be, among others, an infrared device, sonar, optical instruments, or radars.

position information

Having detected a target, precise positional information is required. This information can be provided by the same sources which detected the target. Many devices, however, are incapable of providing both the detection facilities for the entire area around the weapon and the precise data required on specific targets. In these cases, a second system must be provided to track specific targets, or the overall detection facilities must be discontinued while the precise data on specific targets is determined. This positional information determines the line of sight to the target which is the line connecting the target and the viewer. Although the eye may not be capable of seeing the target, the positional data will determine where the line of sight would be if visual observations could be made.

relative target velocity

Not only is the present position of the target required, but its relative velocity must be ascertained. One method which may be used is to plot the position of the target at specific intervals, and to determine target velocity from this data. The direction of the target will be evident from the path of the plot. If the intervals are short enough, the target can be assumed to be traveling in a straight line between the individual points on the plot. The target speed can be determined by dividing the distance traveled during the interval by the length of the interval.

$$\cos Br_1 = \frac{b}{R_0}$$

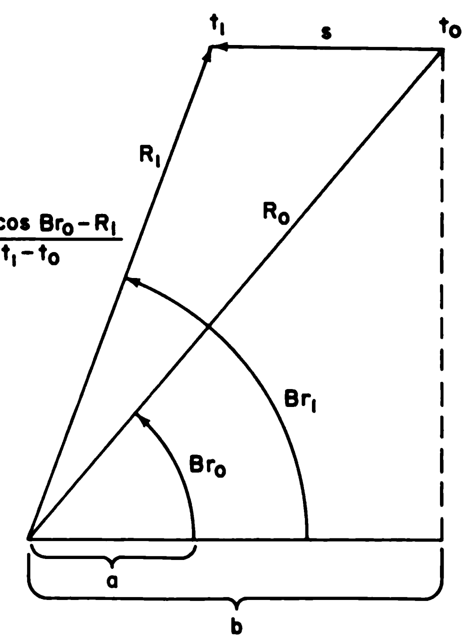
$$\cos Br_1 = \frac{a}{R_1}$$

$$s = b - a$$

$$b = R_0 \cos Br_0$$

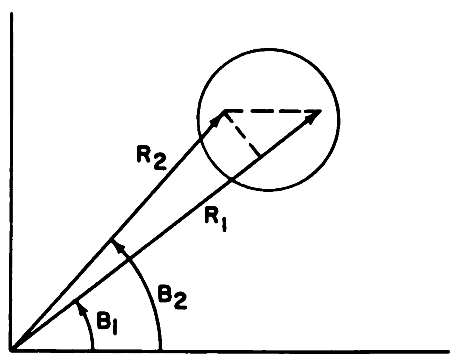
$$a = R_1 \cos Br_1$$

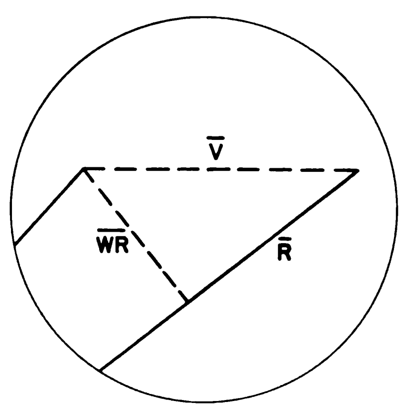
$$V_{Tgt} = \frac{s}{t_1 - t_0} = \frac{R_0 \cos Br_0 - R_1}{t_1 - t_0}$$



PROBLEM

An alternate method of determining target velocity is to use the angular velocity of the line of sight and the rate of change of range. The rate of change of range is the component of target velocity along the line of sight. The component of target velocity perpendicular to the line of sight is the product of the angular velocity of the line of sight and the range to the target. From the velocity components, target velocity can be readily determined. These procedures, both of which involve keeping track of the target, are called tracking.





ancillary data

Information other than target data is often equally important for weapon line determination. Ballistic corrections require such environmental data as gravitational forces components, wind velocities, air temperatures, current conditions, sea temperatures, etc. for precise calculations. Some guided missiles require data on the position of the delivery vehicle in addition to the position of the target. The velocity of the weapon launcher, its relative position, and stabilization characteristics if not correctly intergrated into the computations, can cause severe deviation in missile interception trajectory. Information on factors other than target data is often called ancillary or auxiliary data and is of extreme importance to correct computation of interception flight paths.

usable data

All this information or data will be transmitted to an area which serves as the weapon control station for the weapon. Usually this data cannot be used directly in computations. Some data will be received continuously, and other data may be received only intermittently. Certain data will be erratic, and fluctations will occur in the quantity being measured. Some measurements may be taken at one spot, while others may be taken hundreds of feet or many miles away. Because of these and other problems, the raw data must be processed to arrive at a unified set of usable data.

With usable data, the necessary calculations must be performed to determine the prediction angle. These calculations, if they are simple, and if ample time is allowed (as in the case of a fixed weapon and a fixed target), can be made by man using pencil and paper. However, errors can be made easily, and are difficult to find and correct. For a moving target or moving weapon (or for both), the calculations become longer and involved, and the allowable time for making them is decreased. It soon becomes necessary for man to have some help to perform the required mathematical operations. This help can be a simple adding machine, or complex digital or analog computing devices. These calculating devices, or computers, simply may perform certain operations and aid man, or they may be set up to perform all the necessary calculations when the required data is provided to them.

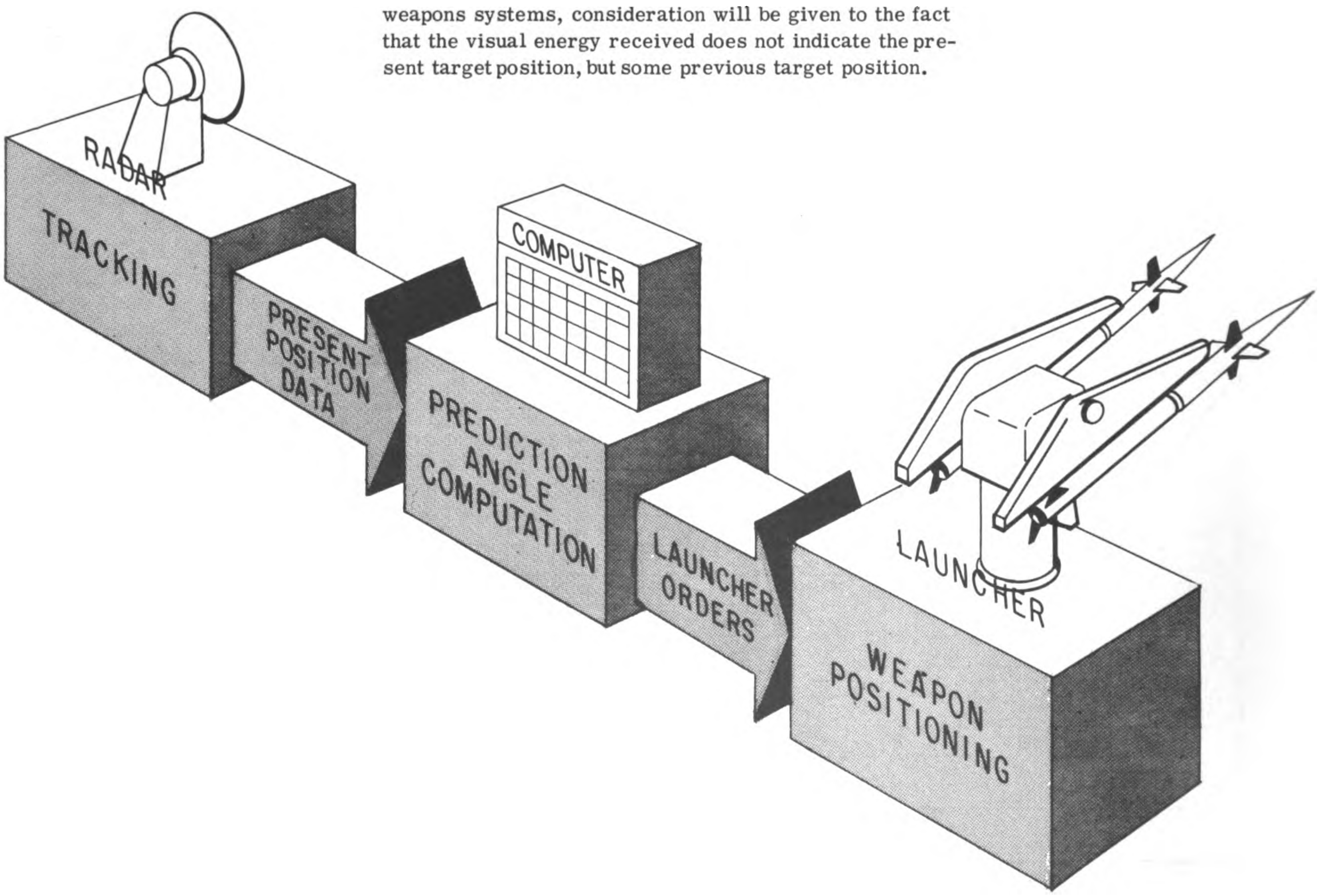
FIRE CONTROL COMPONENTS

The fire control problem then is reduced to
three separate components

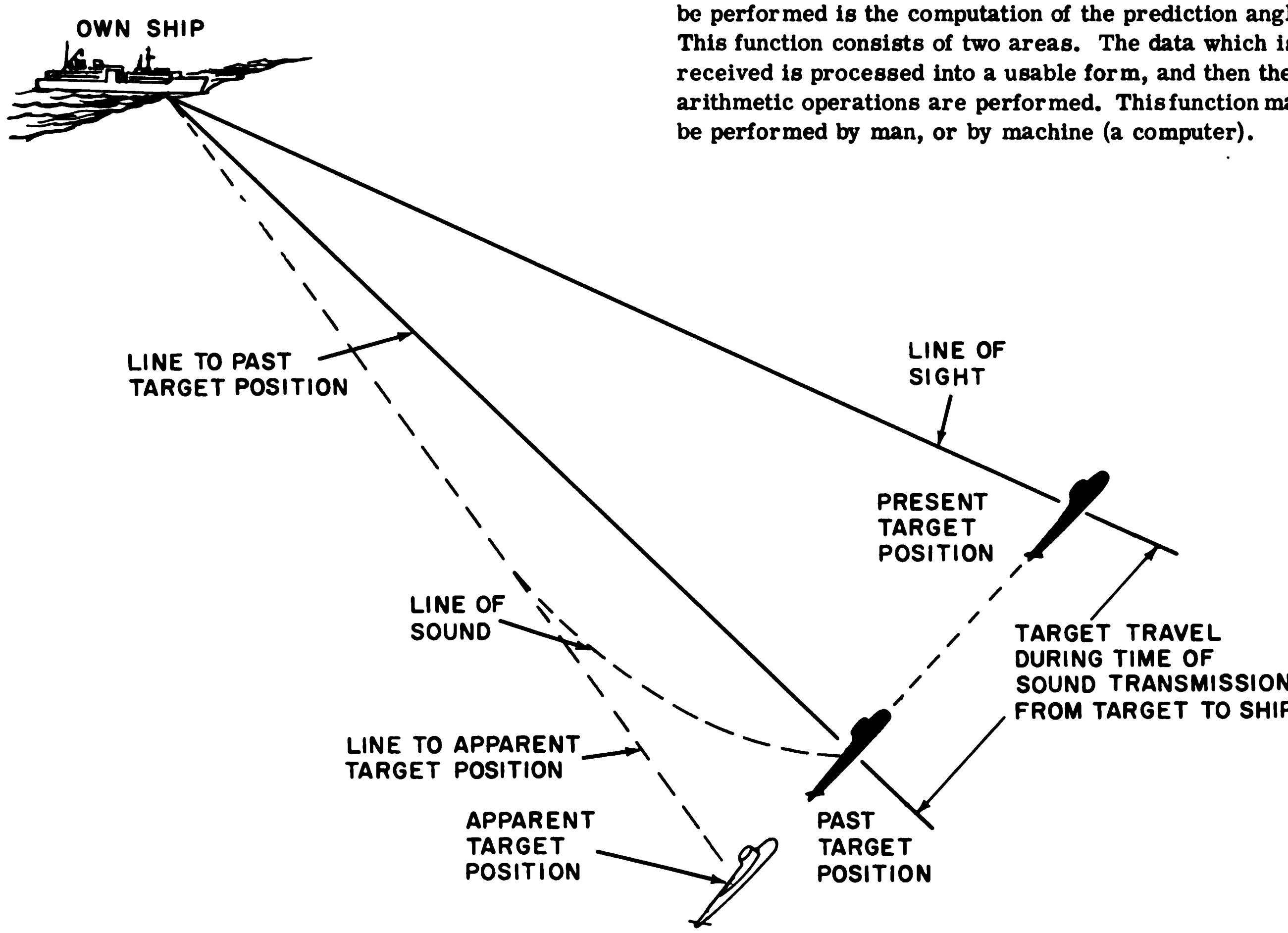
TRACKING COMPUTATION POSITIONING

detection and tracking

The first component, the detection and tracking system, detects the target and provides all necessary data on the target. The tracking device performs this function by establishing a tracking line (LOS) along which it receives the energy from the target. Since the speed of light is about 186,000 miles per second and the distances involved are relatively small, the time for light energy to travel from the target can be considered instantaneous, and the visual indications of the target can be considered the present target positions. When interstellar distances (usually measured in light years) become important in weapons systems, consideration will be given to the fact that the visual energy received does not indicate the present target position, but some previous target position.



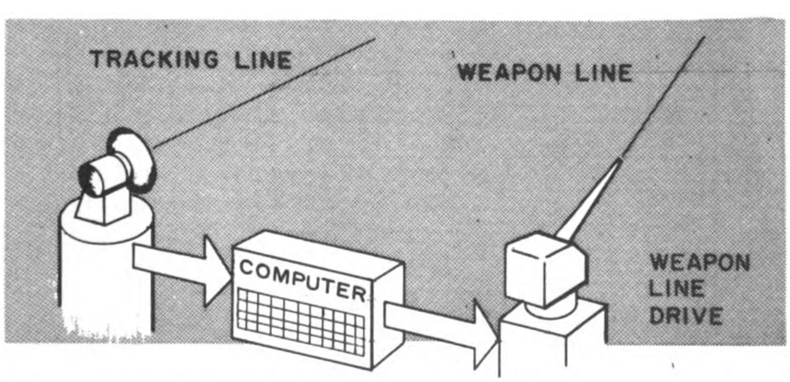
A common example of something similar to this occurs in antisubmarine weapon control techniques. Sound energy traveling in water takes an appreciable time to travel from the target to the sensing device. During this time duration the submarine may move. Complicating the prdblem is the fact that sound does not necessarily travel in a straight line. Therefore, the sensing device gives us a line to an apparent target position. From this, the target position at the time the sound was emitted and the present target position can be determined. From the data on present position, the true line of sight and the prediction angle must be calculated, and the weapon line positioned.



A similar problem would be encountered if we still tracked aircraft with sound detecting devices. Probably all of us have tried to find a supersonic aircraft flying overhead whose sound we heard and discovered that the plane was a considerable distance ahead of the apparent location indicated by its noise. In order to facilitate understanding of the ensuing development of the fire control problem, it is assumed that the tracking line does indicate the true present position of the target. Any error will be corrected in the computation of the prediction angle.

computation

The second function of the fire control problem that must be performed is the computation of the prediction angle. This function consists of two areas. The data which is received is processed into a usable form, and then the arithmetic operations are performed. This function may be performed by man, or by machine (a computer).



positioning the weapon

The third function that must be performed is the positioning of the weapon in accordance with the calculated prediction angle. This amounts to offsetting the launcher axis from the LOS by the amount of the prediction angle by means of a weapon line drive. If the missile has in-flight guidance, a weapon line drive operates to reposition the missile during its flight.

Each of these three functions must be performed in the solution of the weapon control problem. The procedures may be made by one man who detects and tracks the target by eye, mentally determines the prediction angle, and positions the weapon line by aiming a rifle or a shotgun, or by weapons systems that use vast complexes of machinery to perform each of the tasks. However, all type systems must perform the same basic functions. The variations are results of the speed and precision required in the solution of the fire control problem.

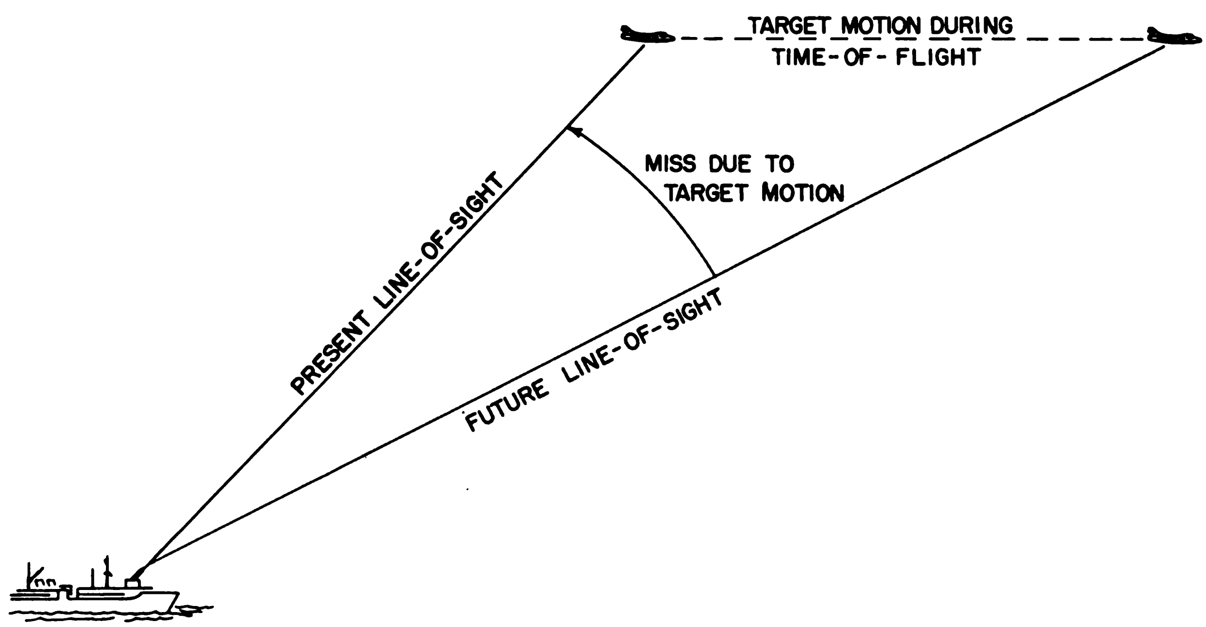
MISS-PRODUCING EFFECTS

Many factors serve to complicate the weapon control problem. Among them are the inherent unpredictability of the motion of any specific target and the inaccuracies possible in the missile flight path because of variations in the launcher and propulsion systems and to external causes such as gravity, wind, drag, etc.

relative motion of target and weapon station

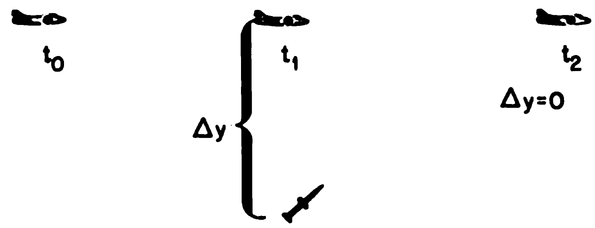
The first of the miss-producing phenomena is the motion of the target with respect to the weapon station. If the case of a fixed weapon and a moving target in inertial space is considered, as in section 6 of Appendix A, it is seen that the missile cannot be fired at the present position of the target. It is necessary to aim the weapon at a future position where the missile and target positions will coincide, i.e., leading the target. The purpose is to have the missile and the target paths cross so that the missile and the target are both at the same point at the same time. The point of intersection of the target path and missile path must be calculated and the weapon line must be directed toward this intersection point and not toward the target's present position.

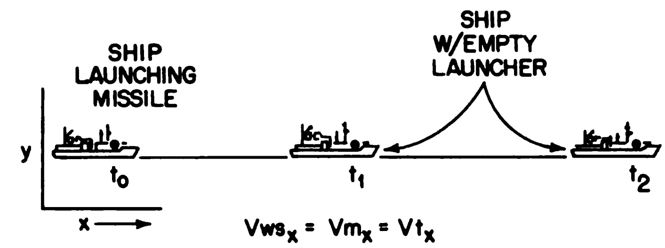
The amount that the weapon line must be directed away from the line of sight from the weapon to the target is called lead correction (sometimes called kinematic lead). Kinematic lead takes into account the relative motion of a target with respect to the delivery vehicle. The missile must be aimed at the target's future position, and not at its present position as indicated by the line of sight. This effect may be more generally defined by the term relative motion. To illustrate this, let us assume that a target is moving at the same speed as the weapon station and that both target and weapon station are moving in parallel paths in the X direction.
Now, let us suppose that the weapon station launches a missile directly at the target in the Y direction. The velocity imparted to the missile is composed of two parts, the muzzle velocity and the velocity of the weapon station. This second component of velocity keeps the missile moving at the same speed in the X direction as the target while the first component closes the distance in the Y direction so that when the Y distance closes to zero, target impact will occur. From this example, it can be seen that there was no relative motion between the target, missile, or weapon launcher.
In other words, an observer placed at any of these three points would not perceive any motion between the other two and himself.



target motion as a miss-producing effect

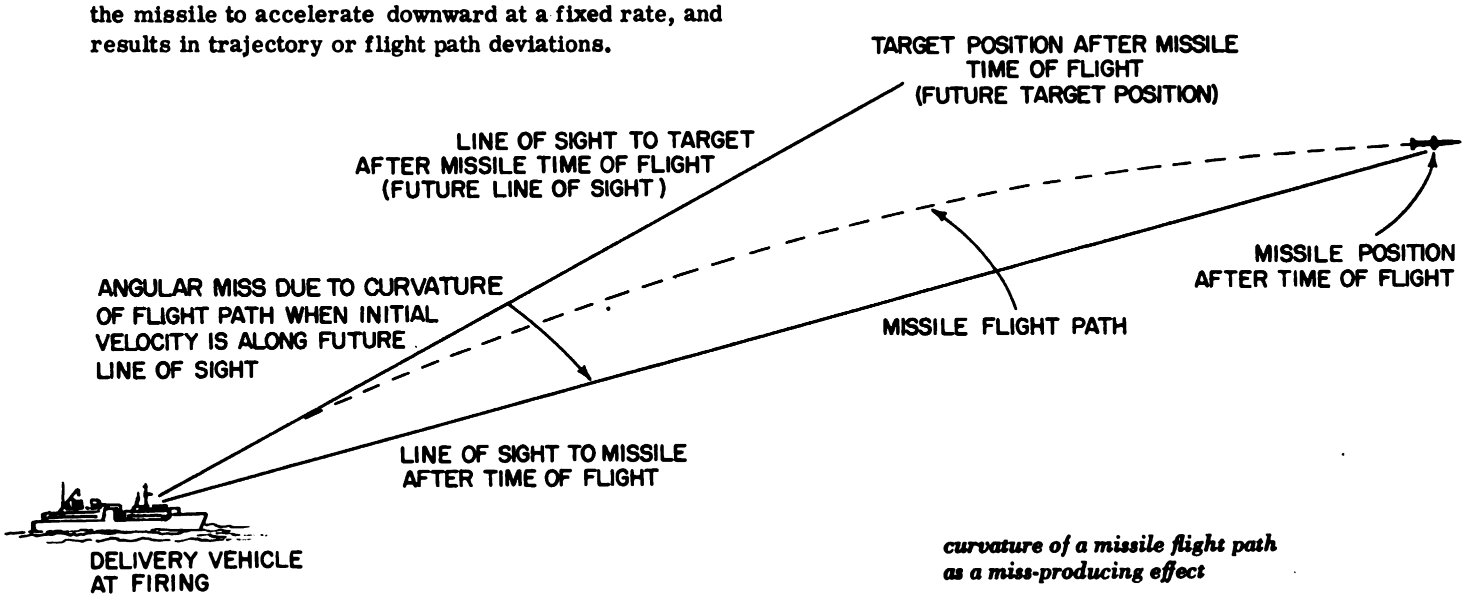
However, if the target were to change its velocity, thereby causing a relative motion with respect to the weapon launcher, this motion would become a miss-producing effect in the same manner as the stationary weapon station - moving target case. In other words, the X vector of the projectile would be no longer equal to the X vector of the target, as in the previous case, and, therefore, a positional difference in X coordinate for missile and target would occur. This positional difference, of course, would cause the missile to miss the target. To sum up the effects described above, it may be stated that any target motion relative to the weapon station tends to produce a miss and therefore may be categorized as a miss-producing effect.





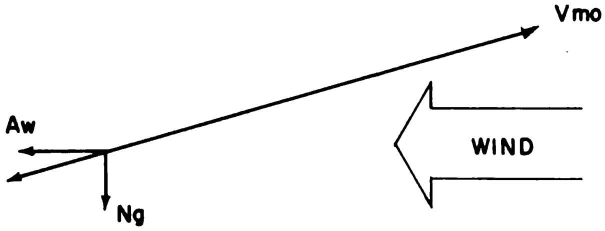
forces acting on a missile during flight.

A second miss-producing phenomenon is caused by the various forces which act on a missile in flight, such as gravity, wind, drag, etc. The effects of gravity cause the missile to accelerate downward at a fixed rate, and results in trajectory or flight path deviations.



curvature of a missile flight path as a miss-producing effect

Wind produces a force which causes the missile to accelerate. The magnitude of this acceleration is proportional to the force of the wind. Frictional drag on the missile adds an additional acceleration opposite in direction to the motion of the missile; the force causing this acceleration is dependent on the nature of the medium traversed by the missile as well as on the aerodynamic or hydrodynamic characteristics of the missile

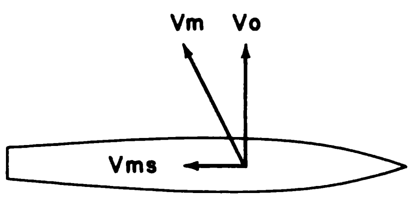


In section 5 of Appendix A the fire control problem is described as it occurs when a fixed target and a fixed weapon are located in outer space. There the missile path is a straight line since no effects such as gravity or air resistance exist, and all that is required to assure a hit is to fire point blank at the target. In this case, the requirement is to make the weapon line (the direction in which the missile is aimed) coincident with the line of sight from the weapon to the target.

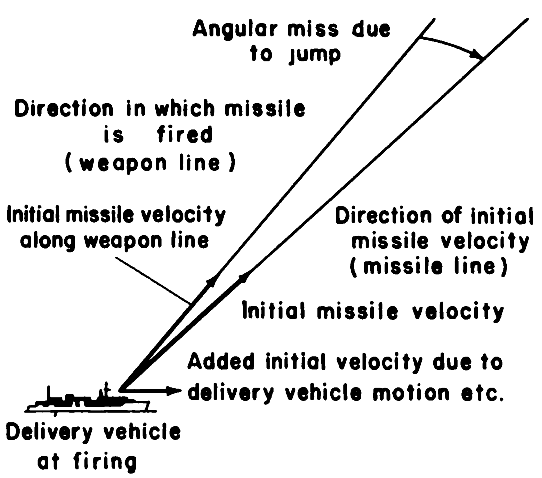
However, if this same case of fixed target and fixed weapon were located on Earth, the various ballistic effects would change the flight path from a straight line to a curved path which may be computed. The amount the weapon line must be offset from the line of sight to the target to compensate for this path curvature is called curvature correction and can be predicted. The solution to this problem is essentially one constant value.

missile velocity vector deviations

A third phenomenon, very similar to the first is weapon station motion. The effect of the motion of the weapon station upon the missile velocity vector may be a result of the speed and course of a craft, the cross traverse motion of a ship due to pitch and roll, the yaw motion of a missile, or many other motions or combinations of motions. Any of these motions will provide an additional missile velocity vector, causing the missile to vary from the desired course.



As was discussed in the chapter on launching systems, if the delivery vehicle is moving, the delivery vehicle velocity is imparted to the missile in addition to the velocity projecting it along the line. To compensate for the additional velocity, the actual weapon line must be offset from the ideal position by an amount sufficient to cause the delivery vehicle velocity and the initial missile velocity combined to be along the desired weapon line.
The velocity imparted to the missile which is not directed along the weapon line is called jump correction which accounts for any velocity imparted to the missile at launch which would cause the missile to move along a line other than the weapon line.



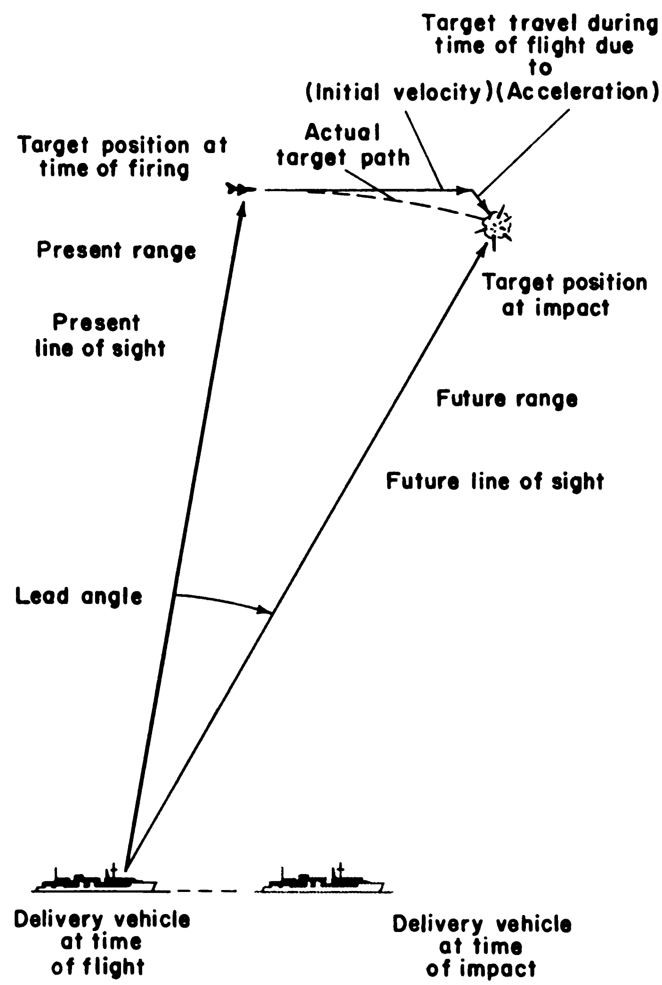
JUMP AS A MISS-PRODUCING EFFECT

PREDICTION

Because of the effects of the miss-producing phenomena, it is necessary to apply a correction to the weapon line angular position to compensate for the errors introduced. This correction is known as the prediction angle and is composed of the sum of the individual corrections for the various miss-producing effects.

The corrections for the miss-producing effects may be listed in the same order as the effects themselves;
1. Kinematic lead, to compensate for target motion during the time of flight.
2. Curvature correction, to compensate for in-flight forces acting on the projectile.
3. Jump correction, to compensate for initial velocity (jump) effects.

These corrections are made in the form of angles to compensate for the miss-producing effects. The sum of these angles is the overall correction angle or the prediction angle. The weapon line is offset from the line of sight by this amount prior to launch, thereby imparting the proper initial velocity to the missile to insure a hit.



GEOMETRICAL FEATURES OF LEAD ANGLE

ANGLES

prediction angle components

The three important prediction angle components are lead, curvature correction, and jump correction. These components are discussed in detail in the following paragraphs.

lead

Lead is the angular correction needed to compensate for the effects of target motion between the time of firing and interception. In other words, it is the angle between the tracking line and the predicted line of sight at intercept. The predicted line of sight is the direction in which the observer would see the target at the time of interception from the location of the weapon system.

Lead correction is determined from a knowledge of both target velocity and acceleration. Since a target with constant velocity presents an easier and more basic problem, it is discussed first. The problems in computing lead when accelerations are involved are discussed in subsequent paragraphs.

lead with constant velocity vector

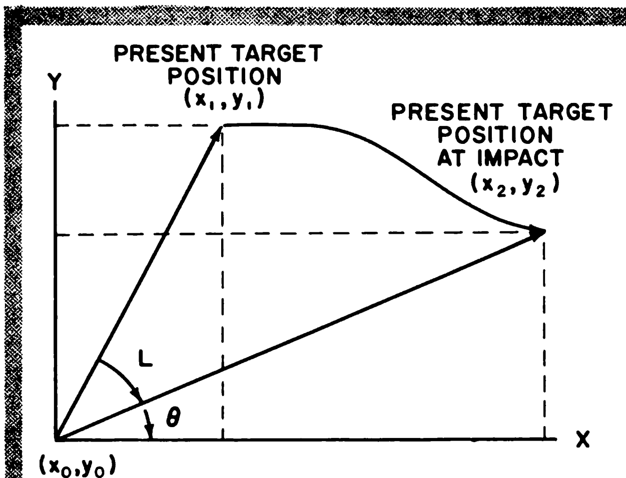
An example of the lead angle is shown in the accompanying illustration. The chosen reference frame is an appropriate set of axes in the inertial plane containing three points: weapons system present position, present target position, and future target position. Knowing the location of these three points in this plane simplifies the solution of the lead angle. The horizontal plane may be considered such a plane for short-range land or surface ship operations. It is then necessary to determine the path of the target and its speed. Knowing this, the point of impact can be determined; it will take as long for the target to move from its present position to its future position as it will for the missile to move from the launcher to the intercept point. From design data and experimental firings, the average velocity of the missile to a target can be determined.

Unfortunately, when this method is tried with targets such as aircraft and submarines that can move in three dimensions, the location of the plane cannot be determined until the future target position is known. The future target position can be determined in an appropriate three-dimensional reference frame, and transformed into this inertial reference frame where the lead angle is solved.

target acceleration considerations

If the velocity of targets were constant, the problem of fire control would be greatly reduced. However, targets are subject to accelerations which are unpredictable in the ordinary sense, since they may depend on arbitrary actions of the target after the missile is fired. Hence, certain assumptions about the nature of target accelerations must be made before any analysis leading to computation of a lead angle may be made.

There are two basic assumptions which can be made concerning the nature of target accelerations. The first is that these accelerations are regular in nature and that, therefore, the pattern established prior to launching will remain constant during the period from launch to intercept. The second assumption considers target accelerations to be composed of certain elements which are random in nature. The treatment of accelerations in this case must be statistical. In the following paragraphs both of these assumptions are discussed in detail.



$$\theta + L = \text{arc tan } \frac{x_1 - x_0}{y_1 - y_0}$$

$$\theta = \text{arc tan } \frac{x_2 - y_2}{y_2 - y_1}$$

$$\therefore \quad L = \text{arc tan } \frac{x_1 - x_0}{y_1 - y_0} - \text{arc tan } \frac{x_2 - x_0}{y_2 - y_0}$$

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regular accelerations

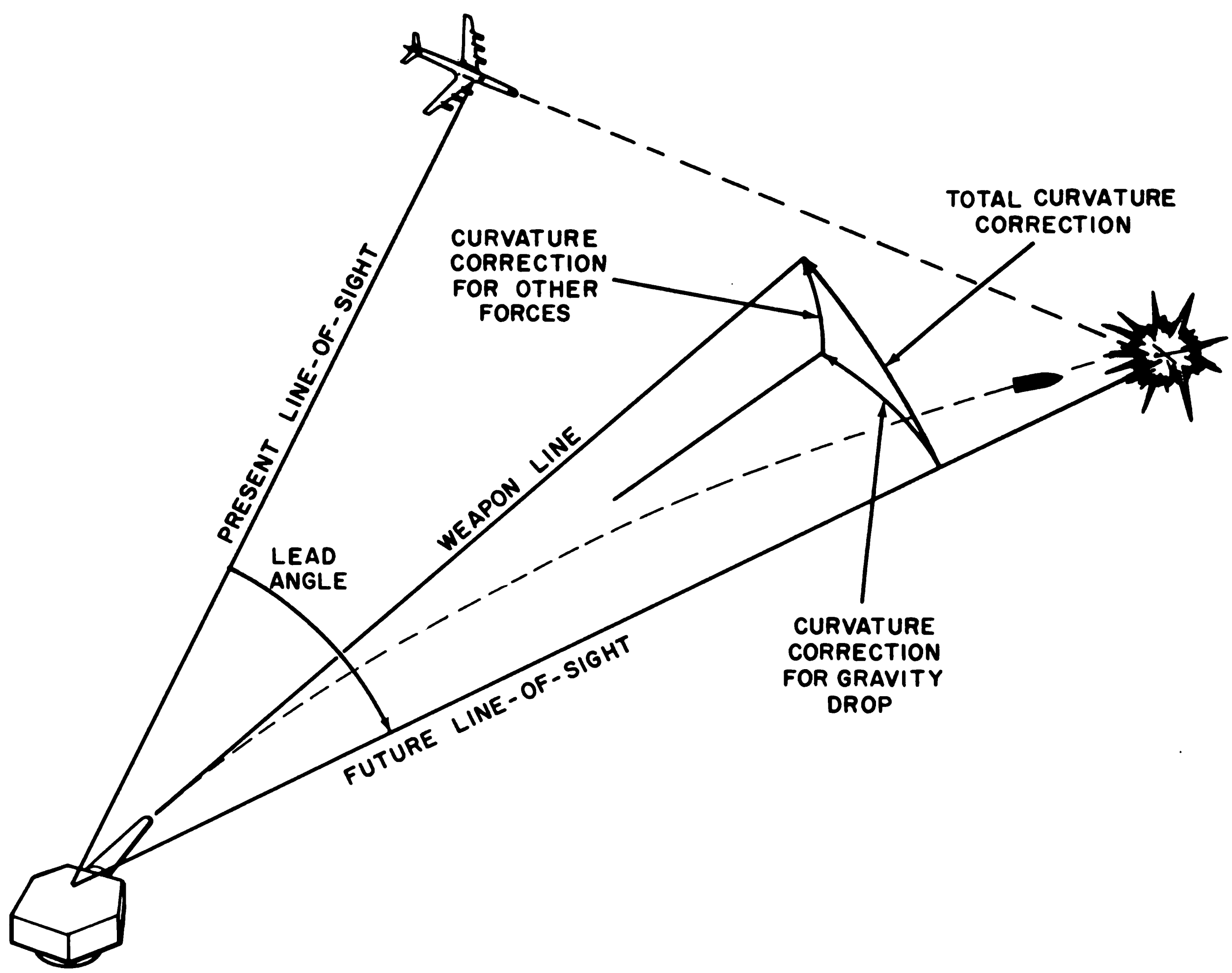
If the target acceleration is assumed to be regular in nature, it may fall into one of two general categories; i.e., zero acceleration, or constant acceleration. In the first case, the problem is the constant velocity problem discussed earlier. For the second case, we may consider a target with constant acceleration and constant speed. Since the velocity vector is changing at a fixed rate but its magnitude remains constant, the target must be following a circular course. Again, because of the regularity of the target motion, a lead angle may be easily predicted. A situation in which the magnitude is of constant value is not a normal one. However, if short enough periods of time are considered with respect to the expected magnitude of target acceleration, the assumption is valid and provides a simple means of computing lead angles.

random accelerations

Generally, random target motion is assumed to be superimposed on a regular motion of the types just discussed. Here, again, there is a wide selection of possible assumptions and the chief difficulty is in selecting an assumption that is both realistic and manageable in calculation.

There are many possible causes of random target accelerations. These may include atmospheric turbulence, response time of both pilot and vehicle if a manual aircraft or vehicle is used, and the maneuverability of the vehicle itself. However, these accelerations are not as likely as those made by the target performing evasive maneuvers upon realization that it is under surveillance or attack. In this case, the multitude of possible target maneuvers must be considered and probabilities assigned to each in an effort to predict target motion. However, this requires that there be a reasonable knowledge of which maneuvers are more probable than others and this question has yet to be solved.

Some military planners believe that as a basic military tactic every piloted or guided aircraft must attempt to press home the attack as quickly as possible to save fuel, resulting in straight line movements. However, there is probably just as much reason to believe that the straight line is the last course a pilot would fly. A second point of disagreement would be over the relative priority of each target type and its relative military potential. We might obtain a clue to target priority by examining history, but who will claim that the targets of previous wars accurately reflect the action of targets of today or ten years from today.



geometry of curvature correction

It might be suggested that the best description of the target attack probability can be obtained by selecting a number of representative samples. Selecting probability constants from among the archives presupposes a number of implied assumptions about the population of attack courses to be met. Some of these assumptions might be well founded, and some entirely unrealistic. Of course, in establishing the types of target attacks, we would have to make the obvious modification in target speed, particularly in case of aircraft targets.

However, it is highly debatable that speed alone represents the difference between the old and new attack patterns. Unquestionably, all military establishments are devising new tactics not possible with earlier delivery vehicles. If we assume that the flight paths remain the same while only the speed changes, we are ignoring the probability of system-confusing tactics that can be employed. It could also be pointed out, by using an assist from the Darwin theorist, that, if any one target course becomes easy to intercept, pilots who took that course would soon be a lost generation, and so would that course.

The second class of hypothesis is inherently more reasonable since it operates on actual courses of targets, and not on mathematically presumed courses which are never actually used by any target. However, determination of the actual course of a target is so difficult and inexact that it is often assumed that the target is on a mathematically defined course, and only nominal attention need be paid to the possible randomness in target acceleration. In practice, this assumption is usually valid, since the results of predicting target path are often closer to being accurate than assuming a target course would be.

curvature correction

Curvature correction is an angular correction needed to compenstate for the effects of forces that act on the missile during its flight. These forces may consist of gravity, drag, wind or current, etc. The effects of these forces on the missile's trajectory and the resulting miss-producing motion may be examined by taking the simplified case of a missile fired with an initial velocity being acted upon by the force of gravity.

GRAVITY EFFECTS

Let us assume that a solidly emplaced antiaircraft gun has fired a missile directly at the predicted impact point without taking into account the force of gravity. At the instant that the missile leaves the gun barrel, the force of gravity begins to act on the missile, thereby providing an acceleration.

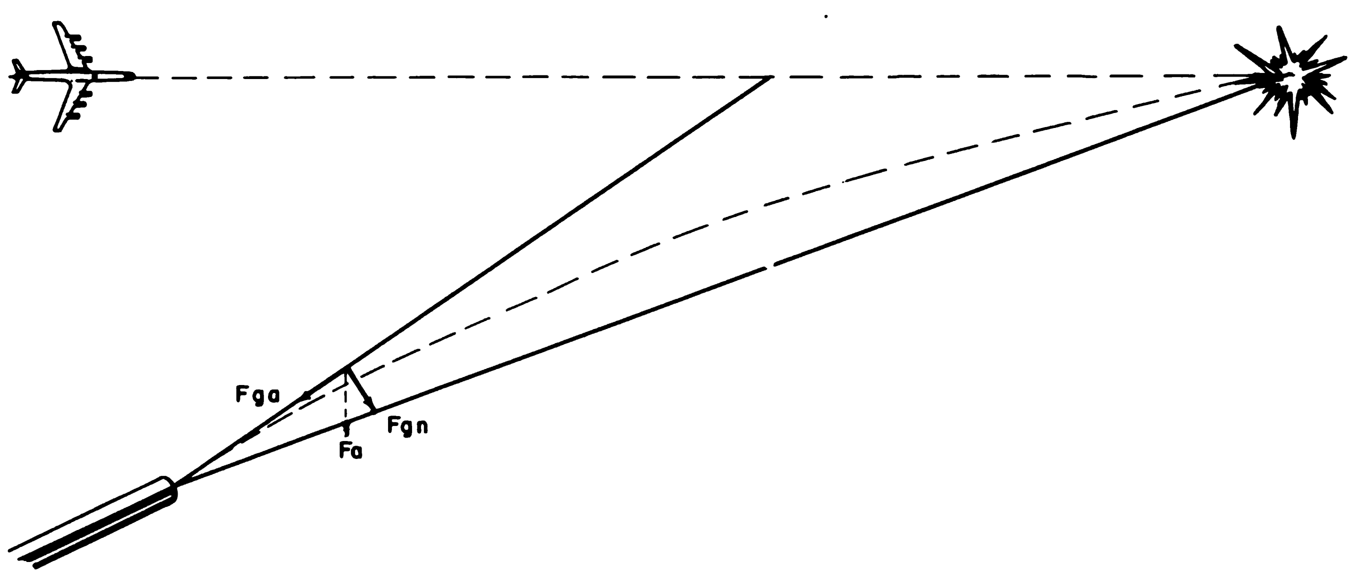
As seen from the illustration, the force of gravity, Fg, always acts in a vertical direction. However, this force may be resolved into two components, one acting along the missile line, Fga (in this case, in the opposite direction to missile motion, thereby slowing the missile down), and one acting at right angles to the missile line, Fgn. The force component acting at right angles to the missile line accelerates the missile in the direction of the force but does not affect the missile's speed. Therefore, the missile line moves in the direction of this force component, causing the missile's path to curve.

The magnitude of this normal force component varies, increasing as time passes, while the force acting along the missile line decreases. This relationship holds true until the trajectory peak is reached, at which time all the gravitational force acts at right angles to the missile line. Should the trajectory continue after reaching its peak, the reverse situation applies.

As seen from this discussion, a missile being acted upon by the force of gravity will accelerate because the component of this force acts normal to the missile line. Since Fg is equal to mg, where m is the mass of the projectile and g is 32 ft/sec^2, and since the initial angle of the missile line is known, it is possible to predict the total curvature angle of the missile prior to launching. In this manner, the weapon line may be varied to compensate for this effect.

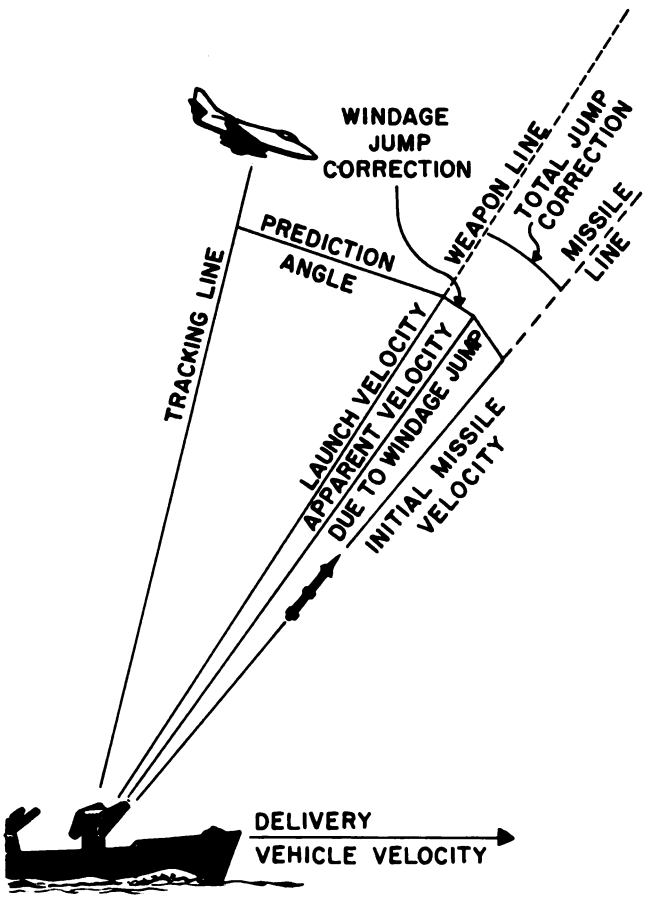
LIFT, DRAG, WIND

In the same manner as the force of gravity affects the trajectory, various other forces acting on the missile with components normal to the missile line causes curvature. In most cases, these forces, such as lift, drag and wind effects, must be measured prior to launch so that accurate curvature angles are obtainable.



jump correction

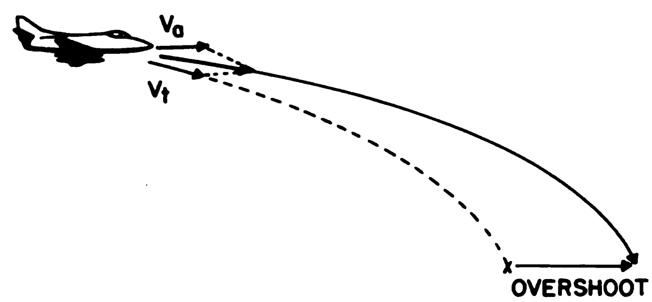
The jump effect results from an initial velocity component imparted to the missile in addition to its normal initial velocity. The miss-producing effects of this component may be seen readily in the following example.



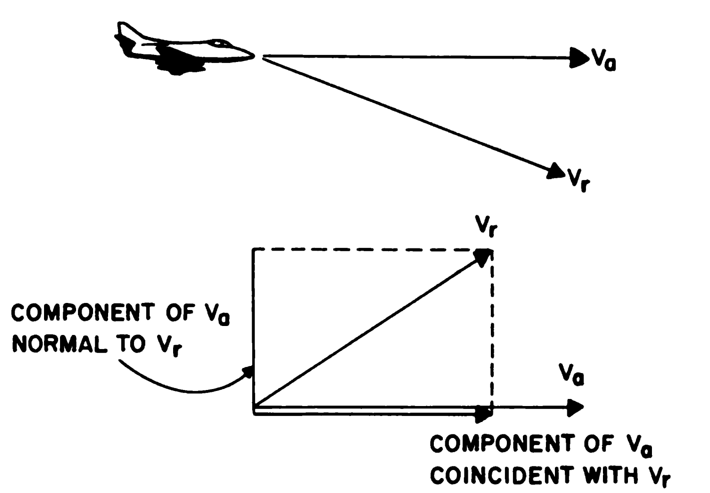
geometry of jump correction

Let us assume that a high speed aircraft is attacking a fixed surface target with rocket fire. In addition, let us assume that the rocket thrust alone produces an initial rocket velocity of V_r, and that the aircraft's speed is V_a. Also, let us neglect all curvature corrections except gravity. Now let us examine two situations: one in which the aircraft launches its rocket along its own heading line; and, second, when the aircraft launches its rocket at an angle with its own heading line.

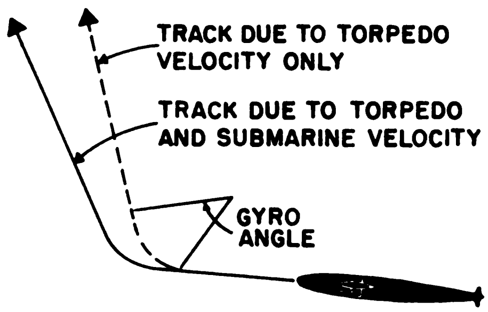
In the first situation, the aircraft computes total gravity curvature, which depends on the total time of flight, using the rocket's thrust velocity only. However, since the aircraft's own velocity is acting in the same direction as the rocket's thrust velocity, the two velocity vectors must be added. This increases the actual initial velocity of the rocket, and, since the aircraft is high speed in nature, the increase is significant. The result of the increased initial velocity is a decreased total time of flight. Now, since the force of gravity is constant, and, therefore, curvature due to this force depends upon the total time during which the force acts on the rocket, the total curvature will be significantly less than that originally expected. This causes the rocket to overshoot the target.



In the second case, as seen in the illustration, in addition to the overshoot problem resulting from the extra initial velocity along the weapon line, there exists a second component of initial velocity acting at right angles to the weapon line. This component changes the direction of the overall initial rocket velocity vector, thereby causing a miss.

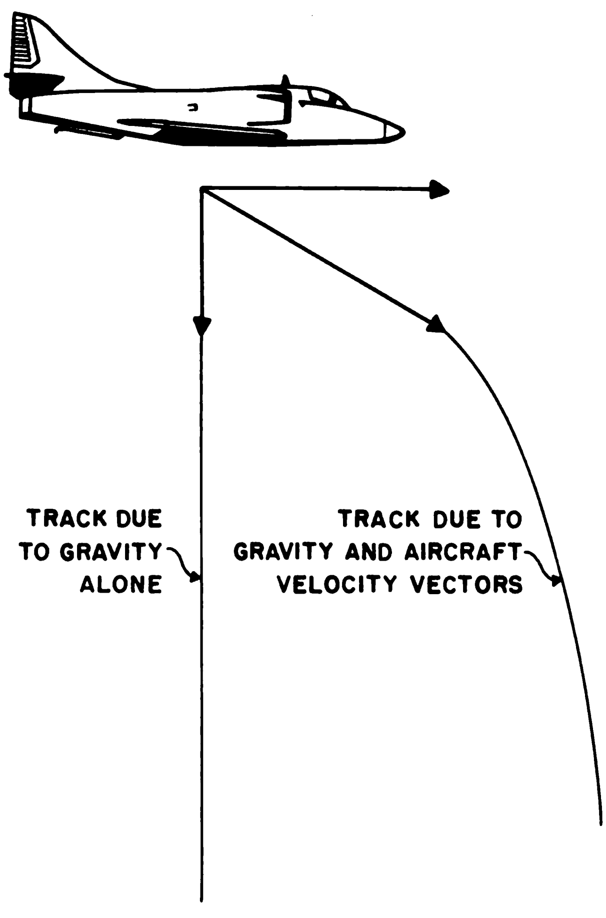


An initial velocity vector may be added also to the thrust velocity vector resulting from the force of wind or current at the instant of launch. Other examples of jump because of weapon station motion are illustrated.

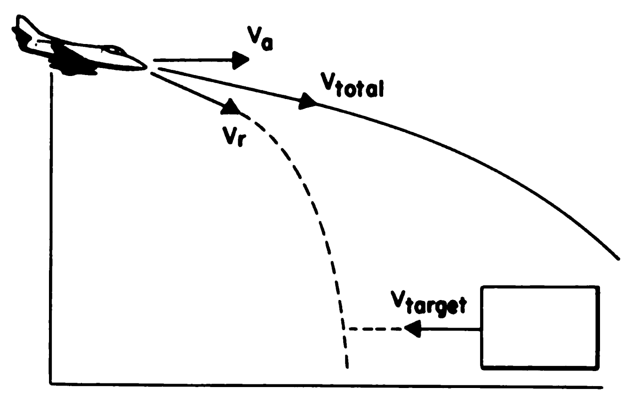


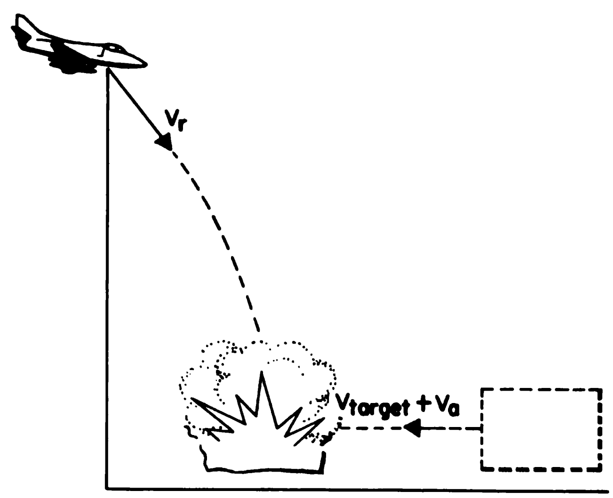
SUBMARINE

In many cases, it is possible to eliminate one or more of the miss-producing effects from the prediction angle computation merely by selecting a different reference frame. One such example is the elimination of the jump consideration, which resulted from an aircraft's velocity vector, by tying the reference frame to the aircraft. This, in effect, made the stationary target move and the resulting prediction angle included a lead correction. The advantage to this method in this particular problem may be readily seen if one considers a moving target rather than a stationary one. A lead correction must be computed regardless of whether the reference frame chosen is the aircraft or some fixed Earth reference. It is possible to limit the prediction computation here only to lead and curvature corrections and completely eliminate jump by selecting the aircraft as the reference.
Since the delivery vehicle is motionless in this reference frame, there is no jump and no jump correction. Then, the prediction is the lead angle. Note that the prediction angle is the same in both reference frames.



BOMBING





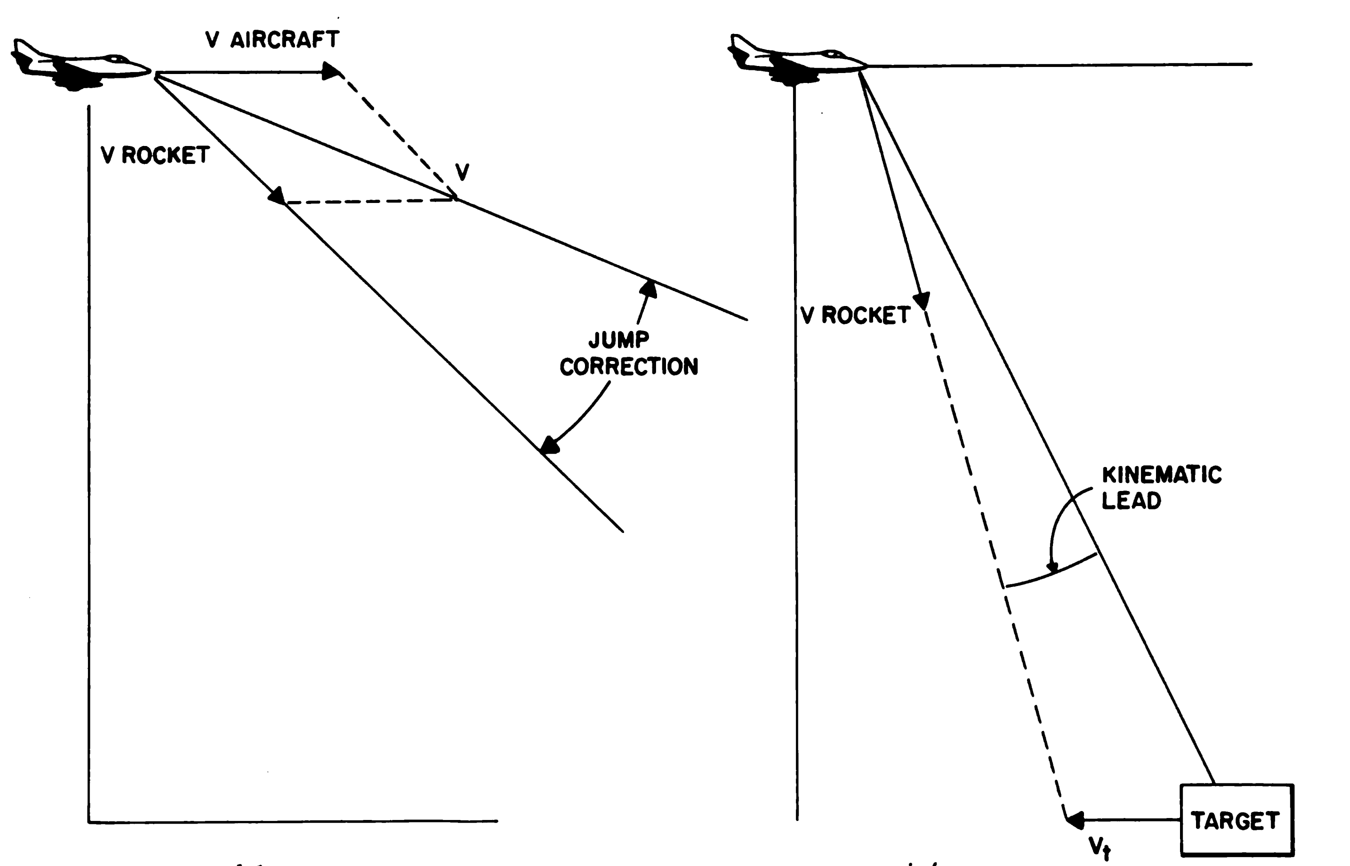
invariance of prediction angle

Even though we measure position and motion, and, consequently, the values of miss-producing phenomena with respect to a reference frame, it is interesting to note that the value of the prediction angle remains the same no matter which reference frame is used even though the corrections for miss-producing phenomena vary in value from reference frame to reference frame. However, this fact may be easily understood when one considers that no matter which reference frame is used, the prediction angle always refers to the angular difference between the tracking and weapon lines.

The tracking line has a unique position in space because it is determined by the line connecting the weapon station and the target and because these two points are at unique positions at any instant. Similarly, the weapon line, the line along which the missile must be fired in order to hit the target, is unique for any instant in time. The position of the weapon line in space is independent of the manner in which it is computed because this line may be aimed in only one direction in order to hit a specific target under specific conditions at a specified time. Therefore, it follows that since both the tracking and weapon lines have unique positions in space, the angular difference between the two must be unique regardless of the reference frame in which they are measured.

A simplified example which illustrates this dependence of the components on the reference frame but independence of the prediction angle itself is the case of an airplane firing an air-to-surface rocket against a surface installation. If the reference frame chosen is an Earth-based frame, the surface target has zero speed while the plane has a high speed relative to the ground. Kinematic lead will be zero, since the target is stationary, but there will be a considerable jump correction, a result of the high speed the aircraft imparted to the rocket. For simplicity, curvature correction is assumed zero, and thus the prediction angle consists solely of the jump correction.

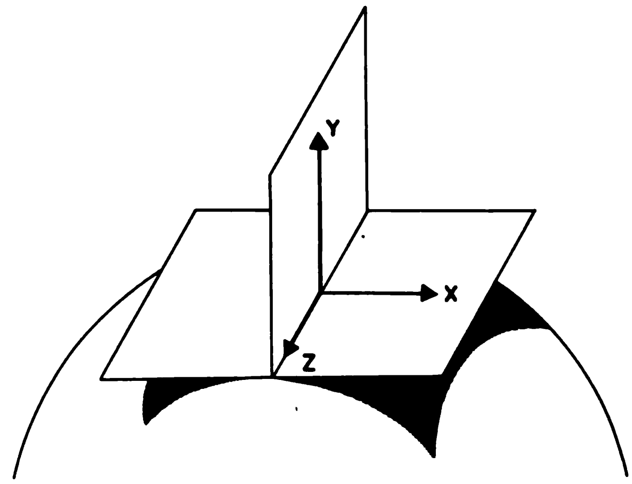
If the reference frame chosen is based on the delivery vehicle, the surface target will have a high speed relative to the plane, while the plane has zero speed. Since the delivery vehicle has no velocity, it can impart none, and the jump correction is zero. The motion of the target results in kinematic lead. Again, curvature correction is assumed to be zero, and the result in this instance is that the prediction angle consists solely of the kinematic lead. However, the prediction angle is the same in both cases. If curvature correction were considered, it would also change when a different reference frame is used. A different prediction angle would result but, like the prediction angle shown, it would be independent of the reference frame selected.

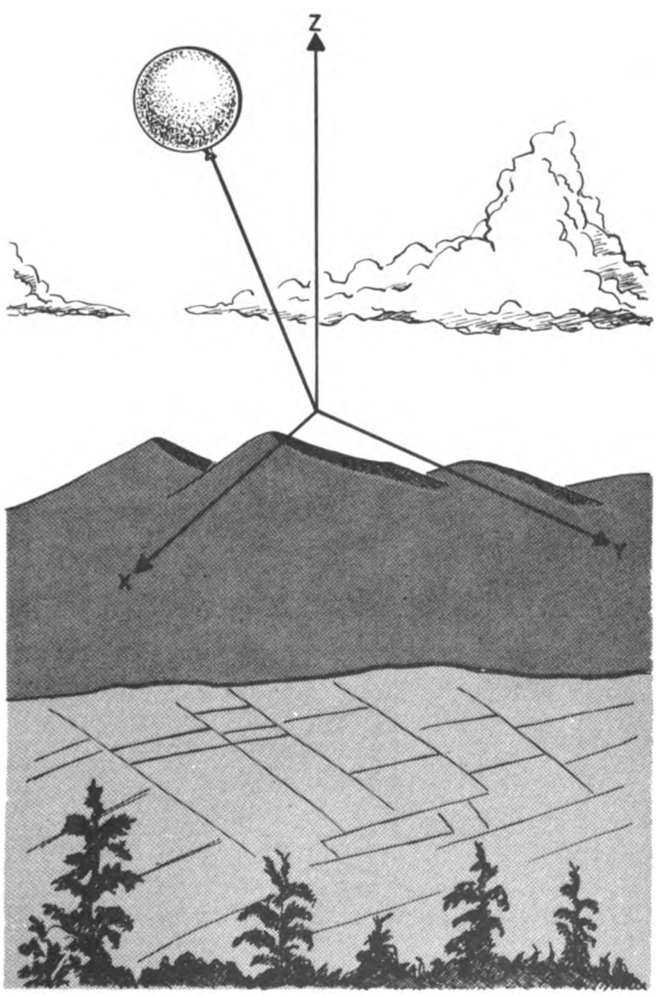


earth frame *air frame*

REFERENCE FRAMES

Before the fire control problem can be completely described, a set of references must be established. Angles cannot be discussed or even measured unless a reference line is established from which to measure them. Similarly, distances cannot be discussed until a point from which the distances are measured is specified. Velocity requires a coordinate frame reference to which it must be specified. For example, air speed and ground speed both correspond to velocities, but they are different because their reference frames (air mass in one case, and the ground in the other) are different.





As indicated previously, the basic parameters of the fire control problem are the line of sight, the weapon line, and the prediction angle. The tracking device will operate to keep a controlled line (the tracking line) in coincidence with the line of sight. There are then two driven lines that are of primary concern, the tracking line, and the weapon line. Each may be regarded as inscribed on physical equipment, and as such may be part of an established reference frame. This reference frame will be the one in which the tracking equipment and the weapon operate. In general, the reference frame will be delivery vehicle based and the coordinate system will be determined by the method used to indicate the position of the driven line.

The most common coordinate system used is spherical coordinates in which the train and elevation of the driven lines are known. The computed prediction angle must be generated in a reference frame; it is this computing frame that largely dictates the choice of other coordinate frames used in the solution of the fire control problem.

basic groups

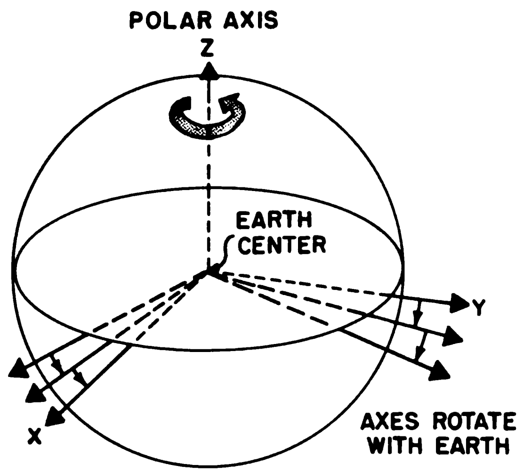
Computing frames fall into two basic groups:

1) frames (usually stabilized with respect to Earth) in which both the tracking line and the weapon line may be rotating,

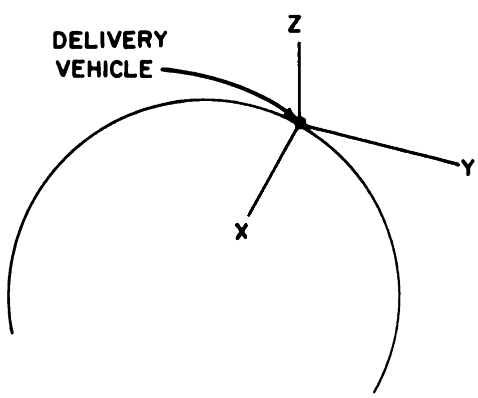
2) frames in which either the tracking line or weapon line is chosen as one coordinate axis; the line not chosen as an axis is measured relative to the other line.

earth frames

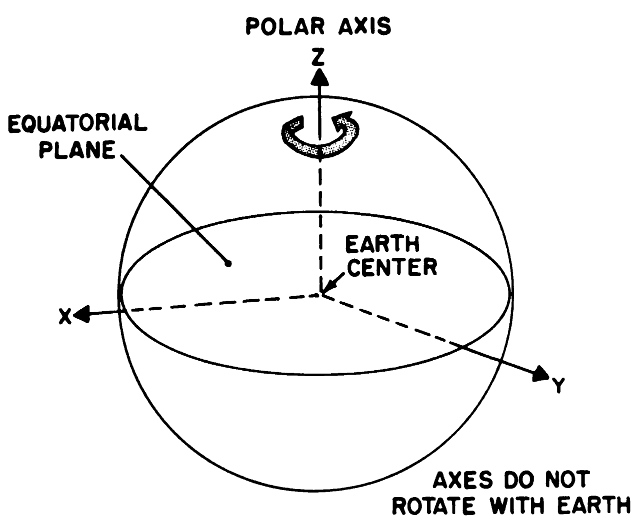
In the first type of reference frame, the coordinates are usually established by a gyro compass and a stable element to indicate north and vertical, respectively. All measurements are then made or transformed through coordinate conversion into this reference frame for computation. Also, the tracking member and the launcher are usually driven relative to this reference frame.



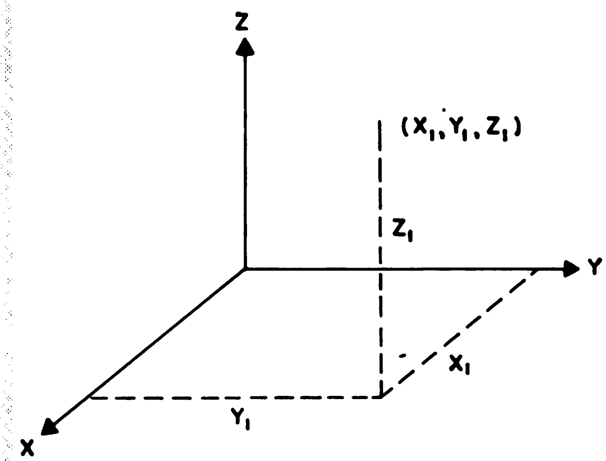
geocentric reference frame

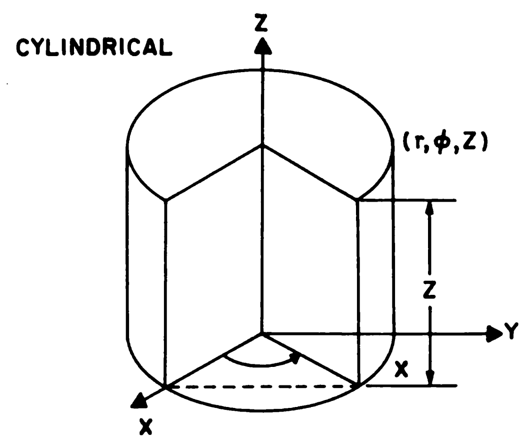


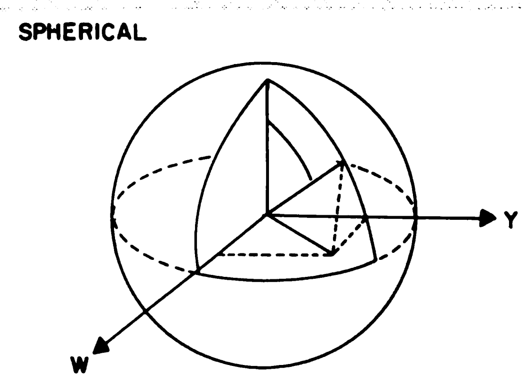
vehicle-centered reference frame

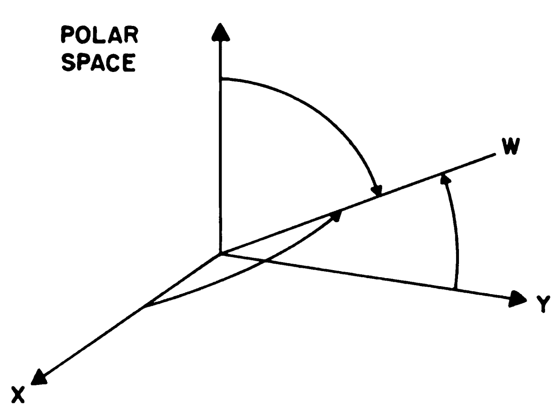


geocentric inertial reference frame



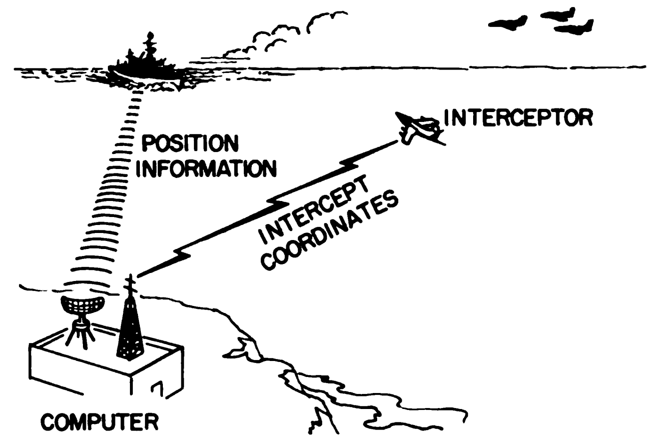






To illustrate the use of the reference frame and also the process of coordinate conversion, assume the following situation:

A shipboard tracking radar on a destroyer patrolling a friendly coast measures aircraft to get coordinates in spherical coordination relative to its stabilized deck plane and its own ship course. These coordinates are transformed into an Earth reference frame with north and vertical axes. The new coordinates, plus the ship's own positional coordinates in latitude and longitude, are then fed to a land-based interceptor aircraft system which further processes both sets of coordinates to determine latitude and longitude of the targets. The land-based system then directs its interceptor aircraft to the computed intercept position based on the present target position.



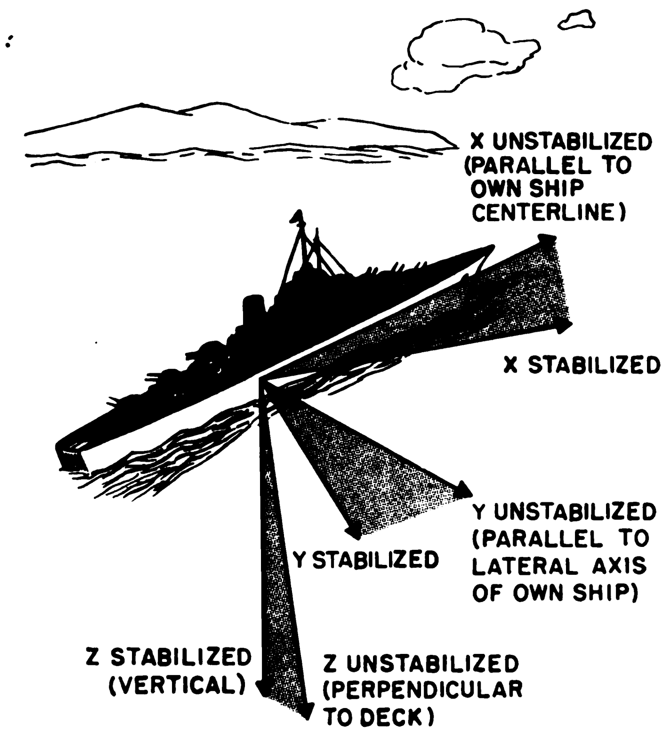
$$x = r \cos \phi \qquad\qquad r = \sqrt{x^2 + y^2}$$
$$y = r \sin \phi \quad \Rightarrow \quad \phi = \tan^{-1}\left(\frac{y}{x}\right)$$
$$z = 0 \qquad\qquad z = z$$

$$x = \rho \sin \theta \cos \phi \qquad\qquad \rho = \sqrt{x^2 + y^2 + z^2}$$
$$y = \rho \sin \theta \sin \quad \Rightarrow \quad \phi = \tan^{-1}\left(\frac{y}{x}\right)$$
$$z = \rho \cos \theta \qquad\qquad \theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$$

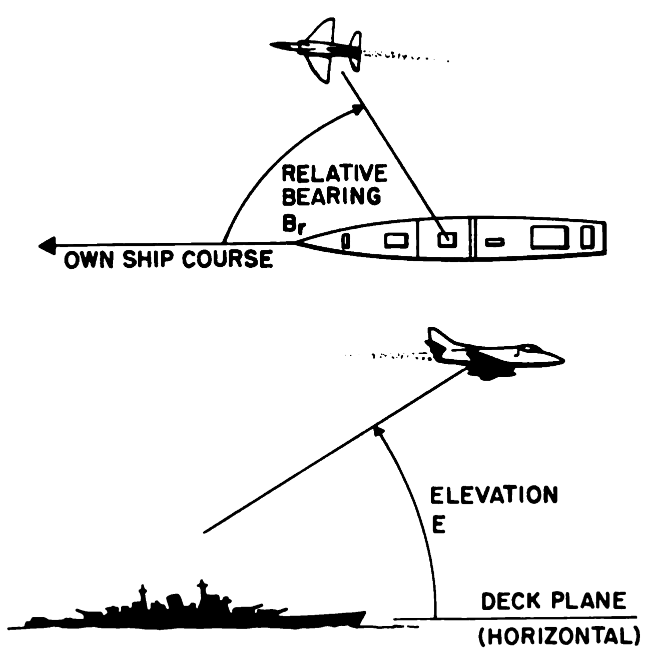
$$\rho = \sqrt{x^2 + y^2 + z^2}$$
$$x = \rho \cos \alpha \qquad\qquad \alpha = \cos^{-1}\left(\frac{x}{\sqrt{x^2 + y^2 + z^2}}\right)$$
$$y = \rho \cos \beta \quad \Rightarrow \quad \beta = \cos^{-1}\left(\frac{x}{\sqrt{x^2 + y^2 + z^2}}\right)$$
$$z = \rho \cos \gamma \qquad\qquad \gamma = \cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$$

STABILIZED DECK REFERENCE FRAME

The deck reference system used by the destroyer's radar has as its coordinates its own ship's course and stabilized deck plane.



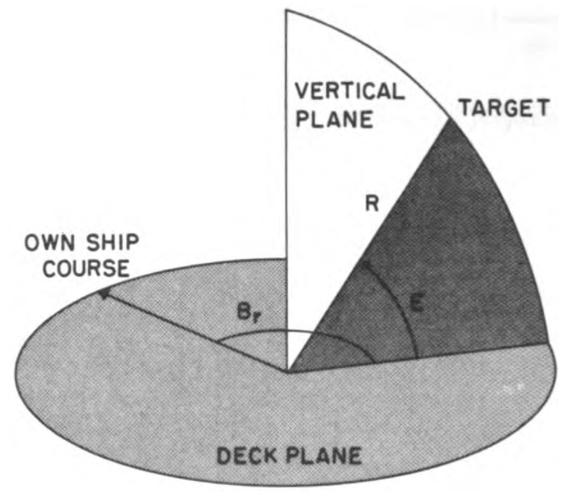
We have assumed that the reference frame is stable; the effects of pitch and roll may be neglected for this example. All bearing angles are measured in the clockwise direction in the horizontal plane, starting from the line indicating own ship course; all elevation angles are measured relative to the stable deck plane (horizontal).

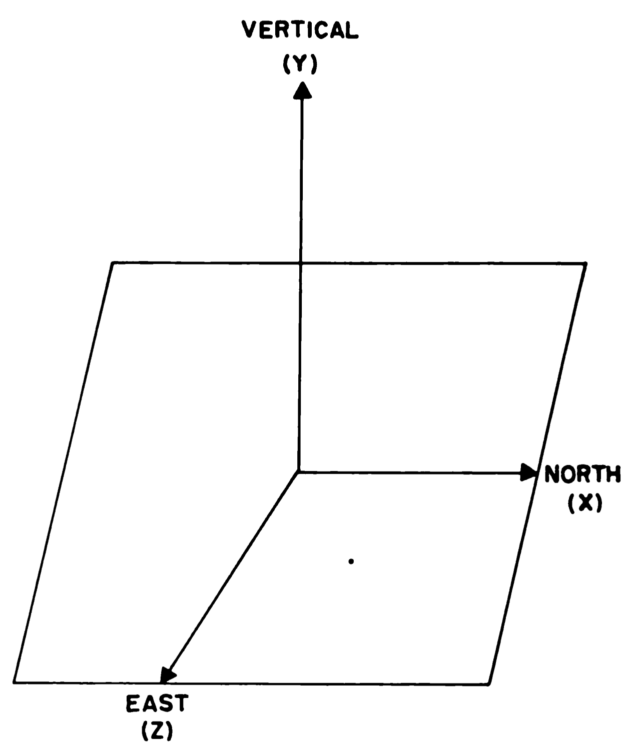


Now let us assume that the target is detected at an elevation angle E, at a relative bearing angle B, and at range R. This information is fed to a computer for coordinate conversion into Earth reference coordinates (North, vertical). The coordinate conversion can be described as two separate conversions:

1) from deck elevation to coordinates along the vertical (y) axis,

2) from relative bearing to coordinates along North (x) and East (z) reference axes.





The first conversion, from deck elevation to y axis, is simple because a direct trigonometric relation performs the entire coordinate conversion, i.e.,

$$y = R \text{ Sin } E$$

Now, in order to derive XZ coordinates, the range vector R, must be resolved into the horizontal plane. The solution to this problem, as illustrated, is

$$R_h = R \text{ Cos } E$$

From the ship's gyro compass, own ship heading, Cqo, is determined. This angle is added to the relative bearing angle to obtain the bearing angle of the target to north, B_n,

$$B_n = C_{qo} + B_r$$

In the particular case illustrated, C_{qo} is a negative angle, so the equation actually reads

$$B_n = B_r - C_{qo}$$

At this point, we have computed a range and bearing for the target in polar coordinates within the horizontal plane. It remains to convert these coordinates into distances along the x and z axes. Using basic trigonometric formulas, the following equations are derived:

$$x = R_h \text{ Cos } B_n$$

$$z = R_h \text{ Sin } B_n$$

Using previously calculated values for R_h and B_n,

$$x = R \text{ Cos } E \text{ Cos } (C_{qo} + B_n)$$

$$y = R \text{ Cos } E \text{ Sin } (C_{qo} + B_n)$$

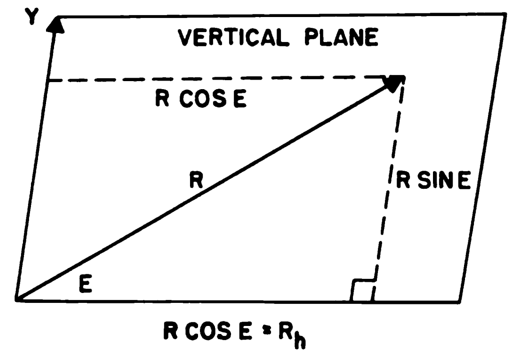
Hence, we can list the entire set of equations necessary to convert from spherical deck (stabilized) coordinates to the Earth coordinates required:

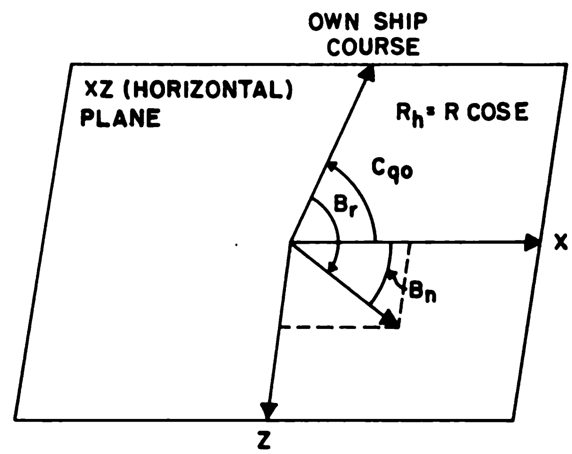
$$x = R \text{ Cos } E \text{ Cos } (C_{qo} + B_r)$$

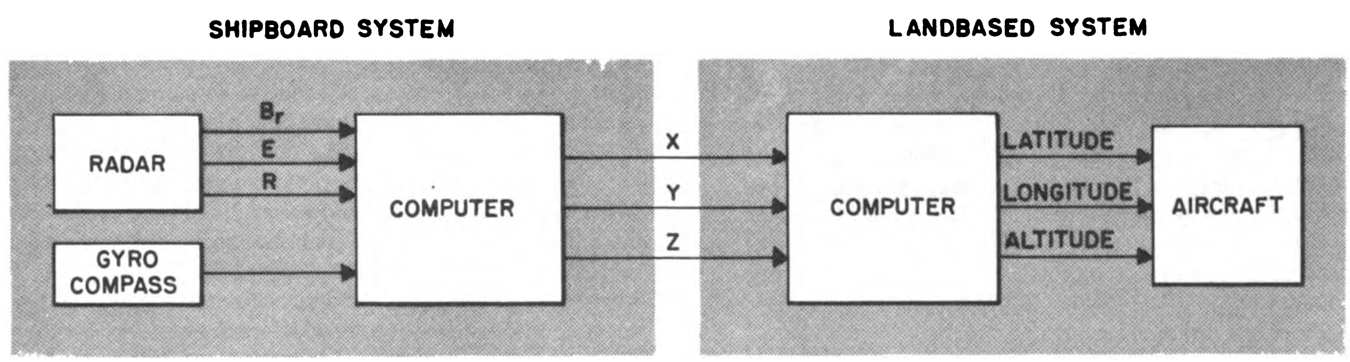
$$y = R \text{ Sin } E$$

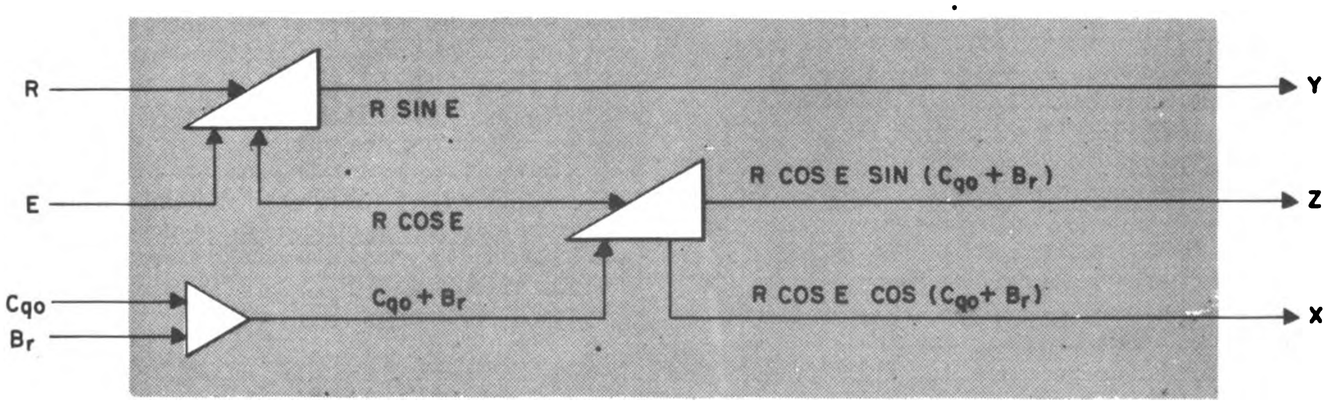
$$z = R \text{ Cos } E \text{ Sin } (C_{qo} + B_r)$$

The accompanying illustration shows a block diagram of the computer required to perform the conversion as well as an overall block diagram of the system described.









UNSTABILIZED DECK REFERENCE FRAMES

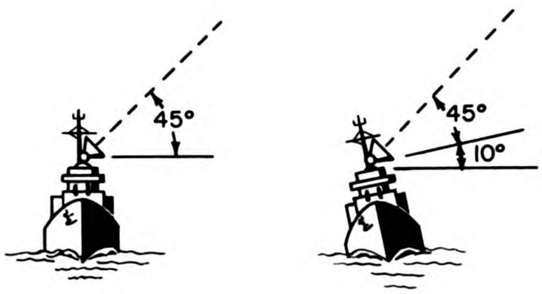
Coordinate frames such as the deck system described may be stabilized, as in the problem, or may be partially stabilized, or not stabilized at all. In an actual stabilized deck reference frame, for instance, the tracking radar would require gyros to detect deck motion due to pitch and roll and to compensate for this motion so that the radar points to the same point in space regardless of the angle between the deck plane and the horizontal.
To illustrate the effects of pitch and roll and the requirements of a stabilized deck coordinate system let us examine what happens if there is no stabilization.
Suppose that at time t_o, the radar is positioned at a relative bearing angle of 90 degrees and an elevation angle of 45 degrees and the deck and horizontal planes are coincident. Now, between times t_o and t_1 the ship rolls through an angle of 10 degrees. It can be seen that if the radar is unstabilized, i.e., maintains a deck elevation angle of 45 degrees, the actual elevation angle with respect to the horizontal is 55 degrees, producing a 10-degree error.
In a stabilized system, however, the amount of roll would be sensed by a gyro or stable element and the deck elevation angle would be changed to compensate for the roll. This process is shown on an accompanying illustration.
In addition, a partially stablized system may be used, in which case some of the ship motion effects on the radar are eliminated.
If measured tracking data are acquired in an unstabilized frame, and the prediction computer operates in a stabilized frame, a transformation from unstabilized to stabilized coordinates must be made at the computer input. Similarly, if the weapon launcher operates in an unstabilized frame, the reverse transformation is required at the computer output.
The tracking line directions may be indicated using an unstabilized reference frame but because it will roll, pitch, and yaw with the delivery vehicle, this motion will disturb tracking. A stabilized reference frame can be made available by using a gyro compass or stable element, and it can be used to measure roll, pitch, and yaw, and to generate signals that will enable compensation of the tracking data for these motions.

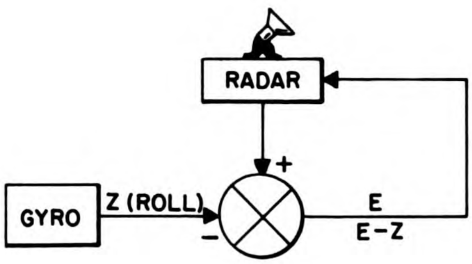
coordinate system of axes

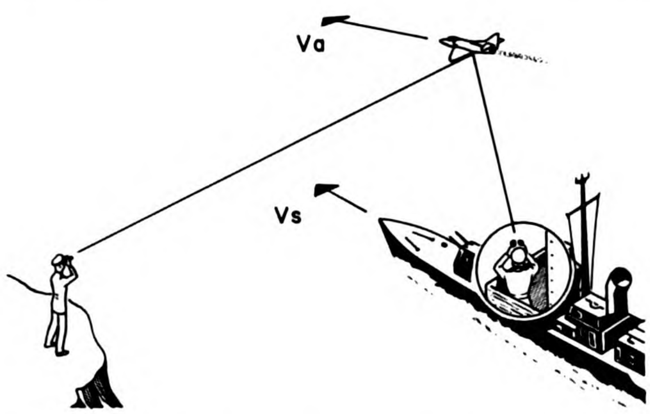
The type of reference frame in which measurements are made is largely determined by the equipment available at the weapons system. Present equipment used to measure speed, direction, or distance of the target (or any similar measurement), operate to determine the quantity with respect to a coordinate system of axes established in the equipment. When a rangefinder determines the range to a target, it measures the distance from the target to the reference point in the rangefinder. Angle of train and elevation of the target are usually determined by indicating devices attached to the train and elevation axes of the telescope, radar antenna, or other detection device. These indicating devices operate with respect to the mounting of the detection device which is normally fixed to the delivery vehicle. Thus the tracking line is usually determined initially in an unstabilized weapon station reference system. If the detection system is fixed on the ground (as in a shore-based weapon system), the measurements are being made in an Earth reference frame.

Our measurements of speed are always with respect to the medium in which we are traveling. A car's speedometer gives us an indication of the speed of the car in relation to the roadway. Since roads are fixed to Earth, the speedometer indicates the speed of the car in an Earth referenced frame. In an airplane, the speed indicator indicates the plane's speed in relation to the air. When there is no way to determine the velocity of the air with respect to the ground, we are unable to know directly the speed of the aircraft with respect to the ground. For this and other reasons, problems of airborne fire control often are solved using air mass reference frames. Shipboard fire control systems often use weapon system reference frames.

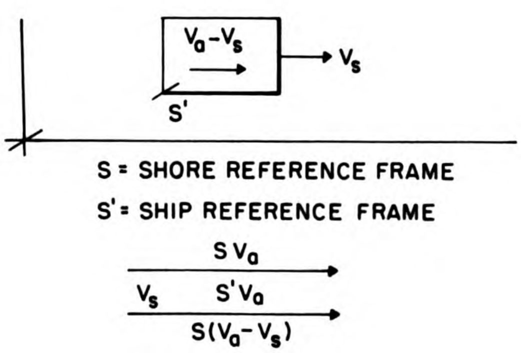
The solution of the fire control problem for long range and intermediate range missiles in which the rotation of the earth is a significant factor is solved in a geocentric inertial frame. Several examples of a fire control problem solved in different reference frames are illustrated in section 5 of Appendix A.
An example of a problem which must be considered when different elements of a system use different reference frames is that of velocity measurement. In order to simplify the discussion of this problem, let us limit it to two-dimensional space.
Motion is always measured relative to some point which is considered fixed. For instance, a man standing on the deck of a moving ship and observing an aircraft flying overhead estimates the plane's speed with relation to himself or, since his position is fixed with respect to the ship, he estimates the plane's speed with respect to the ship. Another man, standing on shore nearby, observes the same plane and estimates its speed with respect to his position fixed on the surface of the Earth. When both men compare their observations, however, they find that these observations do not agree.
Let us assume that the aircraft was moving at a speed of V_a with respect to the man on the coast line, and that the ship was moving at a speed of V_s with respect to the man on the coast line. These, then, are the values that the man on the coast line observes.
Let us further assume that the plane was flying directly over the ship and in the same direction as the ship. The man on the ship sees an airplane speed of $V_a - V_s$ since he can perceive only the difference in speed between the plane and the ship, i.e., the speed of the plane within his own reference plane.

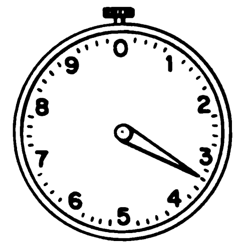






Therefore, if he wanted to convert his estimate into the reference frame used by the man on shore, he would have to consider his own speed (the speed of his reference frame) with respect to the shore reference frame, and add this value to that of the airplane which he perceived within his own reference frame. In this manner, velocity conversion between reference frames may be performed.





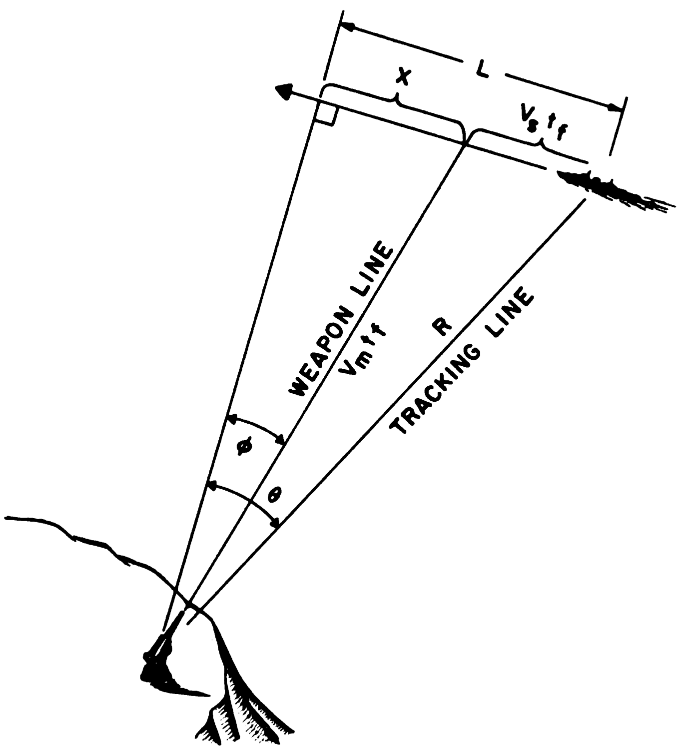
TIME OF FLIGHT

The actual time of flight of the missile is determined first, by the initial missile velocity and, second, by those forces that tend to accelerate or decelerate the missile by aiding or opposing its motion, i.e., that tend to change its average velocity over the flight path. The time of flight can be determined by knowing that for successful target intercept, the target and the missile must be in the same place at the same time. The future target position can be predicted on the basis of its present position, its assumed flight path, its velocity, and the length of time from the present to the impact point. Future position is then a direct function of time of flight. Since the missile flight path is somewhat more under our control than the target flight path, the degree of prediction necessary for the missile flight path is less. From our knowledge of missile paths and the forces acting on the missile, the time it takes to arrive at various points on the path can be determined after the path is determined. However, the missile flight path is a function of the weapon line position which is determined by the tracking line position and the prediction angle. A primary component of the prediction angle is kinematic lead based on the future position of the target which is then a function of the time of flight. Thus, the solution of the time of flight requires the solution of two simultaneous equations, one based on the target's future position and the other on the missile's future position. In many cases, the time-of-flight computation can be simplified by postulating a set of standard conditions. These conditions are that the weapon system is stationary in the reference frame, there is no apparent wind, and the initial velocity of the missile with respect to the weapon system is assigned a standard value. Using these standard conditions, a relationship can be determined between range and time of flight.

To illustrate the time of flight calculation, let us solve a simplified fire control problem. Assume that a shore artillery installation is assigned the task of destroying a ship patrolling the coastline. The velocity of the ship is known, V_s, as well as the ship's present position. The initial velocity of the projectile, V_m is also known.

If the target position and course are known, the angle between the present range vector and a line normal to the target track may easily be computed. From the resulting triangle, the following equation may be written:

$$L = R \sin \theta$$



Since R and θ are measured by the tracking device, L is easily computed.

From the geometry,

$$L = V_s t_f + X$$

$$X = V_m t_f + \sin \phi$$

$$\therefore L = V_s t_f + V_m t_f$$

$$L = t_f (V_s + V_m)$$

Substituting the initial equation for L into this last equation:

$$R\,(\text{pres}) \sin \theta = t_f (V_s + V_m)$$

$$t_f = \frac{R(\text{pres}) \sin \theta}{V_s + V_m}$$

Since all the quantities on the right side of the equation are either known or measured, the value of t_f may be calculated.

summary

To summarize this section on the weapon control problem, let us restate the basic problem: to fire a missile from a moving weapon station at a moving target to obtain a hit on the target. We have seen that various miss-producing effects prevent us from firing directly at the target for a hit. In general, therefore, it is necessary to offset the weapon line from the tracking line in such a manner that the effects of various miss-producing phenomena are compensated for. The total offset angle is the sum of all the various corrections.

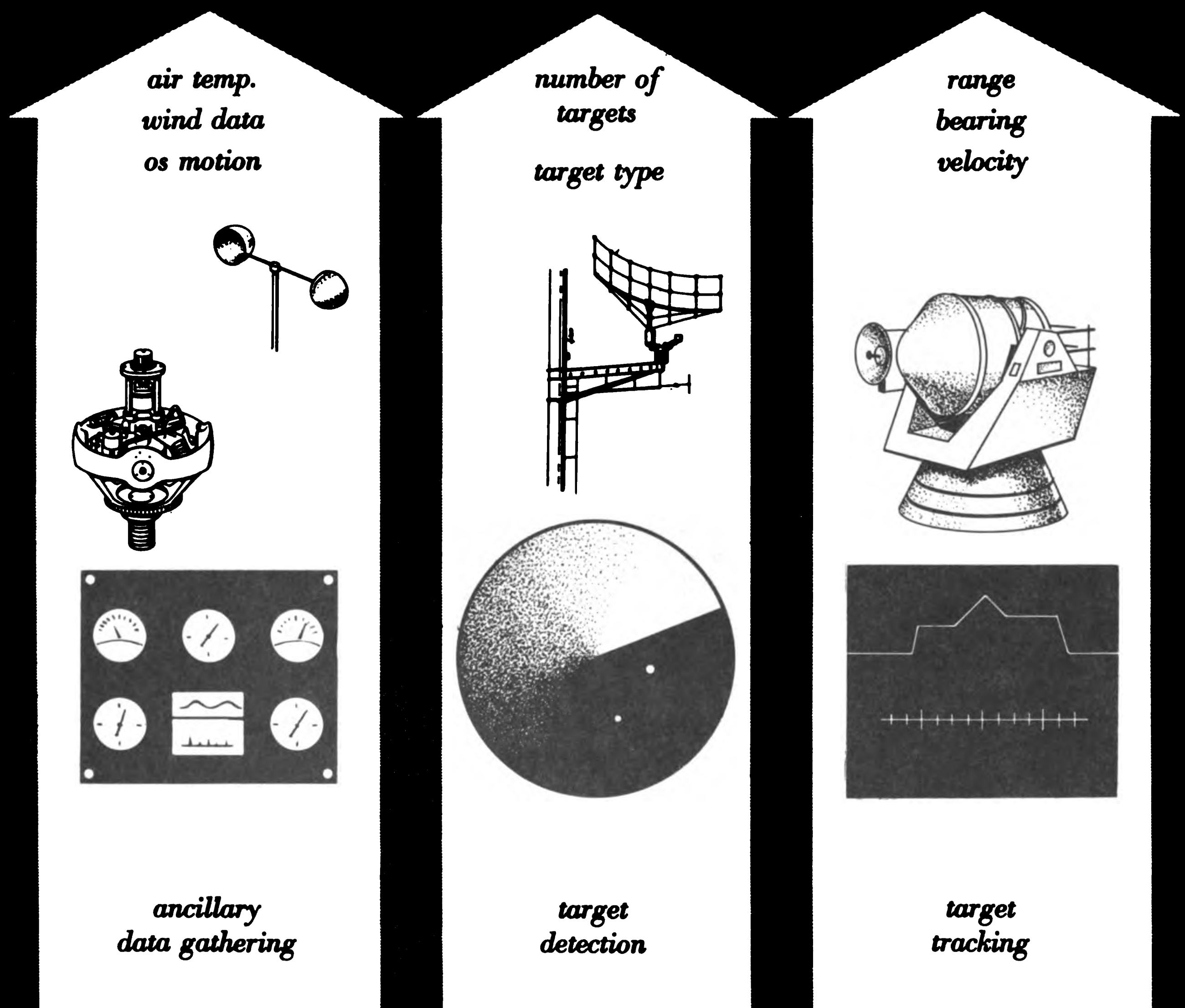
We have seen earlier in this section that the magnitudes and the categories of the various correction angles may change or be eliminated through the choice of reference frame used. However, regardless of how these correction angles change, their sum total, the prediction angle, remains the same. This may be understood easily if one considers that the tracking line is fixed in space at any instant since the target is fixed in space at any instant. Also, the weapon line must be fixed in space for a particular target position and a particular set of conditions. Since the position both of these lines is invariant, the angle between them must be invariant.

The solution to the fire control problem, then, consists of the following procedures:

1. Determine present target position and motion data.
2. Compute the prediction angle.
3. Offset the weapon line from the tracking line by an angle equal to the prediction angle.
4. Fire the missile.

INFORMATION GATHERING

In fire control problems, we are required to find out first that a target does exist. If more than one weapon system is available, information is needed on the target, its characteristics, and the threat it poses so that we can decide which weapon system should be used to kill the target. When a weapon system begins its task, it requires the location of the present line-of-sight. It must be supplied also with sufficient information on the target and the delivery vehicle to determine the components of the prediction angle, lead angle, curvature correction and jump correction. Then, the prediction angle may be calculated from this information. Information gathering procedures are subdivided into detection, tracking, ancillary data, and methods of collecting ancillary data.



DETECTION

The detection of targets may be either a direct function of the weapons system as it is when the weapon normally operates isolated from other weapon systems (such as torpedo weapon systems on submarines), or a function of an overall weapons control system, which is used whenever several weapon systems operate jointly. Examples of this are large ships which are delivery vehicles containing more than one weapon system or formations of ships or formations of aircraft, each carrying one or more weapon system. Regardless of whether the detection system is a component of the weapons system as installed on the delivery vehicle or is a component common to several weapons systems, the operation of the detection system is to initially determine the target position.

methods of detection

The method of determining the target position may be from some form of energy being emitted or reflected by the target or, in the case, of fixed targets (such as airfields, shipyards, and other military installations), they may be detected and their positions noted by various means including aerial photography and intelligence data. In those cases in which the position of the target is determined from radiant energy either emitted or reflected by the target, the line-of-sight often is the line along which the energy is received. This assumes that the energy travels in a straight line with no refracting, bending, or reflecting off various layers of the medium. This is usually valid for light and other electromagnetic waves in air, although many new radars use the principle of tropospheric scatter to increase their range.

Using acoustical energy in water, the bending of the sound wave is appreciable. This bending introduces another error. The relatively slow speed of sound permits the target to move an appreciable distance before the energy emitted is received. Therefore, sonar not only indicates an apparent position of the target, but an apparent past position of the target. By using data on the density of the medium at the various layers, and the location of each of the layers, the true target postion can be determined from the apparent target position. Using this same information, the length of time required for the energy to travel from the target to the detection device can be determined. The prediction circuits can use this data on past target position to predict both present and future target positions.

The quantity in which we are interested is the vector from the delivery vehicle to the target. The magnitude of the vector is the range to the target, and the direction is along the line of sight. Although the complete specification of a vector includes a statement of both magnitude and direction, these quantities may be measured by separate devices. Thus, an air speed indicator measures the direction, but not velocity. A magnetic compass measures the direction, but not the magnitude, of Earth's magnetic field.

The case of present interest is that in which only the direction of the vector is required. When the line of sight to the target is determined, the range can be found readily.

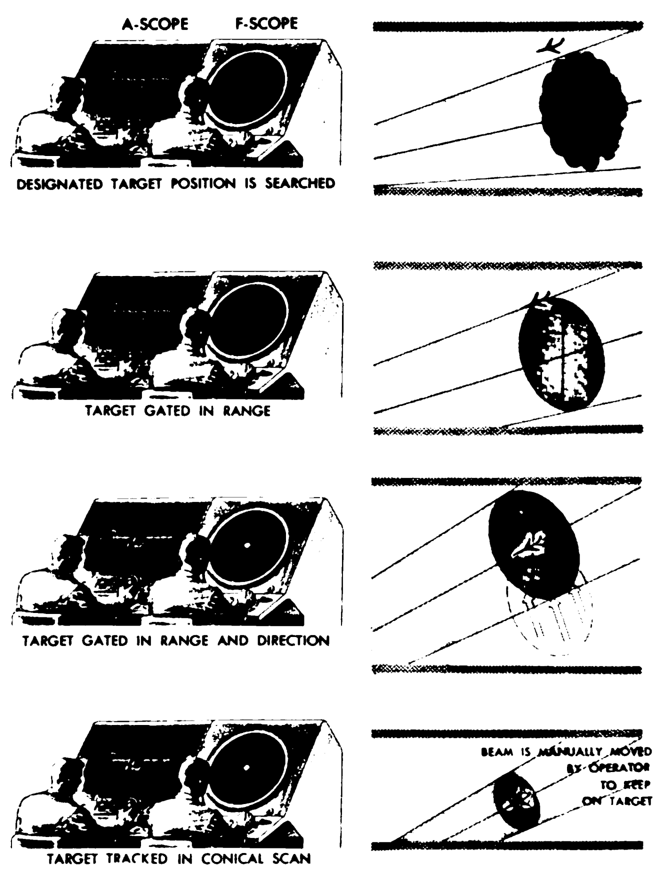
determination of line-of-sight (los)

Knowing the position of the target and the position of the delivery vehicle, the line of sight is determined. If the target or the delivery vehicle, or both, are moving, the line-of-sight is subject to variation, and must be monitored. In virtually all weapon systems, sensors are required to determine the present position of the target with respect to the delivery vehicle. Exceptions to this are weapons systems, such as POLARIS, which employ inertial navigation principles to determine the present position of the delivery vehicle. As locations of the fixed targets the system is to destroy have been determined previously, correct flight paths can easily be determined.

TRACKING

The process of bringing a line on some controlled physical measuring device into coincidence with the energy from the target is called tracking. In fire control, the radiant energy emitted or reflected from a target is tracked by using optical sights in the case of visible-light energy, by heat-sensitive detectors in the case of infrared energy, by angular tracking radar sets in the case of microwave energy, and by sonar devices when sound energy is used.

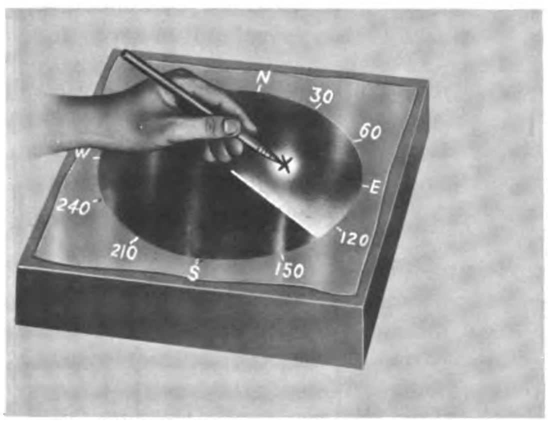
manual tracking

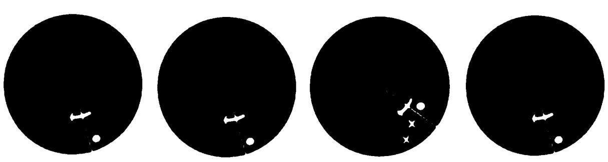


tracking rates

Tracking may be done at predetermined equal intervals, or it may be an uninterrupted process. If tracking is done periodically, the result is a series of points which, when connected closely, approximate the target's actual path. An uninterrupted tracking process produces a continuous curve indicating the target's actual path. Either method is valid, since the plotting and connecting of points in the former solution results in a solution comparable to the second method.

By tracking periodically, one tracking device can be used to track several targets by tracking each one, in turn, for a period. The frequency at which each target must be tracked is a function of many factors, among which are: the characteristics and behavior of the target, the medium in which vehicle tracking occurs, the tactical situation, and the sensing equipment used. If a target is slow, where is little danger of anything occurring between individual plots. Low maneuverability targets are more or less restricted to straight line or slightly curved paths, again not necessitating high frequency plotting.





high speed high maneuverability targets

High speed targets, such as aircraft, must be watched closely because they can travel far between plots. A highly maneuverable target might be capable of executing maneuvers between plots; the result of intermittent plotting would not give a correct representation of the actual target path, particularly if the target were executing evasive maneuvers. A target which has both high maneuverability and high speed (such as many aircraft) must be tracked very closely because it may move far in almost any direction between plots, if the plots are made at widely separated time intervals.

medium

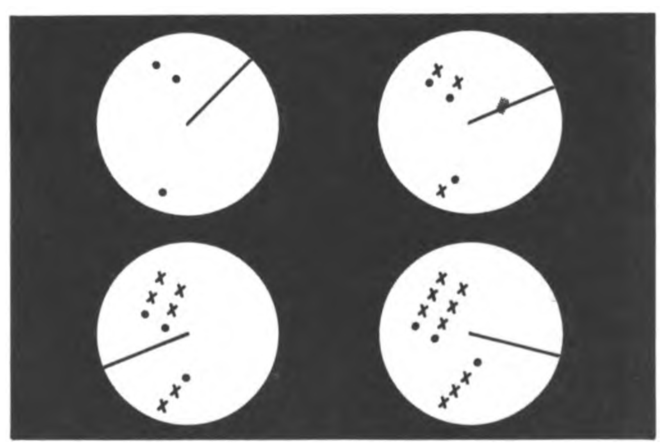
The medium in which the tracking occurs will determine the frequency of plotting necessary for effective tracking. In air, there exists reasonable certainty of receiving energy from the target each time it is sighted. When sonar is operated continuously, there are times when no echo will be heard. If sonar is operated only intermittently, this problem would be accentuated.

tactical situation

Naturally, the tactical situation is a primary factor in determining the desirable frequency of tracking. When only one target is being engaged at any time, that target is tracked continuously. No benefit would be derived from tracking the target only part of the time. Many weapon systems, particularly those intended for use primarily against aircraft, must be able to engage several targets simultaneously. It may not be possible to assign one tracking device to track each target; space is seldom available for all the radar sets which would be required to assign one of each target.

multiple targets

To track several targets at the same time presents a risk. Precise positional information on an individual target exists during only the brief period it is being tracked. During the other times, the weapon system has information on the course of the target, which may launch missiles, activate decoys, or perform other tasks of which we may not be aware until the next tracking period.



The risk can be reduced by increasing the frequency of tracking intervals. However, the frequency of tracking is limited by two factors. The first factor is the speed at which the controlled line in the physical measuring devices can be moved from one direction to another. The second is that active tracking devices, such as many sonar and radar sets, require an appreciable amount of time for the energy to be transmitted to, and reflected from, the target before it is received by the detecting device.

ANCILLARY DATA

The tracking data, received either by plotting or by continuous tracking, gives positional information on the target primarily. Rate information is sometimes measured directly, but is often calculated on the basis of the positional data. The kinematic lead may be predicted from the data on target position and velocity. The remaining components of prediction are not directly based on the target. Curvature correction requires data on medium density, winds or currents in the medium, the force of gravity over the flight path, any acceleration of the reference frame, and the velocity of the missile propulsion system. To determine jump corrections, velocities of the delivery vehicle, apparent wind in the vicinity of the delivery vehicle, and the missile at launch must be calculated. Much of this data is available from other sources. Data on the conditions of the surrounding medium may be available from the sources, or they may be determined by meteorological instruments such as barometers, anemometers, and balloons.

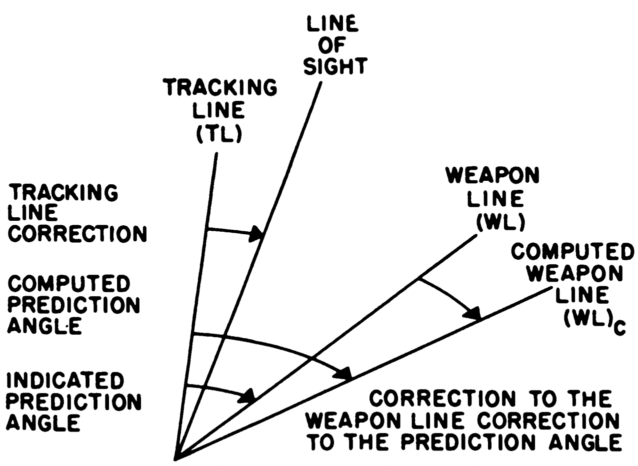
target line

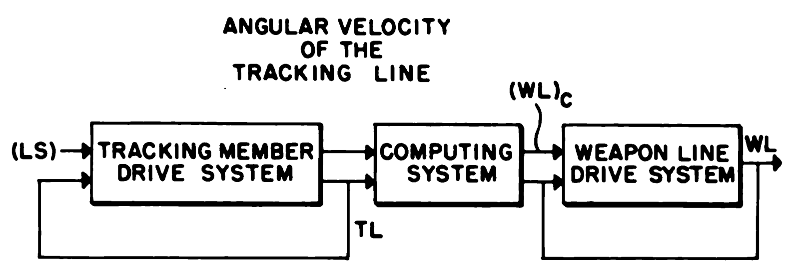
For a fire control system to implement the solution to the fire control problem, it must receive information on target position and motion. This information is the principal input to any fire control system. The line of sight between the sensor and target along which radiant energy from the target is received by the sensor is used to track the target. The line within the fire control system which tracks the line of sight is called the tracking line. Therefore, the tracking line is the line in the fire control system which corresponds to the line of sight, and is used by the fire control system as the solution of the fire control problem.

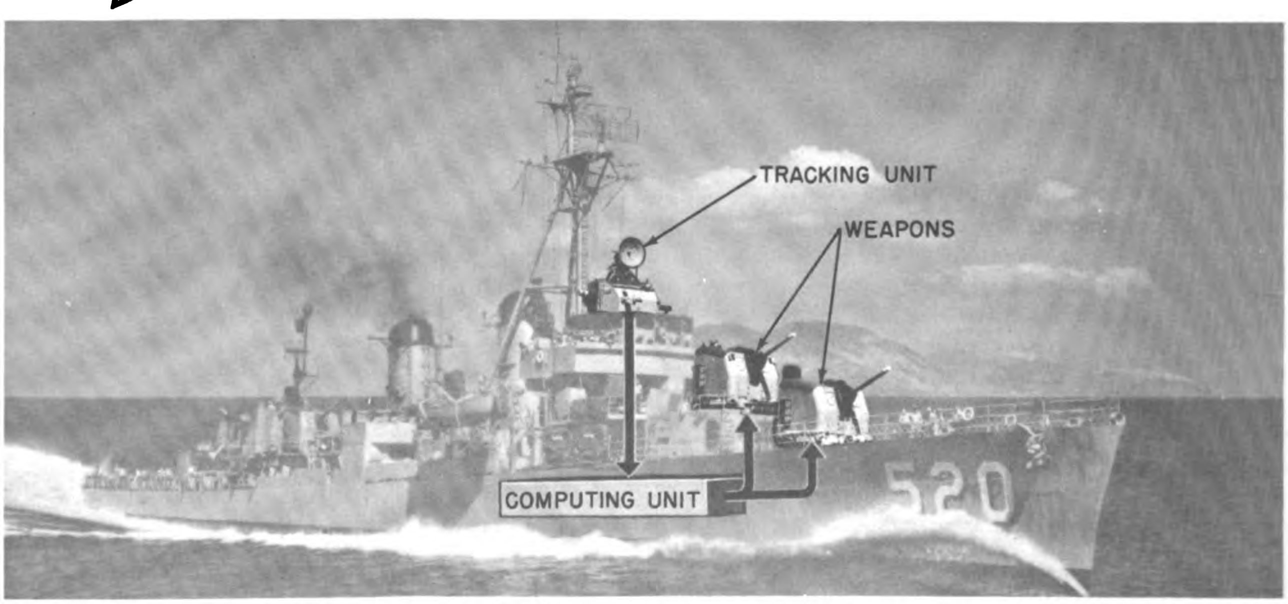
correction to the tracking line

The tracking line must be a controlled line in the fire control system. Therefore, a tracking member drive system is required in every fire control system to orient the tracking line. If the tracking line and the line-of-sight are not coincident, a tracking inaccuracy results, and this inaccuracy requires the computation of a tracking inaccuracy correction. The correction to the tracking line is the angle measured from the tracking line to the line-of-sight and is nulled in the process of solving the fire control problem. Therefore, it is expected to be small.

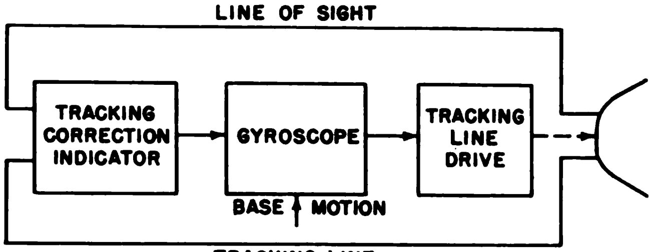
Nulling is used in the servo sense of reducing the correction quantity to its smallest possible value, and does not necessarily mean reducing it to zero. It should be noted that the angle must be nulled as a vector in space, irrespective of any reference coordinate system.







The instrument that detects components of the tracked vector perpendicular to the controlled line is the tracking correction indicator. The tracking system operates to null these perpendicular components, thereby rendering the tracked vector and the controlled line essentially parallel. The angle between them always approaches a null value in the sense that any servo-error quantity does.



METHODS

The methods of nulling the correction to the tracking line may be divided into two major categories. In one method, the detection device, antenna or telescope, is driven directly to null the error. The weapon is then driven separately to maintain its displacement from the tracking line at the prediction angle. In the other method, the antenna is always maintained displaced from the weapon line by the prediction angle. Any tracking error is used to drive the weapon line to null the error. In the former method, the tracking line is controlled directly, while in the second method, the weapon line is controlled directly and the tracking line is controlled indirectly.

The direct method is used in the TERRIER missile system. The tracking radar operates independently to maintain the tracking line coincident with the line of sight. The launcher positions the prediction angle away from tracking line in response to orders from the fire control computer. Fixed aircraft weapons systems usually use the indirect method. If any correction to the tracking line is needed, the aircraft is reoriented to null the correction.

TRACKING DATA

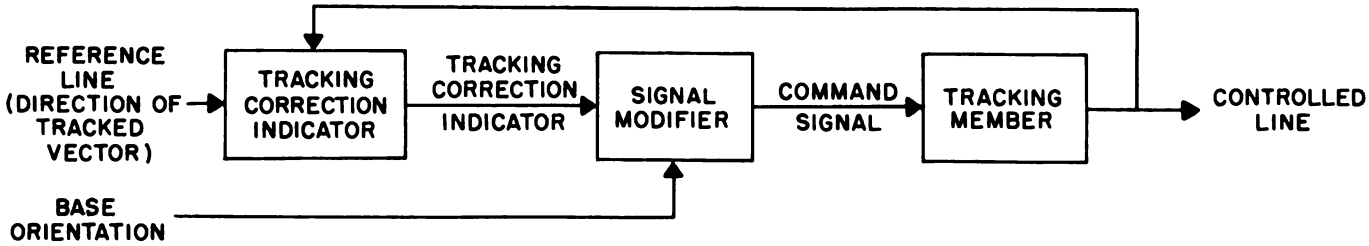
The controlled line in the detection device has no preferred orientation of its own. Information on orientation must be obtained from tracking data that furnishes command to the controlled line drive mechanism which, in turn, nulls the tracking data, thereby closing a loop.

The tracking process is essentially a matching process. When it is performed by a human agency, it is a convergent trial-and-error process. When it is performed automatically, it is a close-loop process employing regenerative feedback, of which a servomechanism is a example. The correction to the tracking line may be considered to consist of two components, one due to the movement of the line of sight caused by target motion or translation of the delivery vehicle, and the other caused by pitch, roll, and yaw of the delivery vehicle.

Therefore, the actual problem of instrumentation is to maintain a direction that remains fixed in inertial space in the presence of interferences, and yet can be changed in response to a command signal. A typical solution is to use a gyroscope coupled into a servoloop, in which the servomotor drives its controlled member to null the output signal of the gyro when disturbing torques act on the member. Also, it drives its controlled member to follow the gyro reference line when a torque signal, applied as a command to the gyro, sets the gyro reference line in motion at an angular velocity proportional to the command signal.

In general, the output angle of a servomechanism is proportional to the angle turned through by a servomotor in response to a command at the input to the servo, over the linear range of the device. This angle is measured between a line marked on the controlled member and a similar reference line marked on the base on which the servomotor is mounted.

The controlled line in this kind of servomechanism thus sweeps out an angle with respect to the base. Volume I of this publication describes the behavior of such a system, as long as the controlled line is meant to indicate position with respect to a base that is implicity fixed in inertial space. However, if the base is not fixed with respect to inertial space, but is moving, the output indication changes if it is desired to have this indication referenced with respect to Earth or to inertial space. Therefore, there are two types of base motions which must be taken into account, i.e., commanded motions and inteferences. Ordinary servo theory does not take interferences into account and, therefore, devices such as gyros coupled to the servomotor are used to filter out the unwanted interferences of pitch and roll, yaw, flexure, etc. In this manner, unwanted motions are removed while desired motions of the tracked vector are taken into account.



characteristics

The characteristics of the tracking loop that are of primary interest are the same ones that are prominent in ordinary servoloops. They are accuracy, speed, and tendency to oscillation. Accuracy in determining the target position is most important; without it all subsequent attempts to obtain accuracy in computation, weapon line drive, etc., are futile. Speed in acquiring precise target data is particularly necessary when dealing with high-speed, high-maneuverability targets. Oscillation, or hunting, must be avoided or kept to a minimum, because the transients set up will hinder the solution to the fire control problem.

The simplest tracking equipment has the tracking index rigidly connected to a member that is directly under the control of the tracker. An early fixed-gun fighter aircraft serves as an example of this type. The pilot maneuvers his aircraft and, consequently, his guns so that it is directed toward the target as indicated by his gunsight. In effect, then, he is directly controlling the tracking line as established by his gunsight, since the gunsight is rigidly fixed to the aircraft.

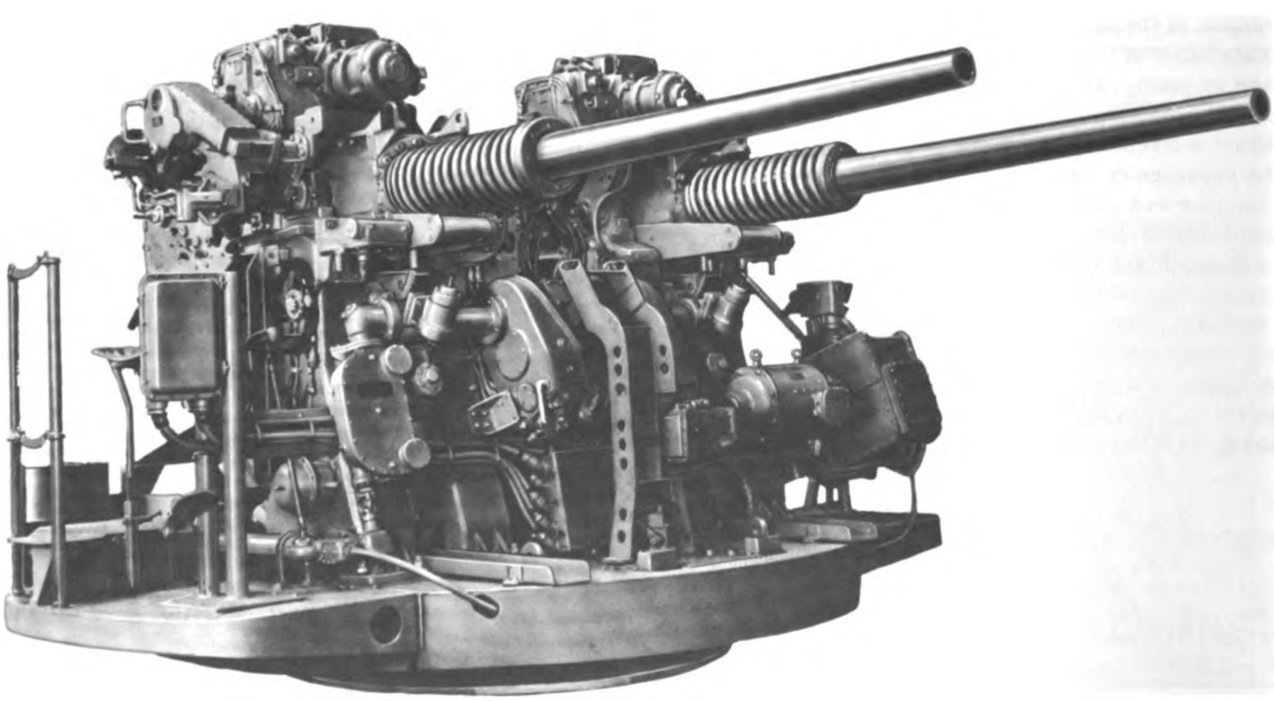
In such a system, the pilot estimates the prediction angle and offsets his angle of attack accordingly. In this type of simple system, its effectiveness is a direct function of the skill and experience of the pilot.

types of operation

The operation of the tracking loop may be manual, semiautomatic, or automatic. In manual systems, man notes the positions of the line of sight and the tracking line, and moves the tracking line. This is the case with the simplest weapons systems - pistols, shotguns, and other small arms.

Larger weapons can also use manual control. However, for larger, heavier weapons, and as the requirements for accuracy and slewing speed become more severe, the use of machinery to supply the power for moving the tracking line is desirable. In semiautomatic systems, man still notes the positions of the line of sight and the tracking line. The tracking correction supplied by man is used by machinery to drive the tracking member. An example of this is the 3"/50 rapid fire gun.

The tracking line is fixed with the gun and so, in this case, the tracking line drive is also the weapon line drive. Any error between the tracking line and the weapon line is noted by the gunlayer in his telescope; then, by operating a handwheel, he indicates the magnitude of the error. In response to the signal from the handwheel, the drive mechanism moves both the weapon line and the tracking line. The automatic systems, commonly used by tracking radars, perform the tasks of determining the tracking correction, and a drive mechanism then operates to null the tracking correction. In tracking a target, man's only function is as an overseer. Often, however, man is required in the detection and isolation of individual targets, and in the initial tracking or locking on of a target. After locking on, radars and other automatic detection devices usually maintain that condition.



The system may work well at short ranges; however, as the ranges increase, the accuracy of the pilot's estimated prediction angle will decrease.
This fact is the basic reason for the development of fire control equipment that computes and introduces the necesary prediction angle without the exercise of judgment by the tracker. The tracker is required to use the skill necessary to track the target line, but his duty only is to keep the tracking line index on the target, not to estimate the prediction angle. As the requirement for tracking targets of long ranges increases, the size and complexity of the detection equipment (telescope, radar, or sonar) increases proportionately.

This complexity makes it difficult for a man to control the tracking device to the precision required and necessitates the use of automatic tracking correction indicators. This is particularly important when several targets are being tracked alternately using the same detection device. The tracking correction indicator, whether man or machine, must be capable of noting a sufficiently small error between the line of sight and the tracking line. Having noted the tracking correction, the tracking correction signal and the command signal must be sufficient to displace the tracking line and to make it coincide with the line of sight.

TYPES OF TRACKING

Another aspect of the tracking loop is the manner in which the tracking member is moved in response to the command signal. If the angular coordinate of the line of sight is denoted by θ, and the angle through which the handwheel indicating the tracking correction is turned is denoted by ϕ, the tracking loop may be classified by the manner in which the variables θ and ϕ are related. This classification yields essentially five types types of tracking: position tracking, velocity tracking, rate aided tracking, acceleration tracking, and acceleration aided tracking.

position tracking

In position tracking, the angle through which the tracking line moves is directly proportional to the tracking angle correction. The corresponding relationship between θ and ϕ is $\theta = A\,\phi$, where A is a constant. This procedure is accomplished by having the handwheel turn the tracking line directly by means of gears.

velocity tracking

In velocity tracking the angular velocity with which the tracking line is moving is proportional to the tracking angle correction. In symbols, $\theta = B\phi$ (B is a constant). Tracking of this type can be effected by having the tracking line driven by a variable speed motor, the speed of the latter being regulated by positioning the handwheel. Velocity tracking enables the tracking line to be slewed through a large angle onto a new target merely by giving the handwheel a larger displacement.

rate aided tracking

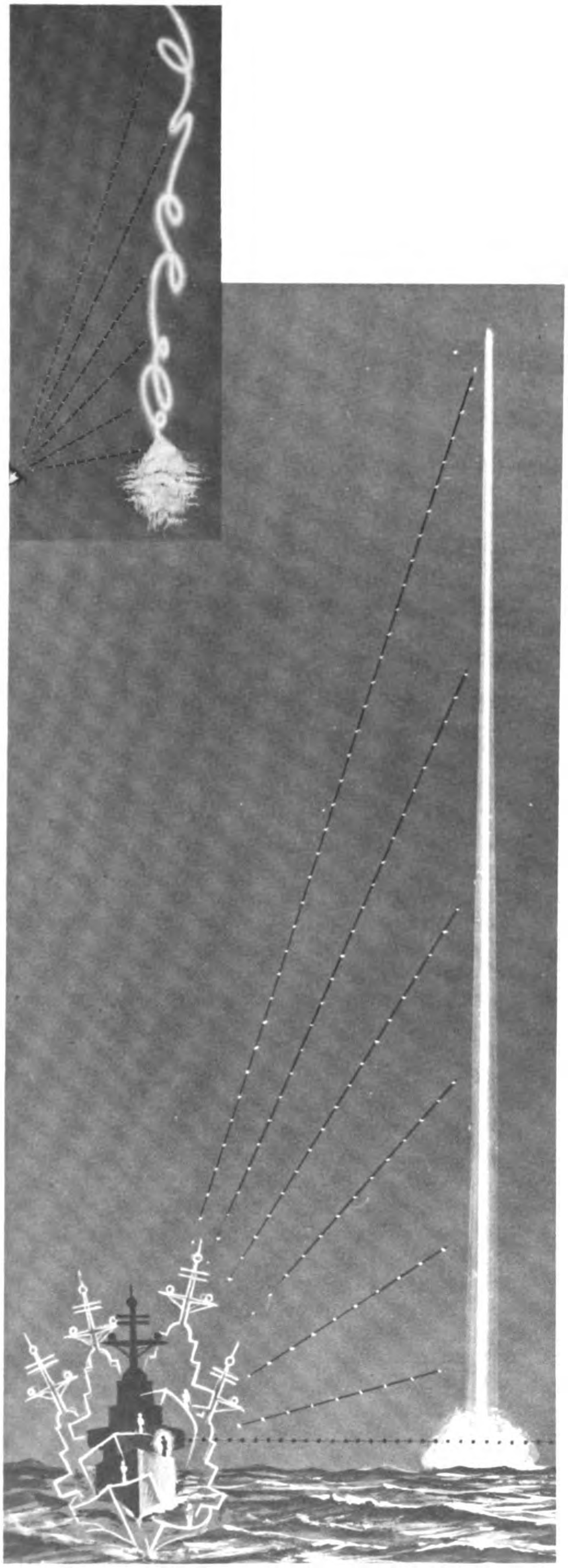
Rate aided tracking combines the first two methods; any displacement of the handwheel not only positions the tracking line, but gives it a velocity. The equation of control may be written as $\theta = A\phi + B\phi$. For a unit displacement of the handwheel, angle θ is displaced by A units and changed in velocity by B units. The ratio of A to B is the ratio of position to velocity control. By varying this ratio, the velocity control can be made more or less important, relative to the direct control of the telescope.

acceleration tracking

In acceleration tracking, the acceleration at which the tracking line moves is proportional to the angle through which the handwheel has been moved, or $\theta = C\phi$ where C is a constant.

acceleration aided tracking

Acceleration aided tracking is a combination of position, velocity, and acceleration tracking. The equation of control may be written as: $\theta = C\,\phi + B\,\phi + A\,\phi$. For a unit displacement of the handwheel, the tracking line is displaced A units, its velocity changed by B units, and changed in acceleration C units. By changing the values of A, B, and C, the relative importance of acceleration and velocity control with respect to positional control can be varied.
In general, either rate aided or acceleration aided tracking gives, more satisfactory results than position, velocity, or acceleration tracking, alone. One reason is that a target having a constant angular velocity can be tracked merely by keeping the handwheel fixed. In the case of acceleration aided tracking, this is also true for a target having a constant angular acceleration. If positional tracking were used, the handwheel would have to be moved continuously. For targets with slowly changing velocities and accelerations, any angular displacement that the tracking line has fallen behind can be corrected by an additional displacement of the handwheel. This has the effect of simultaneously changing the position of the line of sight, increasing its angular rate, and, in the case of acceleration aided tracking, increasing into angular acceleration. By the time the tracking line falls behind again, all that is needed is another slight increment in the position of the handwheel. Aided tracking helps to continue tracking through a region in which no energy is received from the target. Generally, aided tracking is more stable than velocity or acceleration aided tracking alone because there is less tendency for the operator to hunt with the controls.



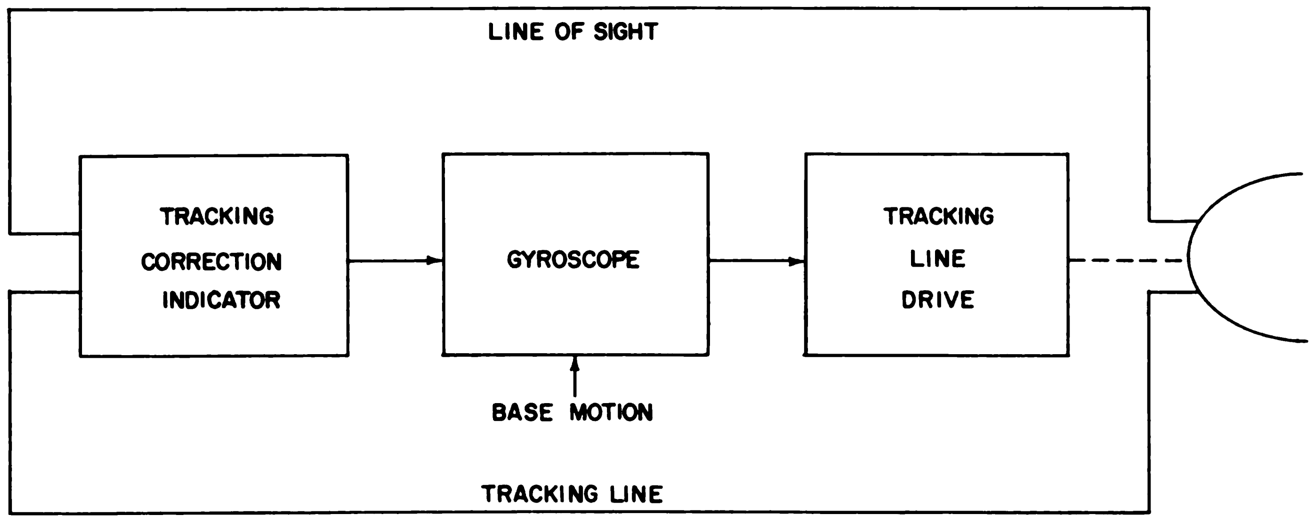
TRACKING INTERFERENCE

In the earlier part of the chapter, the fire control problem was discussed with respect to any arbitrary reference frame. However, the laws of classical mechanic's which allow for the prediction of the behavior of missile dynamics have been defined with respect to an inertial reference frame. Therefore, the noninertial properties of the chosen reference frame must be taken into accounting in implementing the solution of the fire control problem. One of the factors tending to make a chosen reference frame noninertial is geometric interference.

Rotations of the weapon station is one of the basic sources of geometric interference. These motions, such as pitch, roll, yaw, etc., are not part of the actual fire control problem; nonetheless, they almost always occur in a fire control situation. Since these motions do occur but are not a part of the fire control problem, it is necessary to compensate for them in some way other than as part of the computation. This compensation is usually provided by stable elements, such as the gyroscope, so that the interferences are removed from the input data to the fire control computer.

The gyroscope maintains a direction that remains fixed in inertial space in the presence of interferences, and yet can be changed in response to a command signal. The angular velocity of the direction with respect to inertial space is proportional to the command signal. These attributes define a space integrator, of which the gyroscope is probably the most common example. If a space integrator or similar stablizing device were not used, the tracking loop would have to be capable of a rapid response to enable it to follow the apparent motion of the target.

With delivery vehicles such as destroyers and aircraft, which are subject to high and rapid motions of pitch, roll, and yaw, and high speed targets, the apparent motion of the target would be quite high. Since the tracking loop is a closed loop, when feedback of the response of the overall system is made high, the danger of oscillation occurs. By having two loops (one which accounts for target motion, and one which cancels interferences), the overall gain can be lower, reducing the problem of oscillation. The total response of the tracking loop can be made high enough to permit the tracking line to follow the line of sight.

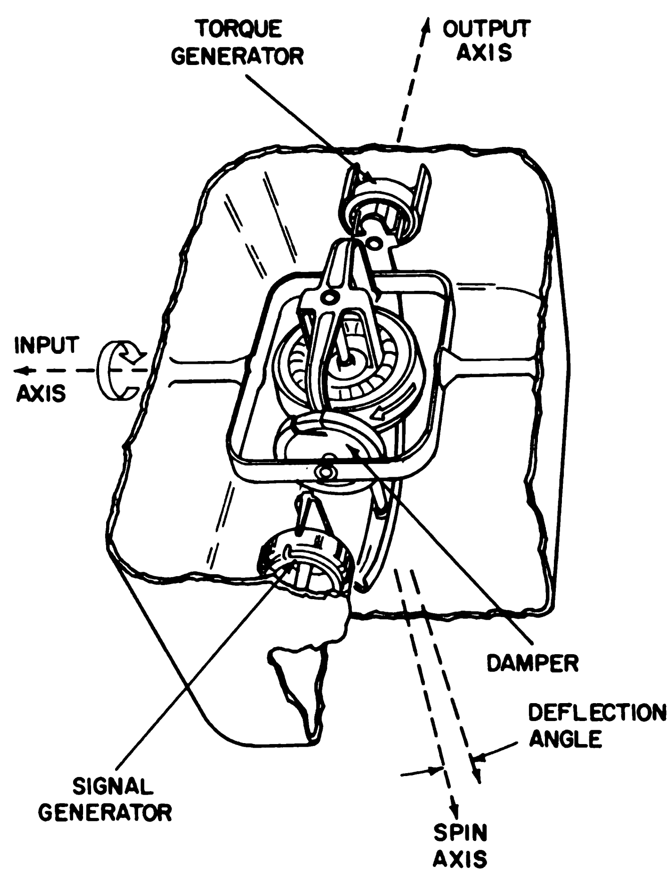


As an aid in understanding the operation of gyros in a tracking loop, a simple tracking loop which can track a target in a vertical plane will be analyzed. The basic theory of operation of gyros is described in Volume I. An illustrative single-degree-of-freedom gyro is shown. When this gyro is used in a tracking loop, the configuration that results is similar to that illustrated. The antenna is the device which receives the radiant energy from the target. In practice, it may be a radar or sonar antenna or a telescope. No loss of generality occurs, since the case may easily be extended to both bearing and elevation and used with detection device.

When the target and the delivery vehicle are motionless, the system is at rest. The tracking line will be nulled with the line of sight, and therefore the command signals from the tracking correction indicator will be zero. The gyro will remain fixed with reference to inertial space, and the signals to the antenna drive signal will be zero. Also, if the delivery vehicle were to pitch downward, the antenna, being mounted on the delivery vehicle, would tend to move downward also. This provides a rotational velocity of the gyro case which causes the gyro to precess at a rate proportional to the downward pitching motion. The precession is detected by a signal generator, the output of which activates the antenna drive system to move the antenna upward at the rate equal to the downward pitching motion. In this manner, the antenna is maintained stabilized with respect to inertial space as represented by the position of the gyroscope.

When the target moves, the line of sight will be displaced from the tracking line as indicated by the antenna position. The tracking correction indicator will produce command signals to drive the torque motor. The action of the torque motor will have two effects. First, it will cause the signal generator to produce a signal causing the antenna to move the follow the target, and consequently, causing the tracking line to follow the line of sight. Second, it will cause the gyro to reposition itself with respect to inertial space, that is, to reposition the tracking line which is always driven to correspond with the gyro position in space.

The command signal may be to provide for any one of the five types of tracking: position, velocity, acceleration, rate aided, or acceleration aided tracking. Since no conditions have been placed on this tracking loop, it can be used on any delivery vehicle subject to pitch, roll, and yaw, including ships, aircraft, and guided missiles which track their targets.

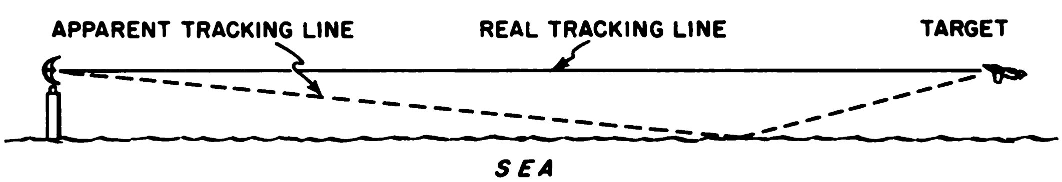


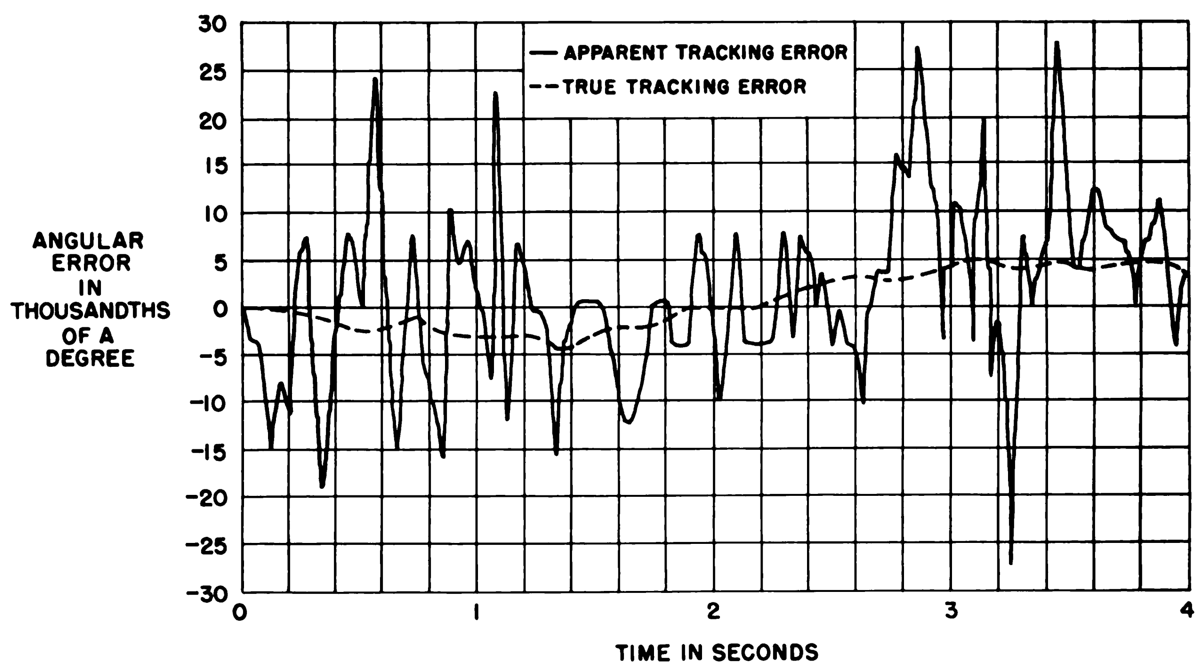
ERROR

Two major error sources are noise and a noncontinuous flow of data noise. Tracking is done under whatever conditions that prevail here on Earth. In an actual tracking system, there is a roughness in the tracking-line angular position, caused by a tendency of the various elements in the tracking loop to follow spurious signals imposed on the true signals. These unwanted signals are called noise, and have several sources. One source is thermal noise generated in the electronic devices, both tubes and transistors, used in the detection device. Another source is in the various servo gear trains. The final roughness of the tracking line is the net effect of the transmission of all the various kinds of noise in the system.
Special conditions which cause problems are the tracking of low angle targets in which reflections from the sea cause varying results. If a target is large, such as a surface ship or shore installation, the tracking radar may not lock on to any one part of the target, but may receive strong signals alternately from the bow, the mast, or other area of the target. This causes the radar set to indicate a motion of the tracking line where none exists.

noise

A predominant noise source in radar tracking systems is radar noise. This noise has its origin in many different elements. A complex target consists of a number of reflecting elements randomly oriented in space. When illuminated by a radar wave, each element reflects a portion of the incident energy. Because the various elements differ in size and shape, they will reflect differing amounts of energy with varying phase relationships. If an aircraft is considered to be a rigid body made up of a large number of such elements in combination, the echo signal consists of the summation of these randomly phased reflections. As the aircraft rotates relative to the line of sight, the individual reflected signals are phase modulated because of the changing path length. This varying echo signal indicates an apparent target motion over and above any actual target motion which may exist.





SOURCES

The effect of this noise on an actual radar tracking loop is illustrated in the graph. Tracking correction, the difference between the tracking line and the line of sight, is measured in mils or thousandths of a radian. The true tracking correction is the actual difference between the tracking line and a line connecting the target present position and the delivery vehicle present position. The apparent tracking correction is the difference between the tracking line and apparent line of sight indicated by the radar returns which include noise. The true tracking correction has a range in this case of only ± 5 mils, while the apparent tracking correction is six times that, approximately ± 30 mils.

ncn-continued flow of data

Another source of difficulty in tracking occurs when the tracking data on the target is available only at discrete times. This is the case when targets are tracked stepwise, or when the target is often lost to observation, as could happen if planes were observed optically on a day with many clouds and fog patches. When radar and active sonar are used, no target information is available except during the brief periods that an echo is received. If the tracking correction indicator could operate only in these intervals, even the results of rate aided or acceleration aided tracking would be erratic and cause tracking errors resulting from hunting.

The tracking process, then, has two major sources of errors. One is caused by various types of noise, and the second results from the lack of a continuous flow of data. Since the first error is a result of random items, the noise is a fluctuating signal of high frequency, superimposed on the slowly changing, true tracking correction signal. Step-wise tracking results in this same phenomena with gaps in both the noise and the true tracking correction signal. The method of compensating for these errors is called smoothing.

smoothing

Smoothing in the fire control problem is similar to the smoothing done at the outputs of d-c power supply in which all high frequency ripple voltages are filtered out and any slight interruptions of the signal are smoothed to present a steady d-c signal to the load. Many of the smoothers used to smooth the tracking have the same configurations as the filters used in electronics - π-filters, L-filters, and the others.

Mechanical filters are also possible. In a mechanical filter, the input signal is used to drive a high inertia motor. The high frequency signals are not of sufficient duration to affect the output of the motor; only the low frequency signals indicative of the true tracking correction cause a change in the motor output. Any interruption of the signal does not immediately affect the motor. In this way, the motor output represents the smoothed tracking correction.

Since smoothing is so effective against high frequency signals, it will help avoid oscillation or hunting, which usually occur at high frequencies. The operation of smoothing depends on making a circuit element resistant to change. This element can be the capacitor which tends to maintain a constant voltage, the inductor with a constant current, and a high inertia motor with constant speed. This resistance to change is an advantage when it hinders the higher frequency changes which are noise. However, it also acts (although not to the same extent) to hinder the low frequency changes which compose the signal. In a-c circuits, the current in a capacitor, which tends to be a constant voltage device, lags the voltage, and the voltage in an inductor, which tends to be a constant current device, lags the current.

In a similar manner, the speed of a high inertia motor, which tends to be a constant speed device, lags the input signal. This lag is often referred to in terms of phase angle and its relationship with a reference signal. However, this lag is a time lag. In the case of repetitive signals, such as sine waves, the lag is not a major problem. If the signal lags far enough, it will lag by 360° and will be in phase again. In a tracking loop, the signal is not of a repetitive nature, as can be seen by examining the figures illustrating the tracking correction. This indicates that the smoothed tracking correction output of the smoothing device is not the present true tracking correction (as indicated by the apparent tracking correction less any noise in the input). Rather it represents a past true tracking correction. Obviously, this limits the speed at which the tracking loop operates. This lag will usually remain as a fixed quantity, indicating that the tracking loop is following a past target position.

The prediction circuits utilize the data on past target position to predict the present target position in the same manner as sonar, which always indicates past target position. The static accuracy of the tracking system is improved, since the noise is removed and a static signal is unaffected. To improve the dynamic accuracy, it is necessary to use a feedback arrangement. Whenever feedback is used in a system, oscillation is a possibility. But, by proper design sufficient feedback can be used to provide beneficial results without resulting in oscillation or hunting.

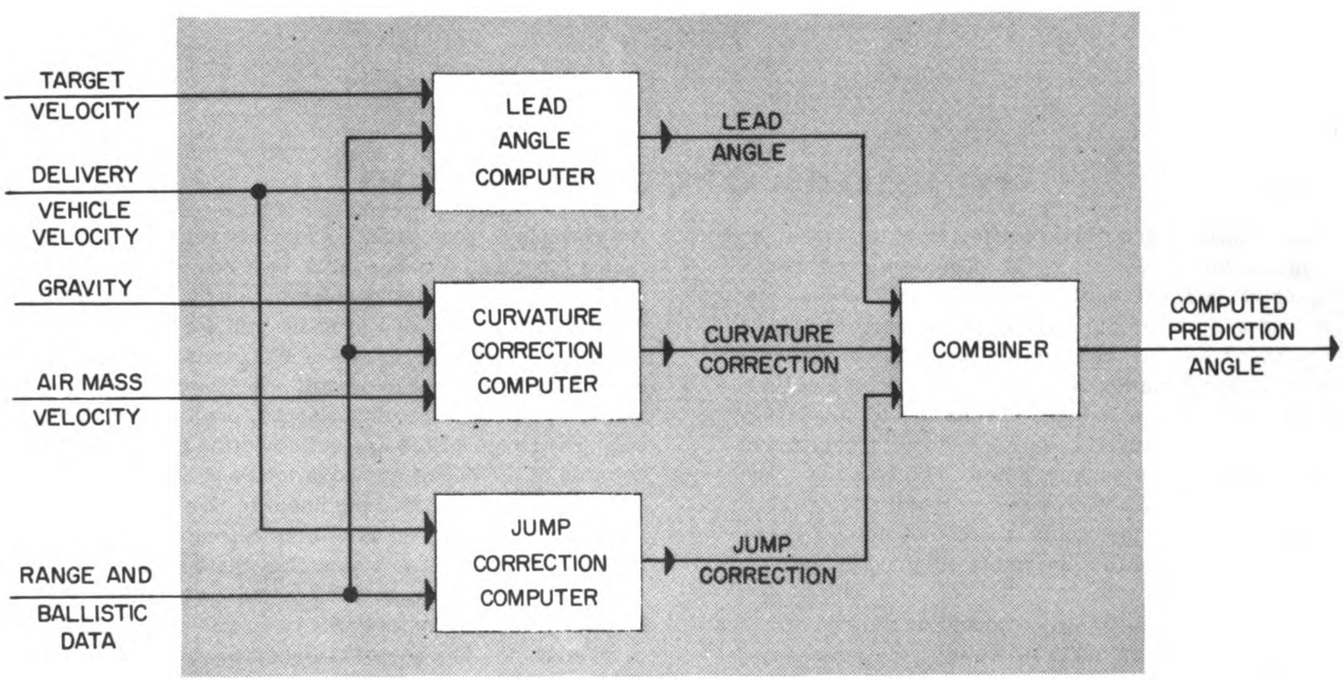
SUMMARY

Information gathering is the first and one of the most important procedures that must be followed in any fire control problem. On the basis of the information gathered, the tracking line is found, the prediction angle computed, and the weapon line positioned. The data necessary first is that there is a target. Then precise data on the target, its position and motion, and any ancillary data required on delivery, vehicle motion, position, and medium must be gathered. Delivery vehicle position and motion are usually available from the navigation equipment. Data on the medium temperature, density, and currents are available from appropriate instruments such as thermometers, barometers, and anemometers.
To determine precise data on the target position and its motion is generally the most difficult task, and one requiring the most effort. The delivery vehicle is under our control, and data on it is relatively easy to obtain. Data on the medium can be obtained or, in many cases, approximations can be sufficient. The data on the target must be obtained to a high order of precision, but, in general, the target is operating to complicate the acquisition of this data. The methods the target uses range from countermeasures to evasive tactics, such as remaining outside the range of tracking devices or being too high to be detected, or too low to be distinguished from the ground.
The procedure generally used in locating a target is to follow the radiant energy reflected or emitted by the target. This radiant energy indicates the line-of-sight from the delivery vehicle to the target. The tracking device then attempts to maintain a controlled line in coincidence with the radiant energy. Then, measurements are taken of this controlled line, and the position of the line-of-sight is known.
The system which maintains this coincidence of the controlled line with the line-of-sight is the tracking loop. The concept of a tracking loop is similar to that of a servo loop.
In a tracking loop, there are two items of interest; one is the reference input, the line of sight as indicated by the radiant energy received, and the output, the controlled line. Any deviation between the line of sight and the controlled line is detected as an error. A signal proportional to this error is used to move the controlled line to null the error. The error signal can operate to simply cause the controlled line to move to a new position, or it may give the controlled line a velocity or an acceleration. If velocity aided or acceleration aided tracking is used, the controlled line will not only be moved to a new position, but it will also be given a velocity or an acceleration.
Just as in the case of the servo loop, extraneous random signals cause undesired motions of the controlled line which result in false measurements. Noise is random; therefore, it can be expected to neutralize itself over a long period. That is, any positive noise will be equal to the negative noise. Noise also has a higher frequency than the error signal. Knowledge of this provides a means of reducing noise. By building a device which is relatively insensitive to the high frequency noise, but sensitive to the low frequency signal which indicates the error between the positions of the controlled line and the line-of-sight, the noise is removed from the input signal. The high frequency signals have no immediate effect because of the insensitivity of the device. The overall effect of noise is zero since it is both positive and negative, thus summing up to zero. However, the insensitivity of the device also affects the response to the true signal, since, if the signal changes, only the fact that it persists distinguishes it from noise. Thus, the procedure for smoothing is to ignore a change unless it remains. This procedure results in a time delay, which is a cause of inaccuracy in the system. The inaccuracy resulting from the time lags is usually much less than the inaccuracy caused by the noise.

Introduction to

DATA PROCESSING

The basic function of a computing system is the computation of the prediction angle in the fire control system. The inputs to the computer are comprised of the various target positional data, the required ballistic and environmental data, and positional data concerning the weapon station itself. The information received is a series of data received continuously for some variables, intermittently for others, and for some only occasionally. From this data flow, the computer must be able to determine the prediction angle and forward its decision to the correct area of the weapon system in a comprehensible manner.



For simplification and accuracy of operation the computations must be performed in some reference frame which has been selected to permit a simplified solution to the fire control problem. Data received by the data processing unit are rarely measured in the reference frame and coordinate system which are used for computation. It may occur when the weapon system is land based (such as field artillery) but seldom occurs with shipborne systems. Because different frames and systems are used, the input data, particularly the tracking data, must be converted to the frame in which the data will be employed. When the prediction angle is determined, it must then be converted to the frame in which the weapon line drive equipment operates. One of the major functions of the computer, then, simply is to convert the data to useful forms. In some computers, more equipment is used for data transformation than for the actual prediction computations.

COMPUTER SYSTEMS

desired characteristics

In a computer we desire speed, accuracy, high capacity, and reliability. Since we cannot achieve all of these characteristics at once, particularly because the desire for reliability is so great, some practical compromise is necessary. The tactical situations for which the system is to be used will determine the minimum requirements for the computer characteristics. If the target will not be an immediate danger and the problem will not change rapidly (such as with a fixed shore target), the computer does not need the speed that is required to measure a high speed maneuverable aircraft. Guided missiles do not require as great a degree of accuracy, since slight errors can be corrected in flight. The number of targets that the weapon system will have to deal with at the same time will determine the capacity of the computer.

If the system will be confronted with several targets which must be dealt with immediately (as an antiaircraft system), the computer may be required to solve several different fire control problems simultaneously. Other systems will deal with only one target at a time, and then it requires the capacity to solve only one problem. A new target will be a new problem.

Speed, accuracy, and capacity all can be increased by adding components to the computer and increasing the complexity. However, greater complexity results in a less reliable computer. By using various computer techniques, the required speed, accuracy, and capacity can be achieved while maintaining a high degree of reliability.

The computer should be mechanized in such a manner to limit its size and cost while maintaining the reliability and accuracy required for its particular application(s).

methods of computation

Computer characteristics are affected by methods used to compute the prediction angle. The computer, considering all possible variables, can solve the complex equations describing the missile flight path. The solution is theoretically accurate for the problem, but will require a vast complex of equipment. The use of extensive equipment introduces errors due to noise, and the inaccuracies caused by the possibility of errors in manufacturing parts such as gears and resistors. Such errors can offset the advantages gained by using the accurate solution. Inevitably, the use of a more complex computer leads to a decrease of speed and a lowering of reliability.

A way to avoid the difficulties caused by the use of exact relationships when an accurate solution is necessary is to perform the calculations before they are needed, and to have the data stored. Although all possible combinations of variables in the fire control problem cannot be solved and readily stored in advance, a sufficient number of problems can be solved and enough data can be made available to correct the general problem for the specific problem at hand. This permits the complex equations to be solved in advance at shore installation where space, time, and reliability are not serious problems, and where the computations are made under improved environmental conditions.

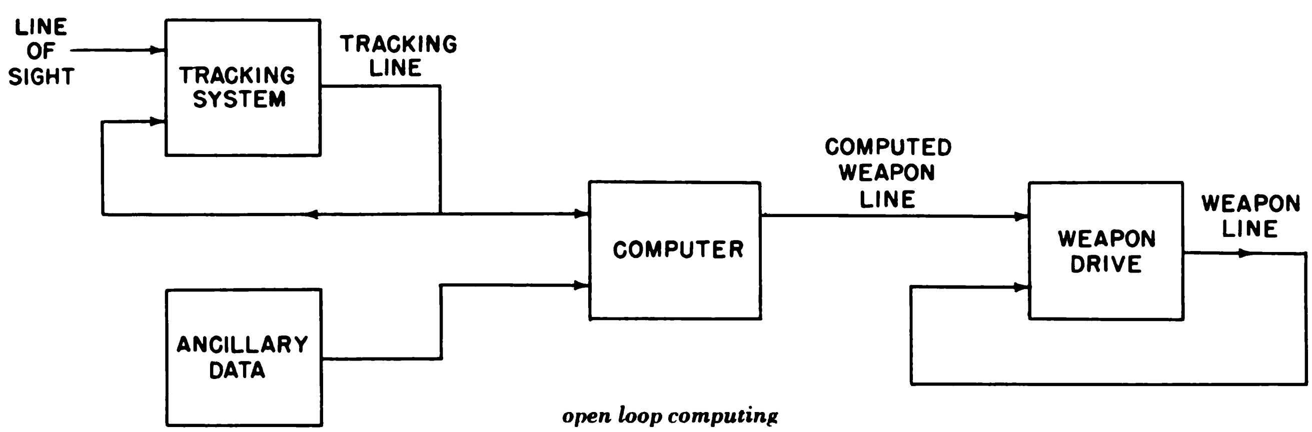
An example of such precalculation is the use of range tables for guns. A range table is a convenient set of data giving the necessary information as to elevation time of flight, etc. for a specific gun for specific ranges, often at increments of 100 yards. Sufficient data is also provided to permit adjustment of these values for changes in temperature, projectile weight, etc. The range tables are set up on the basis of the solution of the necessary equations and actual test firings. The solution to the specific fire control problem can then be solved by a simple computer.

If this method is not practical for the specific weapon system, and an accurate solution of the problem is not feasible, then approximations or empirical solutions may be used. The use of simplifying approximations, either theoretical or as discovered empirically by testing, will greatly reduce the complexity of the equations. The theoretical accuracy of the solution is somewhat impaired by the use of approximations, but the use of less equipment reduces machine-caused errors so that the actual overall accuracy of the computer may be improved. The use of less equipment will result in a faster and more reliable computer. The method of computation that is used will be a prime factor of the computer system, also.

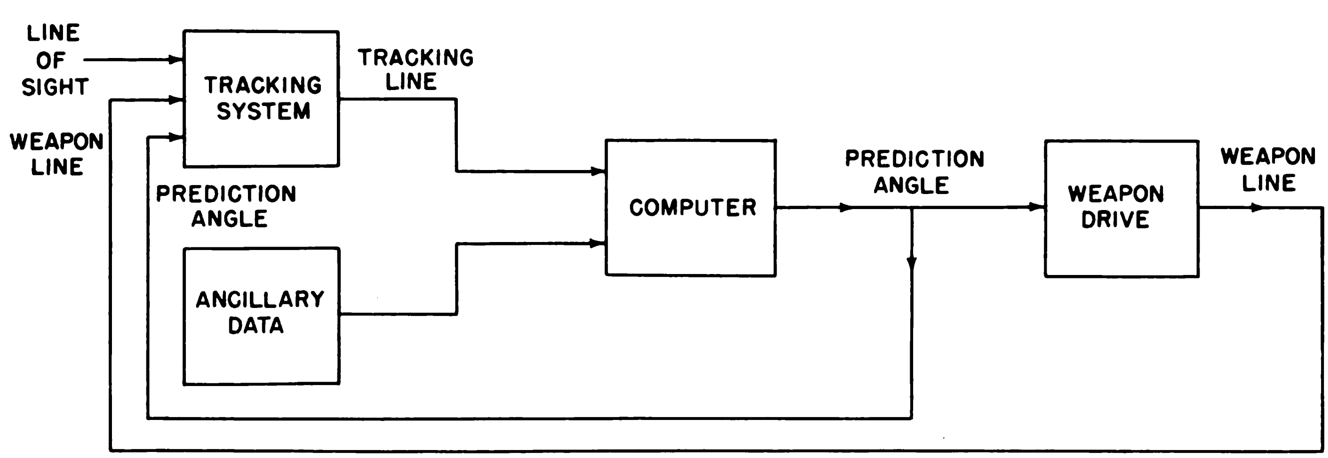
open or closed loop systems

Feedback is necessary to determine that the weapon line is separated from the tracking line by the prediction angle. The arrangement of this feedback provides two computing system arrangements. One, called open loop computing, does not use any feedback from the output to the input. As its input, the computer receives the position of tracking line and the necessary ancillary data. The position of the tracking line is compared with and maintained coincident with the line of sight by the tracking system. From the input data, the computer generates the prediction angle and the computed position of the weapon line. The weapon drive system then drives the weapon line in response to the signal, until the weapon line and computed weapon line are coincident. Since there is no overall feedback, the setting time of the system is small and thus it is a high speed system. However, requirements placed on the components of an open loop are stringent since the output results are only as good as the weakest component. Each component of an open loop system must introduce a minimum of noise and yield reproducible results; for the same inputs, the outputs should be identical within desired tolerances.

Closed loop computing provides a comparison of the output with the input. In the example shown, the weapon line is compared with the line of sight to see that they are displaced by the prediction angle. If not, the tracking line is repositioned, causing a new prediction angle to be computed and the weapon line moves. The closed loop computing system then operates as a tracking loop, or a servo loop, to null at the correct prediction angle and weapon line position corresponding to the line of sight. Obviously, this nulling process and computing takes longer than simply computing as in the open loop system. Thus, the speed of the system is reduced. However, the requirements on the individual components are also reduced, since the feedback will compensate for errors in the system. This permits the use of simpler devices in the computer, yielding a higher reliability and less noise, and resulting in increased accuracy. More reliable devices usually require less maintenance. However, the use of feedback increases the complexity of the maintenance, since any tendencies for oscillation or hunting must be corrected.



open loop computing

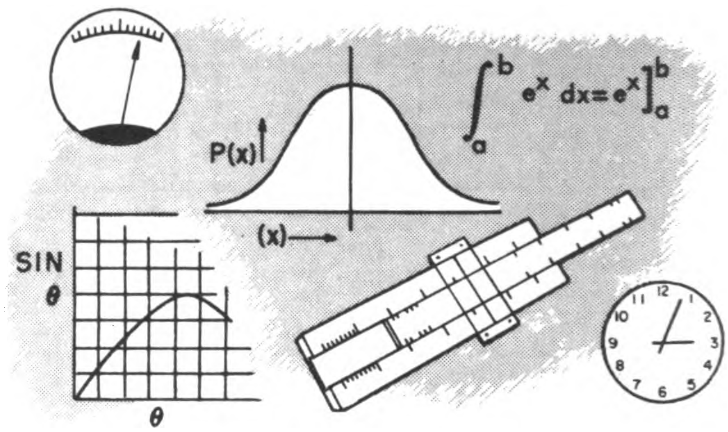


closed loop computing

TYPES OF COMPUTERS

A computer is a device or a machine in which physical operations perform mathematical computations to provide locical or numerical results. Computers are usually classified into two basic types - analog or digital. Analog computers essentially deal with a continuous measurement of a function, while digital computers deal with counting discrete values of a function. Basically the analog computer measures and the digital computer counts.

An analog computer is one in which numbers are represented by physical magnitudes, such as the angle of rotation of a shaft or the magnitude of an electrical voltage or current. The basis of analog computation is the establishment of an analog or scale factor between the data and the computing mechanisms in the computer. The limit of precision of an analog computer is determined by how precisely the scale is read, which is limited by how accurately gears are ground, resistors, inductors, and capacitors are manufactured, etc. The continuous measurement of a variable in an analog computer makes it useful in fire control problems dealing with varying inputs as would be presented by an aircraft problem. On the other hand, because the computer operates on a continuous basis it cannot jump from one problem to another. Switching is possible but it involves a loss of time thus slowing down the speed of a solution. Therefore, analog computers are usually used for solving only one problem at a time.



BASIC PROBLEMS

Two basic problems which complicate the computing of the prediction angle are noise and insufficient information. A basic result of the two types of error sources is to create an unwanted deviation in prediction angle computation.

noise

Noise creates undesired sources of false information that are difficult to detect and isolate. False data intermingled with true data can lead to a complete distortion of weapon line drive positioning. Noise, which occurs in the information gathering equipment, may be smoothed in the measuring equipment, or it can be done by the computer smoothing networks.

lack of information

A lack of information can be due to the inability or the impractibility of determining data. For missiles whose flight paths are in the air, the wind velocity in the vicinity of the delivery vehicle can be continuously measured, and occasionally meteorological balloons can be used to measure winds apart from the vehicle. Aircraft and submarines cannot use weather balloons and it would be difficult for them to find the velocity of the air between them and the target. In many cases, it is not practical to build a measuring device or to attempt to put one on a delivery vehicle such as a submarine or aircraft which have severe limitations on space and weight.

Any accuracy gained by the use of the measuring equipment may be more than offset by the lessening performance of the delivery vehicle. The solution of the fire control problem by the use of available data must then be modified to account for lacks in data. This modification is called CALIBRATION.

Since digital devices deal only with integers representing discrete values of a variable, it is particularly well suited for problems with constant values such as would be the case with fixed targets. The speed of a digital computer is unaffected by what the integers are. This permits the use of a digital computer to solve several problems by computing each one on a time-sharing plan which will not adversely affect computer operation.

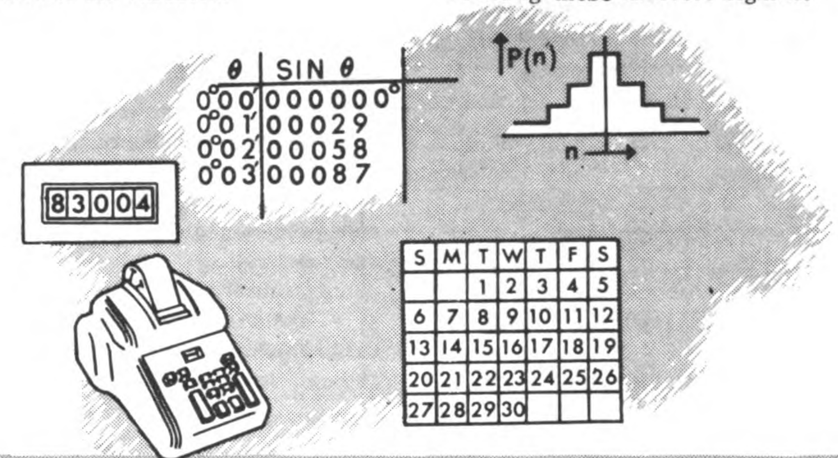
Digital computers can handle many problems simultaneously by providing a switching assignment to control the inputs and outputs of the computer.

The speed of analog and digital computers overlaps. The solution of a problem with fixed inputs requiring one solution can be solved readily by a digital computer. A solution of one point can be done quicker by a digital computer than by an analog. However, if the solution is a curve or a continuous variable, an analog computer often solves the problem faster than a digital computer can solve the large number of discrete problems to yield the overall solution. A compete discussion of data computer types is given in Vol III under computers.

digital computers

Electronic pulses are examples of markers used to represent the digits of numbers on which arithmetic operations are performed. Digital computers possess the capability of a higher degree of accuracy than analog computers. The limitation of precision of a digital device depends only on the size of the smallest unit used to count the value of the variable.

A digital computer processes data consisting of defined numbers, the digits of which represent quantities undergoing computation. The computer sees these numbers as a group of discrete signals, each discrete signal representing one of the digits of the entire number. Digital computers perform arithmetic operations by counting these discrete signals.



smoothing

The issue of noise, which was discussed as it affected the tracking system, will also affect the computer. Of particular interest in the smoothing problem is the digital computer whose output consists of discrete values. The weapon line drive cannot follow an instantaneous change in the prediction angle, which is inevitable with digital computers. In addition, a weapon line drive should operate continuously, but the output of the computer is available only at specific instances.

The prediction angle, the output of the computer, then must be smoothed to reflect a continuous variation. One method of performing this smoothing is end point analysis. A memory system retains the last observation (m-1) and compares it with the present observation (m) to compute the future observation. The present observation is stored to the last observation when the next one is received. Then the output of the smoother is varied from the present observation to the predicted future observation. Since the time between observations is so small, little error between the predicted and actual observations is possible, and the output will represent the continuous solution of the problem.

An alternate scheme to learn the true value of the function is to treat the observed data as necessarily following some curve and operate to find the one that fits. The usual mathematical procedure is to find the particular curve which minimizes the geometric mean of the differences between the observed points and the smooth curve. The geometric mean is defined as the square root of the sum of the sum of the squares of each of the differences. The scheme is known to mathematicians as the least mean square method.

The issue has been discussed for a computer output which is not continuous.

If any of the inputs is not continuous, smoothing will also be necessary. This is true if the computer is analog requiring continuous inputs, or if the computer is digital, since in general the frequency of the input will not be the frequency at which data is selected for use in the computer. The other major use of smoothing is in the elimination of noise. If the smoothing of noise is not done as a part of the tracking system (as was described under information gathering), it can be done in the computer prior to any calculations. Similarly, noise in any of the other inputs can be smoothed in the computer. Noise can be generated not only by external sources, but within the computer itself, usually necessitating some smoothing of the computer output.

The solution of the fire control problem, the prediction angle as determined by the computer will inherently contain certain errors of solution.

These errors are caused by target maneuver or other unpredictable motion, or an unaccounted for change in one of the other variables such as wind, air density, missile velocity, etc. In addition to these errors, the use of approximations and empirical relationships introduces inaccuracies into the computation, like mechanization inaccuracies resulting from backlash, or other losses of precision. Noise which is a random signal superimposed on the true signal is a major source of error in all systems. Smoothing, which reduces the noise, causes an inaccuracy of its own because of the time lag introduced. The magnitude of these errors is kept small and the total inaccuracy is not necessarily the sum of the individual inaccuracies, since some may compensate for others keeping the total error small.

calibration

Calibration is the process by which computer circuits are adapted to meet specific fire control problems. In most cases, it would be impractical to create a computing device which could cope equally well with all possible situations. Hence, computers are calibrated so that they may successfully cope with the most likely situations. Basically, there are two types of calibration performed on a computer, static calibration and dynamic calibration. A few examples will serve to illustrate the use of static calibration.

STATIC CALIBRATION

Assume that we are to fire a surface-to-air guided missile at a target and that we know, for this particular application, that the target will be either a very short range target or a very long range target. For the short range target, it is desirable to compute a prediction angle and solve the fire control problem as though the missile were not guided. However, for the very long range target, it is desirable to fire the missile at the same bearing as the tracking line and also at a fixed elevation angle to get the missile to optimum cruising altitude as quickly as possible. In other words, the prediction angle for the short range target is the normal prediction angle and varies with the situation; the prediction angle for the long range target is fixed regardless of the target position (aside from the fact that it is classified long range).

Since these are the only two situations which we expect to encounter, we may then calibrate our computer so that it automatically gives us the normal prediction angle for short range or the specific fixed prediction angle for long range.

In addition, we may further calibrate our computer by assuming that for short range targets the curvature resulting from gravity will have a nominal value and that the variation of this nominal value will be small enough to be neglected. This is especially true if it is determined that the missile may be guided easily to compensate for this variation without causing excessive acceleration of the missile. In this case, then, we may calibrate our computer for short range problems by adding a fixed curvature correction. This calibration

COMPUTATION OF target motion

In the description of kinematic lead, it was seen that the motion of the target is the result of the initial velocity of the target and the accelerations on the target. If the future range to the target is expressed as a vector, R_2, it can be expressed as the sum of an infinite series of vectors: $\bar{R}_2 = \bar{R} + \bar{V}T_2 + \frac{d\bar{V}}{dt}(T_2)^2 + \frac{d^{n-2}}{dt^{n-2}}(T_2)^{n-1}$

where
- $\bar{R}$ is present range
- $\bar{V}$ is present target velocity
- T_2 is time of flight
- $\frac{d\bar{V}}{dt}$ is first derivative of $\bar{V}$ (acceleration)

If this infinite series were instrumented, a mathematically correct answer would result. This answer presupposes that the initial target velocity, acceleration, etc., actually determine the future range throughout the time of flight of the missile. However, almost all targets have some control; they may be guided missiles or manned vehicles. Therefore, it must be presumed that the target is free to do as it pleases within the limitations of its own capabilities and operating requirements.

BASIC LEAD PREDICTION METHODS

The computer makes an approximation by using a few terms of the series; the simplest approximation uses only the first term. The use of the first term only is called a zero-order approximation. The word order of approximation is determined by the level of the highest derivative used. For instance, if the approximation consists of a positional term only, the 0 derivative, it is a zero order approximation; if the approximation consists of the first two terms, position and velocity, it is a first order approximation, since velocity is the first derivative of position. Use of a zero order approximation is sometimes allowable for magnitude effects, such as time of flight, but cannot be used for directional effects, since it implies a stationary target.

If the target is assumed to be traveling with a constant velocity throughout the time of flight, a first order approximation containing the positional term, indicating initial position upon launch, plus the first derivative or velocity term, indicating the total movement during the time of flight is necessary. Hence, the first order approximation will give accurate results as long as the target continues in a straight path at constant speed. If it is assumed that the future range also includes the effect of the target acceleration, it can be expressed as:

$$\bar{R}_2 = \bar{R} + \bar{V}T_2 + \frac{d\bar{V}}{dt}(T_2)^2$$

In general, it is not desirable to increase the accuracy of the mathematics by instrumenting any higher order terms, since the additional theoretical accuracy is usually offset by instrumentation errors. The time derivative of a sinusoidal function has an amplitude proportional to some power of the frequency. The problem of noise, which is a high frequency signal, becomes increasingly difficult when the higher order derivatives of target displacement are required.

These mathematical approximations must be examined in the light of overall system requirements to see how valid they are tactically. The determination of target motion actually is the predicting of the target's action. If we assume that target motion is random, depending only partly on its present course but primarily on any action of the control system in the future, we can preset the flight path statistically. To do this, by using information on past targets with an attempt to extrapolate

practically eliminates the curvature correction computer, thereby greatly simplifying our machine. Of course, using all of these calibrations restricts us severly since we may now attack only very long range or short range targets.

DYNAMIC CALIBRATION

Suppose, now, that we consider attacking targets at all ranges. We can determine a nominal desired value of prediction angle which varies as a function of range. In other words, for this guided missile system we would like to vary the angle between tracking and weapon lines so that we have a complete spectrum of prediction angles varying in a fixed manner from the long range solution, discussed previously, to the normal prediction angle used for our previous short range problem. We would instrument this program by calibrating our computer as a function of target range. Since this is a constantly varying factor, we call it a DYNAMIC calibration. In the same manner, we might calibrate curvature correction as a function of range, thereby simplifying our computer via a dynamic calibration.

Another kind of dynamic calibration is the use of sensitivity control. Suppose we are guiding a missile towards a maneuvering target. There are two prime considerations: one is that the missile follows the target sufficiently close so that it assures a hit; the second is that the missile is not required to perform accelerations large enough to cause it to fly out of its guidance beam if it is a beam rider, or so large that it cannot comply with the command.

It is not necessary for the missile to follow target maneuvers exactly while the range difference between target and missile is large. In this case it is desirable to smooth the missile's response to target motion. However, as the missile approaches the target, it becomes more and more necessary for the missile to follow target maneuvers to avoid a miss. Hence, a program may be implemented, via dynamic calibration, to make the missile's sensitivity to target maneuvers small at large missile-target range differences, but increase according to a fixed program as the range difference decreases.

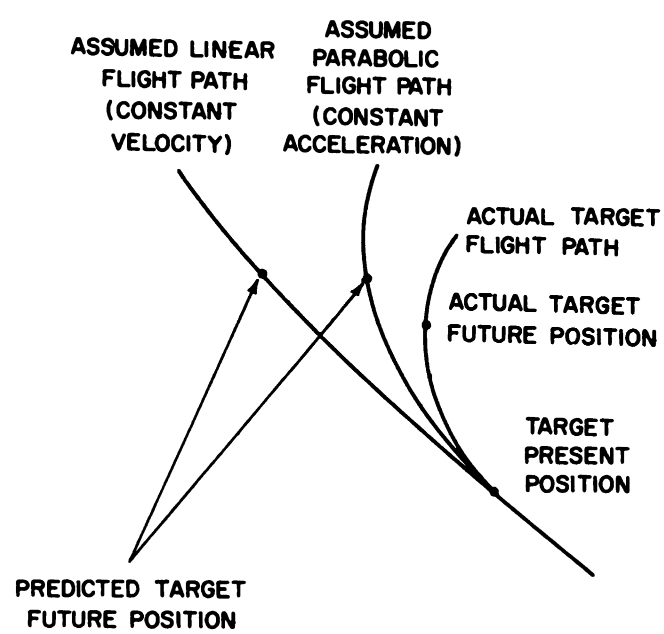
KINEMATIC LEAD

this data to the target of today and the future, is extremely difficult at best. Common practice, then, is to assume that one of the mathematical approximations is valid, and to use warheads with large lethal radii and/or guidance or control mechanisms to compensate for any deviation of the target from the assumed path.

linear predictions

The first assumption corresponding to a zero order assumption is that the target is stationary, or is moving so slowly that its motion is negligible. This assumption is valid for fixed surface installations. Ships and land vehicles operate at slow speeds, so that at short ranges, and short missile times of flight, they can be considered fixed targets. Since most targets have the ability to shoot back, it is desirable to operate at long range. Because of the advantages of operating at long range, and because most targets have high speeds, weapons systems are designed to destroy moving targets. Thus a stationary target is considered simply a target with zero velocity and zero acceleration.

A second assumption that can be made, corresponding to a first order assumption, is that the target will fly a straight line course at a constant speed; that is, the target will have a constant velocity corresponding to its initial velocity. By following a straight line path, the target simplifies our problem of prediction and thus lowers the target's chances of survival. It might be presumed that a target would avoid this path, and use diversionary tactics such as zig-zags, curved paths, etc. Certain factors oppose these choices of target paths. The first is that the shortest distance between two points is a straight line; therefore a target following a straight line course will be exposed to danger for a shorter period of time. The target being in danger for a shorter period of time may be more important than the increased danger for that time. Also, most targets tend to follow a straight line path. Ships the size of aircraft carriers, battleships, and cruisers simply do not have the maneuverability to make sharp turns because of their great mass. Most aircraft have such high speeds that a sharp turn would induce high stresses in both the plane and the pilot, causing a failure in one or the other. Generally, these factors cause targets to follow straight paths, or paths which curve only slightly. This justifies the use of an assumed straight line path for the target in many cases.



QUADRATIC PREDICTIONS

However, many targets are highly maneuverable. Common examples are destroyers and submarines on the ocean, tanks and other armored vehicles on land. These targets travel rarely in a straight line, but instead follow curved paths. If a constant velocity were assumed, an error would occur equal to the distance the target moves during the time of flight because of accelerations.
If the target were assumed to have a constant acceleration equal to its initial acceleration, the predicted target flight path will be a parabola. This corresponds to a second order solution. By using this curved line for the predicted path of the target, the flight path of maneuverable targets is more closely approximated. A predictor of this type is often called a quadratic predictor because of the appearance of the squared term in the equation for the second order approximation:

$$R_2 = \overline{R} + \overline{V}T_2 + \frac{d\overline{V}}{dt}(T_2)^2$$

ADVANTAGES AND DISADVANTAGES OF EACH TYPE

If the target follows a straight line path, the acceleration or change in velocity with respect to time is zero and the computer will operate as a linear predictor.
No disadvantage exists except that the computer is unnecessarily large and complex in comparison to a linear predictor needed to solve this simple problem. If the target begins to follow a curved path as part of an evasive maneuver, the linear predictor will not be capable of yielding an accurate result, but the quadratic predictor will. The accompanying diagram shows a curved target flight path and the results of linear and quadratic predictors.
The quadratic predictor uses information on the present position, velocity, and acceleration of the target. Theoretically, this is possible but, practically, it is difficult. Because of the time delay inherent in the smoothing process, the value of the smoothed velocity will lag behind the smoothed data on the position; the value of the smoothed acceleration will lag behind the smoothed data on the velocity. In some cases, the data might have delays of 1 second for positional data, 2 seconds for velocity data, and 3 seconds for acceleration data. If all this data is used without regard to the different delays involved, the predicted target path would not correspond to the actual path unless the target maintains the constant acceleration that it had three seconds earlier.

One method of resolving this problem is to deliberately introduce delays into the position and velocity data and use the data on the target of three seconds ago. However, this solution presupposes a prediction off the target's present position from which the future position must be predicted.

COMPUTATION OF CURVATURE CORRECTION

The portion of the computer which generates the curvature correction angle receives inputs representing the various forces acting on the missile in flight. Some of these forces are gravity, drag and lift, drift force, thrust, and kinetic reaction. The most significant of these forces is gravity. This force is a constant for a particular missile and causes the missile to accelerate downward at a rate of 32 feet per second per second.
The force of gravity can be resolved into two force vectors, one acting along the missile line in the direction opposite to the missile motion, and a second acting at right angles to the missile line. The first vector causes the missile to lose speed; the second causes it to change direction or to curve downward.
In order to correct for this curvature, the weapon line is superelevated; the angular elevation of the weapon line is increased by the predicted curvature angel to compenstate for the effects of the gravitational force. The magnitude of this curvature correction angle is dependent upon target range, missile velocity, and target elevation.

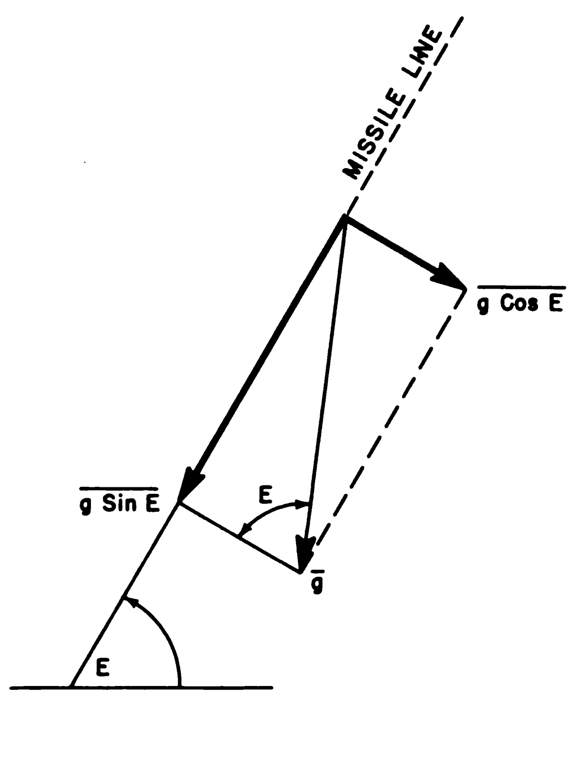
Thus, there are two places for prediction errors to appear, not only in the relationship of the future position to the present position, which all methods must face, but in the determination of the present position. To reduce this latter error, it is necessary to avoid all delays by using up-to-date information and not past data. On the other hand, it is possible to use various weighing factors on the velocity and acceleration data to achieve a reasonably accurate curved path. This path will probably not be a parabola, as was the case in quadratic prediction, but a suitable curve, perhaps an ellipse, which will yield accurate results for the type of target involved and the maneuvers which can be expected. Predictors operating on this principle are called curvilinear predictors. Their results will usually lie between the paths predicted by a linear predictor and a quadratic predictor.

time-of-flight

To solve for the lead angle, it is necessary to know the time of flight, no matter which type of predictor is used. Time of flight is a function of the missile's speed and the missile's flight path, which is determined partially by the prediction angle. Thus, we have two unknowns-time of flight and prediction angle. Since the object of the fire control problem is to have the missile flight path and the target flight path intersect at the same time, both the missile and the target must take the same time to travel from their present positions to the intercept positions.

One equation can be written describing position of the missile on the flight path in terms of the two unknowns, time and prediction angle. A second equation can be written, also, to describe the target position along its flight path in terms of the two variables. When the time is equal to the time of flight, the two positions must be coincident. Then we have two equations in two unknowns: prediction angle and time of flight. The solution of two simultaneous equations may complicate the computer problem and increase solution time.

To simplify computer operation, it may be assumed that the missile will maintain a constant velocity component along the line of sight from the vehicle to the target. The speed is the average speed of the missile as determined by test firings over the usual range of the missile. The present range is usually an indication of the magnitude of the future range. If it is also assumed that the missile travels a straight path to the target, the present range of the target can be divided by the average missile speed to arrive at an approximate time of flight. The computation required is the division of present range by a constant, a relatively simple one to mechanize. The simplicity, reliability, lack of noise, and speed of solution may often justify the use of this assumption rather than to solve the two equations simultaneously. In this case, the solution for the lead angle is simplified, also.

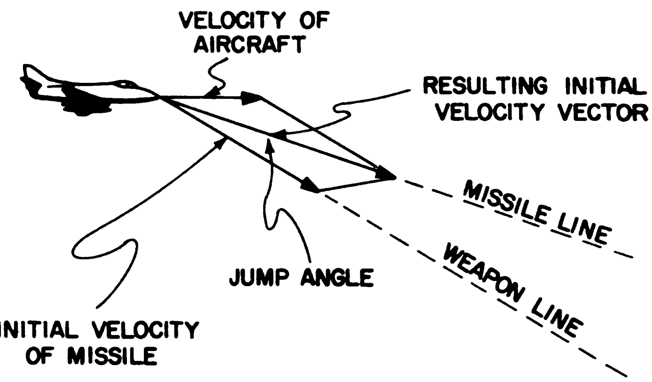


Both range and missile velocity are factors of the time of flight. As the range increases, the time of flight increases; as the missile velocity increases, the time of flight decreases. Since the acceleration resulting from gravity is constant, it follows that the total change in velocity increases as the time of flight increases.

The elevation of the weapon line also effects the magnitude of the curvature correction, since the acceleration vector resulting from gravity, which acts at right angles to the missile line, varies in magnitude depending upon the orientation of the missile line. As the elevation angle of the missile line increases, the acceleration vector normal to that line decreases; therefore, the curvature correction angle or superelevation decreases. From the illustration, it can be seen that the normal component of the gravitational force varies as the cosine of the elevation angle. Therefore, the curvature correction angle varies as the cosine of the elevation angle.

COMPUTATION OF JUMP CORRECTIONS

Velocity jump correction is composed of two separate components. The first is caused by the velocity of the weapon station. The weapon station has a velocity in the reference frame used, a similar velocity is imparted to the missile. When the missile is launched, it has not only a velocity along the weapon line, because of its propulsion system, but also the velocity of the weapon station added to it. This results in the missile initial velocity vector (the missile diagram) being divergent from the weapon line. When the weapon station velocity is in line with the weapon line, the missile line will be coincident with the weapon line. When the weapon station velocity and the weapon line are not aligned, the missile line will be directed along the resultant of the weapon station velocity, and the velocity imparted by the propulsion system. In this case, the missile may appear to jump from the weapon line toward the missile line. Note that this jump may appear, depending upon the reference frame chosen.



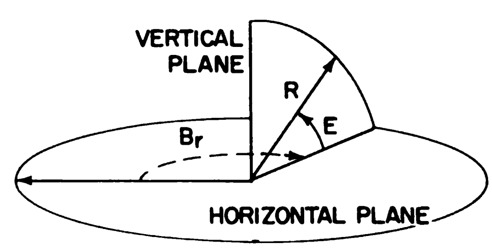
DATA CONVERSION

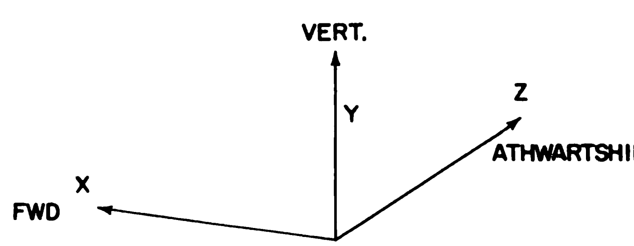
Data conversion is the process of transforming vector components from one coordinate system to another by electrical and/or mechanical means.
A simple example of data conversion is the application of gain control in stabilization loops. For instance, suppose we have a radar mounted on a ship, and it is supplies stabilized data to a fire control computer. It has a gyro unit mounted on it to provide a stabilization reference. If the output of the stable reference is to drive the radar with respect to the unstabilized deck of the ship, the drive signals must first be converted to deck coordinates. In this case, the data converter acts simply as a gain control, either increasing or decreasing the drive signal depending on the position of the deck reference frame with respect to the stabilized reference frame. However, this type of data conversion is not directly applicable to the solution of the fire control problem.
In general, the computation of the fire control solution is accomplished in geometrically stabilized coordinates. Input data are usually stabilized at the sensor or in at a data converter assocated with the sensor. If the computer operates in stable coordinates, its output is naturally in stable coordinates. However, the output may be used to drive a device whose base is not stabilized. In this case, data conversion is required at the computer output to make the drive signals compatible with the base of the driven mechanism.

NEED FOR DATA CONVERSION

Data conversion is required because different reference frames use different measuring equipment which inherently make measurements with respect to its own reference system. The data in any reference system used can be converted to equivalent data in any other specified reference frame by considering the differences in the data to consist of three separate differences. The first difference between data is that the data may be measured in different coordinate systems within the same reference frame.
The second difference is caused by reference frame displacement, and the third is a result of reference frame rotation. It is relatively easy to convert from data from one coordinate system to another in the same reference frame; no other information is necessary. The relationships between the coordinate systems (rectangular, cylindrical, and spherical) are known and relatively easy to instrument.

DIFFERENT COORDINATE SYSTEMS WITHIN THE SAME FRAME
An example of coordinate conversion within a reference frame is the conversion of data in spherical coordinates to data in rectangular coordinates. Let us assume that a shipboard radar antenna measures target coordinates in a stabilized deck reference frame as illustrated, i.e. B_r, E, R





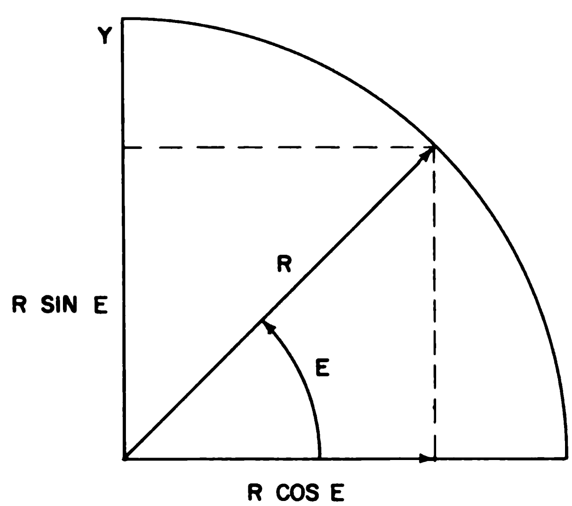
The second component of velocity jump is caused by the difference between the initial velocity of the missile, determined by the attitude of the weapon station and launcher, and the apparent velocity of the medium. The aerodynamic qualities of a missile which act in our favor to keep the missile in flight, also act against us to produce forces tending to cause the missile to line up with the airstream. This is of particular importance for missiles using wings and fins as aerodynamic lift and control surfaces.

Fixed launchers for finned rockets mounted on aircraft provide a particular example of this component of jump correction. The rocket launching tube is mounted so that the expected angle of attack on an average firing will align the rocket with the airstream. If the rockets are fired when the aircraft is flying at this average angle of attack, they will head directly into the airstream, and will leave the vicinity of the aircraft in the same direction as the aircraft velocity.

Generally, the aircraft does not fly at the average angle of attack. Therefore, the rockets will be at an angle with respect to the apparent wind or oncoming airstream when they leave the launching system, and when the fins become effective, the rockets will weathercock into the airstream. Thus, the rockets usually leave the vicinity of the aircraft along a direction that differs from the direction of the aircraft velocity by some angle. In the rocket jump correction, the angle can be assumed to be equal to a constant times the difference between the actual angle of attack and the preselected average angle of attack.

Other miss-producing effects which were discussed previously include wind curvature, drift curvature, and windage jump. Jump resulting from wind may be eliminated by computation in an air-mass reference frame. Drift can often be neglected as a flight path influence for guns carried by fixed-armament aircrafts; even in naval fire control, drift correction is only about two percent of the gravity drop correction.

Now it is necessary to convert this data into a coordinate system whose coordinate references are also stabilized deck referenced with one axis along the ship centerline in the forward direction (X), a second axis directly athwartship (Z), and the third axis vertical (Y).

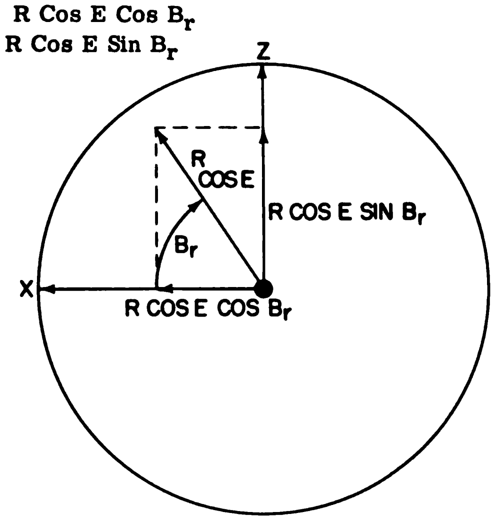


To derive the relationships for this coordinate conversion, let us first resolve the range vector into a component along the vertical line and one along the horizontal line formed by the intersection of the vertical and horizontal planes. The component along the vertical is then equal to R sin E; the component along the horizontal is equal to R cos E. Since the vertical is the same for both coordinate systems, the first equation for our coordinate conversion can be stated as follows:

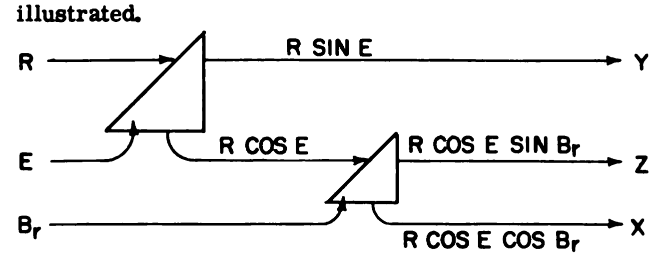
$$Y = R \text{ Sin } E$$

Now it remains to resolve the horizontal component R cos E into components along the X and Z axes. To do this, let us examine the horizontal plane. Since the X axis corresponds to the line from which relative bearing was measured in the spherical coordinate system, the horizontal range vector may be resolved through the angle B_r to obtain the X and Z coordinates as follows:

$$X = R \text{ Cos } E \text{ Cos } B_r$$
$$Z = R \text{ Cos } E \text{ Sin } B_r$$

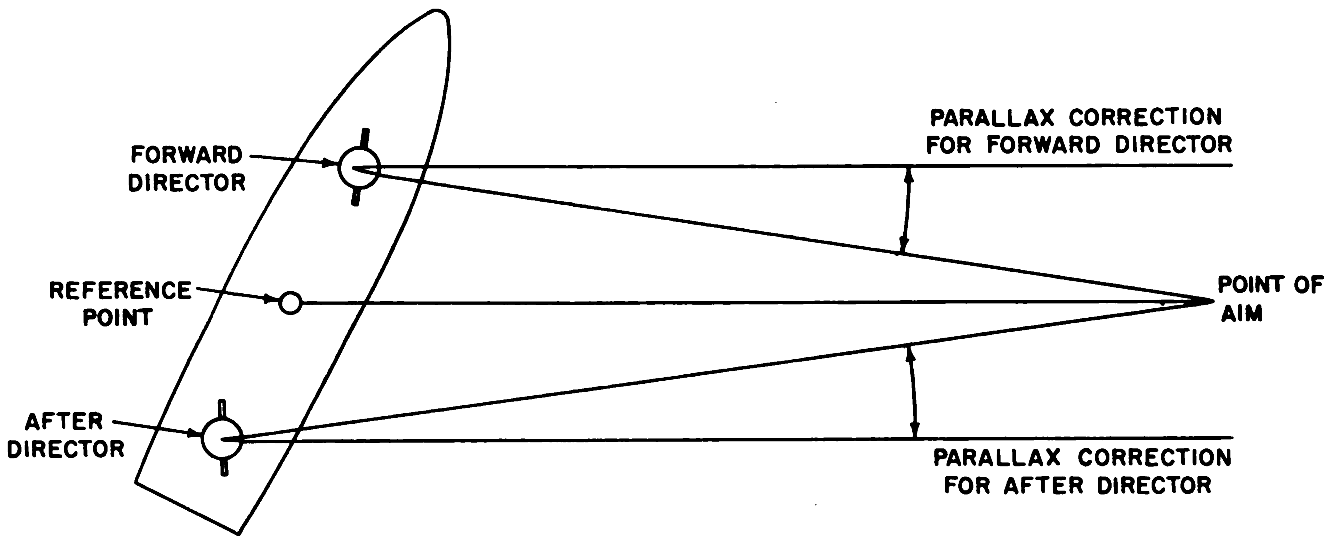


A fractional diagram implementing these equations is illustrated.



DATA CONVERSION BECAUSE OF FRAME DISPLACEMENT

The second difference between data is caused by reference frame displacement. In some instances, the tracking equipment is mounted directly on the weapon; in these cases, the frame displacement is negligible. In many cases it is desirable to have a separate tracking system or systems. For example, in many systems it is advantageous to have two directors, each capable of performing the tracking function, one located forward, and the other located aft. In this way, one director will always be able to establish a line of sight to the target even though the other is blocked by the ship's superstructure. The displacement of radar directors on cruisers is sometimes on the order of 400 feet; on aircraft carriers it might be much greater.



In some cases, tracking can be done by one ship or aircraft, while the weapon system is based on a second vehicle which may be several miles away. The tracking frame is centered on the tracking equipment, while the weapon frame is centered on the launcher. Most shipboard launchers and trackers can be trained or elevated, but are fixed in the sense that they cannot be moved from spot to spot on the ship. The displacement between the launcher, the tracking system (such as a radar set, sonar, or optical tracker) and the center of the ship is determined when the ship is designed. These values remain constant throughout the term of service of the vessel unless a major reconstruction program takes place, at which time the directors might be relocated.

If tracking data is supplied by one ship while the weapon is on the other, some way must be found to monitor the position of the assist ship along the tracking with respect to the computing ship. A similar situation occurs in the use of shore artillery, where the displacement between the director and the launcher depends to a large extent on the terrain and the tactical conditions. The problem of measuring these distances is simpler than measuring target data, since no countermeasures are used.

DATA CONVERSION BECAUSE OF FRAME ROTATION

The third difference between data results from the rotation of the reference frames. The rotation may be a fixed amount, such as that caused by the impossibility of making a perfectly vertical or horizontal part. On shipborne installations, for example, it is not possible to set the plane in which a gun, missile launcher, or radar, etc., rotates exactly parallel to the deck plane. Close tolerances are possible, but absolute precision cannot be achieved.

Since these rotations are kept as small as possible and are essentially constant, they are not a great problem. The rotations due to pitch, roll, and yaw of the delivery vehicle are much larger, particularly on vehicles such as tanks, aircraft and destroyers. A destroyer may roll 20 or 30 degrees. Even in relatively calm air or sea, or on comparatively flat land, small, highly-maneuverable vehicles still undergo considerable rotation.

The rotation of a reference frame based on the delivery vehicle with respect to an inertial frame or an Earth reference frame can be found by using a stabilized device like a gyroscope, and comparing the position of the vehicle-based reference frame with it. The mathematical process of transforming data is a coordinate system in one reference frame to equivalent data in the same coordinate system in a second reference frame, knowing the angle of rotation between the reference frames, can be illustrated by a simplified two dimensional analysis.

Let us assume that we wish to convert data x and y, measured in the XY frame, into data x′ and y′ measured in the X′ Y′ frame, and that the X′Y′ frame is offset from the XY frame by the angle 0. First, knowing x and y, we also know the magnitude of the range vector

$$R = \sqrt{x^2 + y^2}$$

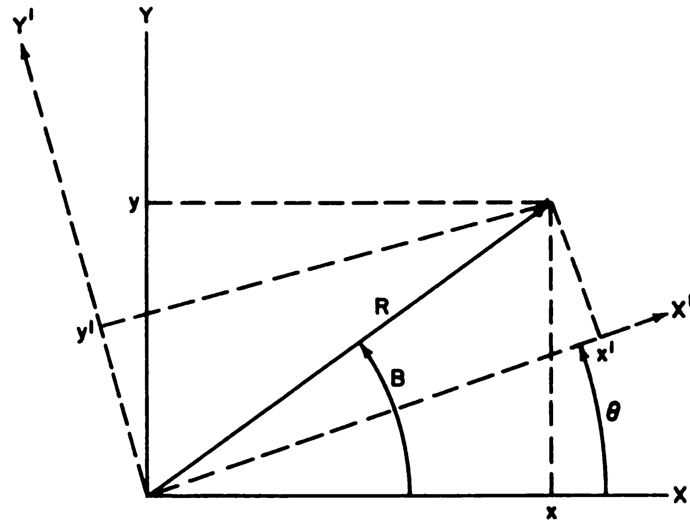
and the magnitude of the bearing angle

$$B = \mathrm{Tan}^{-1}\, {}^{y}/{}_{x}$$

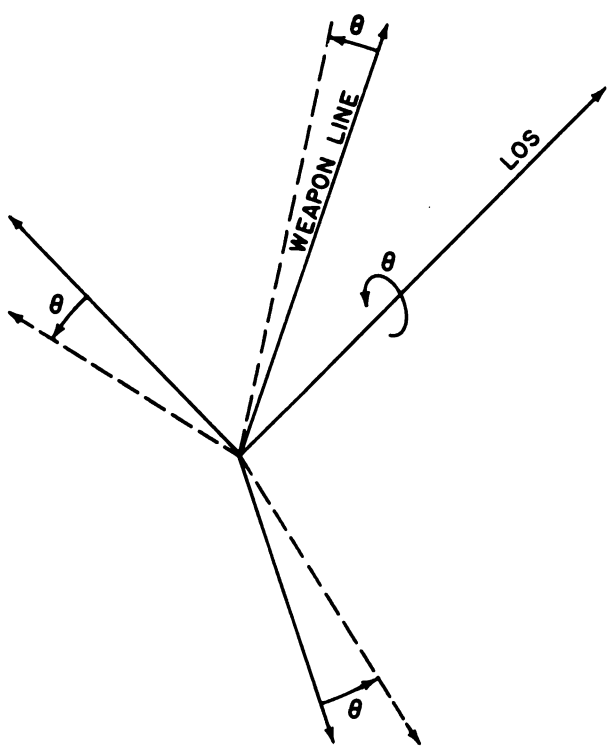
If we resolve this vector into X′ and Y′ coordinates, we get

$$X^1 = \sqrt{x^2 + y^2}\ \ \mathrm{Cos}\ \left(\mathrm{Tan}^{-1}\ {}^{y}/{}_{x} - \theta\right)$$

$$Y^1 = \sqrt{x^2 + y^2}\ \ \mathrm{Sin}\ \left(\mathrm{Tan}^{-1}\ y/x - \theta\right)$$



Simpler methods are available for performing this transformation. However, this example is tendered since its mathematics are relatively simple and illustrative.
Thus, the data in the tracking frame can be transformed to its equivalent in the computing frame, and then the output of the computer transformer to the weapon frame. An alternative to this is to use the tracking line or the weapon line as one of the axes establishing the computing reference frame. This simplifies the problem of stabilization, but may introduce an error when velocities are considered, since there will be an angular velocity component about the controlled line called cross-roll or cross-traverse error.



As an example of the effects of cross roll error, let us assume that a shipboard radar is tracking a target, and that the computation space is determined by a coordinate system with one of the coordinates being the line of sight of the radar. If at one instance the prediction angle is computed, and, in the time between this instance and the time the launcher slews to the proper weapon line position, ship pitch and roll create a cross transverse angle about the LOS of θ, the weapon line will slew to a position θ degrees away from the desired position.
One method of correcting this deficiency is to establish a stabilized reference frame using the LOS as by freezing the coordinate frame at some instance at the start of the tracking operation. From this moment, a data conversion device must continuously monitor the total cross transverse angle and convert the data from the radar into the stabilized reference frame by using this total cross traverse angle.

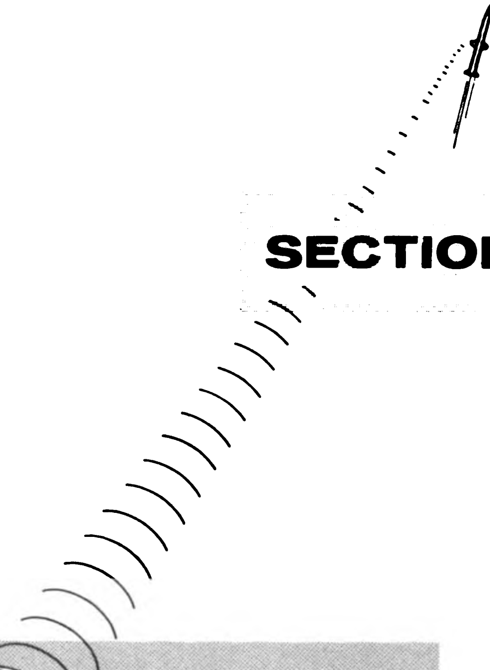
alignment

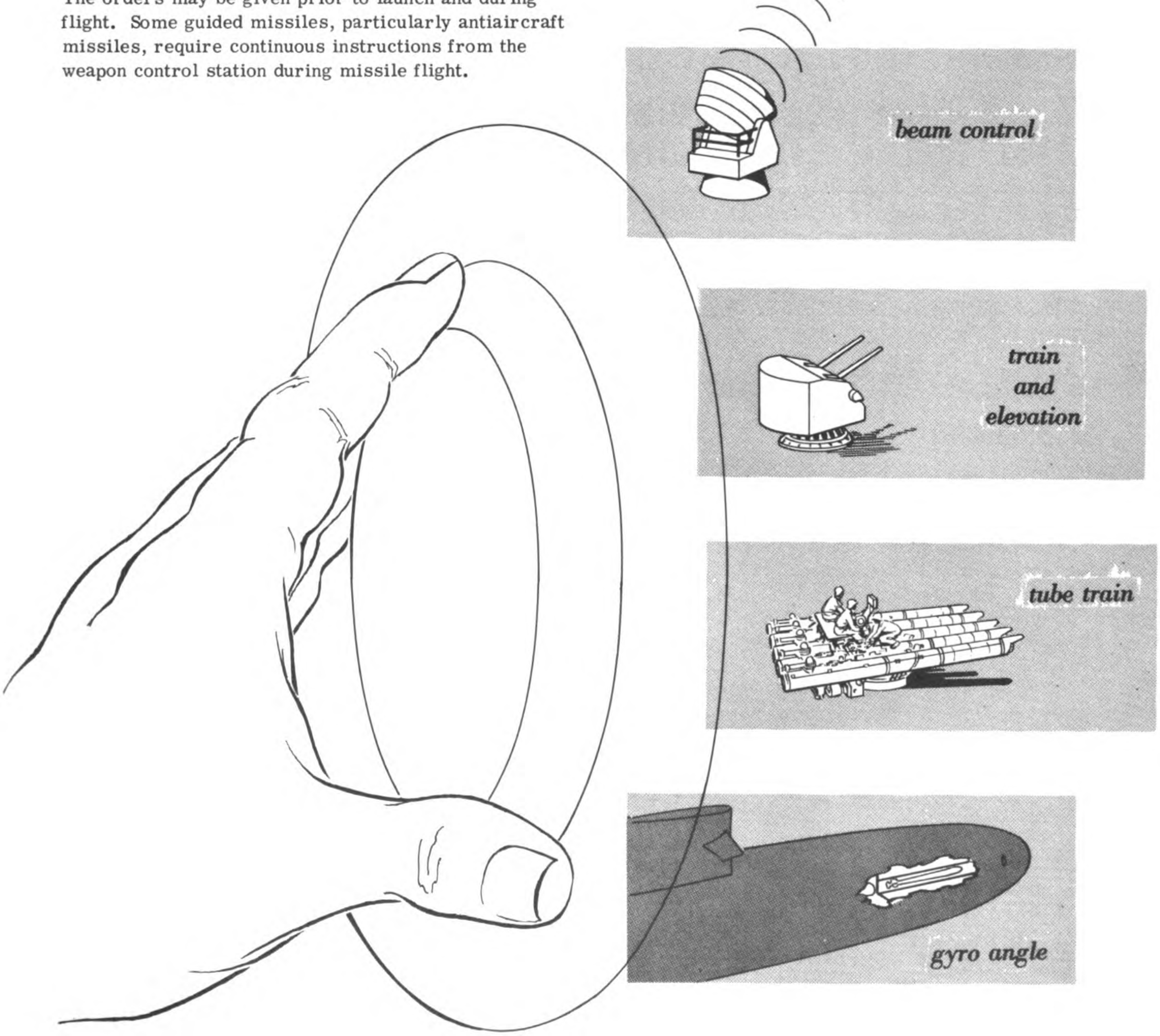
To minimize data conversion and transformation we try to use the same reference frames. This is not actually possible since the equipment must be separated in space, but if the separation is small, this may be ignored. For equipment mounted on the same delivery vehicle, it is possible to ensure that no rotation occurs between frames. Although it is true that rotations aft lag rotations forward, particularly on large vehicles, they are usually small enough so that the vehicle can be considered a perfectly rigid body.
The setting up of the coordinate axes on the physical equipment to agree with the theoretical coordinate axes is called alignment. In establishing the coordinate system to be used, it is common pratice to make one of the axes theoretically parallel to the centerline of the vehicle. All the coordinate transformations will be based on this theoretical position of the coordinate axes. The actual data received from the tracking system or sent to the weapon line drive equipment is based on the actual position of the physical coordinate axes.
The problem of aligning these axes is similar to the problem of zeroing synchros described in Volume I of this publication. In many cases, it is the identical problem, because synchros are used by many systems to transmit data, and the synchro zero position is the position of one of the coordinate axes.

WEAPON LINE CONTROL

SECTION 4

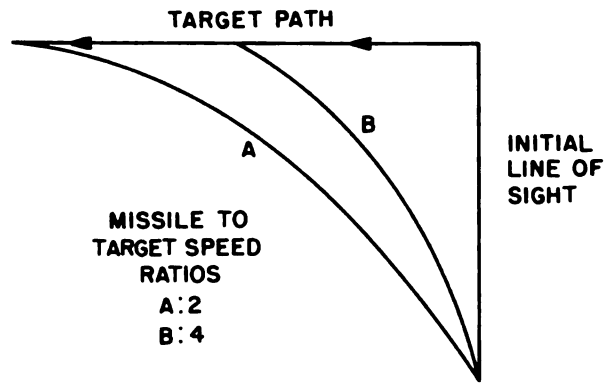
When the line of sight to the target has been established and the prediction angle has been determined, the fire control problem has been solved. Next, it must be put in effect. The weapon line must be positioned from the line of sight by an amount equal to the prediction angle. If the weapon launcher is fixed, the weapon line positioning must be done by the delivery vehicle. If the launcher is movable, weapon drive equipment to position the launcher properly is required. If the missile has inflight velocity control in addition to that provided at launch, the missile must receive orders to follow. The orders may be given prior to launch and during flight. Some guided missiles, particularly antiaircraft missiles, require continuous instructions from the weapon control station during missile flight.





INFLUENCE OF TARGET

The type of target and attack course which can be followed by the delivery vehicle will affect the weapon line control type. A lead pursuit course is highly desirable; the delivery vehicle always moves so that it leads the target by the appropriate angle at which a missile can be launched, thus keeping the delivery vehicle in an attack position for a considerable period of time. Therefore, there may be time to launch several missiles, and even to wait for evaluation of the effect of the first missile, and then fire a second or third if necessary.



ACCURACY

The specific choice of a system is based on the speed and accuracy requirements of the solution and the size and complexity of the equipment that can be accommodated on the delivery vehicle. An unguided missile has severe accuracy requirements on the weapon line drive because the initial orientation is the only time that the weapon line is subject to control. Time lags or any delay in determining the solution must be minimized since they represent errors in the solution as of the present moment.

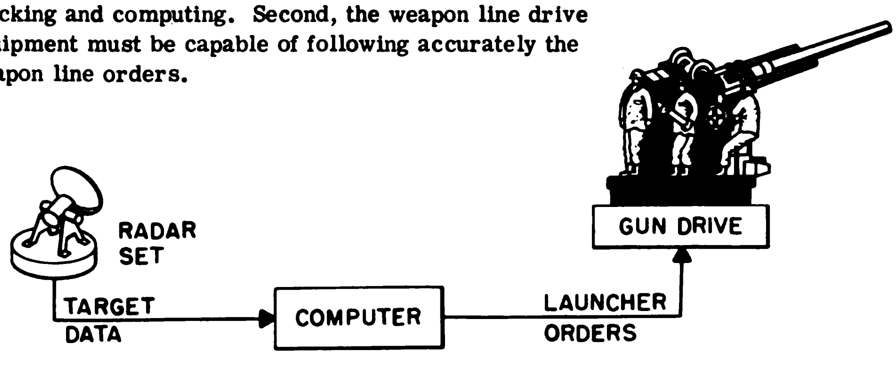
INFLUENCE OF MISSILE ON

Certain missiles receive a weapon line orientation at the time of launch only and are called unguided missiles. For movable launchers, orientation is provided by the launcher positioning gear. If the launcher is fixed, the thrust of the missile propulsion provides the initial velocity as modified by speed and direction. Other missiles have self-contained guidance or remote guidance which provides some degree of inflight control. The inflight control may exist only for a short period during flight, or throughout the entire flight. While the missile is in controlled flight, the weapon line drive is the missile flight control system. For guided missiles and weapons using fixed launchers, it is not simply a problem of finding the direction to aim the weapon, but

unguided missiles

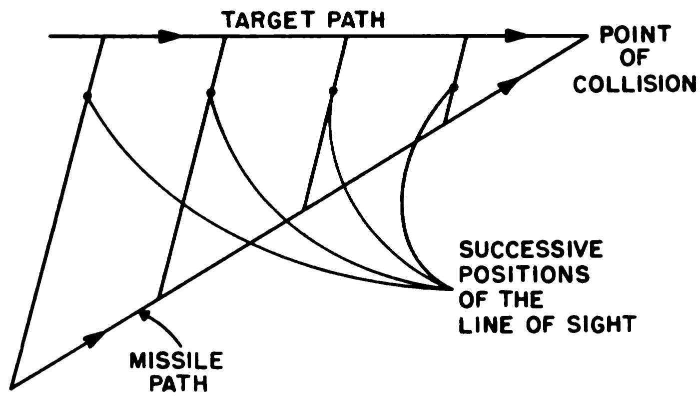
Unguided missiles such as bombs, projectiles, and free rockets receive their only velocity vector control from the initial weapon line position. Any inaccuracy in the weapon line position causes a loss of precision in missile intercept. Since loss of precision is a direct function of range, long range unguided missiles require a greater accuracy unless loss of precision is acceptable. The weapon line accuracy requirements consist of two distinct requirements. First, it is necessary to have accurate weapon line orders, which requires accurate tracking and computing. Second, the weapon line drive equipment must be capable of following accurately the weapon line orders.

To illustrate this point, let us examine an antiaircraft gun system whose function is to engage low flying high speed aircraft. This weapon system contains a radar set, a computer, and the gun itself, which has a capability of firing 50 rounds a minute.



AND ATTACK COURSES

If the target is highly maneuvable, it may not be possible to maintain a pursuit course, and a constant-bearing course may be followed instead. This course requires much less performance in the delivery vehicle but permits missile firing for a brief interval of time only: Under these conditions, missiles with a higher kill probability will be required. Since one of the factors directly affecting kill probability is the accuracy of the weapon line control, the type of target and the type of flight path of the delivery vehicle have a direct effect on the accuracy requirements of the weapon line control system.



REQUIREMENTS

Guided missiles can be divided into two groups, one in which the control is set at launch, and the other in which the missile is subject to variable control, either internal or external, from the delivery vehicle. If the missile follows a preset program, the setting of the weapon line prior to launch must be as accurate as for unguided missiles since errors cannot be corrected after launch. Missiles subject to variable inflight control do not have so serious an accuracy requirement since slight errors can be corrected during the controlled portion of the flight path.

WEAPON LINE CONTROL

one of navigating the delivery vehicle or the missile. Basically, there are three different categories of missiles; unguided missiles such as gunshells, bombs, mortars, etc.; self-contained guided missiles such as ICBM's certain torpedoes, etc., and remotely guided missiles such as antiaircraft missiles, wire guided torpedoes, etc. In addition, there are missiles which combine the features of more than one of these categories, such as antiaircraft missiles which are remotely guided to the impact area and then guided to impact themselves by their own homing devices. Naturally, the type of missile used greatly influences weapon line control potential.

The fire control system operates in the normal manner, with target data provided to the computer from the radar set, and the resulting launching orders fed to the gun drive. Because the gun itself is the largest moving mass in the system, being larger and heavier than the radar antenna, it also has the largest inertia. Therefore, the gun tends to resist any acceleration ordered by the computer. If there is an appreciable lag in the response of the gun drive to the launching orders, the shells fired from the gun will always cross the target path behind the target, thereby rendering the system ineffective.

Calibrating the computer to increase the lead angle in an effort to compensate for gun drive lag does not correct the error since the calibration may be effective only if the target does not maneuver. Hence, the basic requirement exists that the response of the gun drive be accurate and fast. The servomotors which actually drive the gun mount must have enough power to position the heavy mount swiftly in response to the weapon orders and have enough accuracy to position the mount precisely.

In an aircraft-based fixed-armament fire control system, the weapon line drive is the aircraft propulsion and control system. The accurate positioning of the weapon line requires that the aircraft be held rigidiy on its flight path with no pitch, roll, or yaw. Any motions of the vehicle resulting from turbulence, crosswinds, etc. which tend to change the flight path of the aircraft, also affect the weapon line. The aircraft must be capable of correcting these offsetting forces rapidly or the weapon may not intercept the target.

An alternate solution to the problem is to use movable launchers and have a weapon line drive separate from the delivery vehicle drive. No matter what compensating networks are employed, the inherent weakness of an unguided missile is that once launched, no further weapon line control is possible.

SELF-CONTAINED GUIDED MISSILES

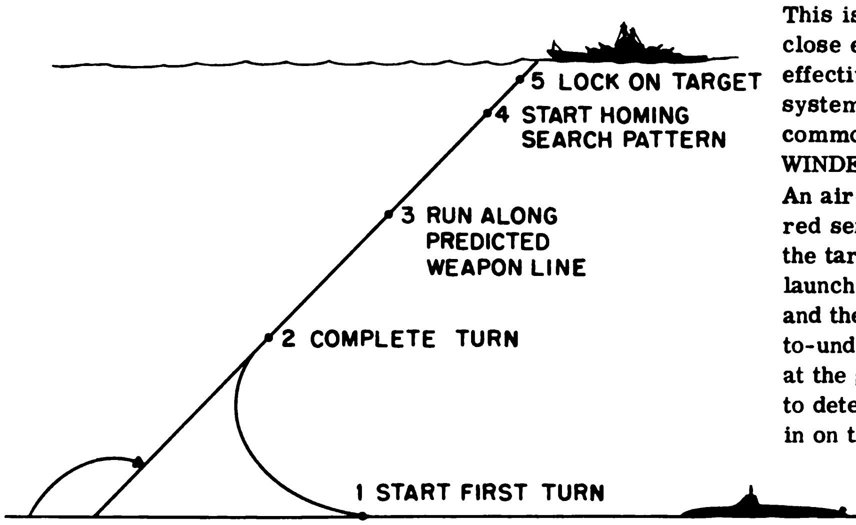
Self contained guided missiles are of two types,one in which the required weapon line is programmed into the missile prior to launch so that the missile's own control system implements this preprogrammed course while in flight, and one in which the missile contains its own sensing device and directs its own movement through its control system to maintain a line of sight course to the target. The first system employs INERTIAL GUIDANCE; the second system employs a HOMING device.

In order to understand the functioning of these two types of self contained guided missiles, let us discuss a missile which employs both types of self-contained guidance, a torpedo. The heart of a modern torpedo's guidance system is its gyro. In most cases, it is impractical for a submarine to maneuver constantly so that its leading is coincident with the required weapon line. Therefore, it is necessary usually to fire a torpedo in some direction other than that indicated by the required weapon line.

In order to compensate for this deviation, a GYRO ANGLE is set into the torpedo, the torpedo is instructed to turn through a specific angle, called the gyro angle, which then sets it on the required intercept course.

How does the torpedo accomplish this maneuver and what does it use as its reference frame? Within the torpedo, a gyroscope is set into motion just prior to launch. It acts as an initial reference, always pointing in the direction established by the gyroscope at this point prior to launch. The torpedo is then launched and runs in its initial direction for a predetermined period of time. Then, the torpedo's guidance system is activated and the torpedo's control system acts to align its axis (missile line) with the directional line established by the gyro. In this manner, it turns through the gyro angle and comes about to its proper intercept course. The torpedo then runs along its predicted weapon line for a predetermined length of time as computed by the fire control computer prior to launch.

At the end of this period, HOMING, the second type of self-contained guidance, comes into play. The torpedo enters a preset search pattern with its acoustic homing device listening for target noise. Upon detection of target noise, the torpedo locks on to the strongest noise signal and activates its control mechanism to steer a course to the source of the noise. In this manner, the HOMING device guides the missile to intercept during the terminal phase of the trajectory.



inertial guidance

From the above example it is clear that many features of the torpedo will affect the system design. First, considering the inertial guidance positionly, it is obvious from the trajectory of the torpedo that for weapon line computation, a parallax factor equal to the distance between the launch point and the completion of the turn must be considered. Essentially, the computer must consider the launch point as point 2 on the illustration. This then affects the time of fire because the torpedo must be fired some time earlier so that it reaches the launch point at the instant determined by the computer. An obvious advantage to the initially guided missile is that it need not be launched in a specific direction.

Other systems, such as ICBM's and IRBM's, use similar guidance tactics. Long-range missiles such as these may be launched vertically thereby easing the launching problem and also allowing the missile to obtain altitude quickly, resulting in longer ranges. In general, this type of guidance permits the use of simple fixed launchers on delivery vehicles which cannot be readily positioned to provide proper weapon line orientation. Of course, many other departures from the standard fire control system may be expanded for this example, but the samples discussed serve to illustrate the effects of a missile of this type.

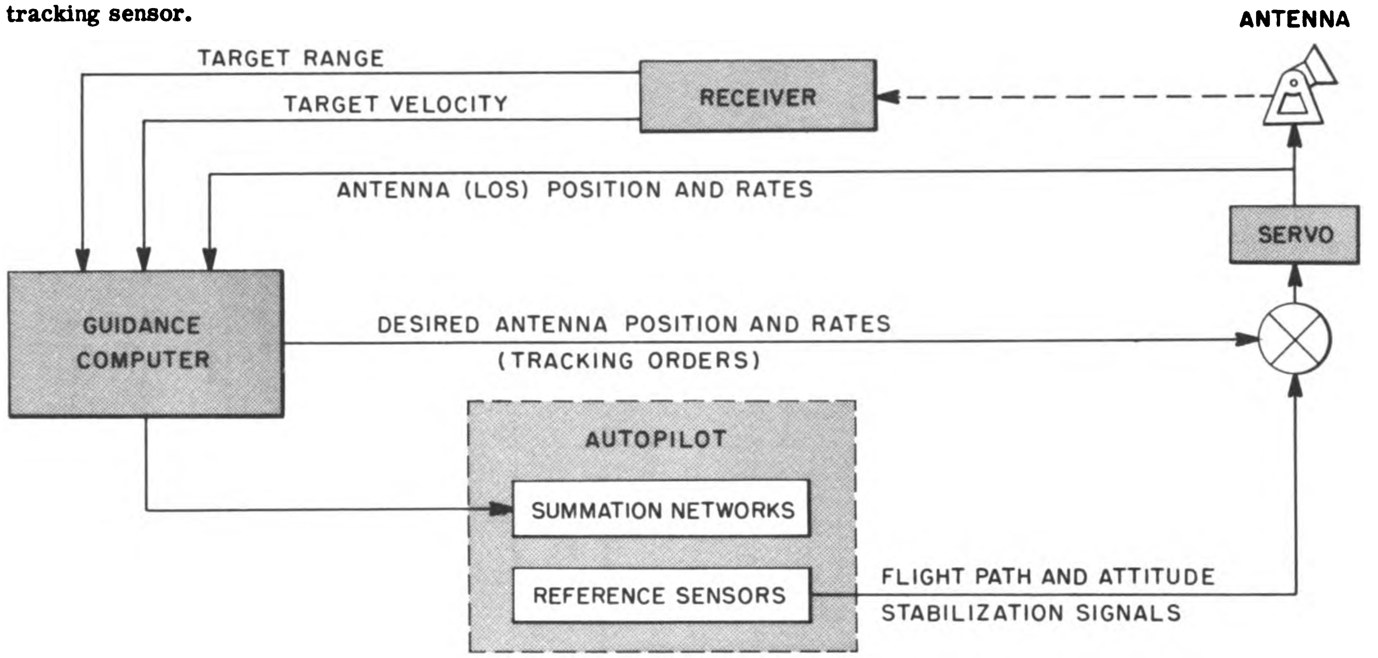
homing

A missile which employs the system of homing guidance does not require as great an accuracy in determining the weapon line as unguided missile systems. This is because the system must get the missile only close enough to the target so that the homing device is effective, whereas in nonguided missile systems, the system must be accurate enough to produce a hit. More common examples of homing missiles are the SIDEWINDER and ASROC.

An air-to-air missile, the SIDEWINDER has an infrared sensor which provides angular positional data on the target. This data is used in the missile after launch to determine the required weapon line position and the weapon line drive orders. ASROC, a surface-to-underwater missile, is fired from the surface ship at the general location of the submarine. Sonar is used to detect and tract the target and to home the missile in on the target's position.

TYPES OF HOMING SYSTEMS

Homing systems can be divided into three types, depending on the source of the energy detected by the tracking sensor.



active homing

In an active homing system, the transmitter of energy which illuminates the target and the sensor which receives the reflected energy from the target are both contained in the missile. A simple homing system consists of a transmitter and receiver of microwave light, heat, or acoustic energy for target detection and tracking of the target, a computer to calculate the required weapon line position, and weapon line drive equipment to direct the missile in response to the computer orders to intercept the target. By putting an energy transmitter in the missile a severe weight and cost requirement is placed on it. Also, the problem of the countermeasures which can be used by the enemy may require counter-countermeasure equipment to be added to all the other equipment in the missile.

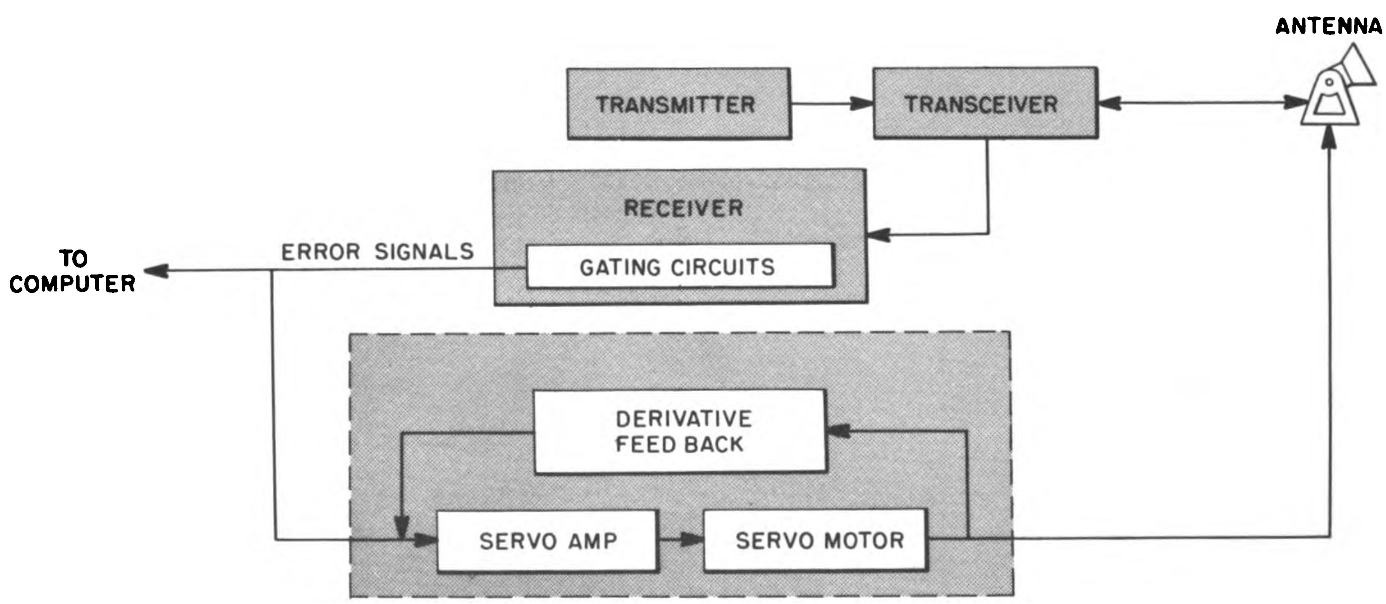
There are several advantages of a missile using active homing.

1) Once homing starts, the missile is completely independent of energy transmitted from an external source and of any externally deviced guidance information.

2) The fire control system of the delivery vehicle may engage another target as soon as homing is initiated.

3) Long range target intercept is possible beyond the accurate discrimination range of weapon station based transmitters.

4) The delivery vehicle has greater flexibility since it can maneuver, retire, etc., once homing is initiated. In a semiactive homing system, the transmitter of energy which illuminates the target is located outside the missile, usually on the delivery vehicle.

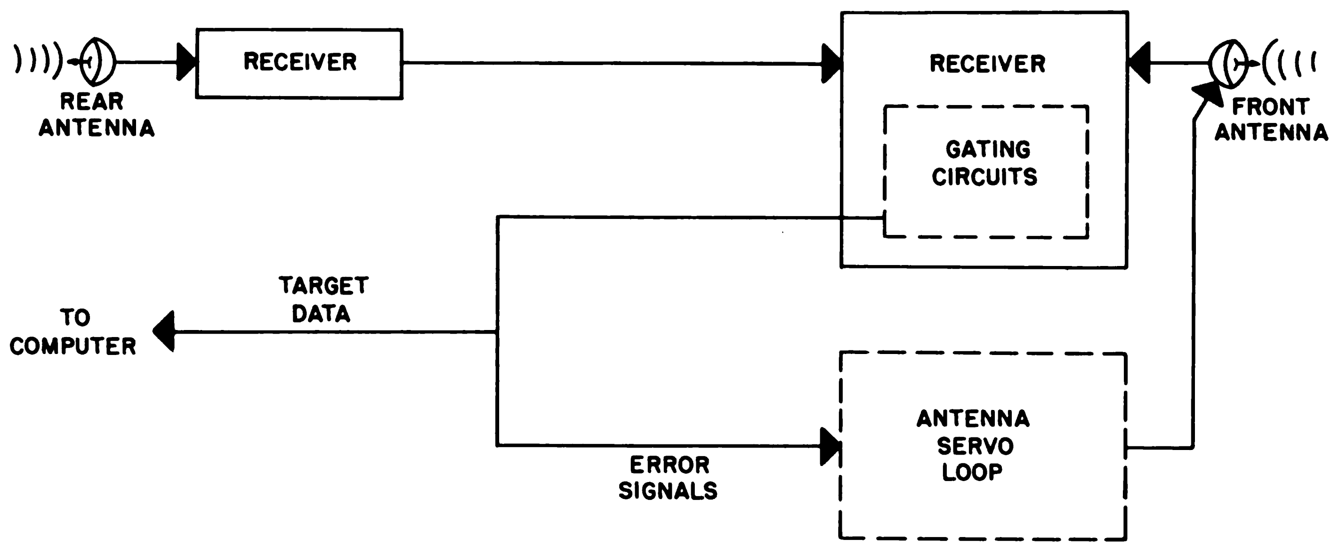


ANTENNA SERVO LOOP

semi-active homing

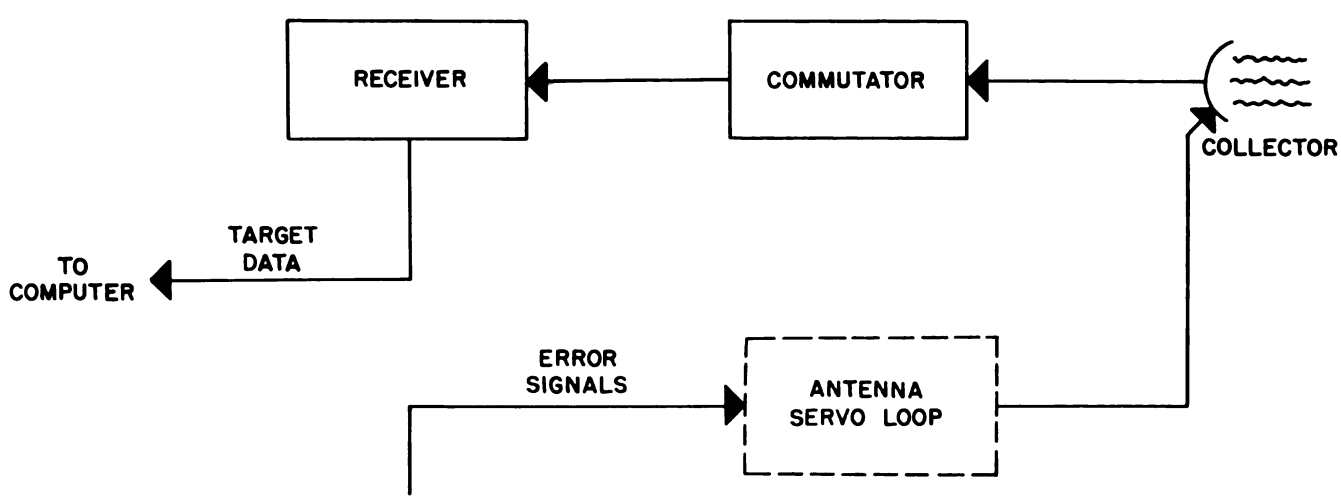
The semiactive homing system operates in the same manner as the active homing system. Because the energy transmitter is no longer in the missile, but on the larger delivery vehicle, the size and weight of the transmitter can be much greater, permitting the use of longer-range, high power transmitters. Without a transmitter, additional material can be included in the missile explosive to increase damage area, propellant to increase range, or guidance equipment to increase accuracy.

However, these advantages are gained at a price. Since the missile operates solely on energy which originates in the transmitter on the delivery vehicle, the delivery vehicles cannot break off the attack on the target once the missile is launched. Once a missile with active homing is launched, the delivery vehicle is free to begin an attack on another target. With semiactive homing, the delivery vehicle must continuously participate in the engagement until intercept occurs.

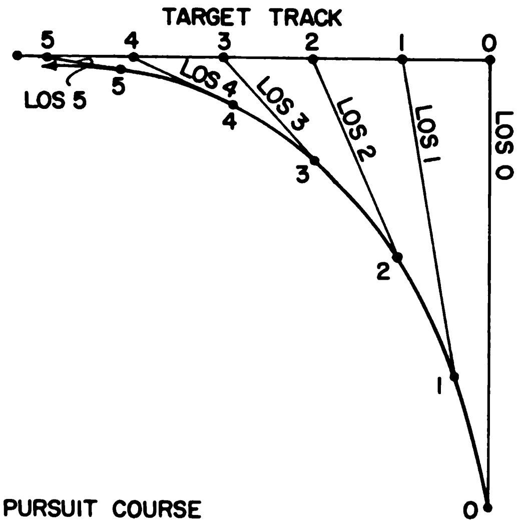


passive homing

The third form of homing is passive homing which relies solely on energy emitted by the target. Because a passive homing system does not require an internal transmitter, it has the advantage of a relatively light weight missile as in a semiactive homing system, and missile independence of the delivery vehicle as in an active homing system. This system has a major disadvantage in that the level of energy received by the missile varies inversely with the square of the range of the target. Inasmuch as the energy level of the target is kept to a minimum by the enemy to render detection more difficult, the passive homing system must usually operate at close range to the target.



Because the sensing, computing, and weapon line drive equipment must be carried in a homing missile, the weapon control problem should be solved as simply as possible. One of the oldest and simplest solutions is the pursuit course, the path followed by a missile, which is always aimed directly at the target. The missile then operates to maintain the weapon line coincident with the line of sight. The sensor can be



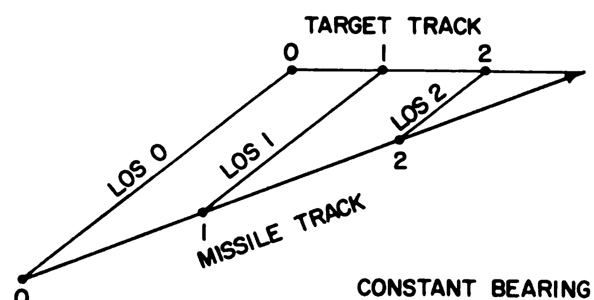
PURSUIT COURSE

a fixed detector whose tracking line is set in coincidence with the weapon line, the missile axis. Any deviation of the target line of sight from the tracking line is the tracking correction signal. Because the pursuit path does not utilize any prediction angle, no computer is required and the tracking correction signal can be used as the weapon drive signal to reorient the weapon drive and the tracking line. The equipment required to perform these tasks is relatively simple, thus accounting for the great popularity of this system for small missiles such as air-to-air missiles.

Unfortunately, in a pure pursuit course the missile follows a curved path even when the target is following a linear course. A missile using a pursuit course must have very high maneuverability not only to follow this curved path, but also to correct launcher dispension, gravity, forces causing flight path curvature, and any target maneuvers.

The requirements for missile maneuverability are greatly reduced if the missile flies a straight line path to intercept. To fly such a collision path, it is necessary to maintain a constant line of sight. In order to compute the prediction angle, the sensor must determine not only target bearing and bearing rate but also target range and range rate. A computer is required in the missile to calculate weapon line drive orders on the basis of data on the target. Once the weapon line is directed properly, it is then necessary to maintain only the line-of-sight stationary, and intercept must occur despite target evasive maneuvers. However, any instantaneous changes in the line of sight direction must be detected and corrected immediately. Because it is not physically possible to build a system which operates instantaneously, some more practical methods must be devised.

One such method is proportional navigation. In this system, as long as the line of sight does not rotate, the missile is maintained on its straight line path toward collision. If the line of sight begins to rotate because of target maneuver or any undesired motion of the missile, the weapon line drive equipment is ordered to turn the missile at a rate proportional to the rate of the line of sight rotation and in the proper direction to reduce the rotation of the line of sight to zero. When the rotation of the line of sight is reduced to zero, the missile is held on its constant bearing path. A proportional navigation system requires data on target beaming and range and the rate of change of each to calculate the initial prediction angle. An additional requirement of this system is a second order control system which turns the missile at a rate proportional to the input signal.



CONSTANT BEARING COURSE

remotely guided missiles

With missiles which have no inflight control or are subject only to a preset fixed or programmed control, no correction can be made for any change in the variables from which the prediction angle was computed. Because of the many high performances and accuracy fire control situations in present day warfare, a definite requirement exists for missiles which may be guided in-flight, thereby enabling new prediction angles to be implemented, thus increasing the kill probability of the missile. This type of missile system is categorized as REMOTELY GUIDED MISSILES.

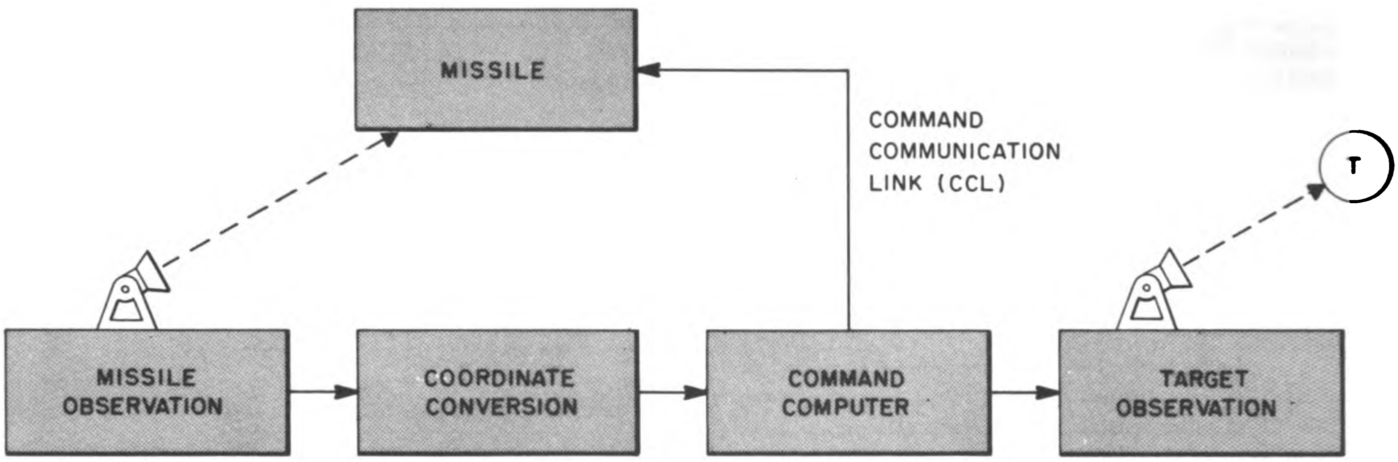
types of remote guidance

There are two types of systems within this category, one in which the delivery vehicle sends weapon control orders via a data link to the missile, which then uses a built-in weapon line drive mechanism to reorient itself; and one in which data are transmitted via a data link to the missile which then determines its own weapon line drive orders. The first type of system is called a COMMAND GUIDANCE system, and the second type is called a HYBRID GUIDANCE system. The most widely used HYBRID GUIDANCE system is the BEAM RIDER system which will be explained later.

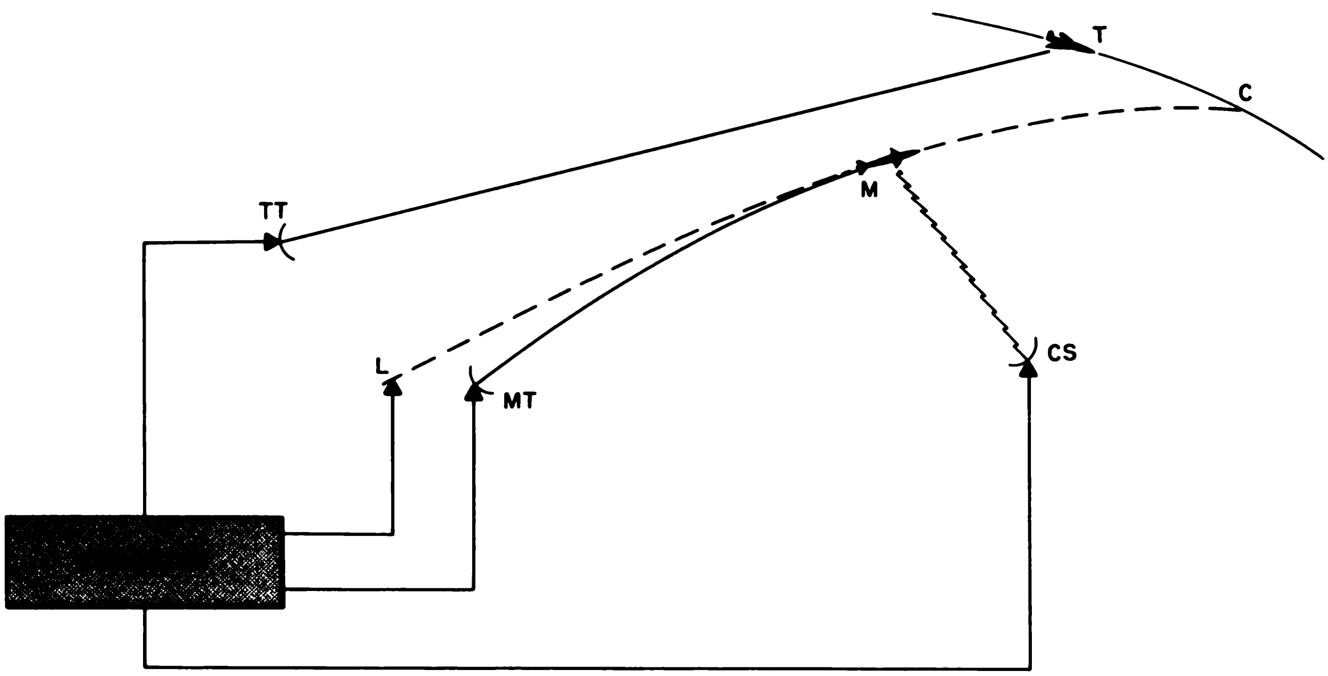
COMMAND GUIDANCE

One of the earliest and still common methods of providing inflight missile control is to provide the weapon control orders to the missile and its built-in weapon line drive equipment by some form of data link. The guided missiles first used by the United States were Azon and "Weary Willie" of the Air Force. Azon was a glide bomb whose yaw was controlled by radio signals from the bomber, permitting control of the missile flight path in azimuth. The "Weary Willie" project made use of bombers which were no longer fit for further service. They were outfitted for remote control, loaded with explosives, and flown to crash into the target. Most modern remotely guided missile systems use microwave data links.

Because these links are subject to jamming and interference and tend to enable detection of the command system site, various antijam and continuous techniques are employed. An alternative is to transmit the necessary data on a wire from the command site to the missile. By attaching a wire to the missile, its range is limited to the length of the wire. The advantages of wire controlled missiles make them desirable for use as relatively short range missiles as in antitank and antisubmarine torpedo weapons.



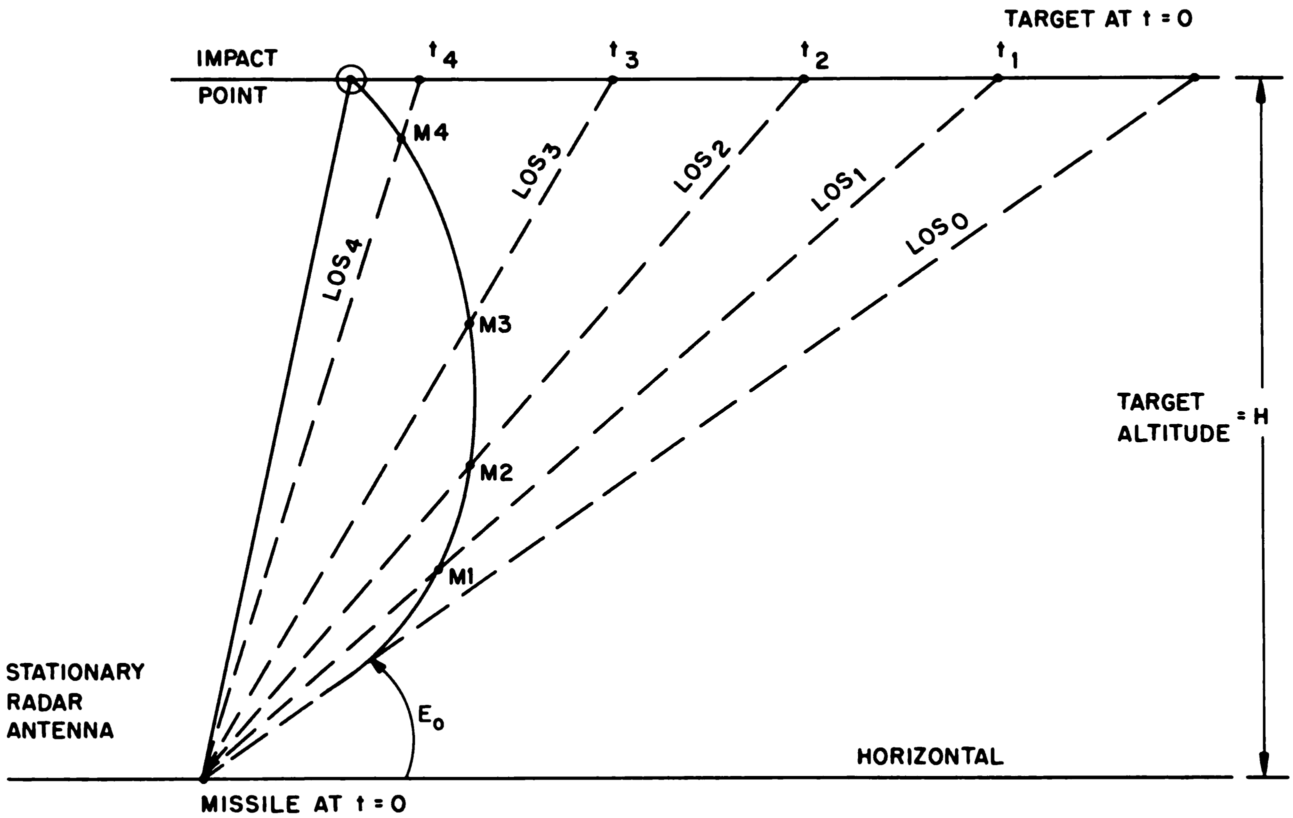
In a typical COMMAND GUIDANCE system, two tracking sensors are used to track continuously the target and the missile until intercept. The sensor tracking the target provides the necessary target position data, while the sensor tracking the missile provides similar data on the missile. On the basis of this data, the computer determines the weapon line drive orders necessary to maintain the missile on a flight path to intercept the target. The weapon line drive orders are usually transmitted to the missile by radar or a wire link.



A third form of guidance, hybrid guidance, combines both command guidance and homing guidance to achieve many advantages of both systems. By maintaining the tracking sensors on the delivery vehicle and transmitting the data to the missile, the long range capabilities of command and semiactive homing guidance can be achieved. By having the missile compute its own weapon line drive orders, the entire mechanization of the fire control problem can be simplified, as it is in-active and passive homing.

beam riding

A common example of a hybrid guidance system is beaming riding guidance system. A radar beam, which may be the beam used for tracking or a separate beam, is used to provide a direction to the target along which the missile must go. The missile must then determine its location with respect to the beam and must move to keep itself in the center of the beam. The equipment in the missile to perform these functions can be comparatively simple. By having the beam for guidance coincident with the line of sight on tracking beam, the missile will follow a form of pursuit course. By using a separate beam, which is made coincident with the tracking beam at intercept, other flight paths can be used which will result in less stringent acceleration requirements on the missile.



other systems

In missile systems which are used against long range targets, it may not be practical to use a guidance system which depends on the delivery vehicle or launching station. Also, the range of active and passive homing systems may be inadequate. Thus, it may be desirable to use an automatic navigation system capable of operating in a fashion similar to the human navigators who chart the course of a ship or plane. The missile may be equipped for celestial navigation in that it tracks certain reference stars to determine its position. The computer then determines the weapon line drive orders on the basis of the missile present position and its required position for intercept.

In a similar manner, the missile may be equipped to compare the terrain of the earth with a map and thus determine its present position, or it may use Loran or other navigational aids to determine its present position. Since each of these methods can be used to find only a geographic location, the target must be such that it can be pinpointed to a specific point and remain fixed during the long time of flight. This limits the type of target to a fixed installation, such as a harbor, storage depot, or transportation center. The complexity, size, and cost of these automatic navigators usually limits their application to extremely long-range missiles with large damage volumes whose use warrants such a guidance system.

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appendix A

REFERENCE FRAMES AND COORDINATES

section

Usually, telling where something is located presents us with no particular problem. From early childhood we have all become accustomed to answering "where" questions, and in ordinary circumstances, we find little difficulty in giving information that is good enough for the purpose.

But there's a big difference between telling Aunt Minnie where to find her cat and giving the kind of position information necessary to pinpoint and destroy a missile hurtling down into our outer atmosphere.

Unfortunately, the problem of describing locations is so familiar and so simple that we ordinarily never find it necessary to think out the basic rules that must be observed to give statements of location that are entirely clear and complete. In every day life, this does no harm.

But in all of weapon control, the problem of exact definition of position is of prime importance. To understand weapon control, it is first necessary for us to re-examine our ideas of just what the question "where?" means in the mathematical sense and to fully grasp the significance of the basic rules for the precise statement of position.

The rules are quite simple but it would be best not to consider them on the earth where we are surrounded by everyday things and may be misled by preconceived notions.

Let us leave the earth and move to some point in OUTER SPACE

Here, unencumbered by familiar ideas, we can start from the beginning to build up, step-by-step and with no complications, the fundamental meaning of the word "where" and the basic mathematical rules for position description.

APPENDIX A

REFERENCE FRAMES FOR POSITION

SECTION 1

We will not resort to simply pointing out the object. Because we are doing this as an exercise in order to learn the principles of position description, let us suppose that the position must be stated with mathematical precision. Here in space, the job is not simple because we have nothing to go by — up, down, north, south, and other such terms are completely meaningless, so we will have to improvise. In working this out, we'll start right from the beginning and do things just as they occur to us, without trying to think too far ahead. We'll solve each difficulty that arises, and in doing so we should get a clear understanding of the rules which must be followed in precision position description.

a definite, clearly defined, geometric point is needed . . .

CONSTRUCTION OF A FRAME IN OUTER SPACE

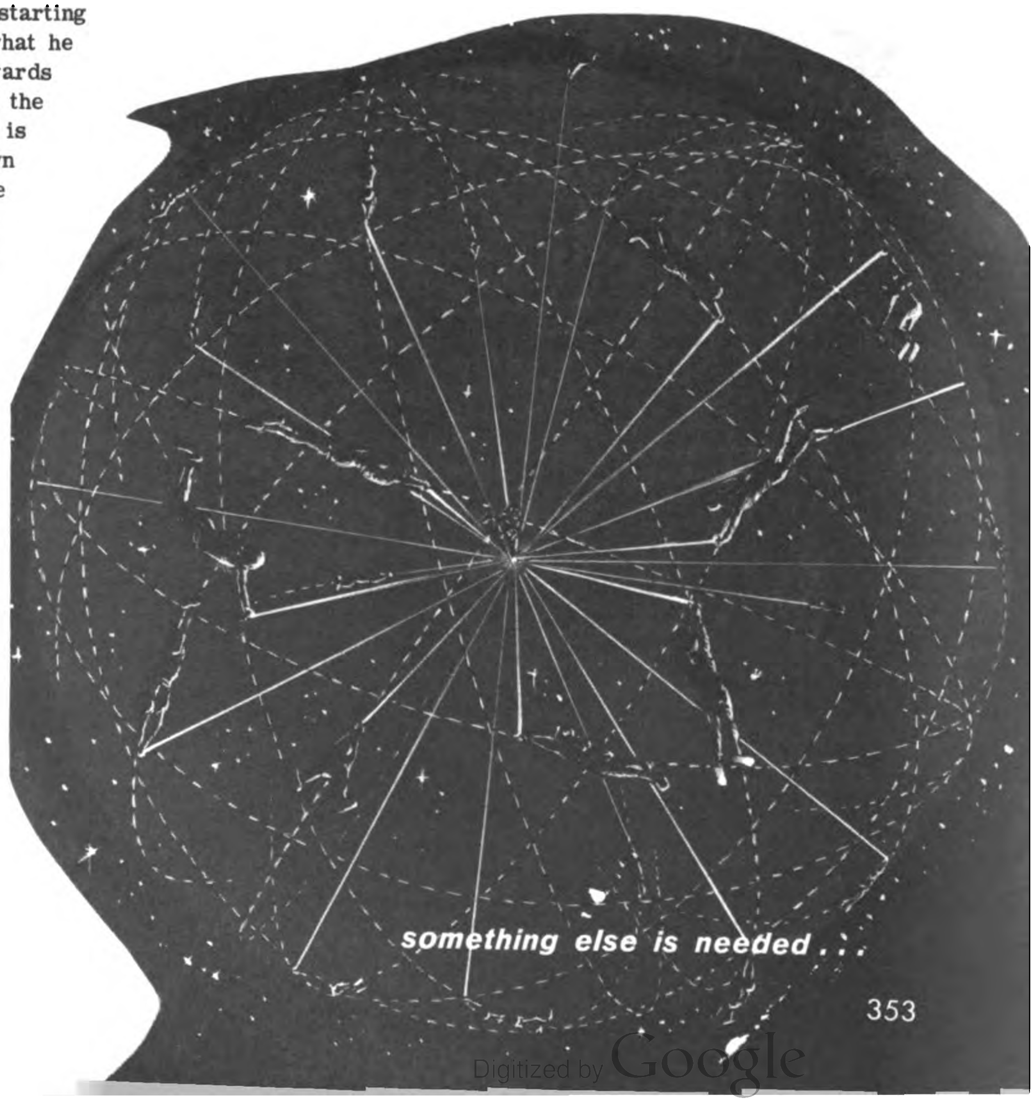
need for a reference point

Suppose that A starts by stating that the object is exactly 100 yards away. This sounds fine until B tries to use the information. He must ask, "100 yards from what?" From B? If so, from what part of him; from his head, his toe?

In order to describe distance we must be able to measure it off. Suppose B wishes to use a tape measure. Before pulling out the tape, he must have some place to locate the zero. Note that he will need some identifiable geometric point. (It wouldn't do, for example, to say that the object is 100 yards away from a large object like the space ship which may itself be many yards long.) Thus, in order for A's information to mean anything, he must identify the exact point for B to start from.

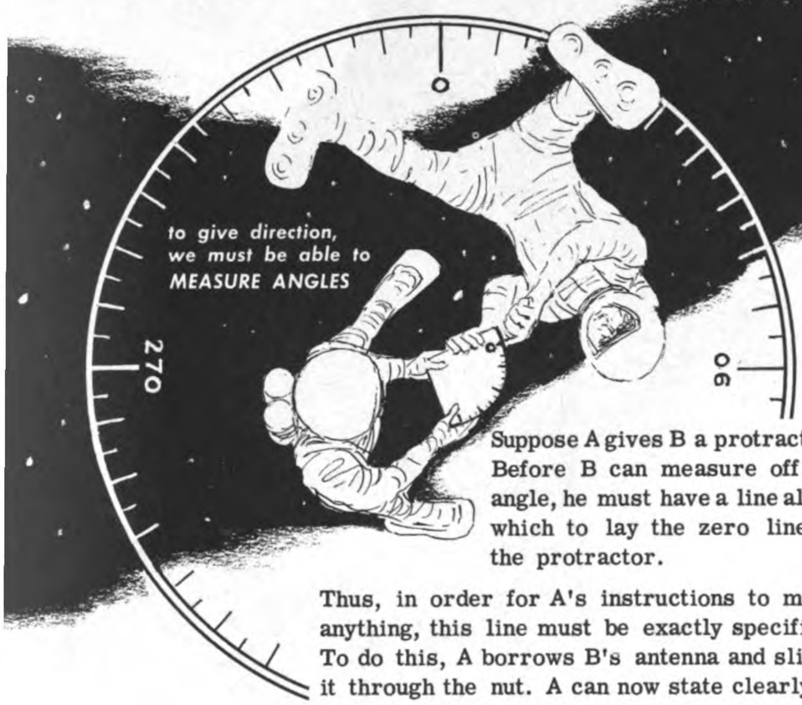
To do this, A could place some small object like a common nut with its center at the starting point. A can now state clearly to B what he means. "The object is exactly 100 yards away from a starting point which is at the center of that nut." As the dimension is "referred" to it, this point will be known as the REFERENCE POINT. It can be seen that establishment of the reference point is of vital importance in starting a description of any position. Without a reference point we can do nothing.

Now B knows that the object is exactly 100 yards from an identified reference point. This seems like useful information and so it is, but only partially. When B attempts to lay out the distance, he finds he can do so in any direction. All that the information really tells him is that the target is somewhere on a sphere 100 yards in radius, with its center at the reference point. With only the reference point given, there is no way to specify which direction is the correct one. . . . . Therefore, . . .



needed . . . some way **TO GIVE DIRECTION**

need for a REFERENCE LINE



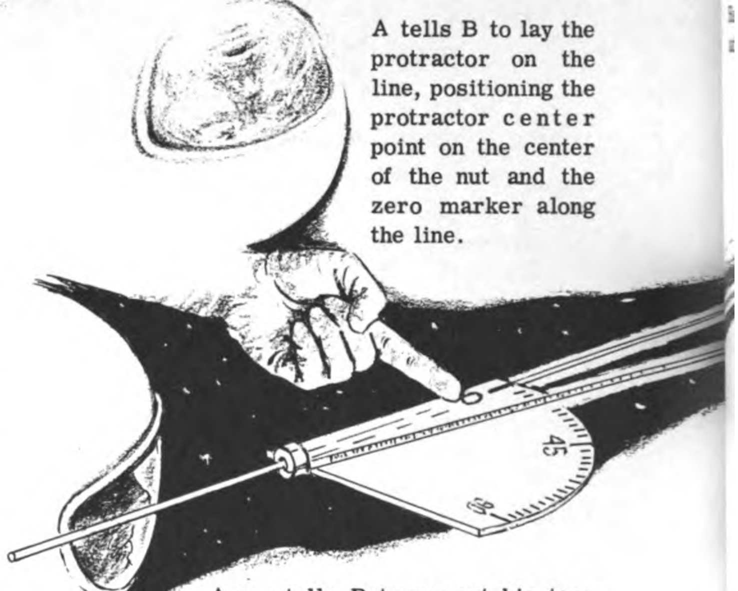
Suppose A gives B a protractor. Before B can measure off an angle, he must have a line along which to lay the zero line of the protractor.

Thus, in order for A's instructions to mean anything, this line must be exactly specified. To do this, A borrows B's antenna and slides it through the nut. A can now state clearly to B: "The object is 17 degrees from the line determined by the antenna".

The angle is measured at the reference point, i.e. the center of the nut.

This line is just as important as the reference point and must be established before we can give any angular values.

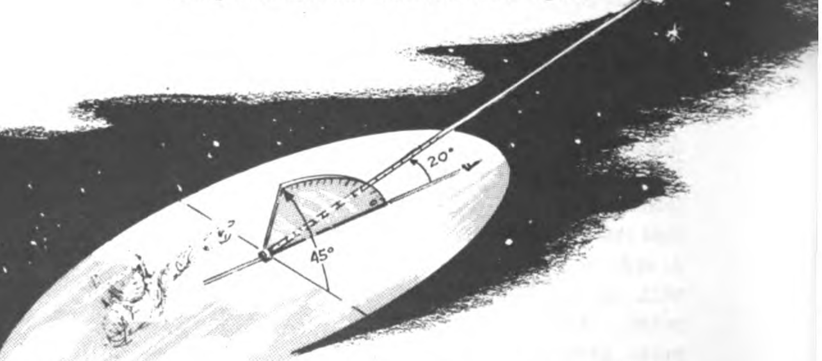
A tells B to lay the protractor on the line, positioning the protractor center point on the center of the nut and the zero marker along the line.



A now tells B to run out his tape measure at an angle of 17 degrees **clockwise from the line.**

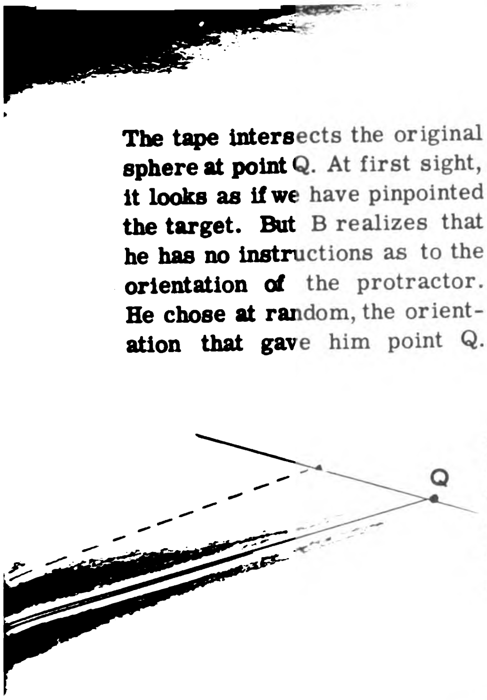
need for a REFERENCE PLANE

What we have so far failed to do is to define the PLANE in which to lay the protractor.
A and B now construct a plane. Any plane, arbitrary but identifiable, will do. We will make the plane so that it contains the reference line and, consequently, the reference point. The plane does not contain the target.



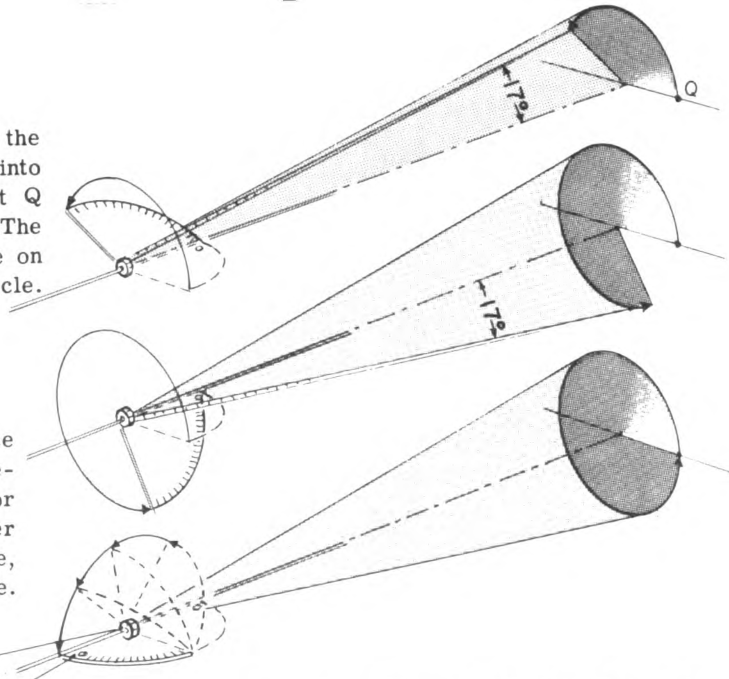
In order to give the location of the target, A can now give B **three instructions. In the course of his first instruction he again tells B to position the protractor with its center point at the reference point and its zero marker along the reference line direction, but he now adds: "With the protractor in the plane lying to the right of the reference line."**

His second instruction is: **"Rotate the protractor counterclockwise about the reference line (facing along the arrow head) through 45 degrees."**
His third instruction is: "Lay your tape along the 20-degree marker for 100 feet; there you will find the target."

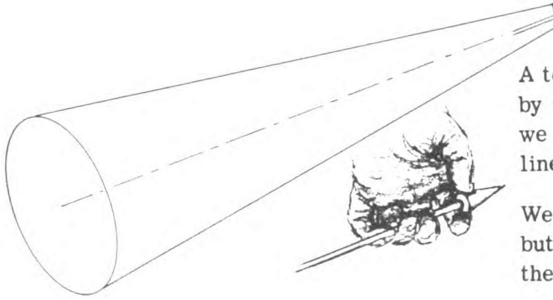


The tape intersects the original sphere at point Q. At first sight, it looks as if we have pinpointed the target. But B realizes that he has no instructions as to the orientation of the protractor. He chose at random, the orientation that gave him point Q.

Actually, he can rotate the protractor about the line into any position. The point Q then generates a circle. The target can lie anywhere on the perimeter of this circle.



Also, since the reference line extends through the reference point, the protractor could be placed in the other direction along the line, creating a second circle.

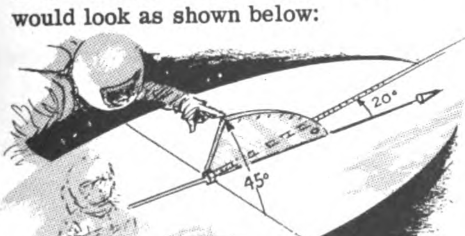


A tells B that he can at least eliminate the second circle by adding an arrow head to the line, giving it sense. Since we can now give an angular direction with respect to this line, we call it a REFERENCE LINE.

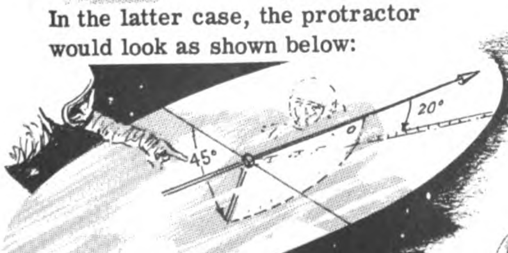
We have narrowed the search from a sphere to a circle but have still not determined the plane in which to orient the protractor. Something more is needed.

When B reaches the third instruction, he finds himself in trouble. The trouble is that he suddenly realizes that A is on the other side of the plane. Did A mean that the protractor should be on B's side of the plane or his own? In other words, should it be laying to the right of the reference line as seen by B or by A?

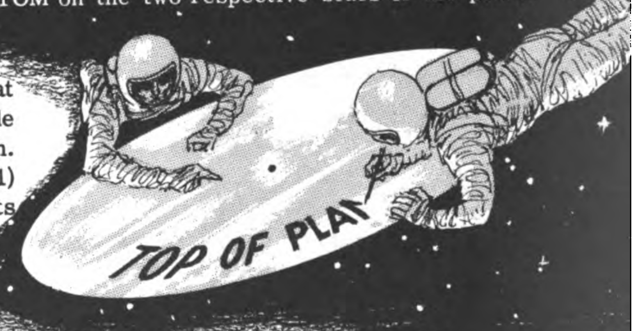
In the former case, the protractor would look as shown below:



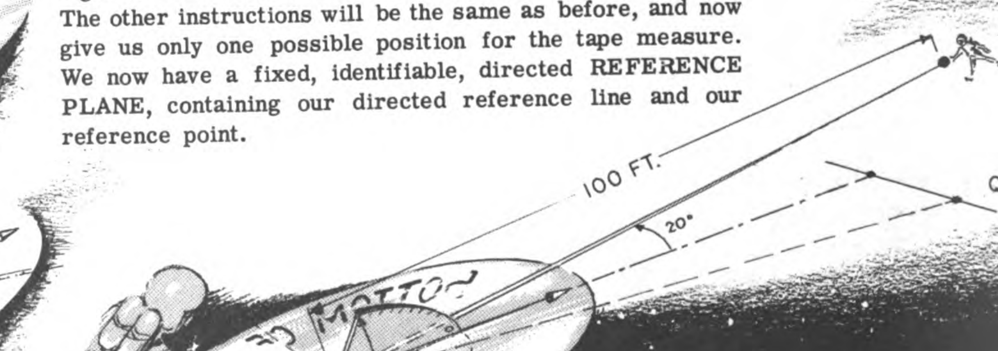
In the latter case, the protractor would look as shown below:



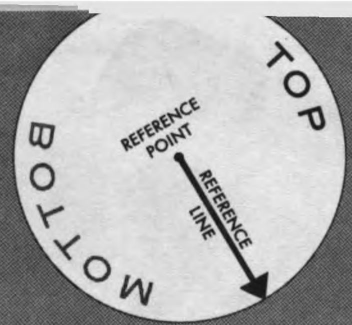
To avoid this ambiguity, A tells B that he can paint the words TOP and BOTTOM on the two respective sides of the plane.



From now on, instructions that he gives to B specify which side of the plane B is to measure on. His instructions now read: (1) Position the protractor with its center point at the reference point and its zero marker along the reference line direction, with the protractor on the top side of the plane, and lying to the right of the reference line. The other instructions will be the same as before, and now give us only one possible position for the tape measure. We now have a fixed, identifiable, directed REFERENCE PLANE, containing our directed reference line and our reference point.



We call this combination a REFERENCE FRAME. This frame can pinpoint any target. Precision locating must embrace those elements that make up a frame of this form or of some equivalent form.



SUMMING UP

COMPLETION OF THE REFERENCE FRAME

To sum up: We now have completed the construction of a REFERENCE FRAME, meaning a framework in which we can pinpoint a target. Without this frame, or its equivalent, we could never pinpoint any target. This particular type of frame consists of a directed plane, a directed line, and a point. We shall discuss some other types further on. Let us first examine some of the ways of using the point-line-plane type of reference frame.

Throughout this part of this section, and to some extent in the later part, we describe the position of a target by imagining that we are describing to someone how to get there from the reference point. Target data will take the form of INSTRUCTIONS. This is to make the position data easy to visualize. Actually, of course, nobody goes to the target; we acquire the necessary data — and try to hit the target. These instructions might take various forms, some being more convenient than others, depending upon the circumstances. Three examples are presented in detail in the following section, under the heading of COORDINATE TYPES AND CONVERSION.

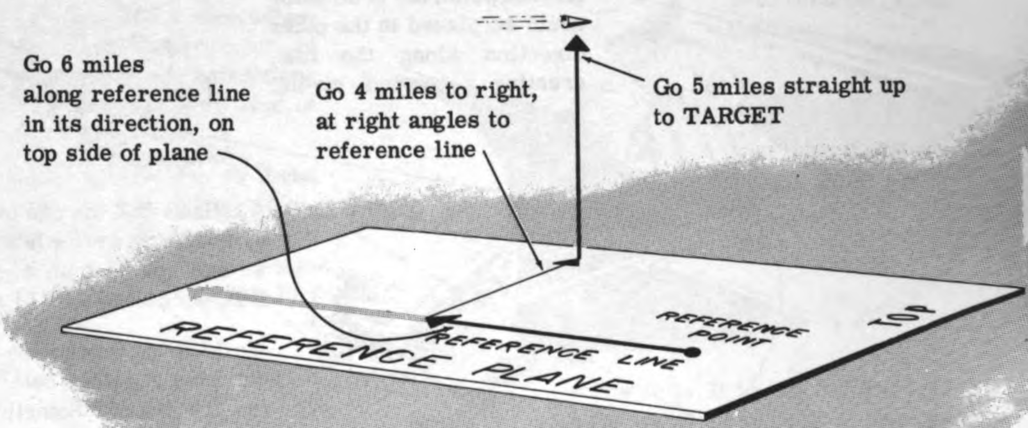
HOW INSTRUCTIONS

general

There are many ways we can use a reference frame to give instructions for reaching a target. On the preceding page we showed one rather special way, which is not much used in fire

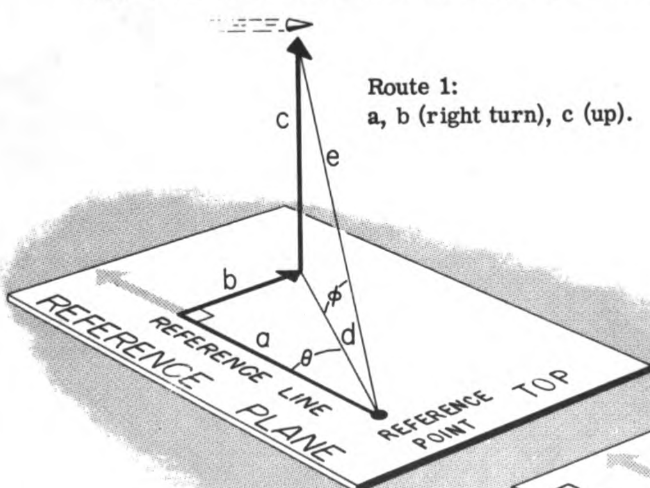
three types of instructions

INSTRUCTIONS GIVING THREE DISTANCES

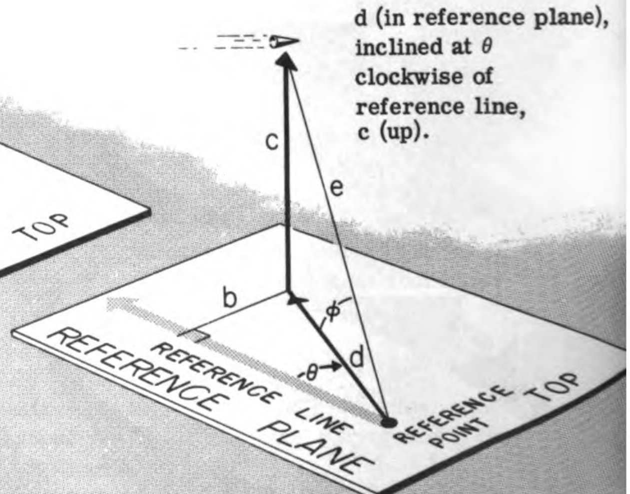


conversion

We have shown three different ways of instructing someone how to reach a target, starting from the reference point, and all in the same reference frame. For variety, we used a different target for each illustration. But we could equally well have used the same target. Here is an illustration of three routes to a target, similar to those in the three previous illustrations:



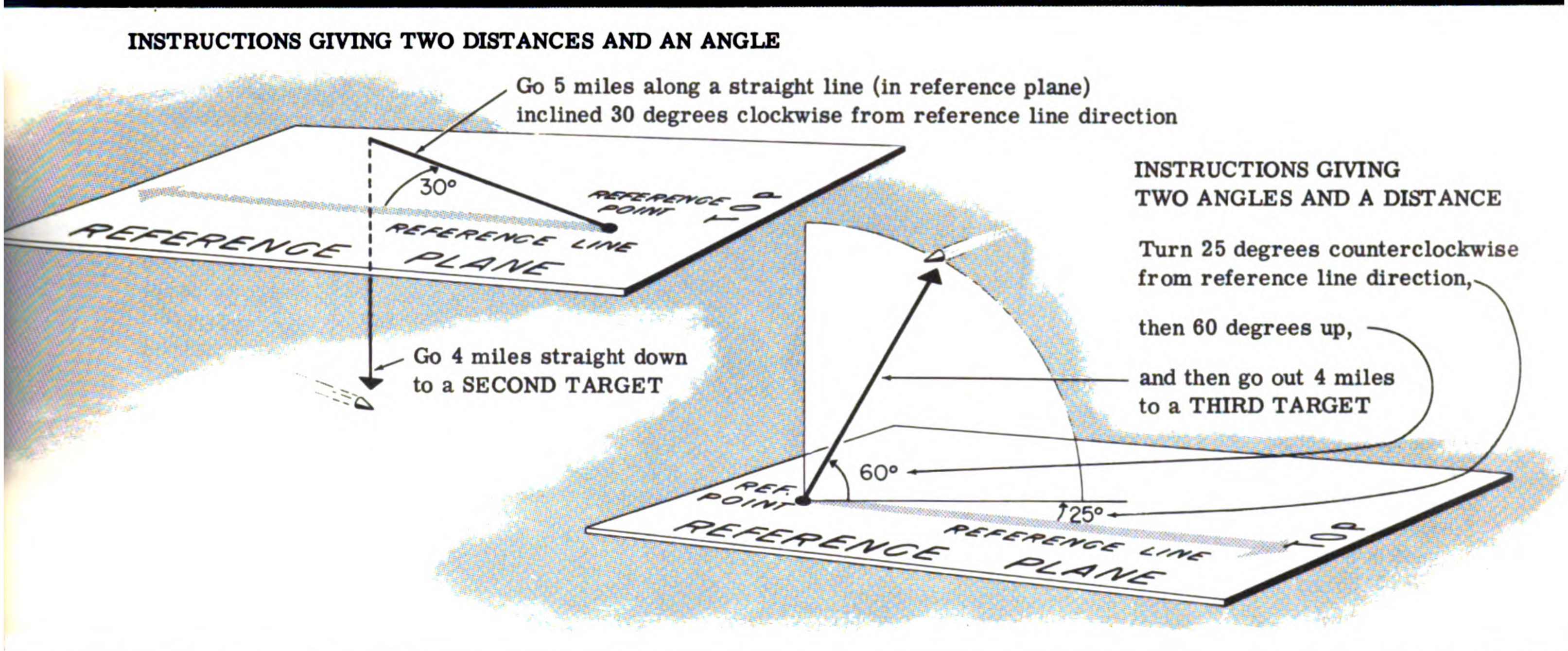
Route 1:
a, b (right turn), c (up).



Route 2:
d (in reference plane), inclined at θ clockwise of reference line, c (up).

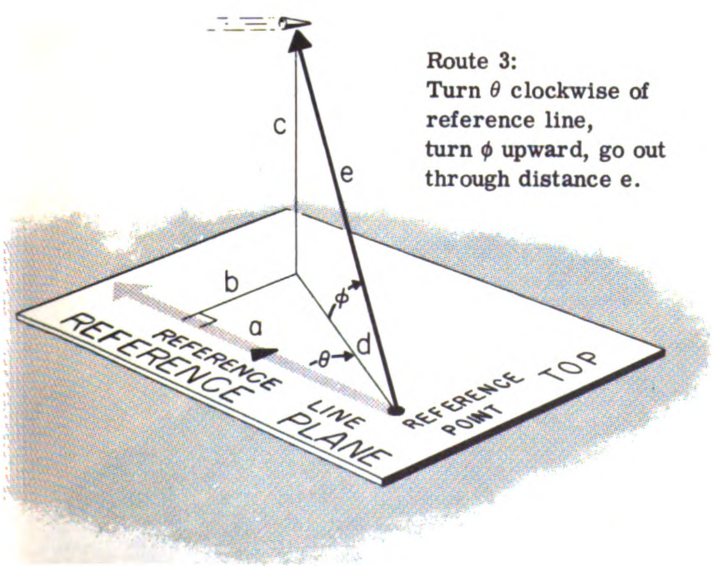
ARE GIVEN

control. This was done to bring the use of a protractor to its logical conclusion. Here are some more typical ways of giving instructions. One of them uses no protractor at all.



From these figures it is apparent that quantities a, b, c, d, e, θ, and ϕ are connected in simple mathematical relationships. For instance:

$$\tan \phi = c/d$$
$$a = d \cos \theta$$
$$d = e \cos \phi$$



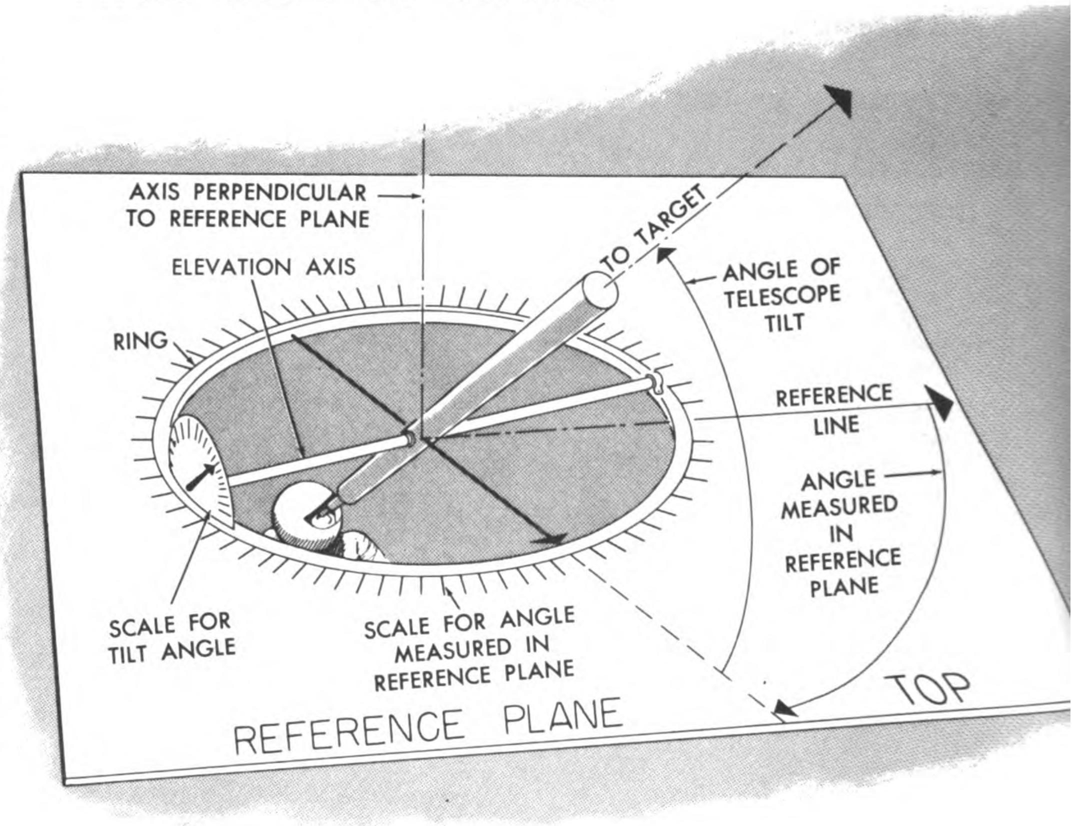
In other words, if we had the data for one set of instructions, we could derive the data for others. This may be necessary in practical problems, because the instructions that are the most convenient to use are not necessarily the most easily measured. The mathematical computations involved, called CONVERSION, are discussed in detail in the following section, under the heading of COORDINATE TYPES AND CONVERSION.

summary

We have shown — in outer space — that, in order to define the position of a target, we need to establish an identifiable reference point in an identifiable, directed reference line in an identifiable, directed reference plane. That is to say, we need this or some systems which are equivalent. Such systems are called reference frames. Precise and unambiguous instructions must be given, within the frame, in order to reach the target. We have shown three different ways of giving such instructions, using different targets, and the same target. When it is desired to change one set of instructions into another, a simple mathematical process, called conversion, is used.

ACTUAL MEASUREMENT AND USE OF DATA

We have explained how a frame may be constructed, and how the observer can locate a target if he is supplied with the necessary data. But we have not explained how this data is measured. Let us begin by approaching this problem in simplified terms. First, imagine that we have cut a circular hole in the reference plane large enough to accommodate a telescope mounted on two axes permitting rotation in bearing and elevation. These axes intersect at the center of the circle. The elevation axis is mounted on a ring which is free to turn about an axis (imaginary) perpendicular to the reference frame. Pointers indicate on scales the angle measured in the reference plane, and the angle at which the telescope is pointed up. The distance of the target is obtained by some ranging device (not shown).



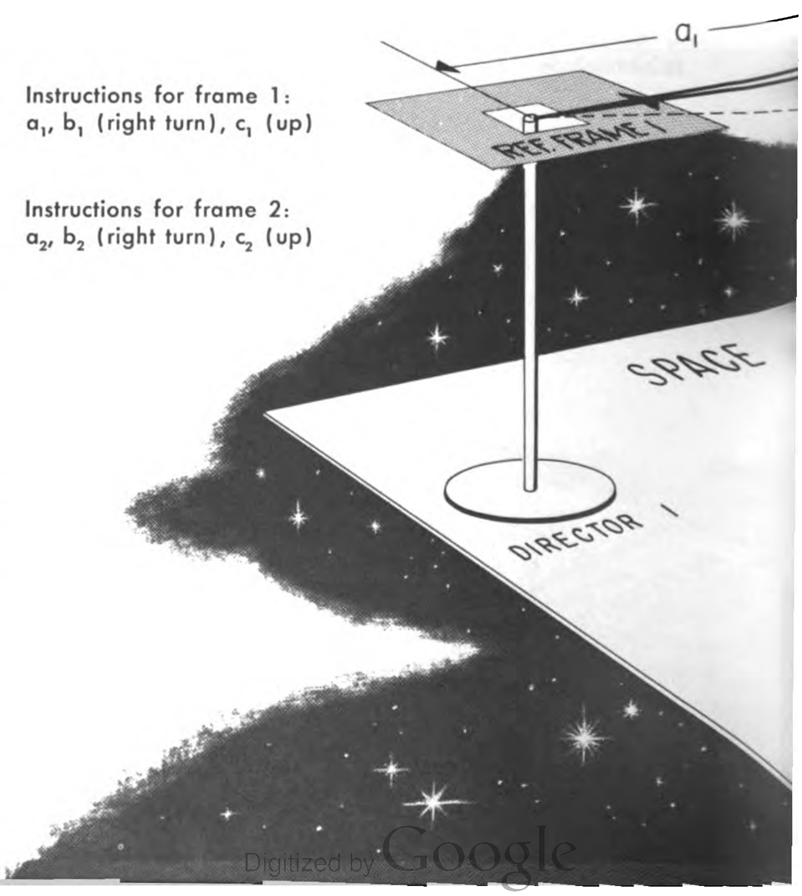
USE OF MORE THAN ONE FRAME

general

In fire control, it is often necessary to operate simultaneously with two or more reference frames. (For instance, these frames might be situated in different parts of a ship.) It may be desirable to measure target data with several directors, because of a need for flexibility in controlling different gun stations, or to obtain a wide range of view, or in case one director is out of commission. In such an event, we are faced with a problem: directors in different frames measure different target data, thereby providing different instructions.

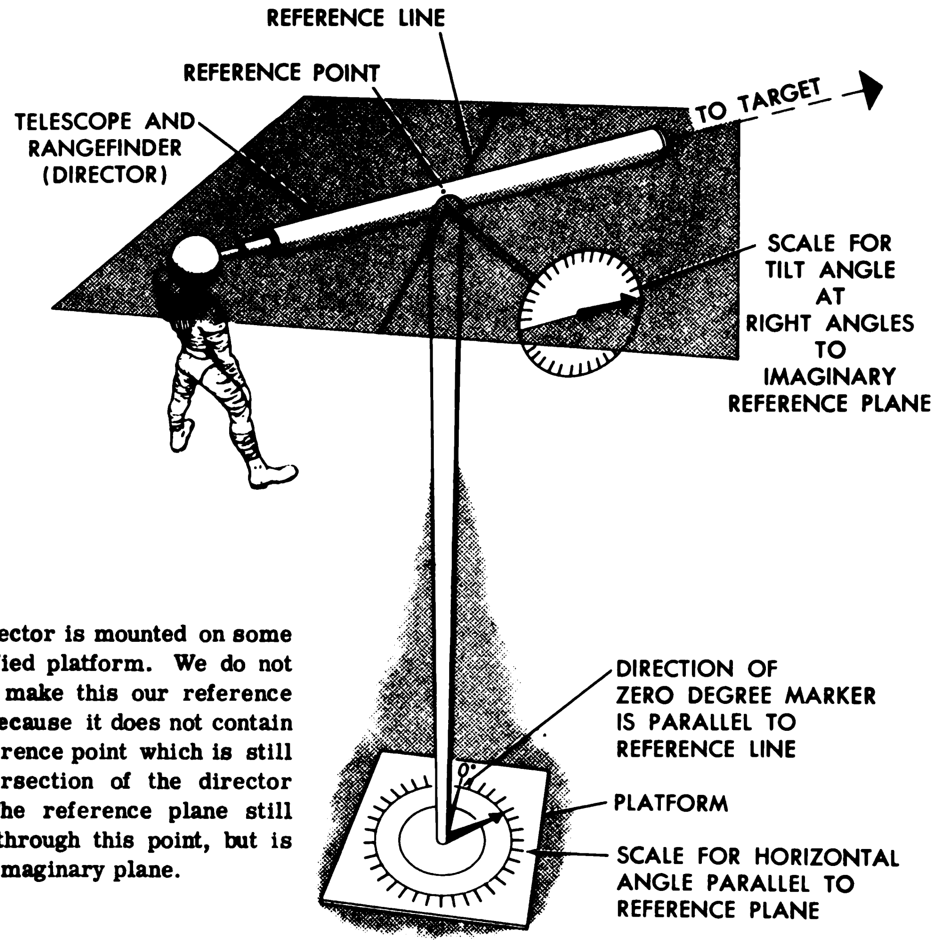
DIFFERENCE IN INSTRUCTIONS

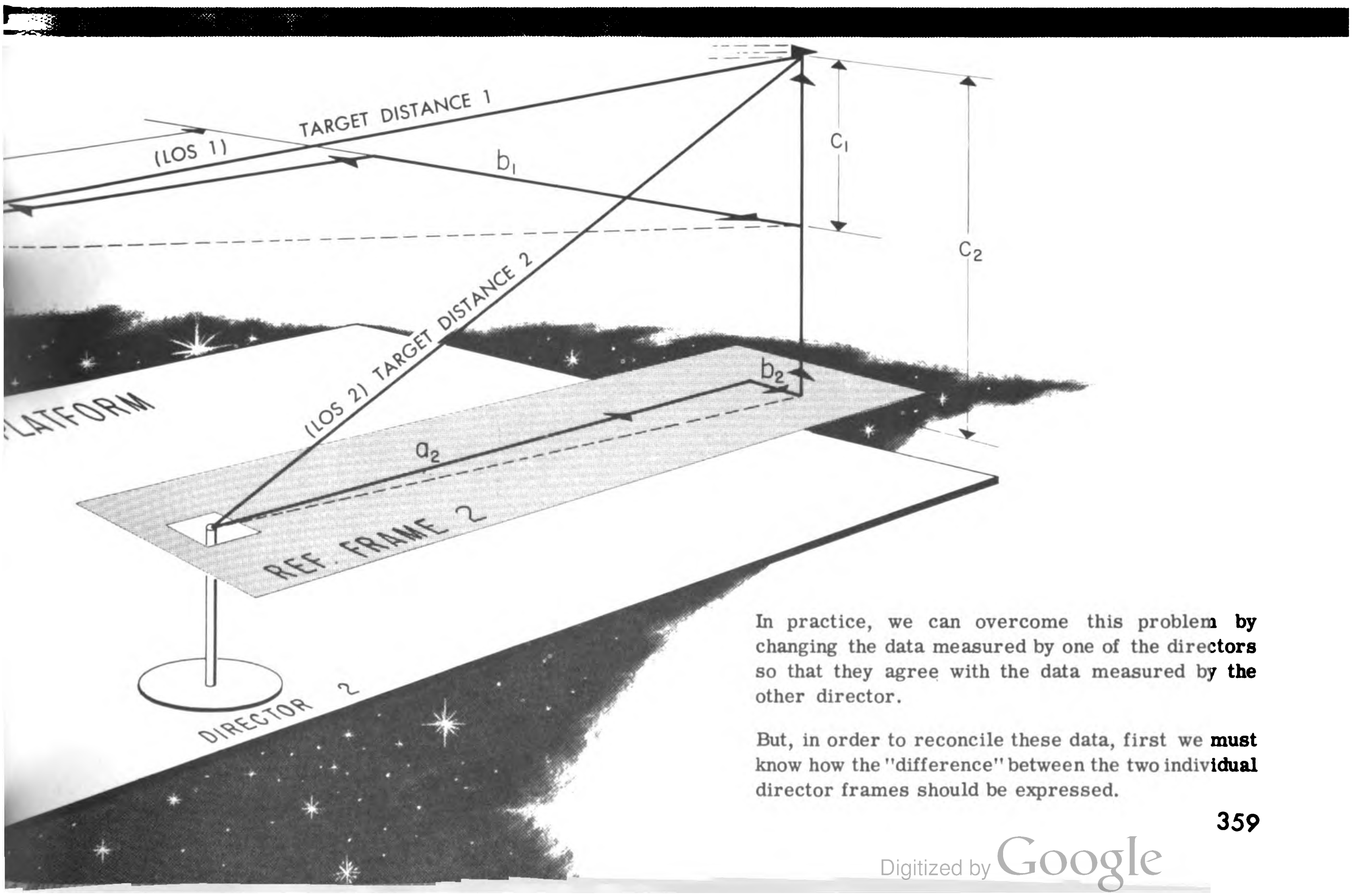
Imagine a space platform on which two directors are mounted and displaced horizontally and vertically from one another. Each one has its own reference frame where it makes its measurements of target data which provide the instructions. Instructions "a_1, b_1, c_1" for frame 1 are different from instructions "a_2, b_2, c_2" for frame 2. The instructions would still be different if they were given in another form; the two target distances and the two target bearings and elevation angles are different.



The combination of telescope and rangefinder is called a "director". For simplicity, we have assumed that the rangefinder is integrated with the telescope. In practice, it may be necessary to separate the two units; for instance, when radar is used. Ideally, the two units should be at the same reference point. However, as the distance between the units is so small, assuming that they are at the same point gives a close enough approximation. In previous examples, we have considered the reference plane as a solid object. Now that we have introduced the director, we need a realistic way of supporting it.

The director is mounted on some unspecified platform. We do not want to make this our reference plane, because it does not contain the reference point which is still the intersection of the director axes. The reference plane still passes through this point, but is now an imaginary plane.





In practice, we can overcome this problem by changing the data measured by one of the directors so that they agree with the data measured by the other director.

But, in order to reconcile these data, first we must know how the "difference" between the two individual director frames should be expressed.

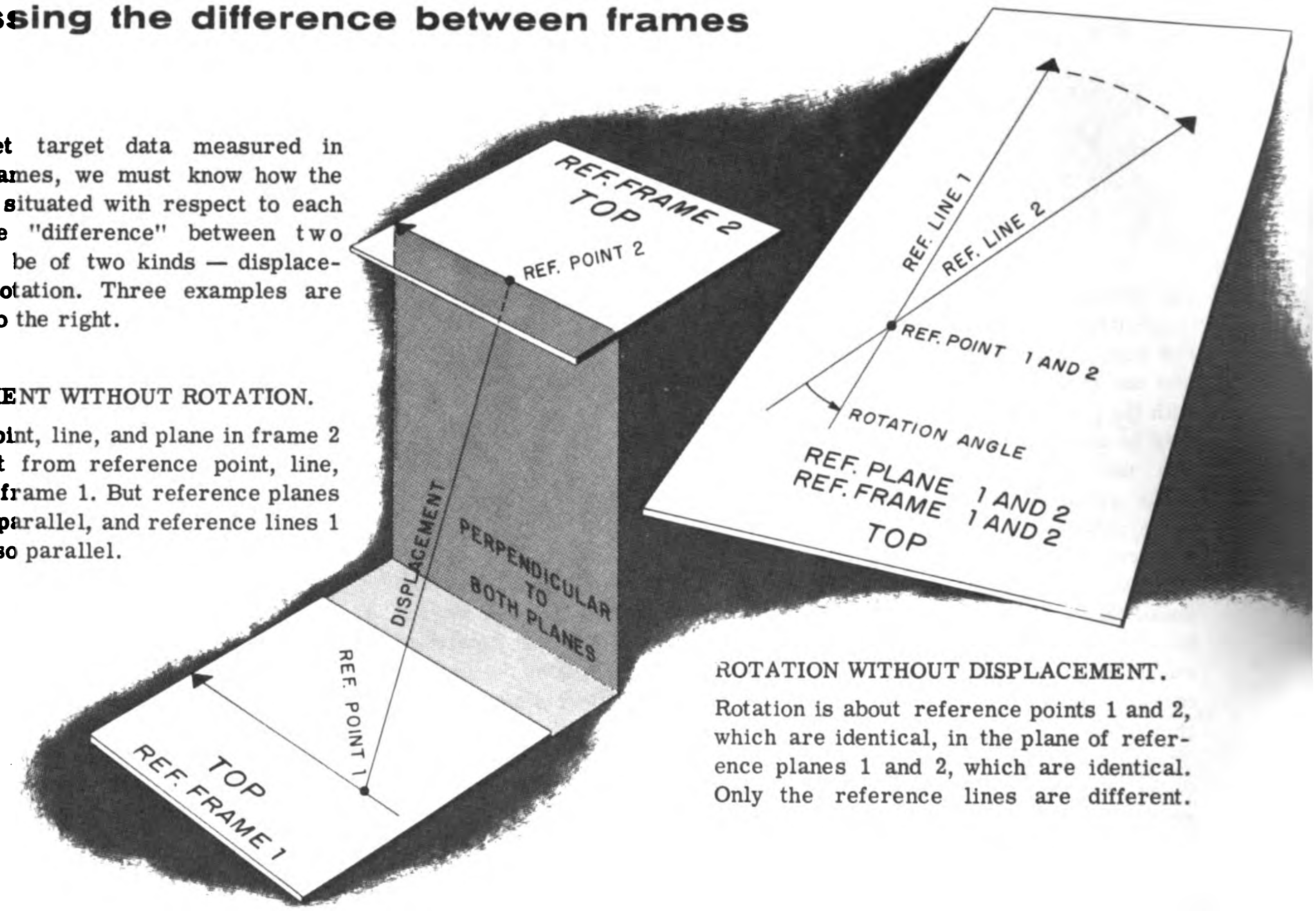
USE OF MORE THAN ONE FRAME

expressing the difference between frames

To interpret target data measured in different frames, we must know how the frames are situated with respect to each other. The "difference" between two frames may be of two kinds — displacement and rotation. Three examples are illustrated to the right.

DISPLACEMENT WITHOUT ROTATION.

Reference point, line, and plane in frame 2 are different from reference point, line, and plane in frame 1. But reference planes 1 and 2 are parallel, and reference lines 1 and 2 are also parallel.



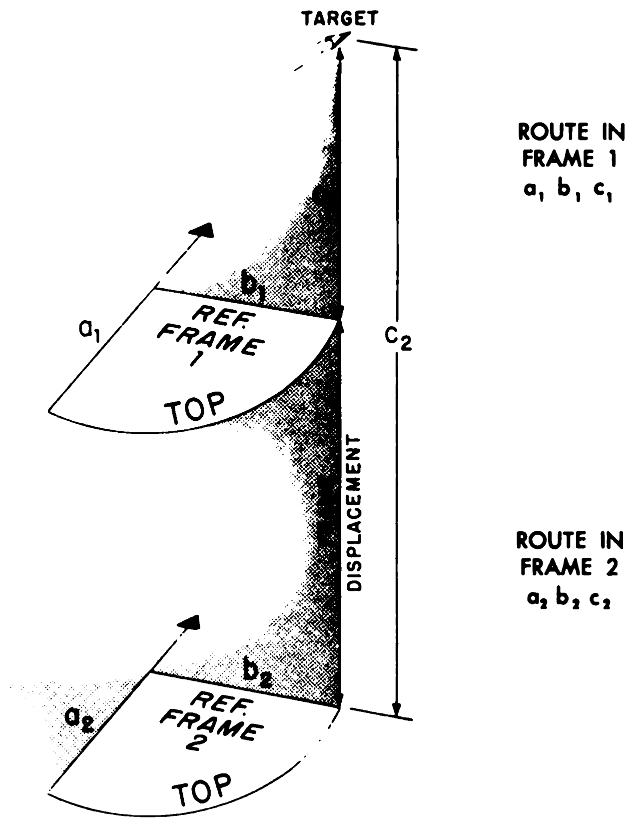
ROTATION WITHOUT DISPLACEMENT.

Rotation is about reference points 1 and 2, which are identical, in the plane of reference planes 1 and 2, which are identical. Only the reference lines are different.

transformation of instructions

In order to use in one frame data obtained in another frame, we must perform an operation called TRANSFORMATION. Consider the two simple examples shown to the right.

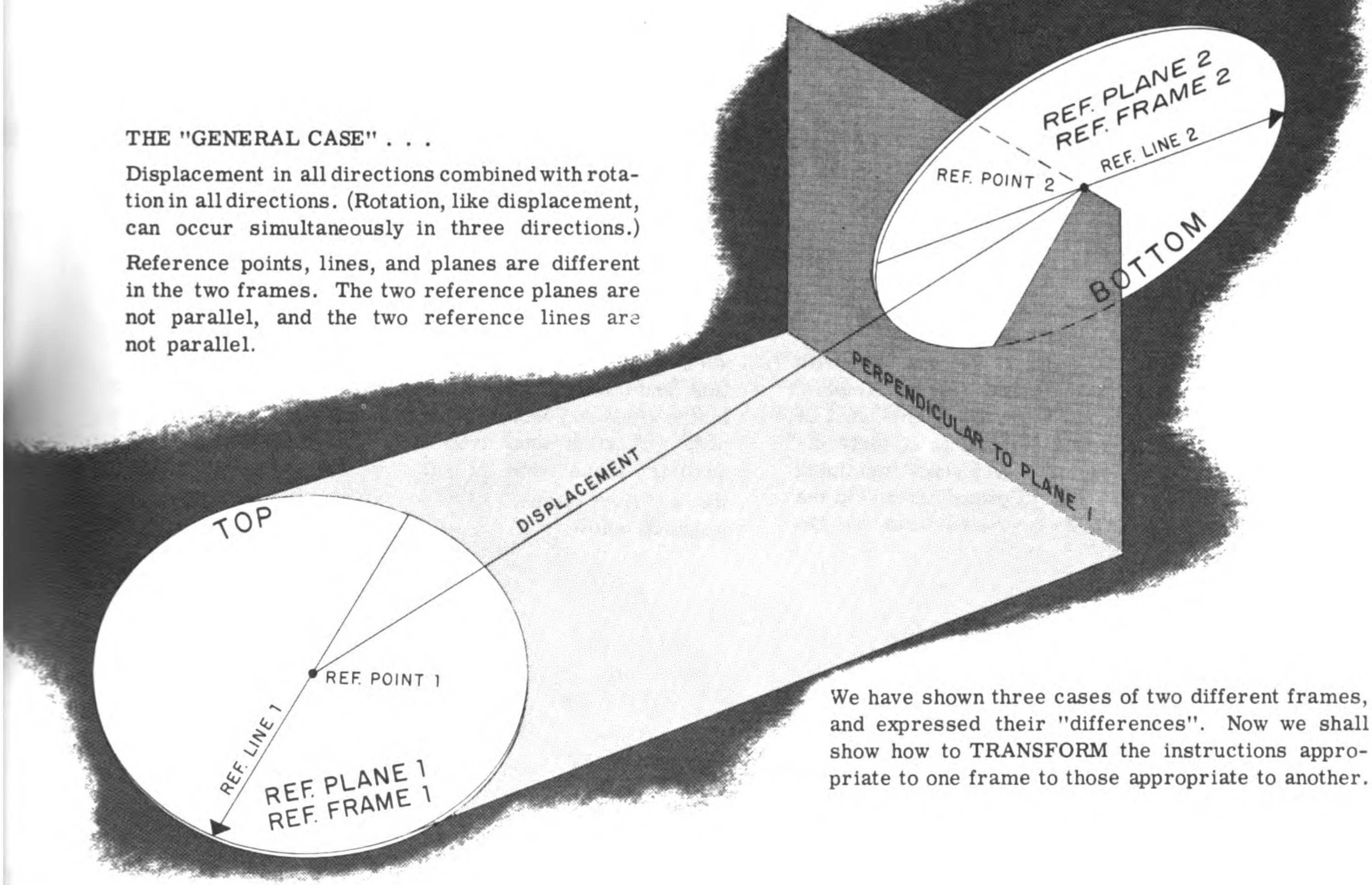
The reference planes are parallel, with one vertically above the other, and one reference point vertically above the other. This is an example of displacement in one direction without rotation. Here, only the "up" instructions differ (by the amount of frame displacement).



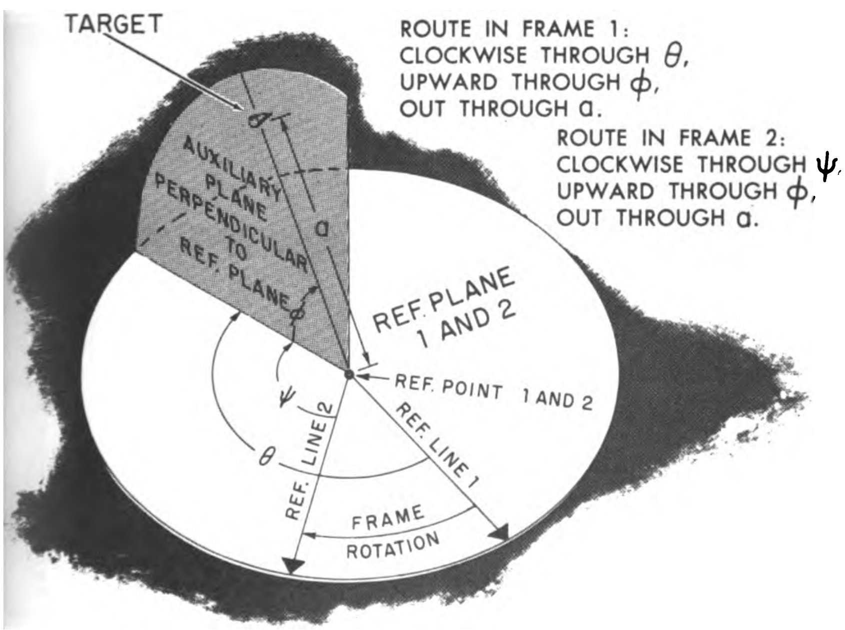
THE "GENERAL CASE" . . .

Displacement in all directions combined with rotation in all directions. (Rotation, like displacement, can occur simultaneously in three directions.)

Reference points, lines, and planes are different in the two frames. The two reference planes are not parallel, and the two reference lines are not parallel.



We have shown three cases of two different frames, and expressed their "differences". Now we shall show how to TRANSFORM the instructions appropriate to one frame to those appropriate to another.



The frames have the same reference plane and the same reference point, but the reference lines are rotated with respect to each other. This is an example of rotation in one direction without displacement. Here, only the in-plane instructions differ (by the amount of frame rotation).

In weapons control we often have on shipboard more than one director, each operating in its own frame, and obtaining its own target data. Our task is to reconcile these various data (which may differ considerably when the directors are widely separated in position or orientation). Consider two directors, obtaining different data for the same target. Our object is to TRANSFORM the data obtained by one director so that they agree with the data obtained by the other. We do this by applying corrections to one group of data. In two simple cases shown at left, the corrections are frame vertical displacement, and frame horizontal rotation, respectively. These are purely additive corrections. In more complicated cases, the target data from one director and the differences between that director frame and the other director frame are fed to a computer, which computes the new required data with reference to one of the directors.

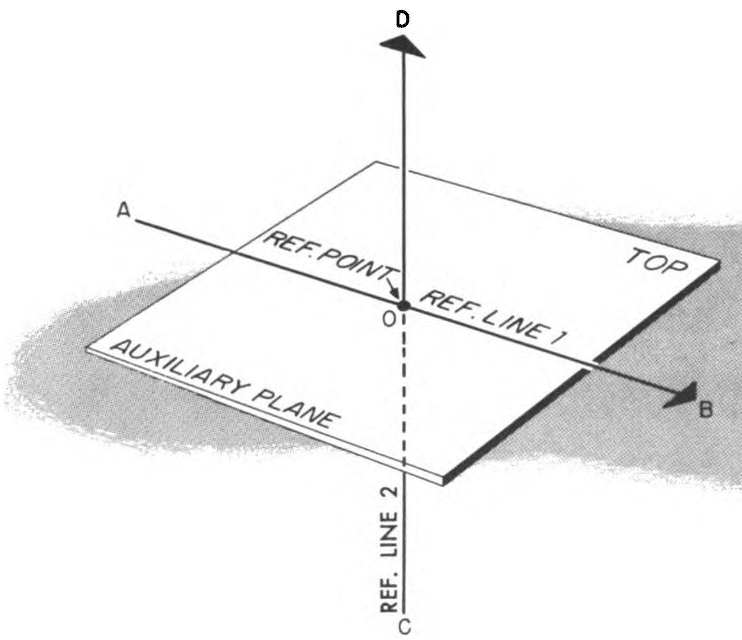
Linear displacement of one frame with respect to another is discussed further in another section, under the heading PARALLAX. Rotation of one frame with respect to another is discussed further in another section, under the heading ROTATION TRANSFORMATIONS.

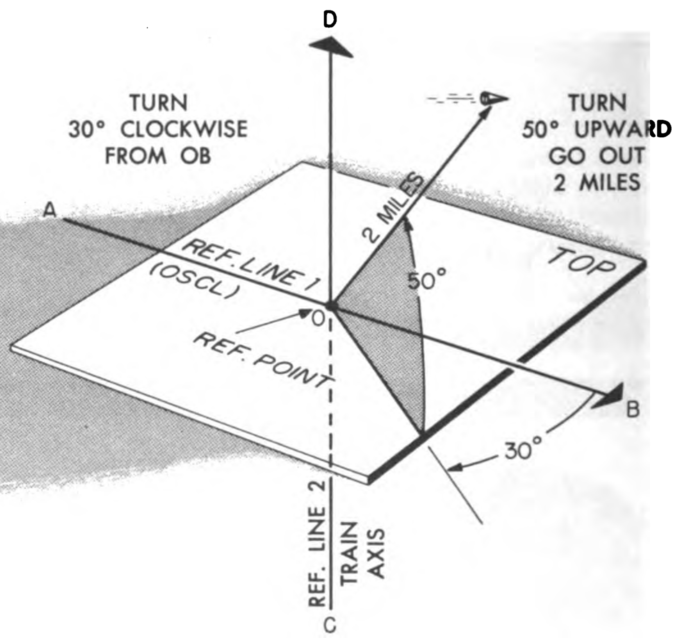
DIFFERENT KINDS OF FRAMES

two lines and a point

A reference frame can be constructed out of any two directed lines AB and CD and a point O, with suitable "sense" indications. A simple arrangement has both lines and the point in the same plane. In the simplest arrangement (shown below), the two lines intersect at right angles, and the reference point O is at their intersection. Use is made of an auxiliary plane containing AOB and perpendicular to COD. "Upward" means in the direction OD. "Clockwise" is measured from the OB direction on the top side of the auxiliary plane. (Sense could be defined in other ways.) A practical example of such a frame is one consisting of own ship's center line, and the train axis of a director.

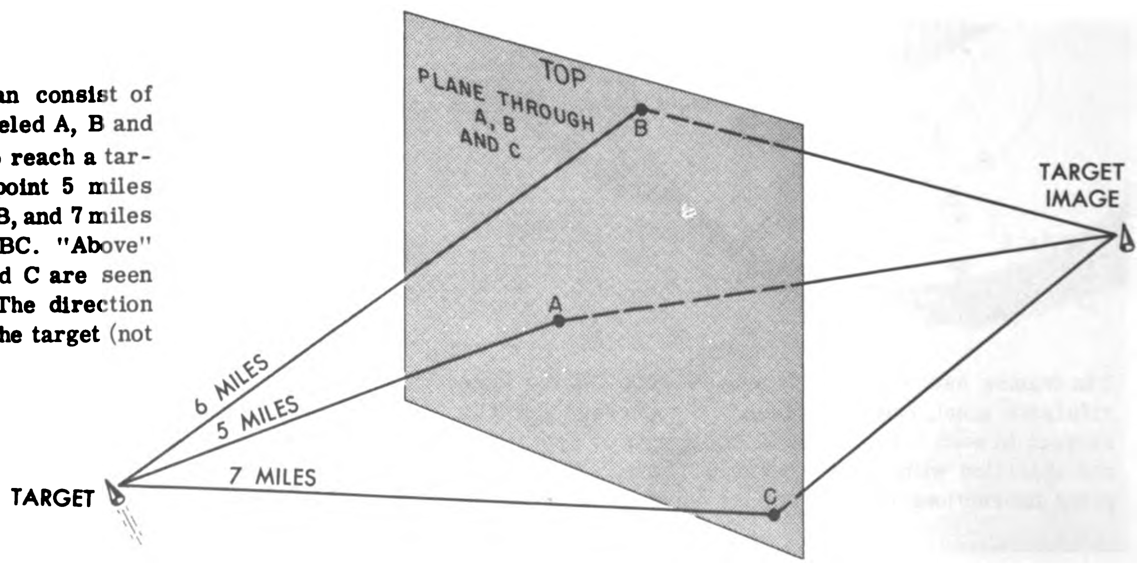
Instructions to reach the target from reference point O are: a rotation about train axis COD, measured in the auxiliary plane from the OB direction, elevation above the auxiliary plane, and target distance. An example is shown below.





three points

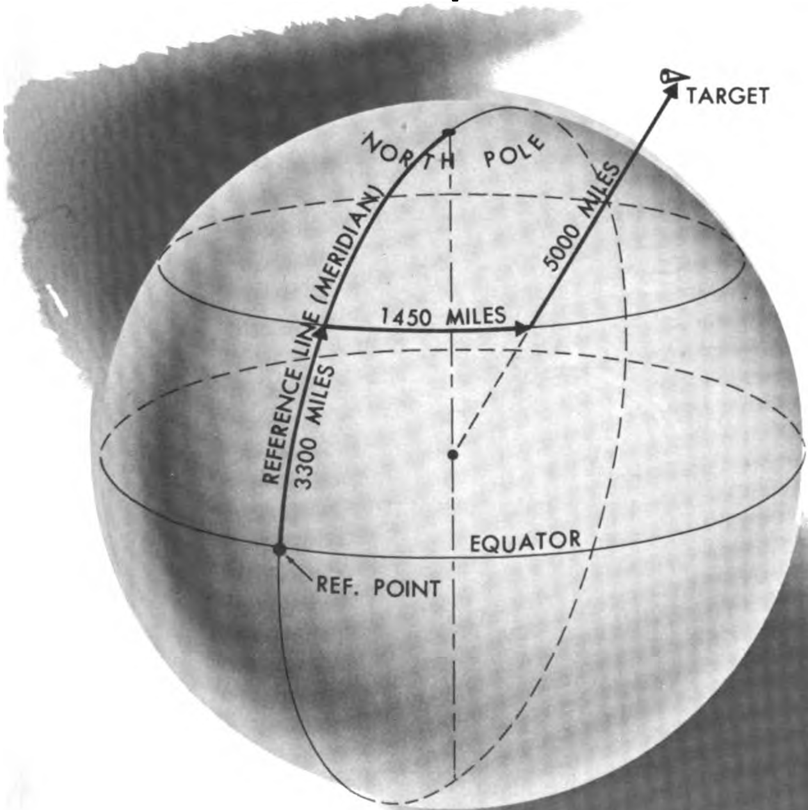
A reference frame can consist of three points, here labeled A, B and C. The instructions to reach a target could be: find a point 5 miles from A, 6 miles from B, and 7 miles from C, above plane ABC. "Above" means where A, B, and C are seen in clockwise order. The direction "above" ensures that the target (not its "image") is found.



We have treated reference frames as if they always consisted of a directed plane, a directed line, and a point. This frame is convenient to discuss because it lends itself to simple instructions, and is widely used, but there are other kinds to be considered as well.
We shall now discuss some of them.

curved references

The frames so far discussed use either a plane or a geometrical configuration which implies a plane — such as the two lines and a point, or the three points. But curved surfaces as well as planes can be used. Any curved surface, a line (straight or curved), and a point can form a reference frame. A practical instance is the use of Earth's surface as a REFERENCE SPHERE. Let us return from space for a moment and consider such a frame. We will take a fixed point on the equator for our reference point, and a meridian through this point for our reference line ("reference great circle"). Here are two ways to give instructions to reach a target:

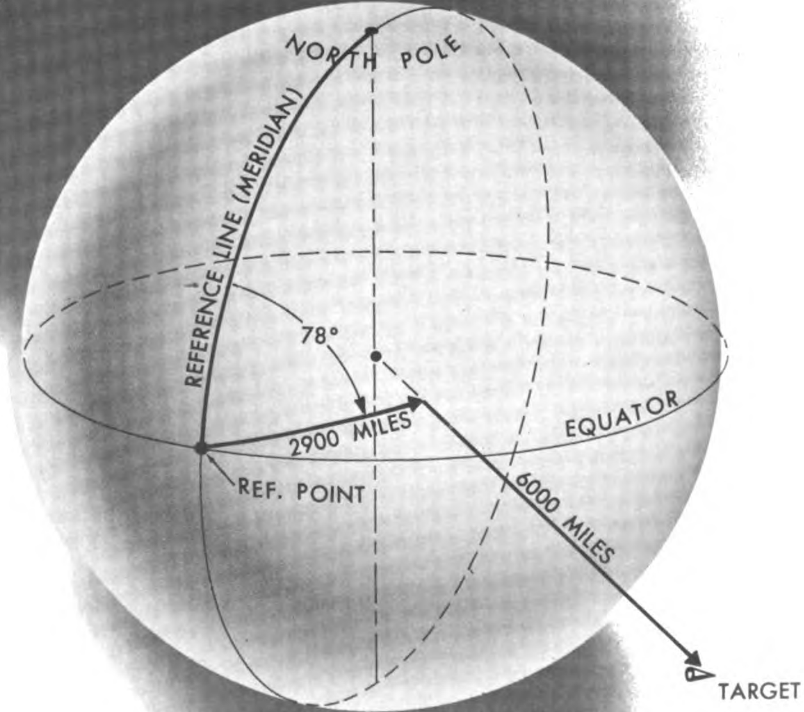


Go 3300 miles north along reference line.

Go 1450 miles due east.

Go 5000 miles straight up.

(First two laps of journey are on circles. Points of compass provide "sense".)



Go 2900 miles at constant bearing 78° degrees east of reference line.

Go 6000 miles straight up.

(First part of journey is a RHUMB LINE, actually an equiangular spiral. Nature of curve does not matter, because instructions are clear, and can be followed with a log and compass.)

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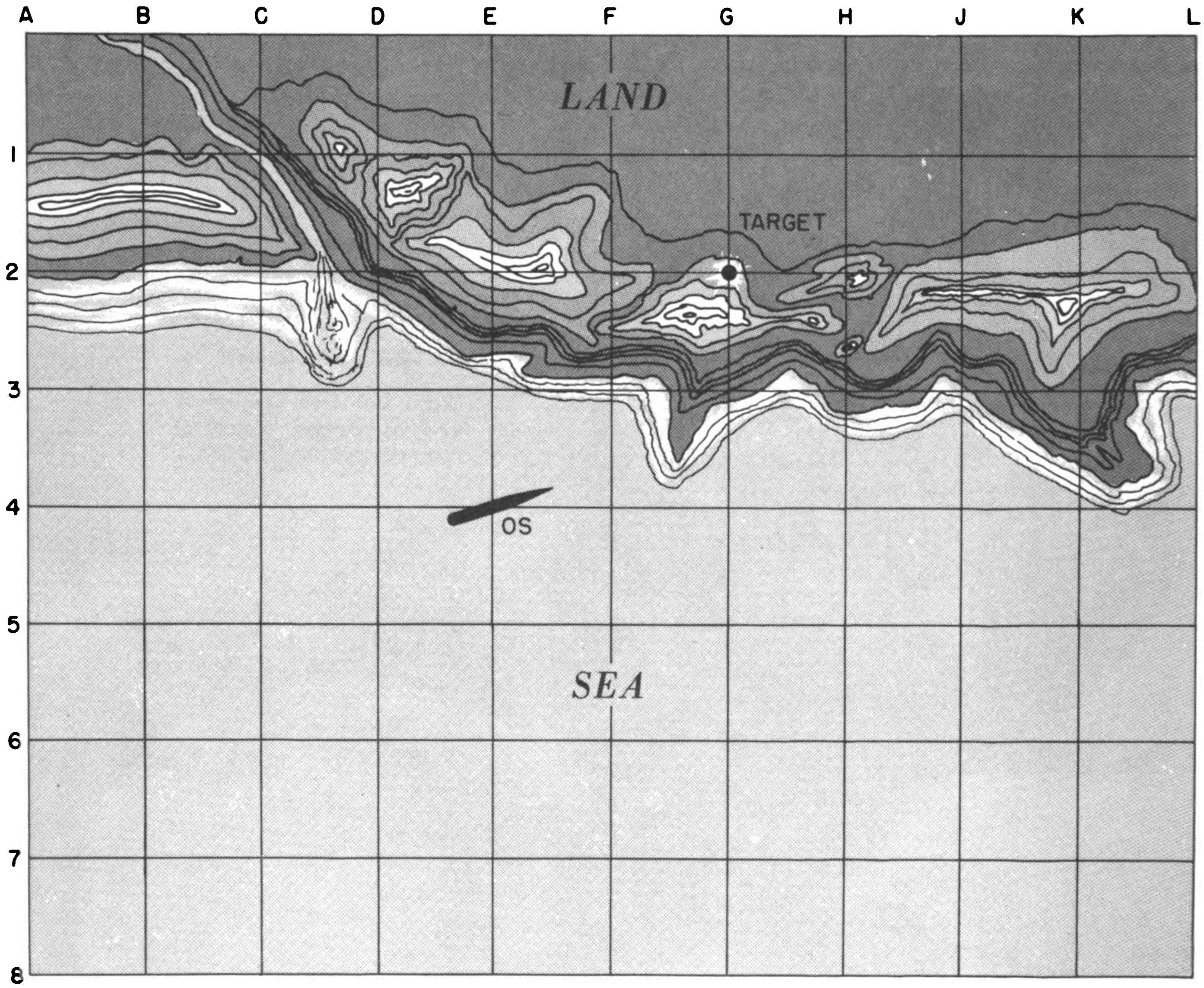
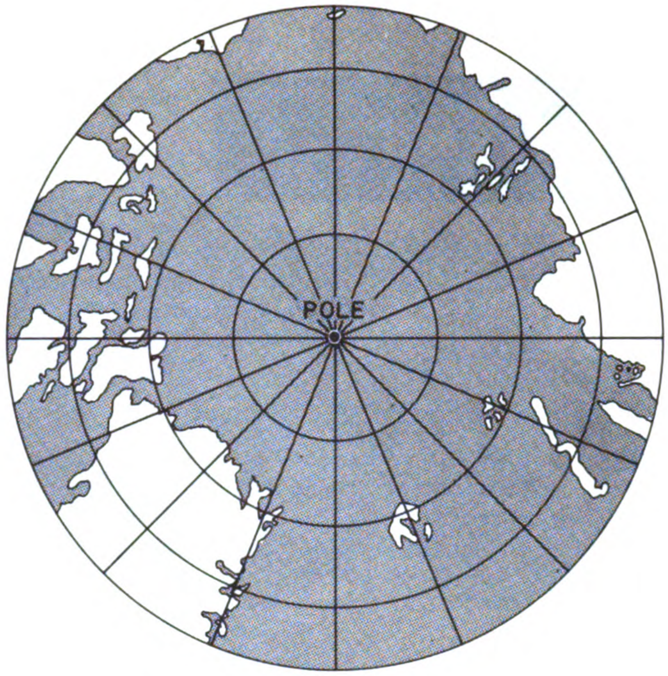
GRIDS

Another type of reference frame, often used when the target is on land, is the GRID. This type of frame is essentially a set of intersecting lines superimposed over a chart. The diagram shown below illustrates the position of a ship at sea close to a shoreline. Behind the shoreline, hidden by hills, a target is located.

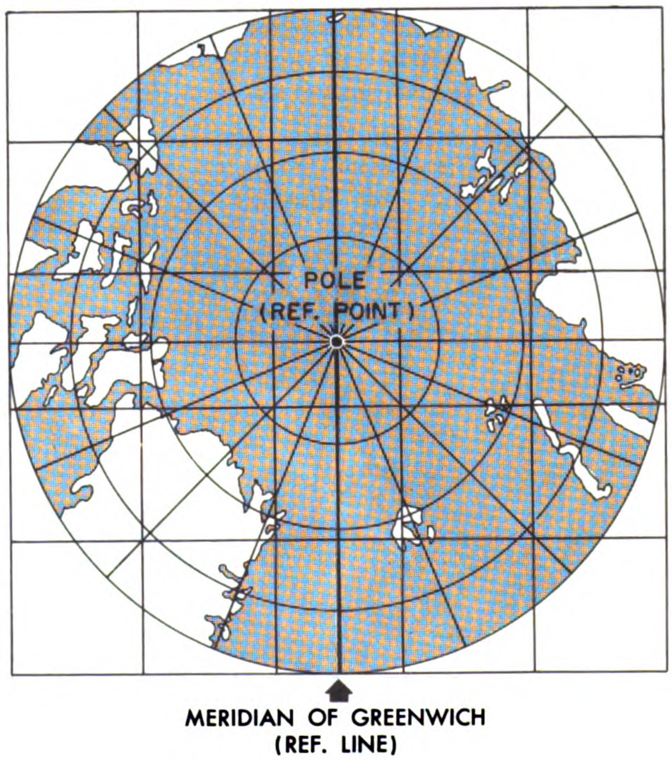


The ship's personnel ascertain the ship position on the grid; that is, which intersecting lines it is on. The ship position here is E4. The ship receives target position G2 from observers on land. These two positions (plus target altitude) are all that ship's personnel require in order to know the position of the target relative to the ship. The plane of the grid is the reference plane. The reference direction is established by means of the grid lines. In such a case, the location of the ship may be regarded as the reference point.

A grid need not necessarily consist of straight lines at right angles. Meridians and parallels of latitude form a grid that is largely and familiarly used. In the neighborhood of the equator, this grid is approximately rectangular, but, as we leave the equator, the meridians become less and less parallel, and the parallels of latitude become more curved. In arctic and antarctic regions, the latitude-longitude grid looks like this:



Such a grid as the one shown above — consisting of radial lines and concentric circles — is not the most helpful for defining the positions of a ship and its target in polar regions. This is because it gives these positions as latitude and longitude, involving the points of the compass, which are difficult to ascertain near a pole. In polar regions it has been found practical to superimpose on the polar map a rectangular grid such as this:



summary

Starting with the question "what does WHERE mean?", we have built up the reference frame as an absolute necessity for stating any position. This frame may take various forms; we have concentrated on a convenient form consisting of a reference point, a directed reference line, and a directed reference plane. Using such a frame, we have given various types of instructions on how to reach a target, starting at the reference point. These types of instructions, and how to change from one type to another, are discussed in the next section under COORDINATE TYPES AND CONVERSION. We have also shown some methods of measuring target data. Changing from one frame to another has also been discussed, a subject treated elswhere under PARALLAX AND ROTATION TRANSFORMATIONS. Finally, we have shown some examples of frames other than the plane-line-point.

PROBLEMS

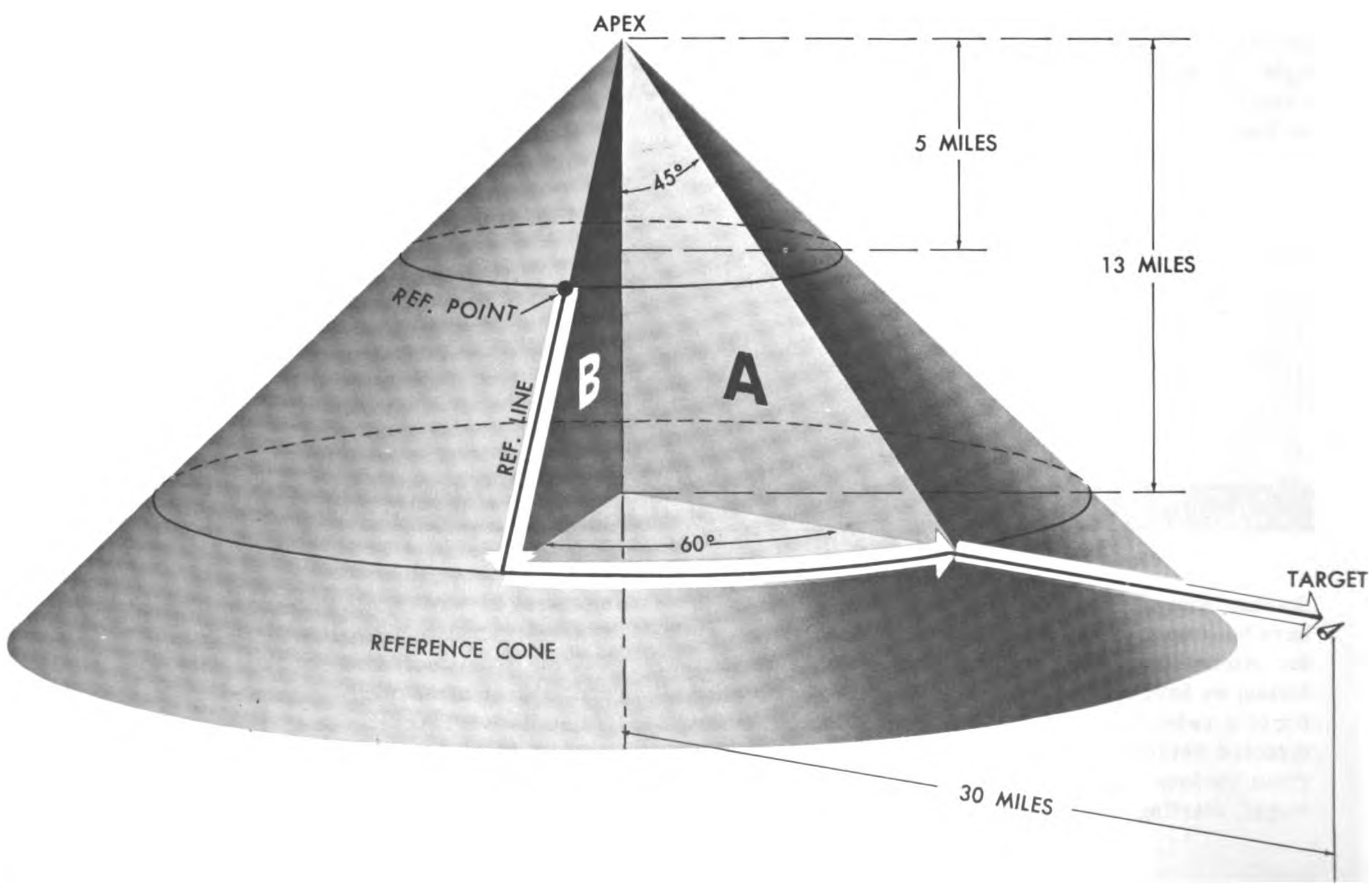
The following problems are exercises in the use, in the abstract, of arbitrary reference frames. Actually, the frames have no practical use in weapons control.

1. We are provided with a reference point and a reference line (through the point) which has no arrow to indicate sense. We are informed only that a target is 5 miles from the reference point, and 3 miles from the reference line. Describe the possible positions of the target, and also illustrate the positions by means of one or two diagrams. How far from the reference point is the projection of the target on the reference line?

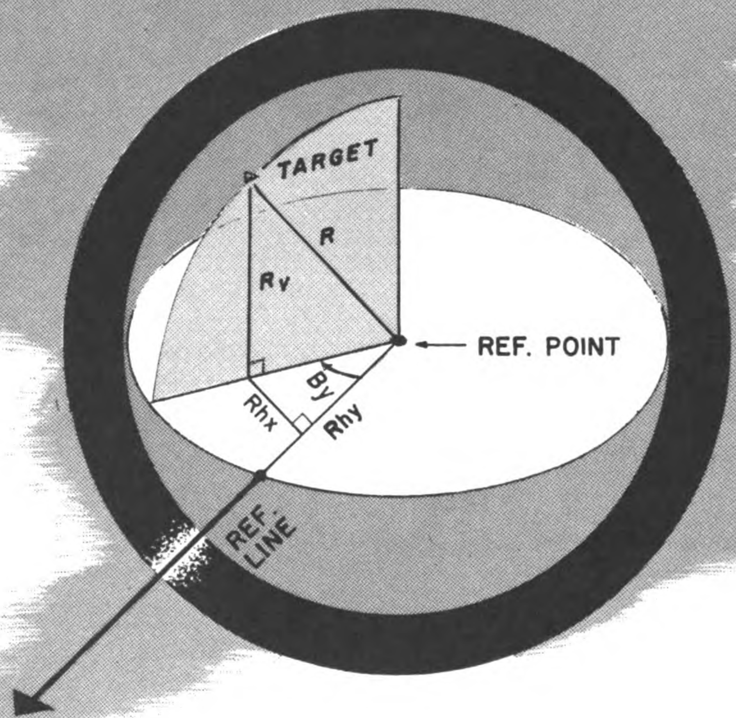
2. A reference frame consists of the following: A vertical reference cone with a 45-degree half-angle. A reference point located as shown on the surface of the cone, 5 miles below the apex (measured vertically). A reference line through the apex and the reference point.

   A target is 13 miles below the apex of the cone (measured vertically), 30 miles from the cone's central axis. The target is in vertical plane A through the central axis. Plane A is inclined 60 degrees counterclockwise (looking down) from vertical plane B through the reference line and central axis.

   Give instructions (in miles) on how to reach the target from the reference point along the path shown. The answer may contain π and radicals, not worked out, but no trigonometric ratios.



COORDINATE TYPES AND CONVERSION



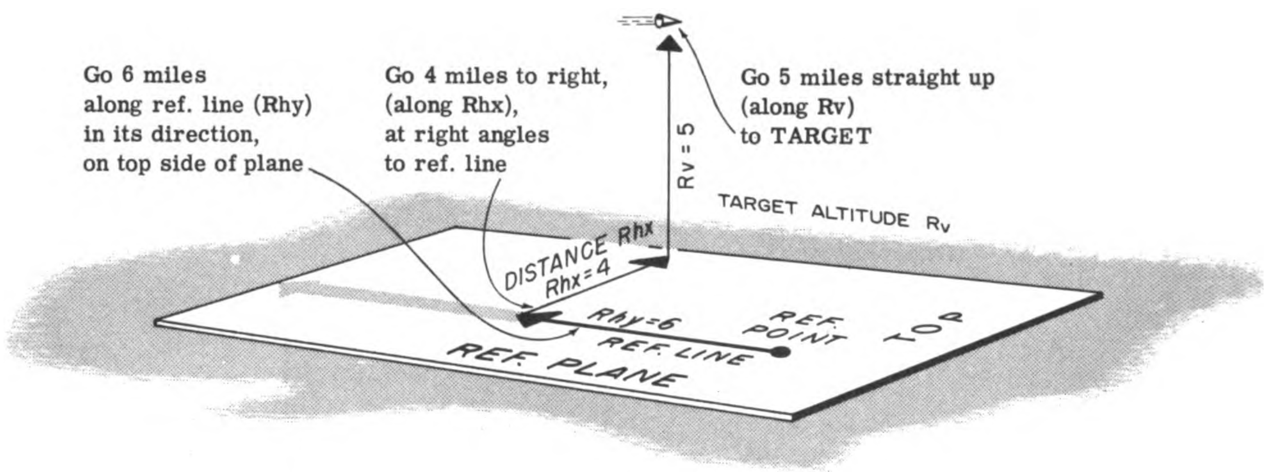
In the previous section we discussed reference frames. In particular we concentrated on the type of frame consisting of a reference point in a directed reference line in a directed reference plane, and showed three different ways of giving instructions on how to reach a target, starting at the reference point, in such a frame. These sets of instructions are only a few among many others that we shall review on the next page. The reader has encountered such instructions in books on analytical geometry, but expressed in different language. There they are called the COORDINATES of a POINT with reference to an ORIGIN and given AXES. In weapons control, the point is the target. The origin is the reference point. The axes consist of our reference line, and two other lines which we shall generally ignore in this section. We have shown that it is possible to give different sets of instructions to reach a target, using the same frame. In the language of analytical geometry we should say that it is possible to use different COORDINATE TYPES in the same frame in order to describe the position of a target.

scope of section

In the first part of this section we show some, of the different COORDINATE TYPES used in the same point-line-plane reference frame. In the second part we consider the relative advantages and disadvantages of different coordinate types, and show how, when we are given target coordinates of one type, we can compute coordinates of another type. This is called COORDINATE CONVERSION.

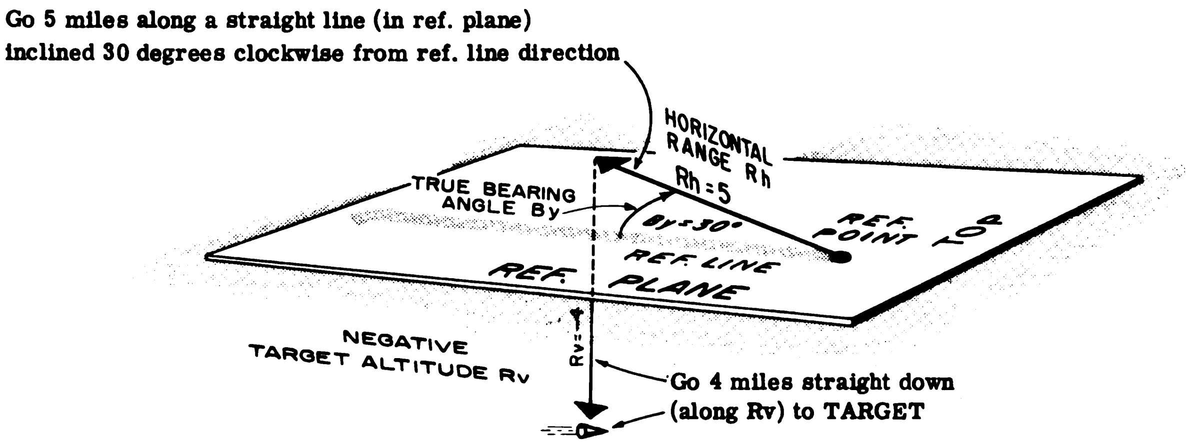
COORDINATE TYPES

rectangular coordinates



Rhy, Rhx, and Rv are the RECTANGULAR COORDINATES of the target. They are positive when along and to the right of ref. line direction, and upward. (On Earth, ref. line direction would be northward.)

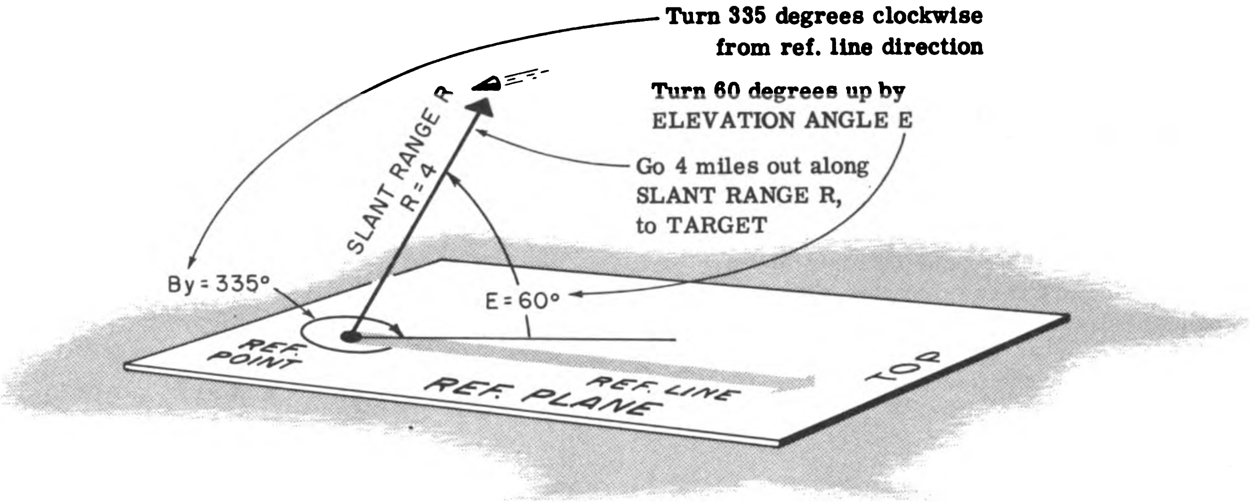
cylindrical coordinates



Rh, By, and Rv are the CYLINDRICAL COORDINATES of the target. Rh and By are always positive. By is always measured clockwise. (On Earth, ref. line direction would be northward.)

Let us consider some types of coordinate systems that can be used with the same point-line-plane reference frame. In the preceding section on reference frames, we showed three ways of instructing someone how to reach a target. We shall now repeat these instructions, this time expressing them in language of COORDINATES; each set of instructions represents a different coordinate type. We shall also briefly refer to another type that can be used in the same frame.

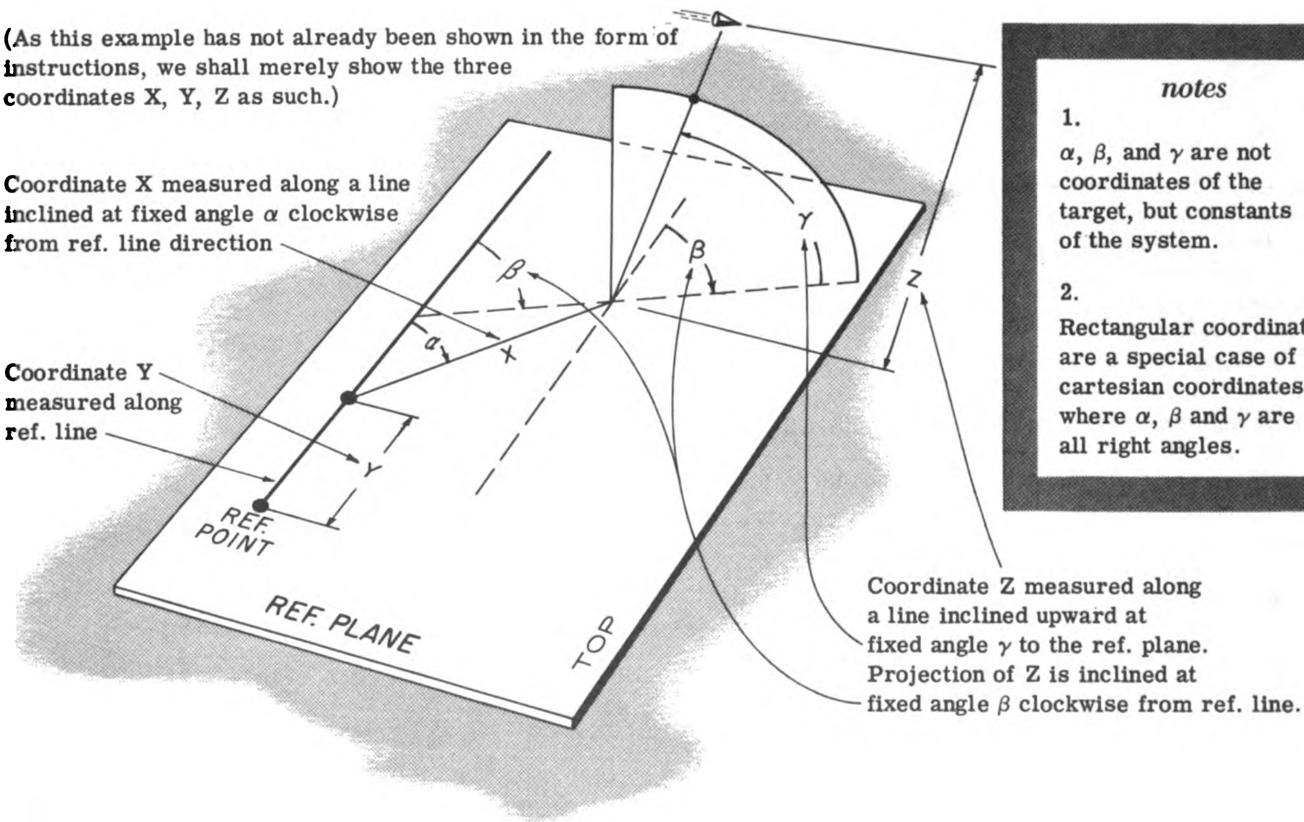
spherical coordinates



R, E, and By are the SPHERICAL COORDINATES of the target. E is positive when upward; negative when downward. R and By are always positive; By is always measured clockwise. (On Earth, ref. line direction would be northward.)

oblique cartesian coordinates

(As this example has not already been shown in the form of instructions, we shall merely show the three coordinates X, Y, Z as such.)



notes

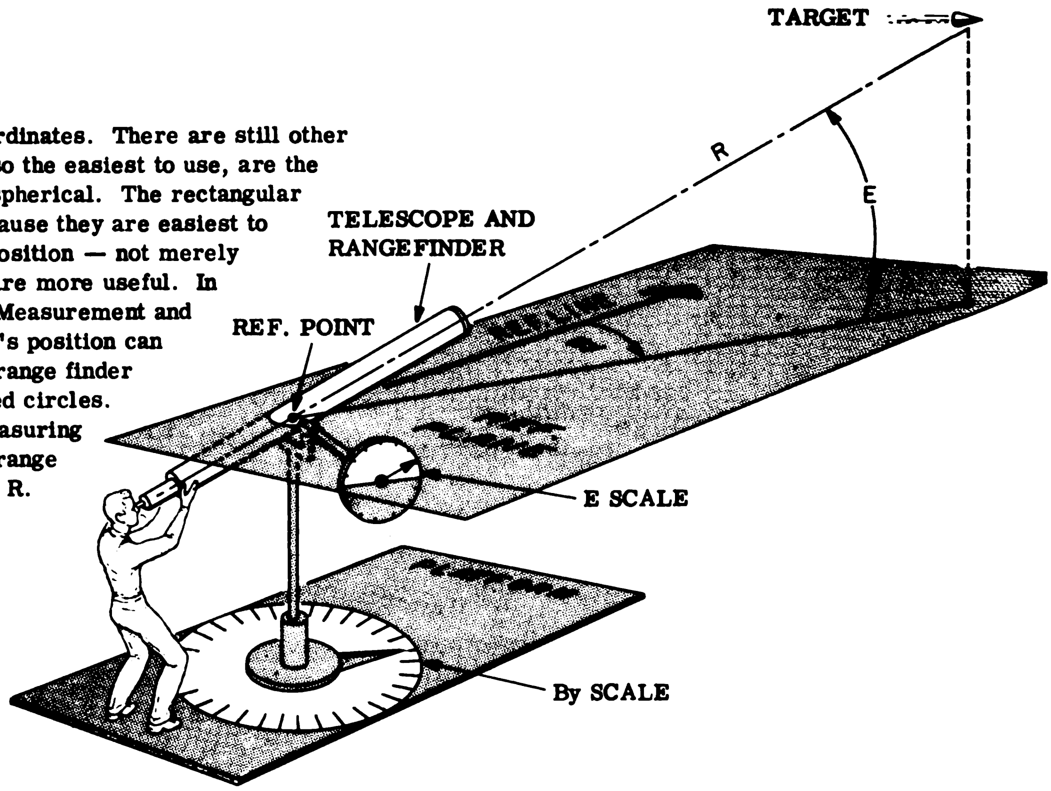
1. α, β, and γ are not coordinates of the target, but constants of the system.

2. Rectangular coordinates are a special case of cartesian coordinates where α, β and γ are all right angles.

SELECTION AND CONVERSION

measuring spherical coordinates

We have discussed some types of coordinates. There are still other types. The three most often used, also the easiest to use, are the rectangular, the cylindrical, and the spherical. The rectangular coordinates were described first, because they are easiest to explain. But, to MEASURE a target position — not merely describe it — spherical coordinates are more useful. In the preceding section, under "Actual Measurement and Use of Data", we showed how a target's position can be measured using a telescope and a range finder mounted on two axes with two graduated circles. The graduated circles are used in measuring elevation E and true bearing By; the range finder is used to measure slant range R.

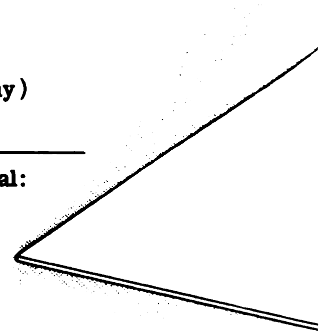


coordinate

This method of measuring rectangular coordinates is much more complicated than the above method of measuring spherical coordinates, and is actually not practical (especially in the case of a moving target!)
An indirect and easy way to obtain the target rectangular coordinates is first to obtain the target spherical coordinates by direct measurement at the reference point; then, change them to rectangular coordinates by coordinate conversion. If one knows the target coordinates of any type, it is easy to convert them to the target coordinates of any other type in the same reference frame.

COORDINATE CONVERSION EQUATIONS

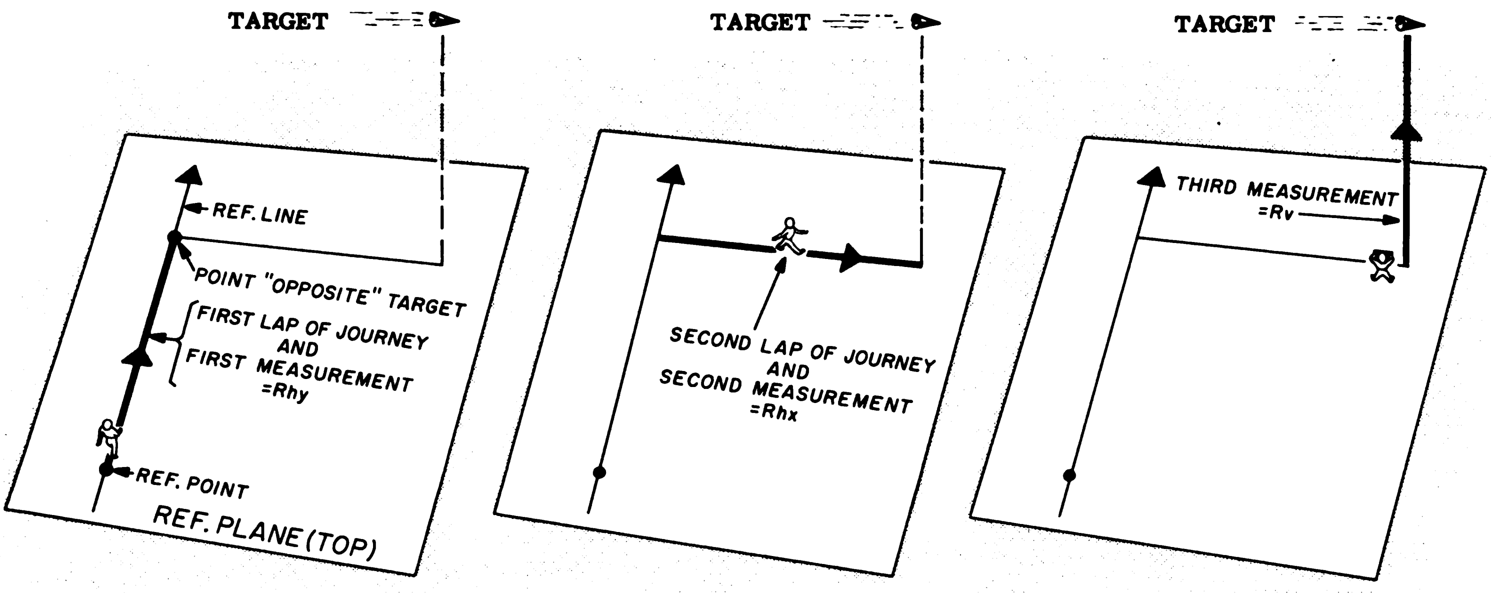
Cylindrical to Rectangular: $Rhx = Rh \sin By$ $Rhy = Rh \cos By$ $Rv = Rv$	Rectangular to Cylindrical: $Rh = \sqrt{Rhx^2 + Rhy^2}$ $By = \tan^{-1}(Rhx/Rhy)$ $Rv = Rv$
Spherical to Cylindrical: $Rh = R \cos E$ $Rv = R \sin E$ $By = By$	Cylindrical to Spherical: $R = \sqrt{Rh^2 + Rv^2}$ $E = \tan^{-1}(Rv/Rh)$ $By = By$
Spherical to Rectangular: $Rhx = R \cos E \sin By$ $Rhy = R \cos E \cos By$ $Rv = R \sin E$	Rectangular to Spherical: $R = \sqrt{Rhx^2 + Rhy^2 + Rv^2}$ $By = \tan^{-1}(Rhx/Rhy)$ $E = \tan^{-1}(Rv/\sqrt{Rhx^2 + Rhy^2})$



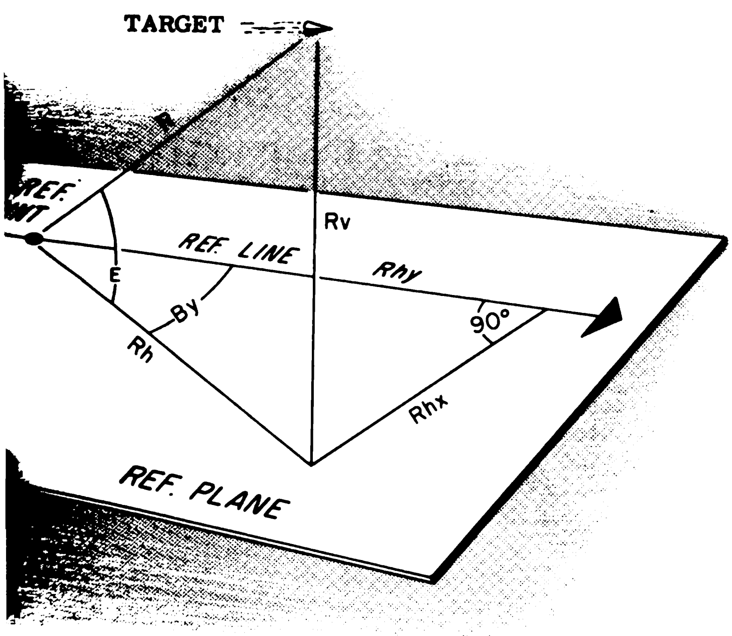
OF COORDINATE TYPES

measuring rectangular coordinates

But suppose we wanted (for some reason) to measure the rectangular coordinates of a target. A direct and hard way is to move along the reference line until one is "opposite" the target (this distance is Rhy); next, move at right angles to the reference line until one is directly under the target (this distance is Rhx); then, measure Rv with a range finder.



conversion



coordinate selection

Now let us consider why some types of coordinates can be more suitable for use than others, depending upon the purpose for which they are required.

Suppose we desire to express the fact that target altitude equals, say, 3 miles. In rectangular coordinates this would be expressed as:

Rv = 3, involving only one coordinate.

In spherical coordinates this would be expressed as:

R sin E = 3, involving two coordinates.

Here, rectangular coordinates are more advantageous. On the other hand, we might desire to express the fact that target distance equals, say, 5 miles. In spherical coordinates this would be expressed as:

R = 5, involving only one coordinate.

In rectangular coodinates this would be expressed as:

$\sqrt{Rhx^2 + Rhy^2 + Rv^2} = 5$, involving all three coordinates.

Here, spherical coordinates are more advantageous. The comparative advantages of different types of coordinates for easy measurement and for use, will be demonstrated more fully when we consider changing the coordinates from one frame to another. This is called COORDINATE TRANSFORMATION.

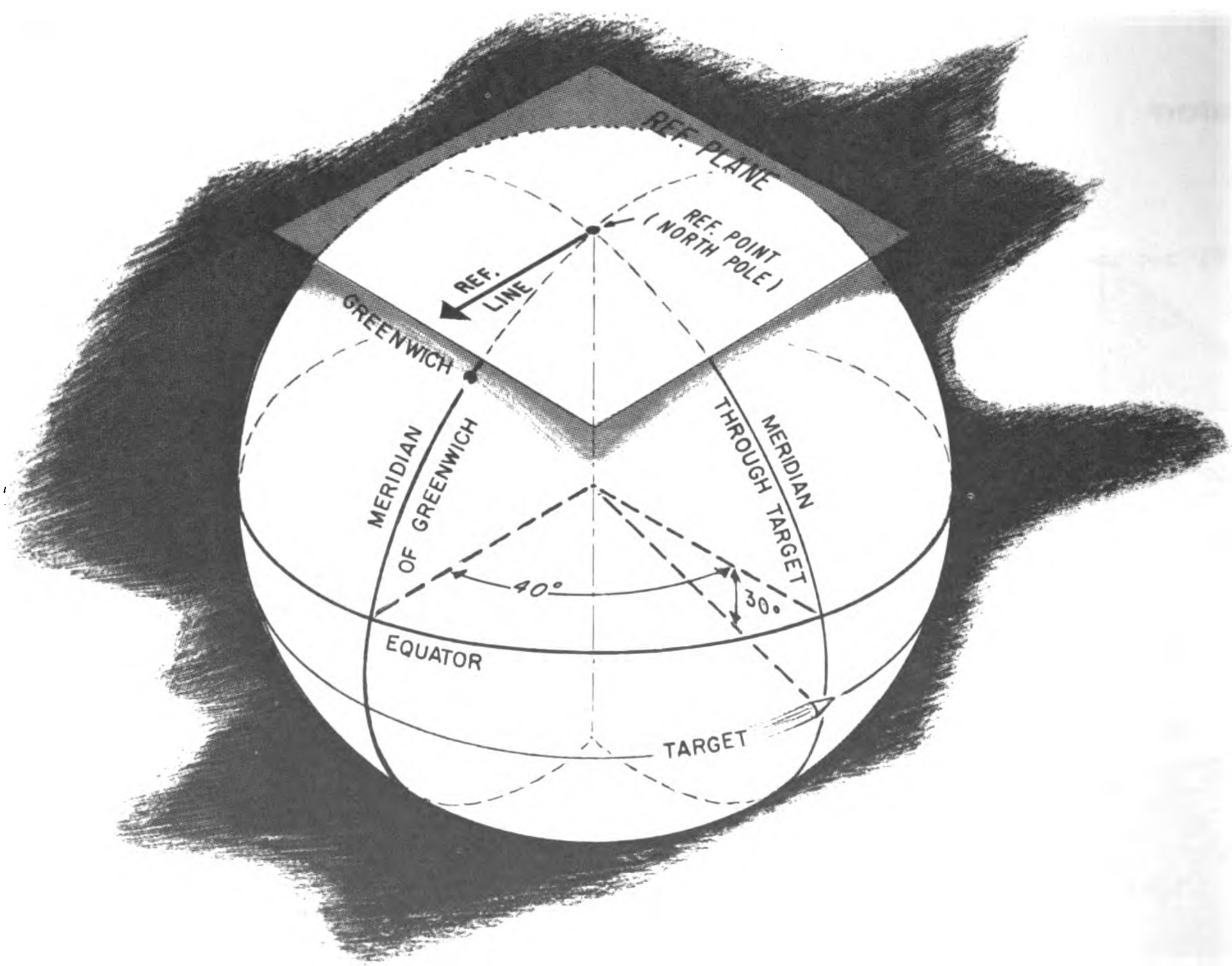
PROBLEMS

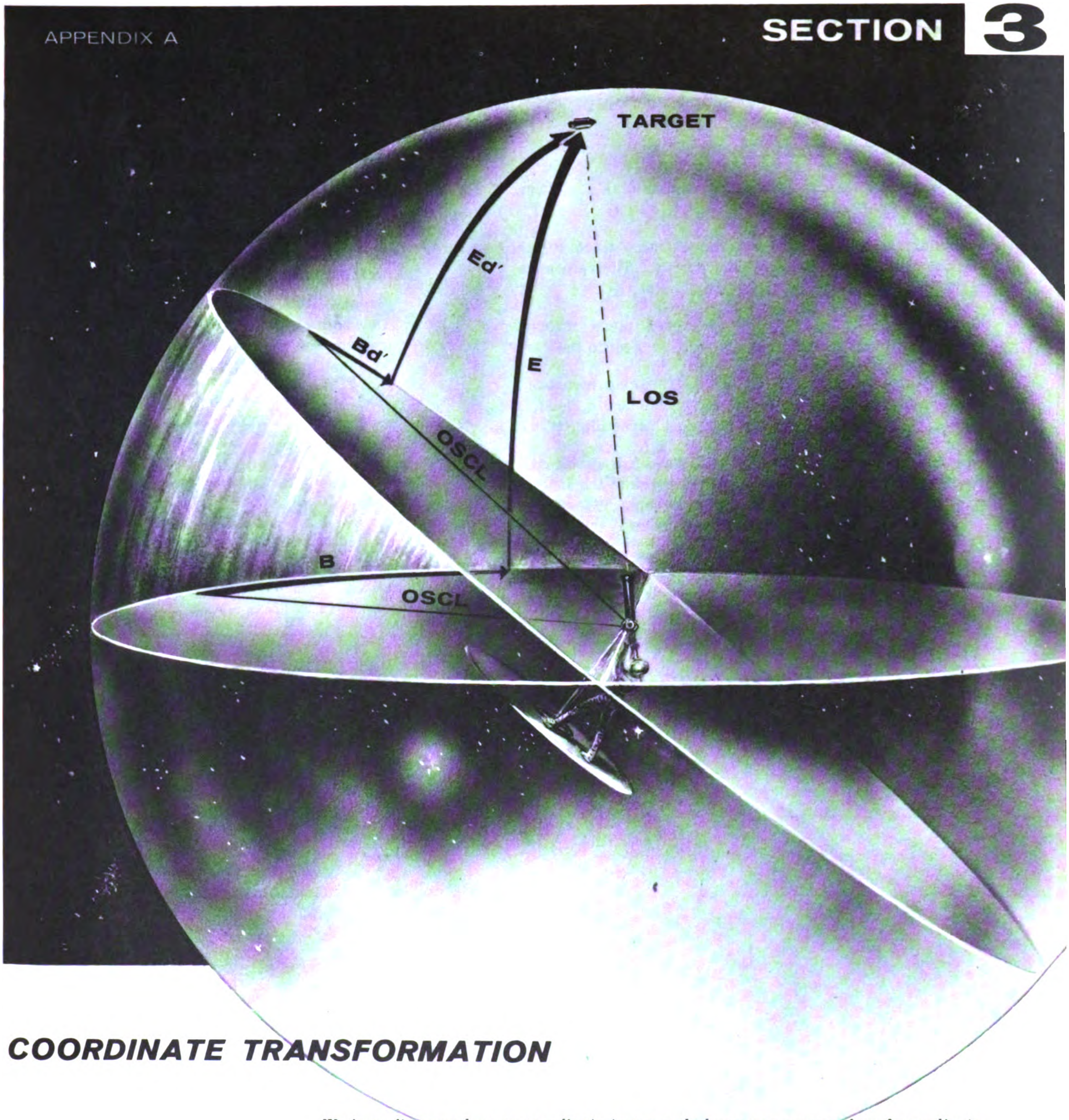
1. A listing of the rectangular, cylindrical, and spherical coordinates of a target, with respect to a plane-line-point reference frame, has had a portion removed. (Plane is horizontal.) All that remains is the information that:

Altitude = 12 miles
Distance west = 3 miles
Elevation angle = 67° 23'

Compute the values of slant range, horizontal range, distance north or south, and true bearing. Give all possible answers. (Distances to nearest integer, angles to nearest minute.)

2. Using the North Pole as a reference point, a horizontal plane at the North Pole as reference plane, and a tangent at the North Pole to the Meridian of Greenwich as reference line (directed over the shortest Earth distance to Greenwich), state rectangular coordinates Rhy, Rhx, and Rv of a target located at 30°S, 40°E. Ry is positive when in the reference line direction; Rhx is positive when to the right, looking along the reference line direction; Rv is positive when in a direction vertically up at the North Pole. (Earth is assumed to be a sphere of 4000-mile radius.) State also the cylindrical coordinates of the target (Rv, Rh, and By), and the spherical coordinates (R, E, and By). Rh, R, and By are always positive. By is always measured clockwise looking down on the North Pole.





COORDINATE TRANSFORMATION

We have discussed some coordinate types, and shown some examples of coordinate conversion; that is, transferring from one type of coordinate system to another within the same reference frame. We will now consider a more complicated subject: coordinate transformation; that is, transferring from a set of coordinates in one reference frame to a set of coordinates in another reference frame. The two sets of coordinates may be of the same or of different types.

This section discusses changes between reference frames of the same types, using the plane-line-point type of frame. Displacement and rotation of frames are discussed in turn. A type of spherical diagram is introduced; this facilitates discussion of "compound rotation", where the tilt of a ship's deck is described in terms of roll and pitch, or level and cross level. In addition to the descriptions of coordinate transformations, some mathematical equations of transformation are obtained.

THE NATURE OF

DISPLACEMENT OF REF. FRAME

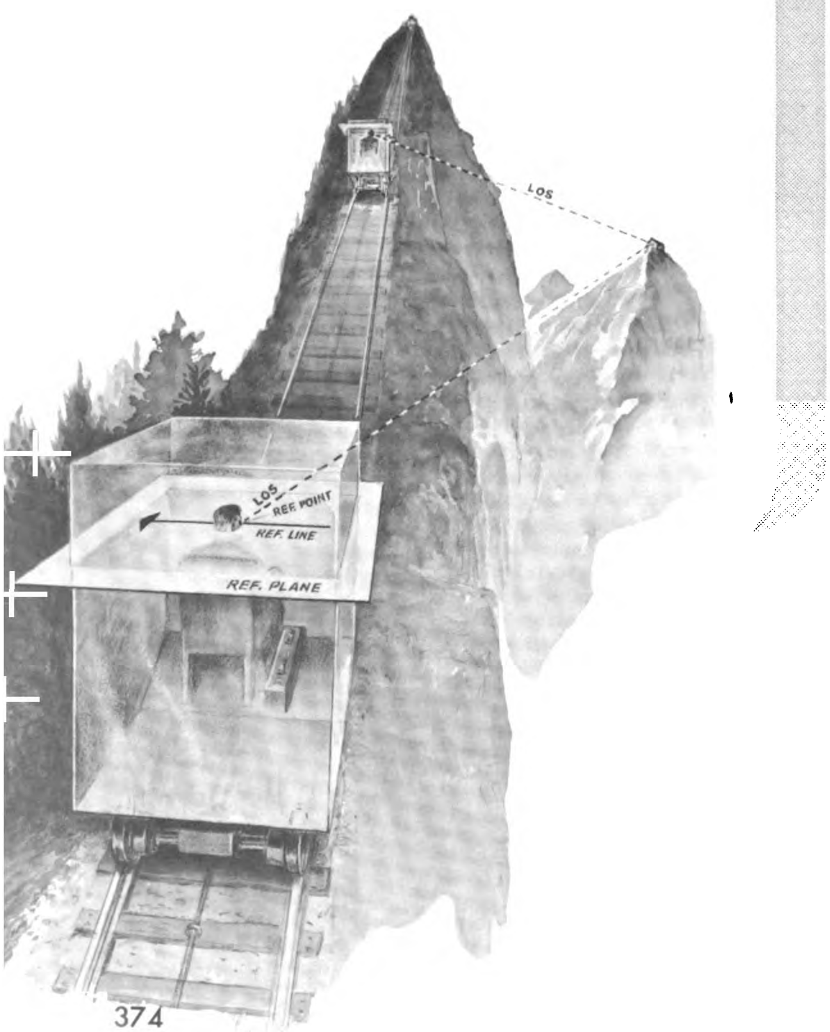
Suppose that a man is going up a mountain in a railway car on straight track.

Consider the man as in a reference frame fixed within the car; his eye is at the reference point. The reference plane is through his eye, and is horizontal.

The reference line is in the reference plane through the man's eye, and athwartships with respect to the car.

He observes a hut on the top of a neighboring mountain. Some minutes later he looks at the hut again, sees it closer than before, and in a different direction. Why?

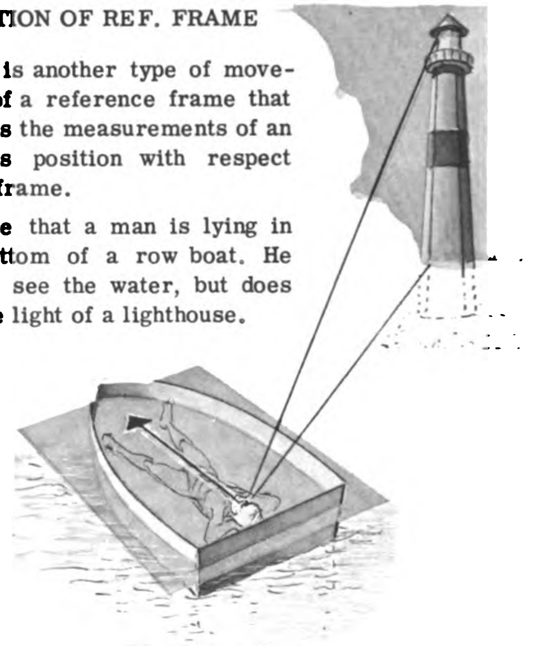
The reference frame has moved and as a result the position of the hut, measured with respect to the frame, has changed -- in distance and in direction. This movement of the frame is called DISPLACEMENT. It consists of a movement in which the reference plane and line remain parallel to their former positions.

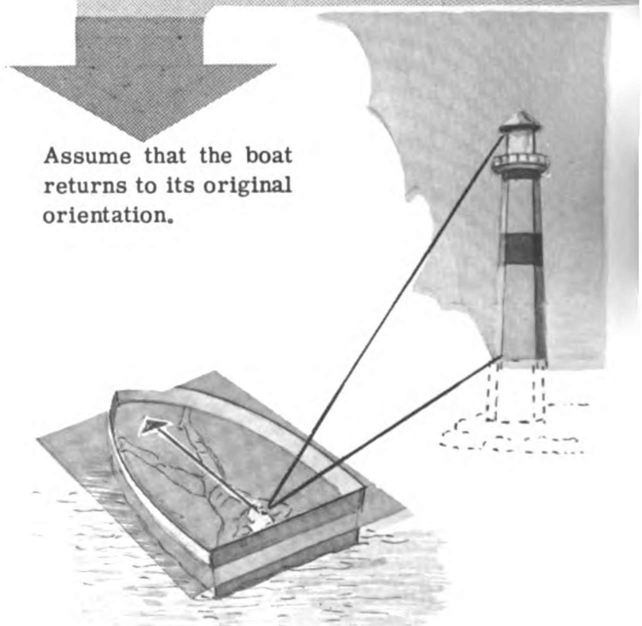


ROTATION OF REF. FRAME

There is another type of movement of a reference frame that changes the measurements of an object's position with respect to the frame.

Suppose that a man is lying in the bottom of a row boat. He cannot see the water, but does see the light of a lighthouse.

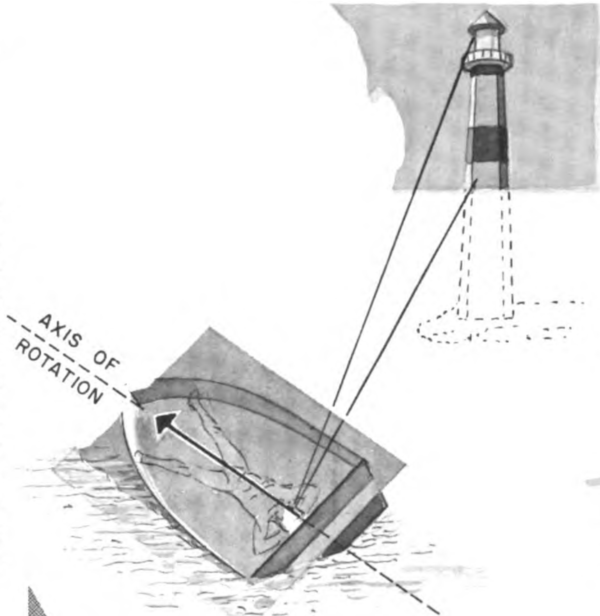




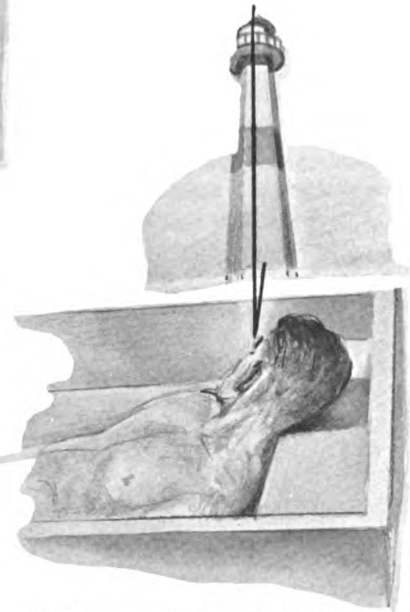
Assume that the boat returns to its original orientation.

THE PROBLEM

Construct a reference plane through his eye and parallel to the deck, containing a reference line fore-and-aft through his eye (the reference point). When the boat rolls, the reference point and line do not move, but the reference plane turns about the reference line. The light's position, measured with respect to the frame, changes in direction but not in distance. This movement of the frame is called ROTATION.

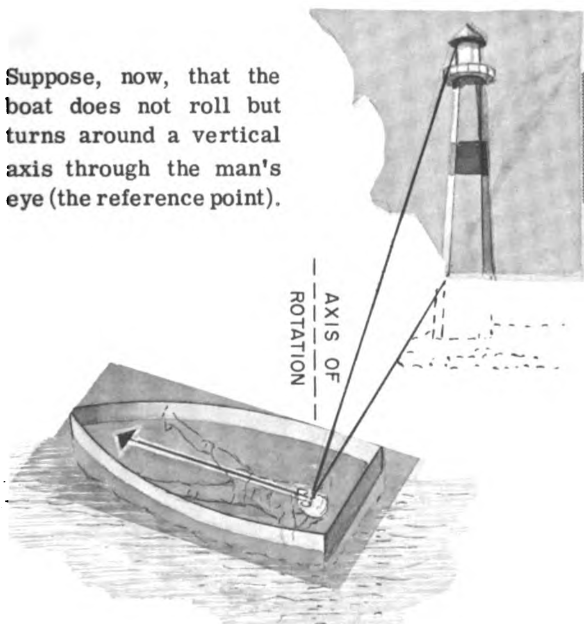


The boat rolls over to the left through a large angle. We will suppose, for simplicity, that it rolls about an axis that is fore-and-aft through the man's eye.

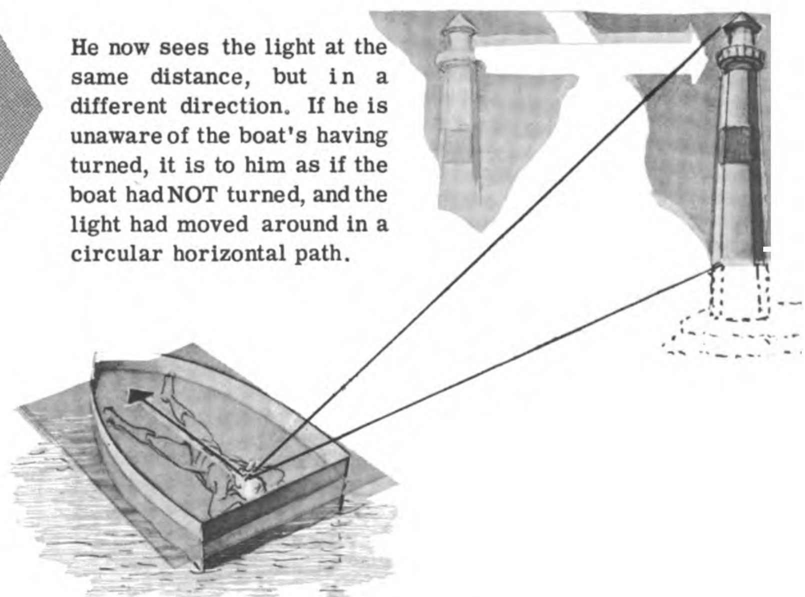


If he is unaware of the boat's having rolled, it appears to him as if the boat has NOT rolled, and the light had moved downward in a circular path.

Suppose, now, that the boat does not roll but turns around a vertical axis through the man's eye (the reference point).



He now sees the light at the same distance, but in a different direction. If he is unaware of the boat's having turned, it is to him as if the boat had NOT turned, and the light had moved around in a circular horizontal path.



Here, the reference line has rotated about the reference point in the reference plane. We have given an example of reference frame displacement, and two examples of reference frame rotation. Of course, many other kinds of displacement and rotation are possible; sometimes they occur in combination.

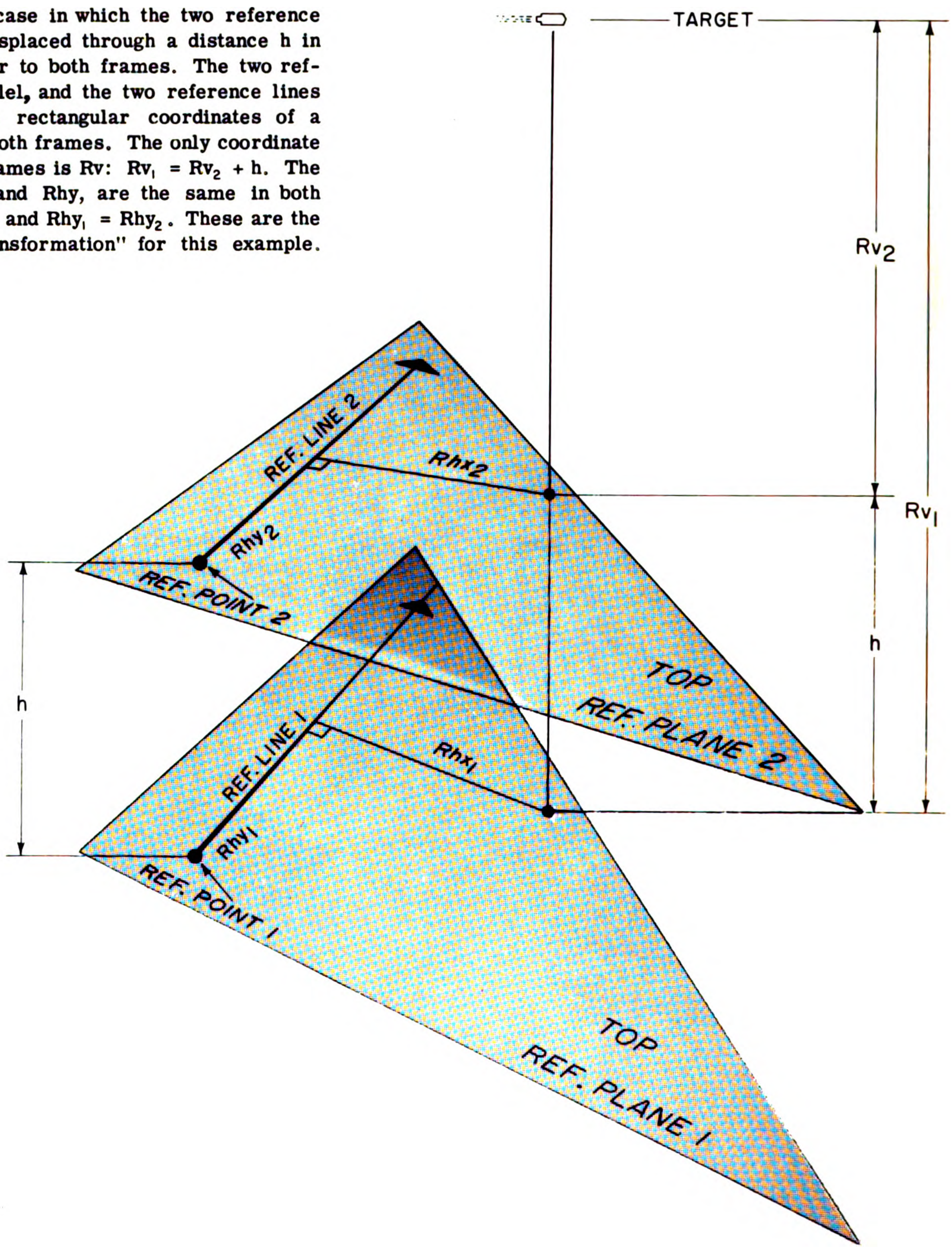
SOME BASIC EQUATIONS

Given a set of measurements of an object (target) position with respect to a reference frame, the problem before us is to find out what these measurements become when the frame is moved — in displacement, or rotation, or both.

This is called COORDINATE TRANSFORMATION. It is used to solve problems arising from frame displacement (problems of PARALLAX); also problems arising from frame rotation (problems of STABILIZATION).

DISPLACEMENT OF RECTANGULAR COORDINATES

Let us first consider a case in which the two reference frames are mutually displaced through a distance h in a direction perpendicular to both frames. The two reference planes are parallel, and the two reference lines are also parallel. The rectangular coordinates of a target are measured in both frames. The only coordinate that differs in the two frames is Rv: $Rv_1 = Rv_2 + h$. The other coordinates, Rhx and Rhy, are the same in both frames, i.e., $Rhx_1 = Rhx_2$ and $Rhy_1 = Rhy_2$. These are the three "equations of transformation" for this example.

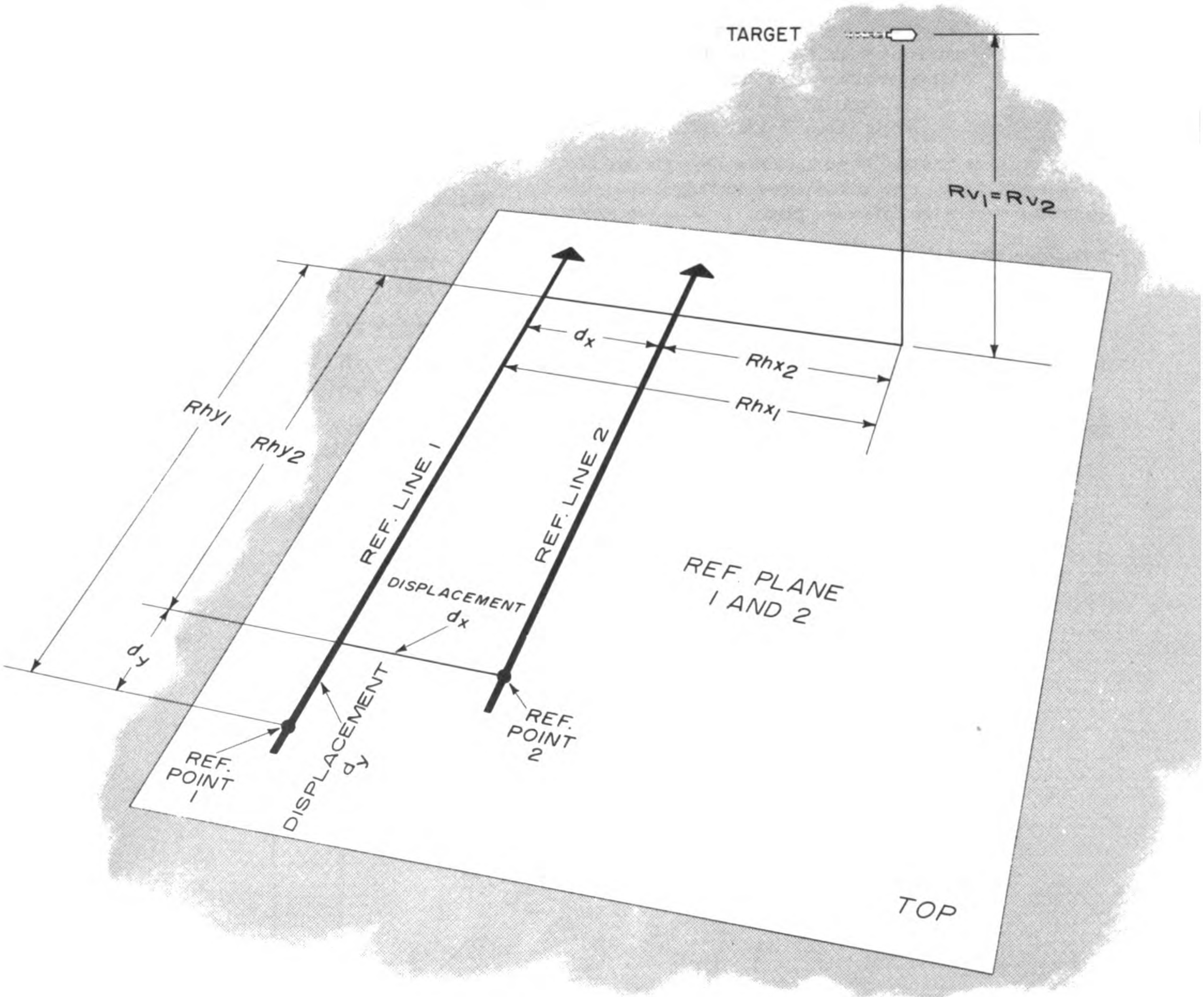


Let us next consider a case where the reference plane is the same in both frames, and the reference lines are mutually parallel. The reference points are mutually displaced by d_y parallel to the reference lines and by d_x perpendicular to the reference lines, in the directions shown.

Here two of the coordinates are changed.
The equations of transformation are

$$Rhx_1 = Rhx_2 + d_x$$
$$Rhy_1 = Rhy_2 + d_y$$
$$Rv_1 = Rv_2$$

If the displacements were in the opposite direction, d_x and d_y would have minus signs.



Transformations of coordinates due to displacement of frames are useful in solving problems of parallax. These are important in weapons control, for the following reason: On shipboard, target positions may be measured in several different frames, by DIRECTORS located in different positions. It is often desirable to transform these measurements to a common "master" frame. In transmitting orders to the weapons, corrections must be made for the displacements of weapon frames from the "master" frame.

ROTATION OF COORDINATES IN REFERENCE PLANE

cylindrical coordinates

Let us first consider a case where the reference plane and reference point are the same in both frames. The reference line is rotated in the common reference plane through angle γ about the reference point to a different position, looking downward on the top side of the reference frame. (See figure to the right.)

rectangular coordinates

Sometimes it is necessary to use rectangular coordinates in cases of transformation by rotation. Then, the equations of transformation are more complex. The figure below shows the same target and reference frames as shown at the upper right, but with coordinates rectangular instead of cylindrical. Here, only coordinates Rv are unchanged; both Rhx and Rhy are changed.

The changes in Rhx and Rhy are shown more clearly in the figure at lower right, a "flat" view, looking downward on the top side of the reference plane.

Reference line 2 is shown rotated clockwise through γ relative to reference line 1.

$$Rhx_1 = DB$$

$$Rhx_1 \cos\gamma = DB \cos\gamma$$

$$= AB$$

$$Rhx_1 \cos\gamma = AC + CB \qquad (1)$$

$$AC = DE \text{ (opposite sides of a rectangle)}$$

$$= DF \sin\gamma$$

$$AC = Rhy_1 \sin\gamma \qquad (2)$$

$$CB = Rhx_2 \qquad (3)$$

Substitute (2) and (3) in (1):

$$Rhx_1 \cos\gamma = Rhy_1 \sin\gamma + Rhx_2$$

Transpose:

$$Rhx_2 = -Rhy_1 \sin\gamma + Rhx_1 \cos\gamma \qquad (4)$$

Also:

$$Rhy_2 = FC$$

$$Rhy_2 = FE + EC \qquad (5)$$

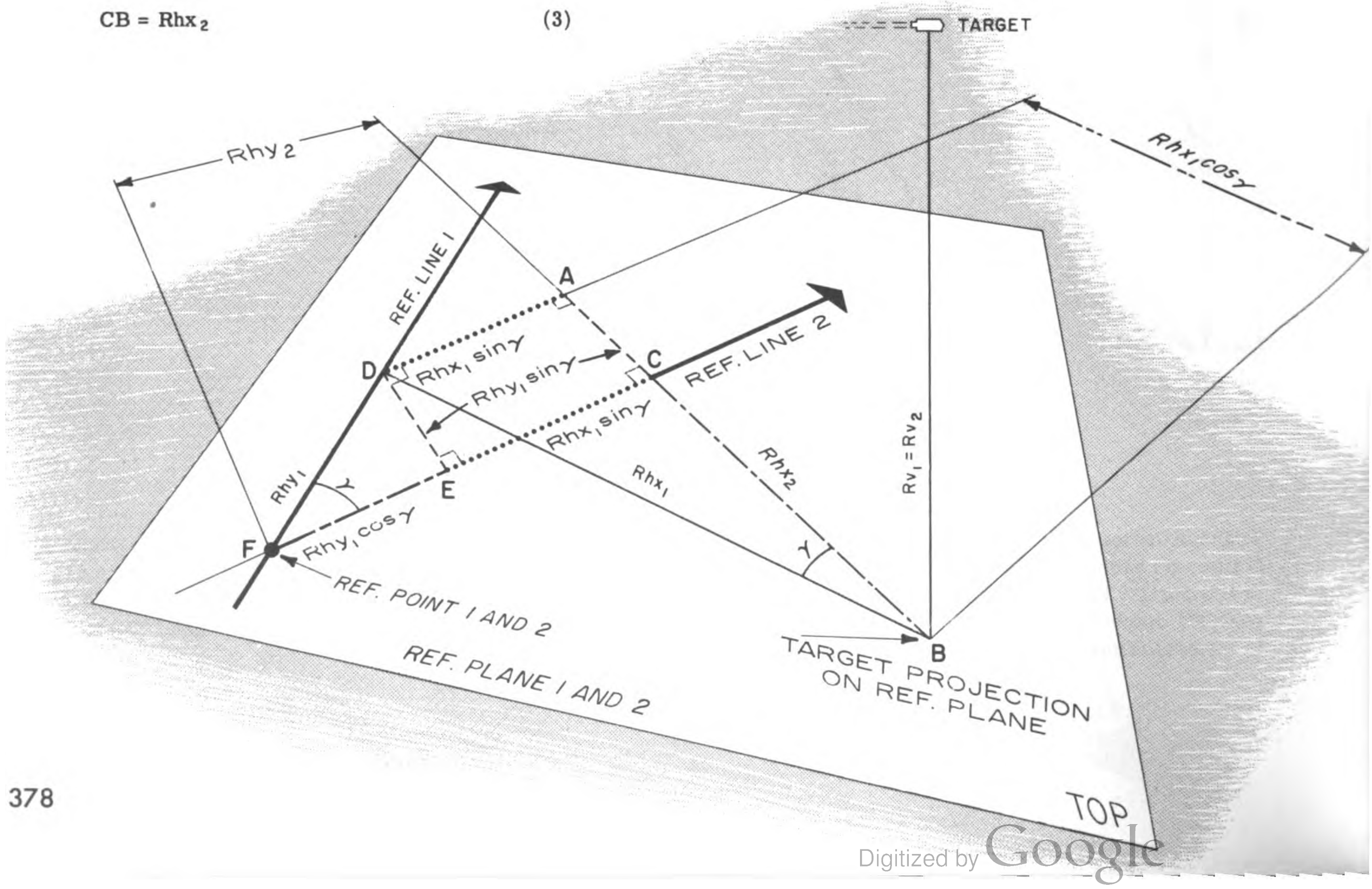
$$FE = FD \cos\gamma$$

$$FE = Rhy_1 \cos\gamma \qquad (6)$$

$$EC = DA \text{ (opposite sides of a rectangle)}$$

$$= DB \sin\gamma$$

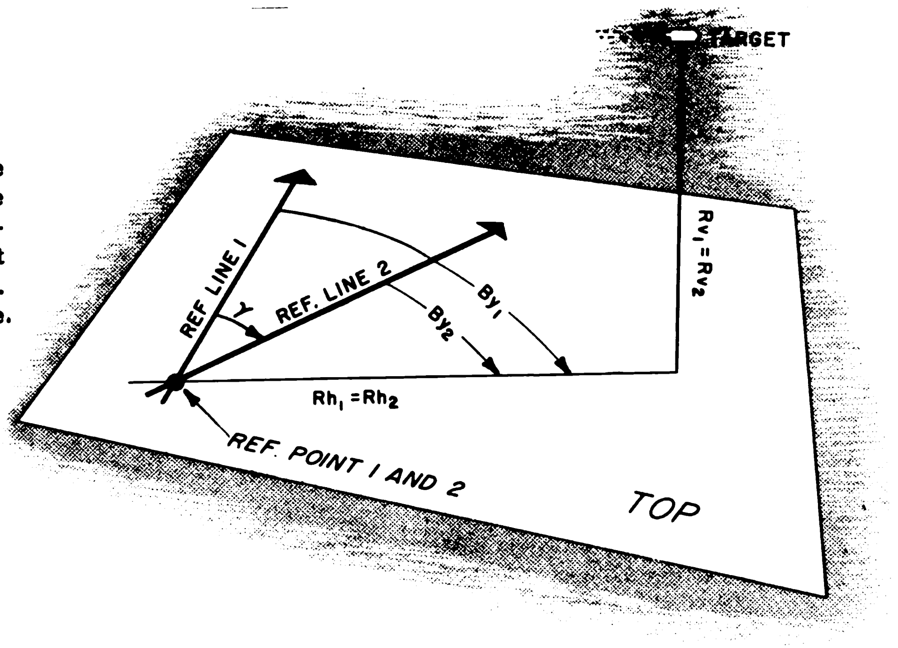
$$EC = Rhx_1 \sin\gamma \qquad (7)$$



The most convenient coordinates to adopt in this case are cylindrical, with "horizontal range" Rh in the common reference plane, and "altitude" Rv perpendicular to this plane. The same figure shows that Rh and Rv are the same, measured in both frames. Only coordinates By ("true bearing") differ (by the angle of frame rotation γ).

The equations of transformation are:

$$Rv_1 = Rv_2$$
$$Rh_1 = Rh_2$$
$$By_1 = By_2 + \gamma$$



Substitute (6) and (7) in (5):

$$Rhy_2 = Rhy_1 \cos\gamma + Rhx_1 \sin\gamma \qquad (8)$$

So, the equations of transformation from frame 1 to frame 2 are (8), (4), and (9):

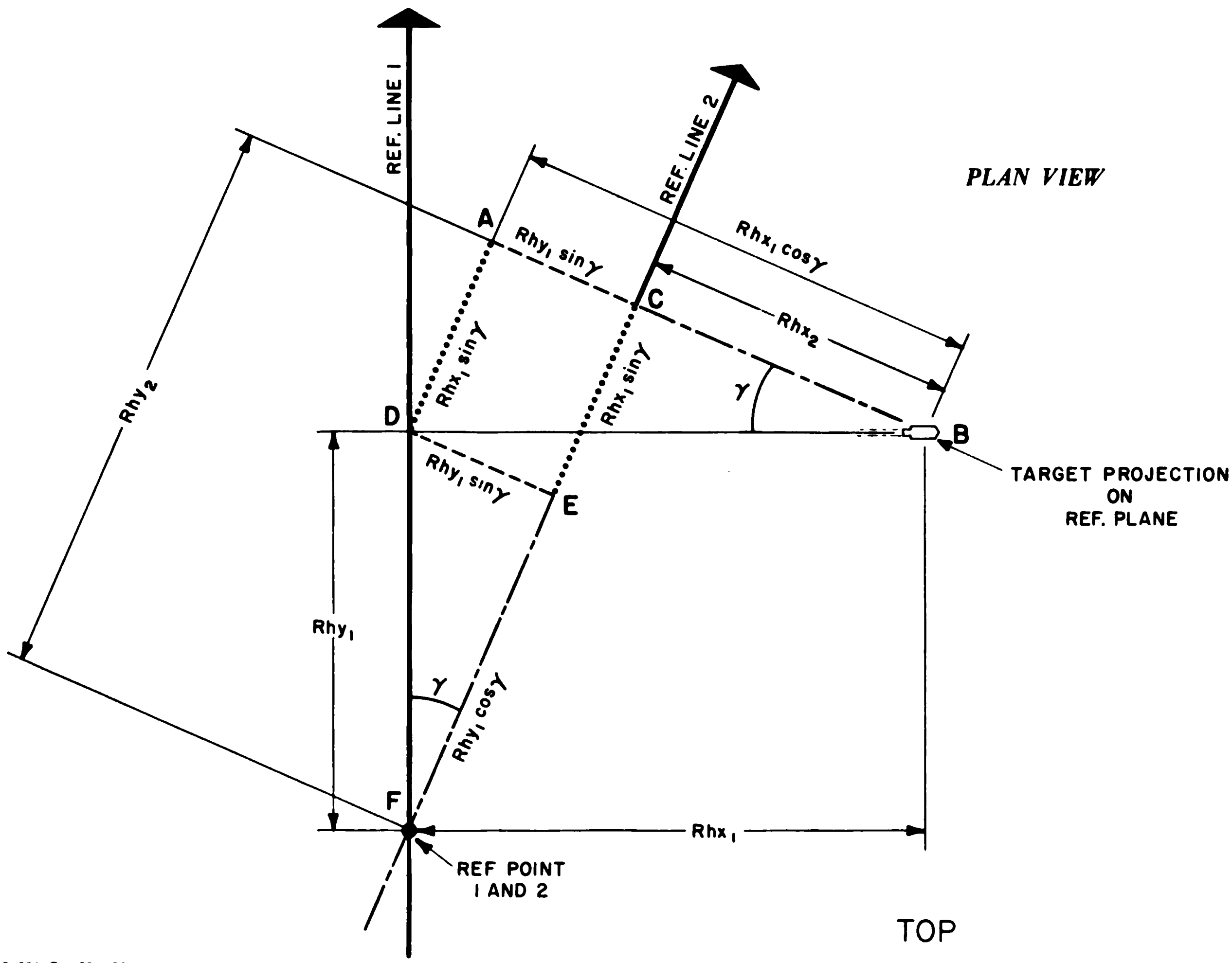
$$Rhy_2 = Rhy_1 \cos\gamma + Rhx_1 \sin\gamma \qquad (8)$$
$$Rhx_2 = -Rhy_1 \sin\gamma + Rhx_1 \cos\gamma \qquad (4)$$
$$Rv_2 = Rv_1 \text{ (from previous figure)} \qquad (9)$$

It is preferable to put Rhy before Rhx because Rhy is along the reference line.

If reference line 2 is rotated counterclockwise through angle γ relative to reference line 1, the equations of transformation are:

$$Rhy_2 = Rhy_1 \cos\gamma - Rhx_1 \sin\gamma \qquad (10)$$
$$Rhx_2 = Rhy_1 \sin\gamma + Rhx_1 \cos\gamma \qquad (11)$$
$$Rv_2 = Rv_1 \qquad (12)$$

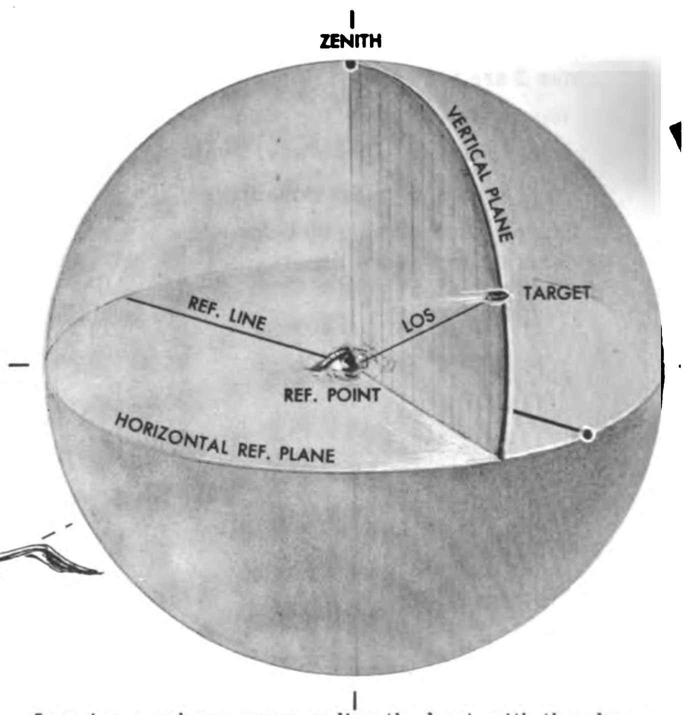
Equations (10) and (11) are derived by substituting $-\gamma$ for γ in equations (8) and (4).

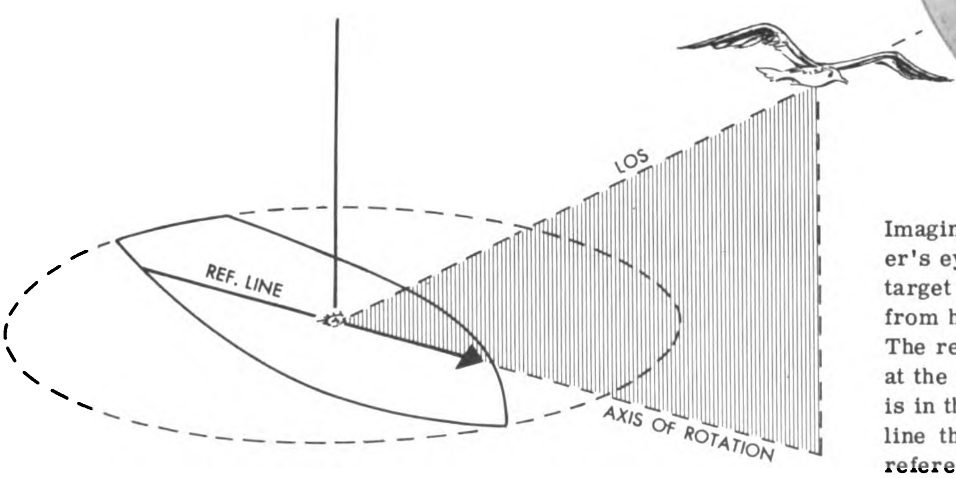


THE SPHERICAL DIAGRAM

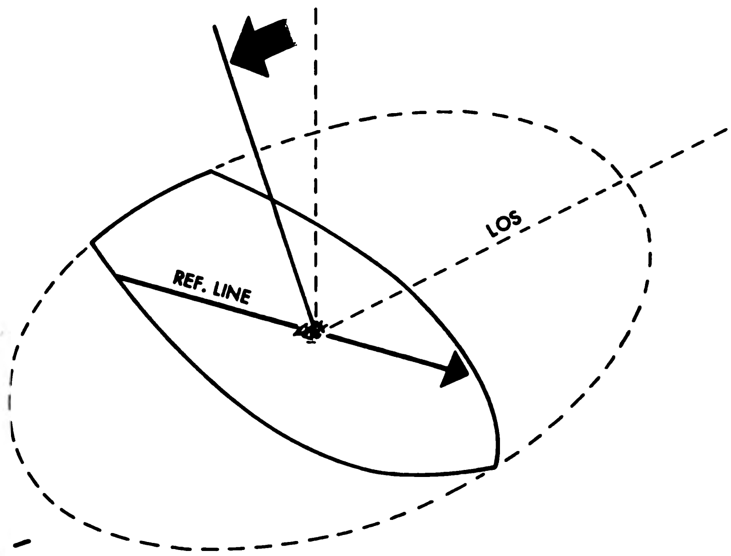
We have obtained some basic equations for the transformation of rectangular and cylindrical coordinates. In weapon control, spherical coordinates are frequently used. Before discussing their transformation (which can be difficult), we will introduce a useful device — the SPHERICAL DIAGRAM. First we shall construct a spherical diagram; then we shall examine frame rotation with the aid of this diagram; then we shall resolve this rotation into two components ("compound rotation"); finally, we shall apply the diagram to the transformation of spherical coordinates.

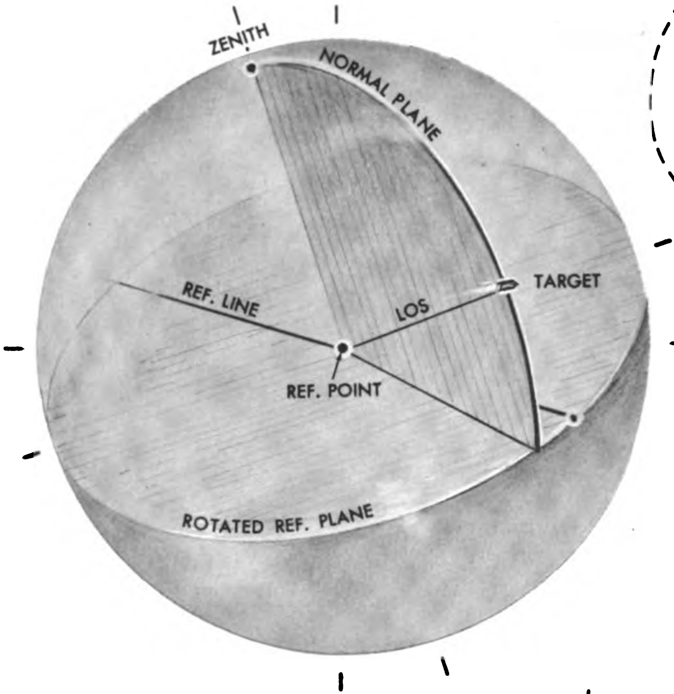
Imagine an observer on the deck of a boat. For simplicity, assume that his eye is in the plane of the deck, whose position is horizontal. Assume that the deck will tilt (owing to the motion of the water) about a reference line in the deck through the observer's eye. For the time, assume that the boat has no "yaw"; that is to say, it does not rotate about a vertical axis.



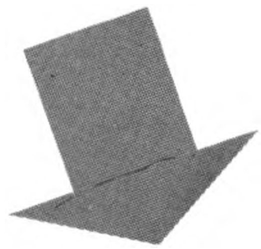


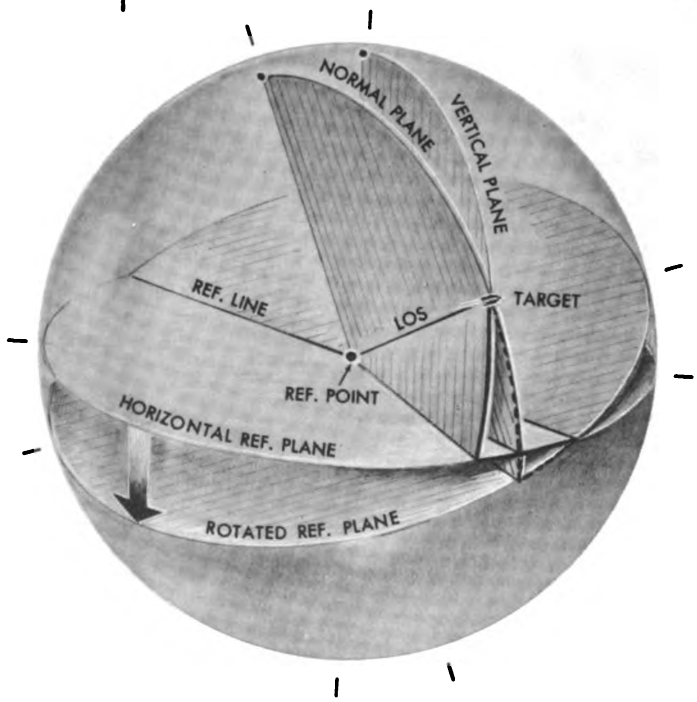
Imagine a sphere surrounding the boat with the observer's eye at the center of the sphere. He is looking at a target which is on the surface of this sphere. The line from his eye to the target is the "line-of-sight" (LOS). The reference plane is the deck; the observer's eye is at the reference point. The reference line (axis of tilt) is in the deck, through the reference point. We draw a line through the reference point perpendicular to the reference plane, intersecting the sphere in the "zenith". **In this case, the line will be vertical. Draw a plane through this vertical and the line-of-sight; this plane is vertical, because any plane through a vertical line is itself vertical.**





Let the deck rotate about the reference line. Let us erect a new plane perpendicular ("normal") to the deck — a plane containing the line-of-sight.





To clarify the relationship between the horizontal and rotated reference planes, the vertical plane, and the "normal" plane, we combine the two previous figures.

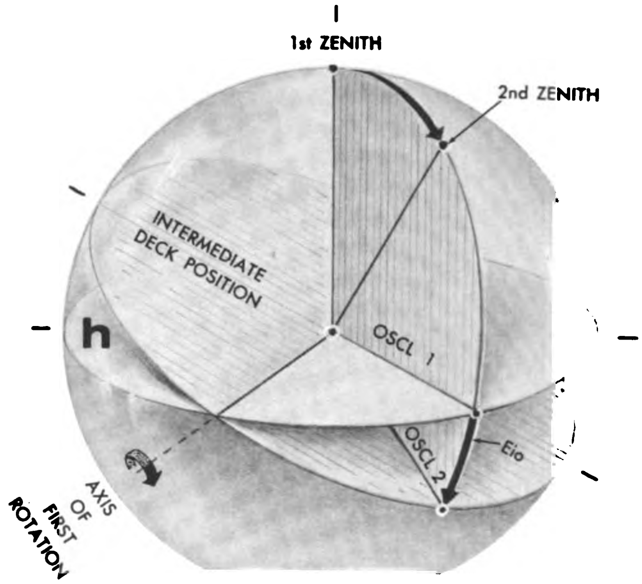
The heavy firm great-circle arcs are a measure of two of the target spherical coordinates (bearing and elevation) in the horizontal reference frame. The heavy, dotted great-circle arcs are a measure of the coordinates in the rotated frame.

Note that the normal plane is not obtained by rotating the original vertical plane rigidly with the deck. It is a new plane.

CHANGE IN NOMENCLATURE

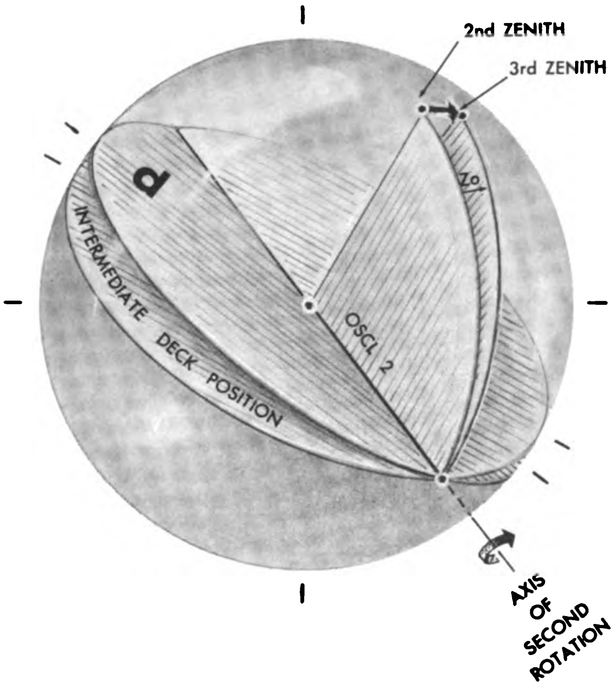
From here on we shall simplify our nomenclature. We shall regard the horizontal reference plane as the desired rather than the actual position of the deck, and refer to it merely as the "horizontal plane". We shall refer to the rotated reference plane as the "deck plane". On figures they will be labeled "h" and "d". We shall cease to refer to the reference point. The reference line we shall commonly allude to by some specific name, such as north, or own ship centerline (OSCL), etc.

Let us forget about the target for a time, and discuss deck rotation by itself. In general, the deck tilts about some axis other than the reference line. When this happens, it is convenient to express this rotation as the resultant of two rotations related to the reference line (say, own ship centerline), or to some other important line (such as the line-of-sight).



pitch Eio

Here is one example: The first rotation is about the athwartship axis, causing own ship centerline to dip downward in a vertical plane from position $OSCL_1$ to position $OSCL_2$. The measure of this rotation is the angle between the horizontal plane and the new (intermediate) deck plane, measured in the vertical plane through own ship centerline. The angle is known as PITCH (Eio).

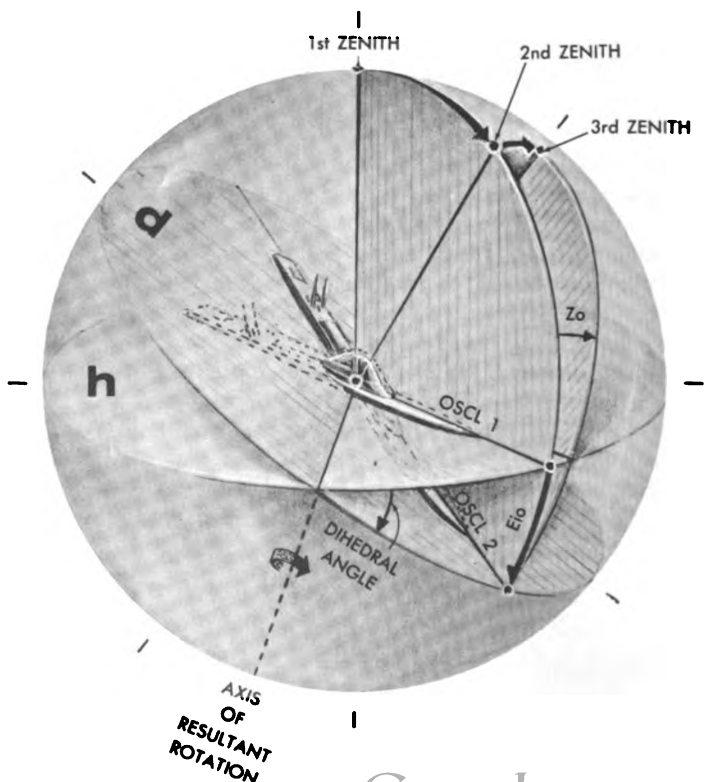


roll Zo

The second rotation is about the new position of own ship centerline ($OSCL_2$), clockwise. The measure of this rotation is the angle between the vertical plane through own ship centerline and the plane perpendicular to the deck through own ship centerline, measured about own ship centerline. The angle is known as ROLL (Zo). The deck is now rotated from its intermediate position to its final position (d).

combining pitch and roll

At the right we see the effects of combining pitch and roll. The inital (h) and final (d) positions of the deck are shown; to avoid complication, the intermediate position is omitted.

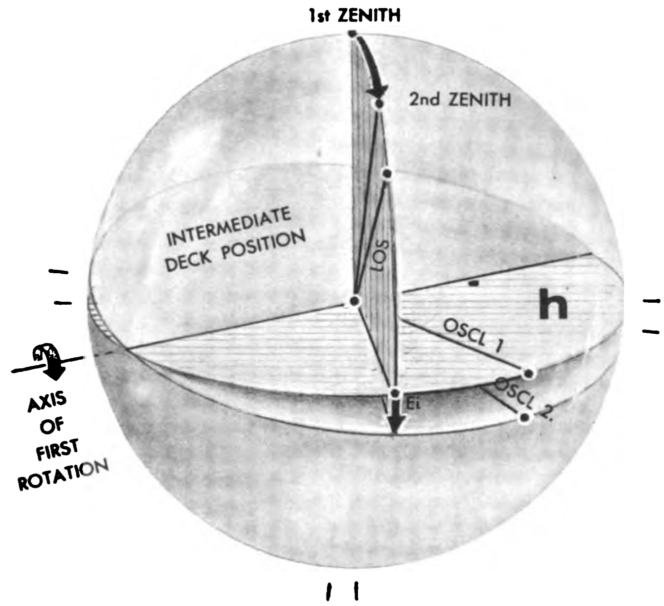


note

When we say "first rotation", "second rotation", and "final position", we are merely assuming — for the sake of convenience in measurement — that the deck makes two rotations in succession. Actually, it makes one rotation, about the axis of resultant rotation. It rotates through the dihedral angle between the horizontal (h) and final (d) deck positions. The axis of resultant rotation is, in general, not the reference line. Therefore, we break up the resultant rotation into two rotations, one of them about the reference line, to facilitate measurement of deck tilt. In this example the two rotations are pitch and roll.

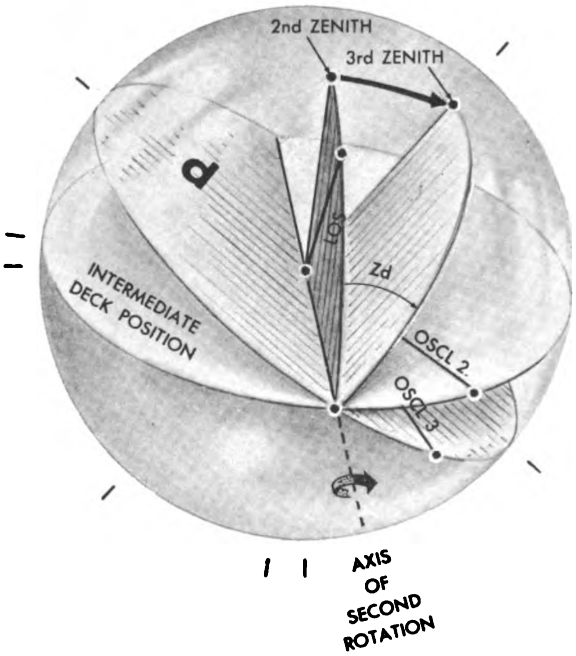
level Ei

The same deck rotation can be expressed as the resultant of two rotations other than pitch and roll. For example, the first rotation can be about an axis perpendicular to a vertical plane through the line-of-sight. This rotation brings the first zenith down in the vertical plane to the second zenith position. The measure of this rotation is the angle between the horizontal plane and new (intermediate) deck plane, measured in the vertical plane through the line-of-sight. The angle is known as LEVEL (Ei).



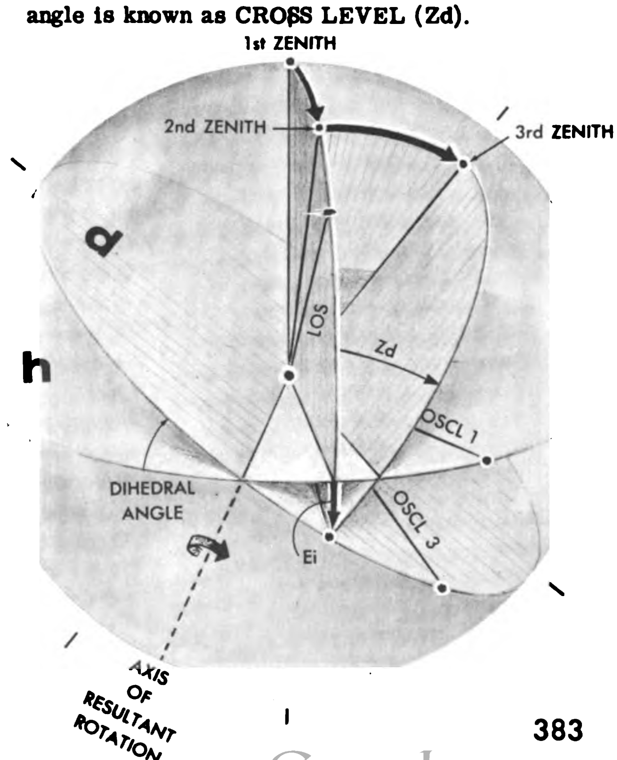
cross level Zd

The second rotation is about the intersection of the vertical plane through line-of-sight and deck, clockwise. The measure of this rotation is the angle between the vertical plane through the line-of-sight, and the plane perpendicular to the deck through the intersection of the vertical plane through the line-of-sight and the deck plane, measured about that intersection. The angle is known as CROSS LEVEL (Zd).

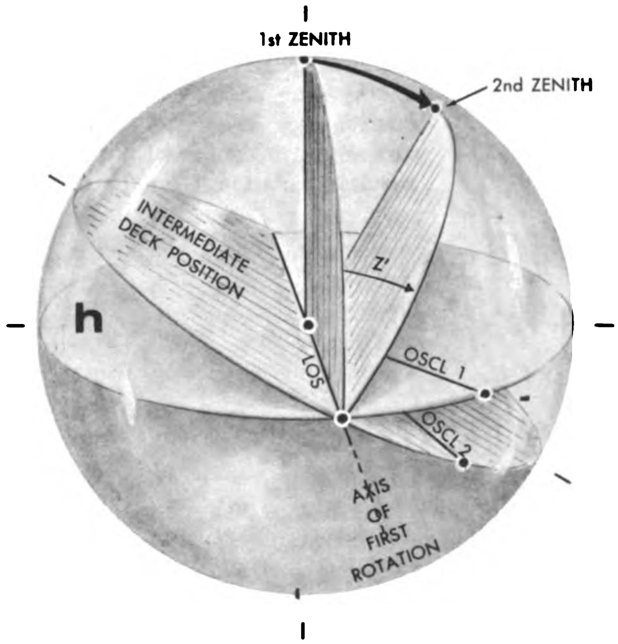


combining level and cross level

At the right we see the effects of combining level Ei and cross level Zd.

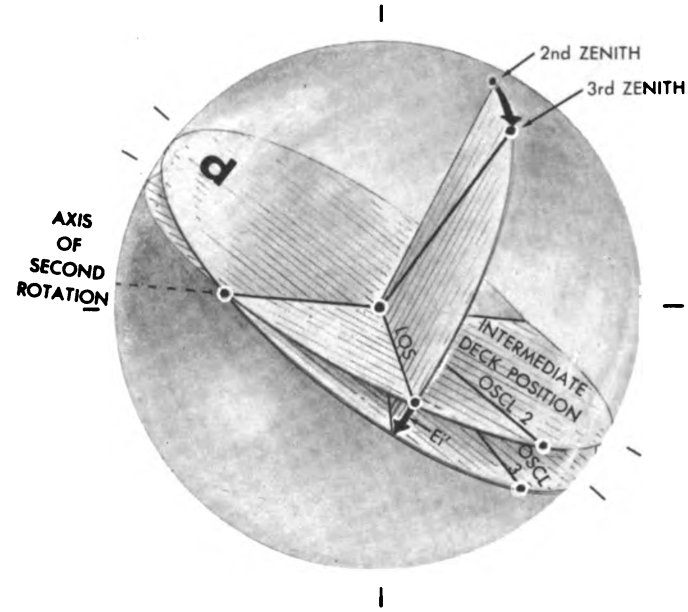


COMPOUND ROTATION



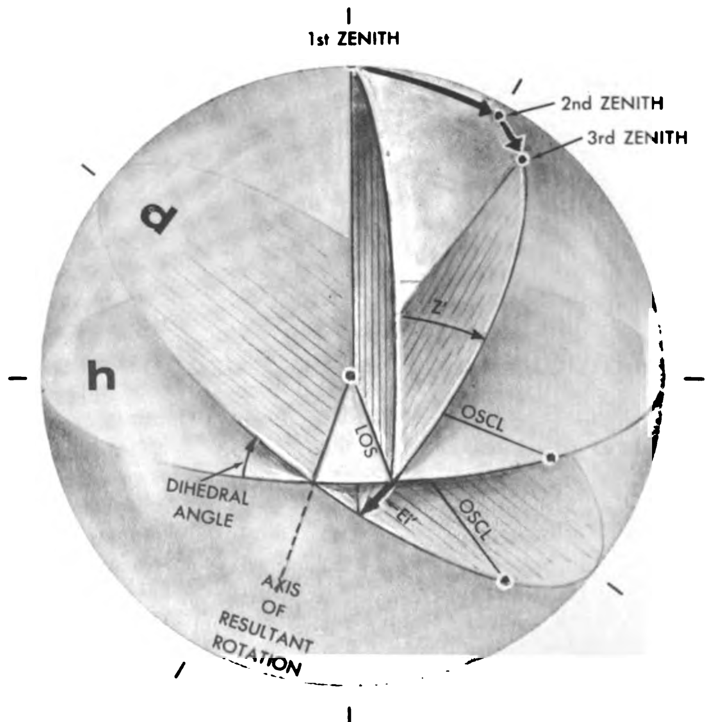
cross level Z'

Still another way of resolving the same deck rotation may be used with surface targets; that is, with a horizontal line-of-sight. We suppose that the rotation is about the line-of-sight, clockwise. The measure of this rotation is the angle between the plane through the line-of-sight perpendicular to the new (intermediate) deck plane and the vertical plane through the line-of-sight, measured about the line-of-sight. The angle is known as CROSS LEVEL (Z').



level Ei'

The second rotation is supposed to be in the plane perpendicular to the intermediate deck plane. The second zenith moves downward to the third zenith position. The measure of this rotation is the angle between the intermediate and final deck planes, measured in the plane through the line-of-sight perpendicular to the intermediate and final deck planes. The angle is known as LEVEL (Ei').



combining cross level and level

At the right we see the effects of combining cross level Z' and level Ei'.

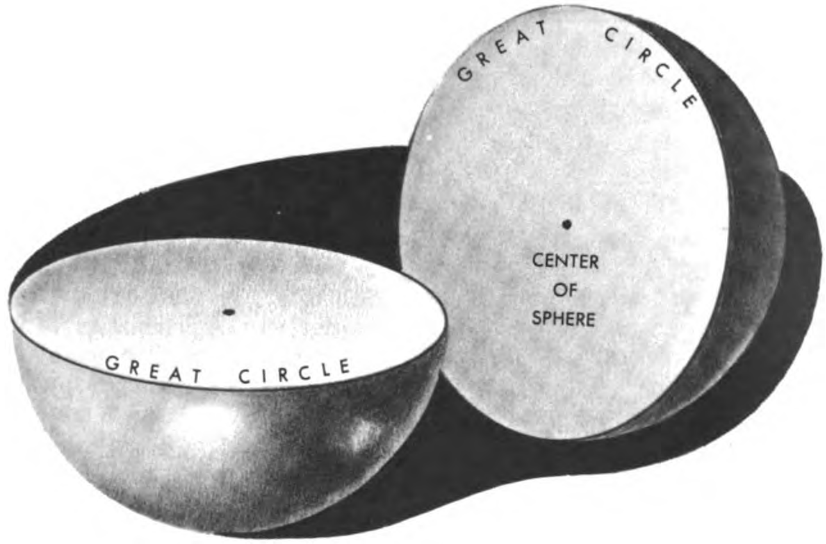
Note that these angles of rotation are different from level (Ei) and cross level (Zd). This is due to our considering the two rotations as occurring in the reverse order; that is, Ei "precedes" Zd, but Z' "precedes" Ei'. The significance of different types of level and cross level involves their utilization by various types of equipment.

general rotations

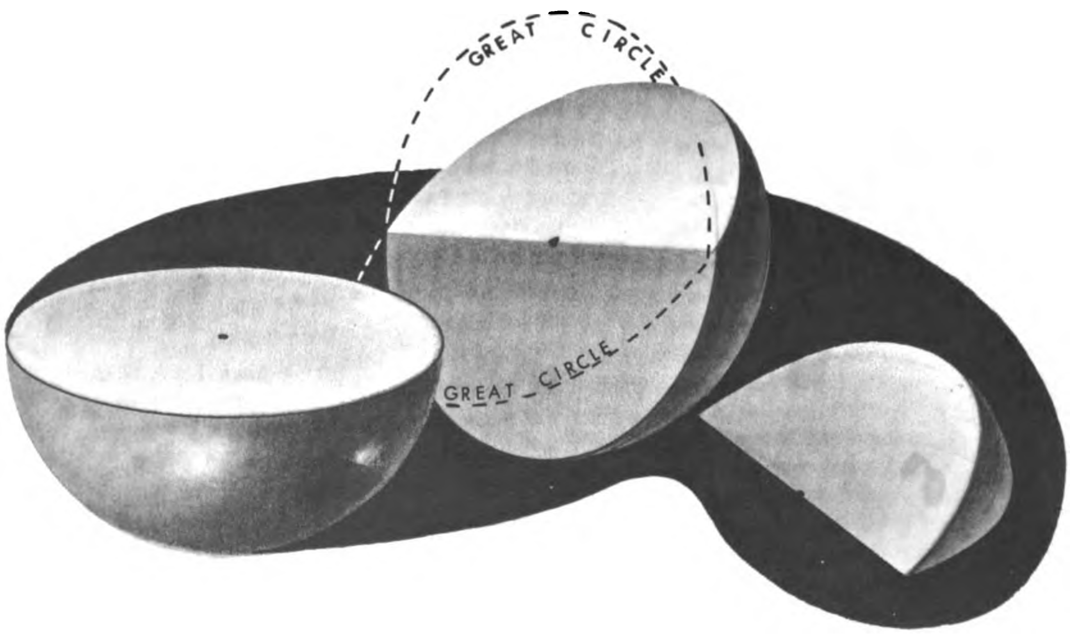
There are many ways of expressing deck rotation as the sum of two rotations. Our objective is to choose two rotations that are convenient to use, and easily measured. From these two quantities (which tell us the new position of the deck), and from the target coordinates with respect to the horizontal frame, we can obtain equations for the target coordinates with respect to the deck frame. Or, given the deck coordinates, we can find the horizontal frame coordinates.

note on great circles

Note that all the arcs drawn on the surfaces of our spheres are arcs of great circles. This is because they are all formed by the intersection of the surface with some plane through the center of the sphere. Consequently, these spherical diagrams lend themselves to solutions of coordinate transformations by spherical trigonometry.



At the right and directly below, are shown two progressive cuttings through the center of a solid sphere, with the resultant formation of two great circles.



On the following pages, we shall study compound rotation from a mathematical standpoint.

TRANSFORMATIONS FOR COMPOUND ROTATION

. . . GENERAL

We have made a brief study of "compound rotation" —— meaning a single rotation expressed, for convenience, as two successive rotations about different axes. Let us apply what we have learned to our original problem: coordinate transformation.

EQUIPMENT

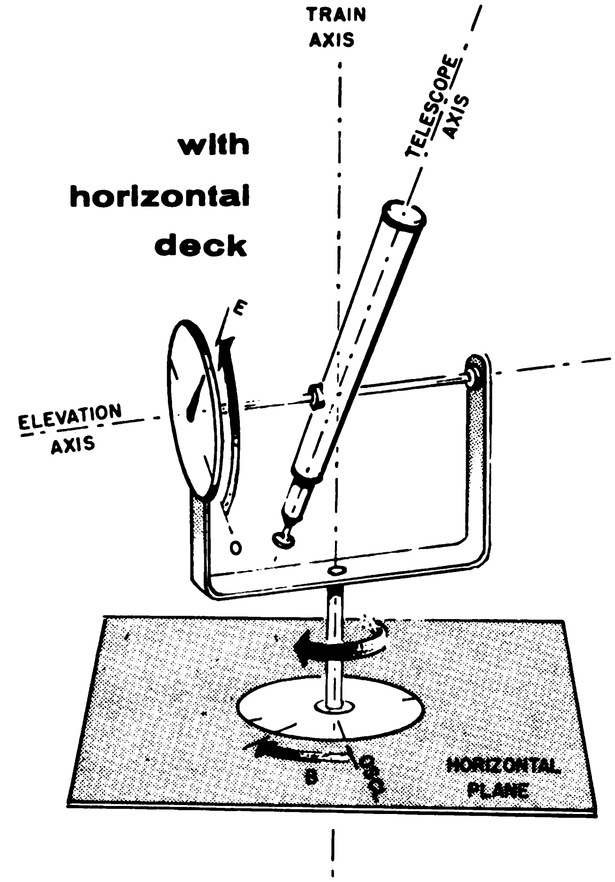
Imagine a telescope, mounted on the deck of a ship, and capable of rotating about two axes, as shown. The train axis is perpendicular ("normal") to the deck. The elevation axis is parallel to the deck. The telescope axis is perpendicular to the elevation axis; consequently, the telescope elevates in a plane normal to the deck (when there is no rotation about train axis). Both modes of rotation can be measured on graduated dials. The zero marker for train is usually own ship centerline (OSCL); the train dial measures RELATIVE TARGET BEARING (B) or DIRECTOR TRAIN (UNSTABILIZED SIGHT) (Bd'), according to the orientation of the deck. Elevation is zero when the telescope is parallel to the deck. The elevation dial measures TARGET ELEVATION (E) or DIRECTOR ELEVATION (UNSTABILIZED SIGHT) (Ed'), according to the orientation of the deck. Quantities B and E are spherical coordinates in a horizontal frame; quantities Bd' and Ed' are spherical coordinates in a deck frame.

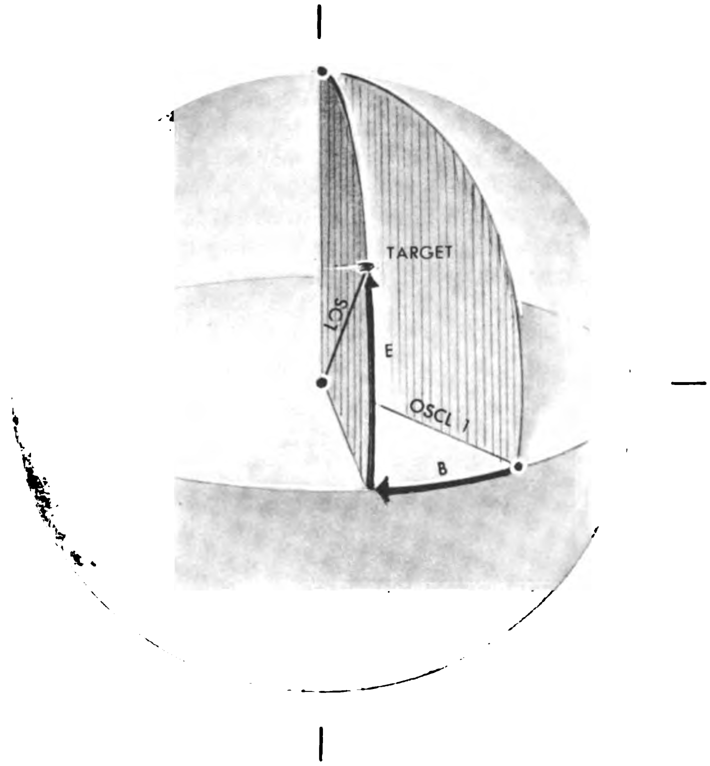
It is often our purpose, in weapon control, to transform coordinates Bd' and Ed' to coordinates B and E, or vice-versa. In some equipments, this operation is simplified by keeping the elevation axis horizontal instead of parallel to the deck, by orders from a gyro measuring device. The "optical axis" (line along which the telescope "sees") then moves in a vertical plane, independant of deck tilt. This is called "sight stabilization" (actually it achieves a partial stabilization of coordinates). The same result may be achieved by continuous adjustment of the trunnion, or of a prism or mirror within the telescope. Let us now examine some transformations between deck and horizontal coordinates, using roll and pitch and level and cross level.

Definitions:

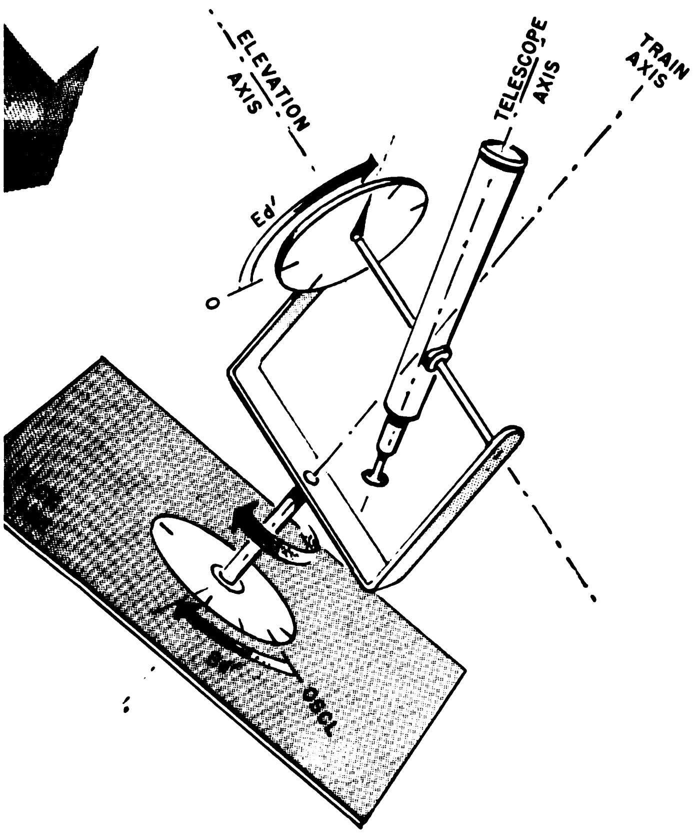
- B: Angle between the vertical plane through OSCL, and the vertical plane through the line-of-sight, measured in the horizontal plane clockwise from OSCL.
- Bd': Angle between the vertical plane through OSCL, and the plane through the line-of-sight perpendicular to the deck plane, measured in the deck plane clockwise from OSCL.
- E: Angle between the horizontal plane and the line-of-sight, measured upward in the vertical plane through the line-of-sight.
- Ed': Angle between the deck plane and the line-of-sight, measured upward in the plane through the line-of-sight perpendicular to the deck plane.

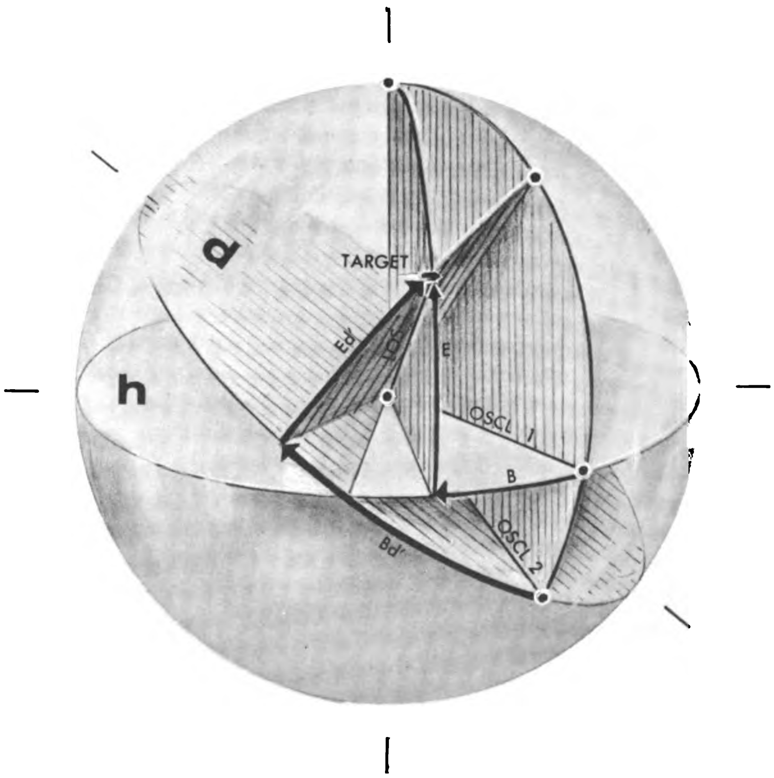
with horizontal deck





with tilted deck





ROLL AND PITCH

Assume that the deck is horizontal, and the telescope is pointing at a fixed target. This target has bearing B and elevation E, indicated on the dials. Now suppose that the deck tilts about some unspecified axis, and that some measuring device continuously measures this tilt as roll Zo, and pitch Eio. The device may be gyroscopic, or it may be some other device capable of measuring the necessary angles. When the deck tilts, the telescope moves off the target, unless steps are taken to prevent this. These steps consist of changing the train and elevation angle of the telescope. We may suppose that Bd' and Ed' are measured, and it is required to compute from them (and Eio and Zo) what the train and elevation angles would be if the deck were horizontal — that is to say, B and E. Coordinates B and E (unlike Bd' and Ed') are constant when the target is stationary with respect to the ship. When the target moves, changes in B and E are due solely to target relative motion; not deck tilt. Previously we mentioned a "master frame". Coordinates B and E are suitable for use in such a frame.

LEVEL AND CROSS LEVEL . . . (FIRST TYPE)

An alternative to measuring deck tilt in terms of roll and pitch is to measure it in terms of level Ei and cross level Zd. Here we "stabilize the sights"; that is to say, the optical axis elevates in a vertical plane. The telescope is trained in the deck plane and elevated in a vertical plane so as to keep on the target as the deck tilts. Coordinates Bd — DIRECTOR TRAIN (STABILIZED SIGHT) — and Ed — DIRECTOR ELEVATION (STABILIZED SIGHT) — are thereby continuously measured. A measuring device continuously measures the values of level Ei and cross level Zd appropriate to the direction of the optical axis. It is required to transform coordinates Bd and Ed to B and E. The value of E is obtained simply by subtracting Ei from Ed. The value of B is computed from the values of Bd, Ed, Ei, and Zd, by spherical trigonometry, or (as is more usual) by an approximate empirical formula. Note that Bd and Ed are not orthogonal coordinates, like Bd' and Ed', or the coordinates discussed in the previous section.

Definitions:

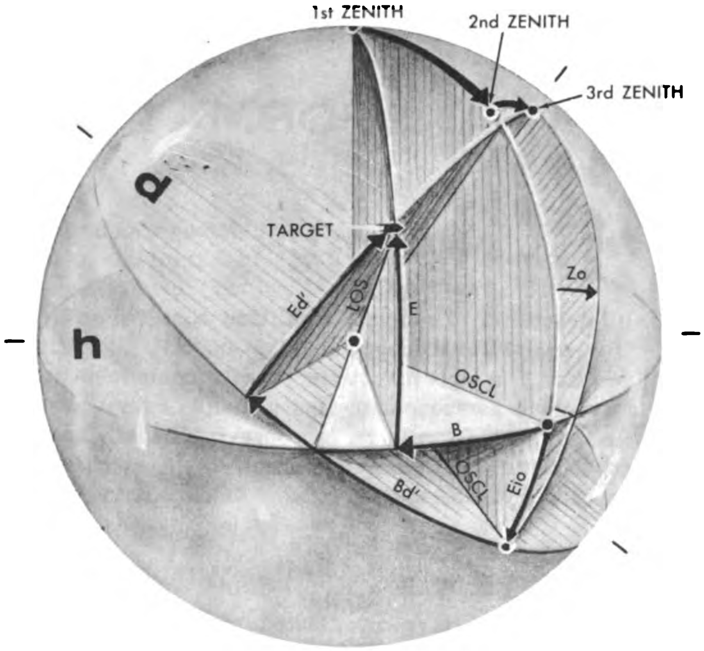
Bd: Angle between the vertical plane through OSCL, and the vertical plane through the line-of-sight, measured in the deck plane clockwise from OSCL.

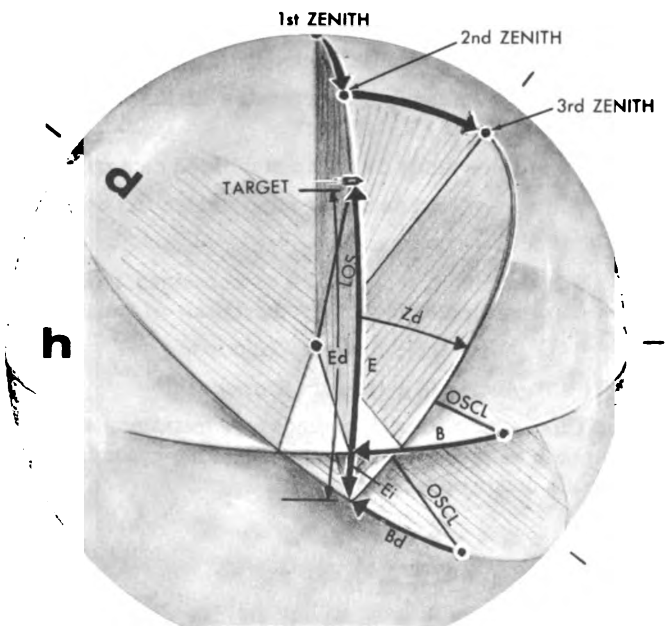
Ed: Angle between the deck plane and the line-of-sight, measured upward in the vertical plane through the line-of-sight.

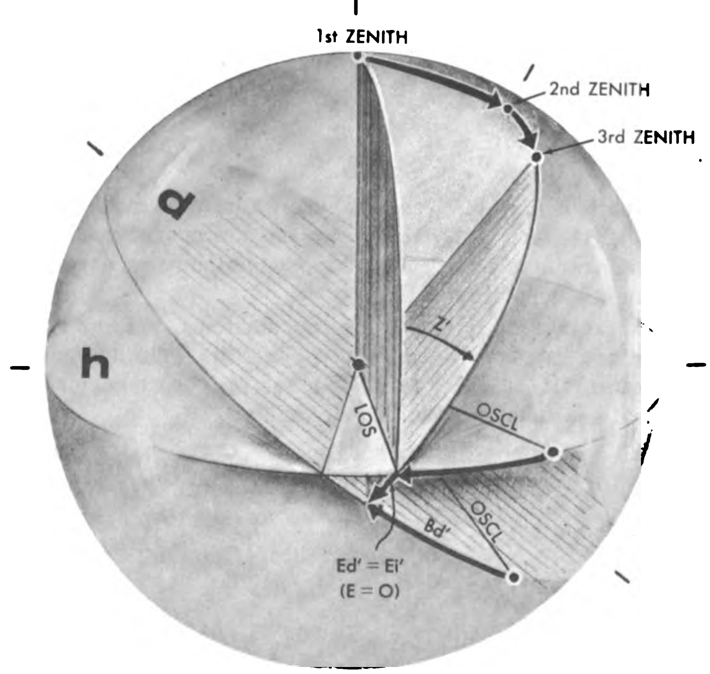
CROSS LEVEL AND LEVEL . . . (SECOND TYPE)

Still another method of coordinate transformation can be used if the target is in a horizontal line with the director, that is, with a surface target. The sights are not stabilized, so the optical axis elevates not in a vertical plane but in a plane normal to the deck. The measuring device now measures deck tilt in terms of cross level Z' and level Ei'. When the optical axis is kept on the target, Bd' and Ed' are measured.

It is required to compute B and E. The value of B is computed from Bd', Ed' (which equals Ei'), and Z'. The value of E is zero. (Note that, in this case, cross level "precedes" level).







note

The diagrams accompanying this discussion suggest large deck tilts followed by large train and elevation corrections. Actually, deck tilts are much smaller; also, correction is a continuous process, designed to keep the optical axis on the target at all times.

summary

We have shown three ways of applying coordinate transformation to measurements made on a tilting deck. A measuring device, such as a telescope, measures target coordinates in a deck frame such as Bd' and Ed', or Bd and Ed. Another measuring device, often gyroscopic, measures deck tilt components (roll and pitch, or level and cross level). A computer receives these four inputs, and computes target coordinates in a horizontal frame: B and E. In many cases, the reverse procedure takes place: Bd and Ed (say) are computed from B and E.

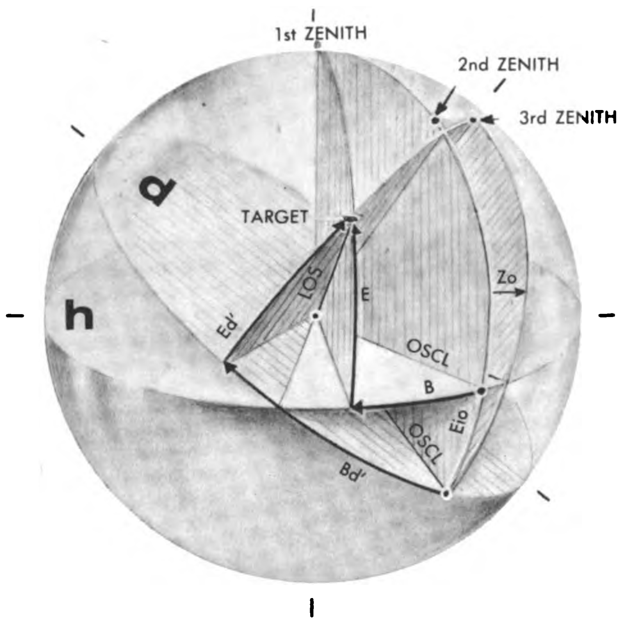
When B and E are computed, the transformation of coordinates is known as STABILIZATION of coordinates. The significance of using stabilized coordinates is discussed in a later section — "Inertial Frames". Just as B and E are stabilized spherical coordinates, and Bd' and Ed' are unstabilized spherical coordinates, so Bd and Ed may be described as partly stabilized spherical coordinates. The director sights are stabilized, but the train axis remains normal to the deck. If the train axis were also stabilized (that is, if kept vertical, as it is in some equipments), the director would measure B and E, regardless of any tilt of deck. Level and cross level may be measured by a device such as a "stable element" (see "Functions of Gyro Devices" in Volume 1). Roll and pitch may be measured by a device such as a ship gyro compass.

Some of the methods of obtaining mathematical equations of coordinate transformation will next be described.

TRANSFORMATIONS FOR ROLL AND PITCH

Let us now solve the three preceding problems of coordinate transformation, using roll and pitch and two different types of level and cross level. (Pitch "precedes" roll, so un-roll "precedes" un-pitch, etc.)

It is required to compute stabilized coordinates B and E from unstabilized coordinates Bd' and Ed', pitch Eio, and roll Zo. The most direct method of accomplishing this is by spherical trigonometry, but it would be very laborious. An alternative would be to convert all these spherical coordinates to rectangular, rotate them as shown on a preceding page, and then convert them back to spherical.



Let the rectangular coordinates of the target in the UNSTABILIZED FRAME be:

- Rdyo — along own ship centerline
- Rdxo — in deck, at right angles to centerline
- Re — normal to deck

Let the rectangular coordinates of the target in the STABILIZED FRAME be:

- Rhyo — along own ship stabilized centerline
- Rhxo — horizontal, at right angles to centerline
- Rv — vertical

The stabilized coordinates are obtained from the unstabilized coordinates as the result of two rotations:

(1) About OY_1, through Zo, ccw

(2) About OX_2, through Eio, ccw

(Clockwise and counterclockwise rotations are always taken as though being viewed looking inward along the axis under consideration).

Rotate the unstabilized rectangular coordinates ccw about Oy , through angle Zo. Let the intermediate coordinates so obtained be called x, y, and z. Refer to equations (10) and (11) on the preceding page, where rotation was about the vertical or "z" axis. Here rotation is about the "y" axis, so we must modify (10) and (11) making our new "y" and the new "z" to the old "x". The equations of transformation then are:

$$x = Rdxo \cos Z0 - Re \sin Zo \qquad (19)$$

$$z = Rdxo \sin Zo + Re \cos Zo \qquad (20)$$

$$y = Rdyo \qquad (21)$$

Equations (13), (14), and (15) use different symbols from (10), (11), and (12), because we are now operating in a deck frame and using own ship centerline as a coordinate axis and a rotation axis.

Now, rotate the intermediate coordinates x, y, and z about the new "x" axis (X), ccw through Eio. A similar modification of (10), (11), and (12) gives us equations for our final coordinates Rhxo, Rhyo, and Rv:

$$Rv = z \cos Eio - y \sin Eio \qquad (22)$$

$$Rhyo = z \sin Eio + y \cos Eio \qquad (23)$$

$$Rhxo = x \qquad (24)$$

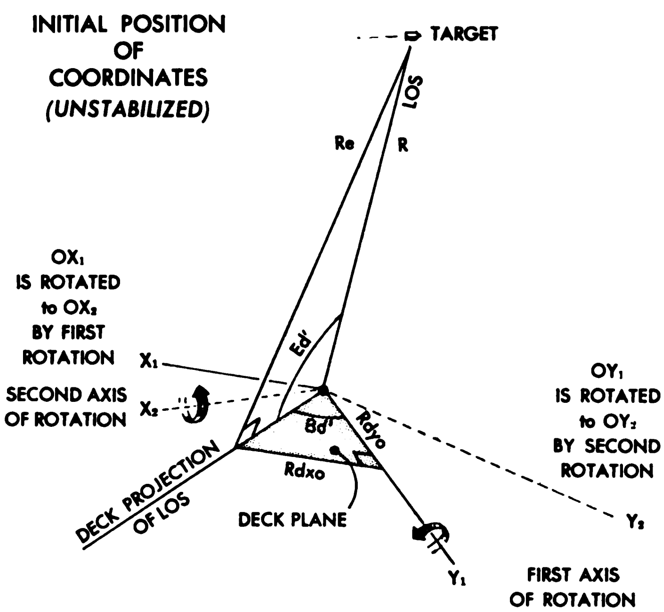
COMPOUND ROTATION . . . *ANALYSIS*

First let us obtain unstabilized rectangular coordinates Rdyo, Rdxo, and Re in terms of unstabilized spherical coordinates Bd', Ed' (and R). (See "Coordinate Types and Conversion").

$$Rdyo = R \cos Ed' \cos Bd' \qquad (13)$$

$$Rdxo = R \cos Ed' \sin Bd' \qquad (14)$$

$$Re = R \sin Ed' \qquad (15)$$

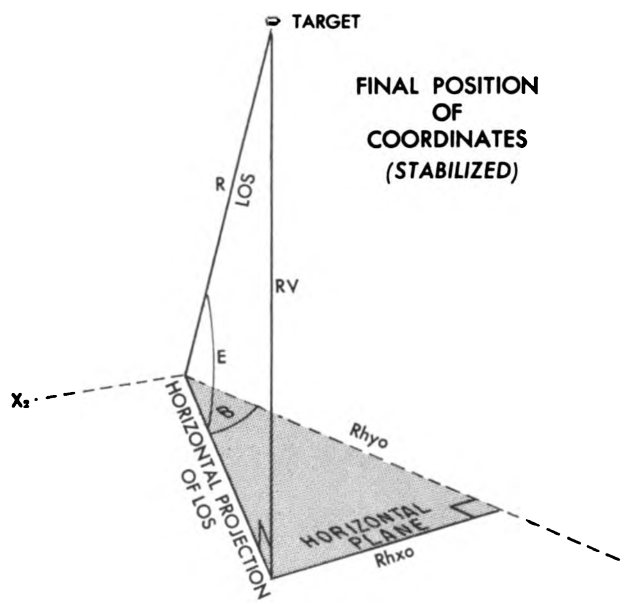


Let us also obtain stabilized rectangular coordinates Rhxo, Rhyo, and Rv in terms of stabilized spherical coordinates B, E (and R).

$$Rhyo = R \cos E \cos B \qquad (16)$$

$$Rhxo = R \cos E \sin B \qquad (17)$$

$$Rv = R \sin E \qquad (18)$$



Substitute (19), (20) and (21) in (22), (23), and (24), finally obtaining:

$$Rv = Rdxo \sin Zo \cos Eio - Rdyo \sin Eio + Re \cos Zo \cos Eio \qquad (25)$$

$$Rhyo = Rdxo \sin Zo \sin Eio + Rdyo \cos Eio + Re \cos Zo \sin Eio \qquad (26)$$

$$Rhxo = Rdxo \cos Zo - Re \sin Zo \qquad (27)$$

Convert to spherical coordinates by substituting equations (13) through (18) in equations (25), (26), and (27). (As "R" occurs in every term, we can drop it out.)

$$\sin E = \cos Ed' \sin Bd' \sin Zo \cos Eio - \cos Ed' \cos Bd' \sin Eio + \sin Ed' \cos Zo \cos Eio \qquad (28)$$

$$\cos E \cos B = \cos Ed' \sin Bd' \sin Zo \sin Eio + \cos Ed' \cos Bd' \cos Eio + \sin Ed' \cos Zo \sin Eio \qquad (29)$$

$$\cos E \sin B = \cos Ed' \sin Bd' \cos Zo - \sin Ed' \sin Zo \qquad (30)$$

$$\sin B = \frac{\cos Ed' \sin Bd' \cos Zo - \sin Ed' \sin Zo}{\cos E} \qquad (31)$$

Obtain E from (28); substitute cos E in (31). Then, (28) and (31) are the equations of transformation for the compound rotation.

We have transformed Director Train (Bd') and Director Elevation (Ed') (Unstabilized Sights) to Relative Target Bearing (B) and Target Elevation (E).

LEVEL AND CROSS LEVEL . . . (Ei and Zd)

Our object now is to compute stabilized coordinates B and E from partly stabilized coordinates Bd and Ed, level Ei, and cross level Zd. Here, a direct solution is easily obtained by spherical trigonometry. (Note that B and Bd are measured from own ship centerline, which is not an axis of rotation. This, in itself, debars the use of rectangular coordinates.) The value of B is obtained as shown below.

Since the horizontal plane is through the "equator" of the sphere, and the great circle arcs Ei and Eio are perpendicular to it, they are parts of "meridians". Produced downward, they intersect at the lower "pole", making an angle equal to B. Also, the lengths of both meridians, measured from equator to pole, are 90 degrees of arc. Subtraction gives us the lengths of the dotted arcs: 90°-Ei and 90°-Eio.

If we apply the "cosine rule" (twice), and the "sine rule" (once) to the spherical triangle formed by Bd and the dotted arcs, and simplify, we obtain:

$$\sin Eio = \sin Ei \cos Bd + \cos Ei \sin Bd \sin Zd \tag{32}$$

$$\cos Bd = \sin Ei \sin Eio + \cos Ei \cos Eio \cos B \tag{33}$$

$$\frac{\cos Eio}{\cos Zd} = \frac{\sin Bd}{\sin B} \tag{34}$$

Remember, we do not know Eio. The measuring device measures only Ei and Zd. For that reason we need all three equations (32) through (34).

From (34):

$$\cos Eio = \sin Bd \cos Zd / \sin B \tag{35}$$

Substituting value of sin Eio from (32), and value of cos Eio from (35) in (33):

$$\begin{aligned}\cos Bd = {} & \sin^2 Ei \cos Bd \\ & + \sin Ei \cos Ei \sin Bd \sin Zd \\ & + \cos Ei \cos B \sin Bd \cos Zd / \sin B\end{aligned} \tag{36}$$

Dividing both sides of (36) by cos Bd, and remembering that sin Bd/cos Bd = tan Bd, and cos B/sin B = cot B, we obtain:

$$\begin{aligned}1 = {} & \sin^2 Ei + \sin Ei \cos Ei \tan Bd \sin Zd \\ & + \cos Ei \cot B \tan Bd \cos Zd\end{aligned} \tag{37}$$

Transposing and simplifying gives us:

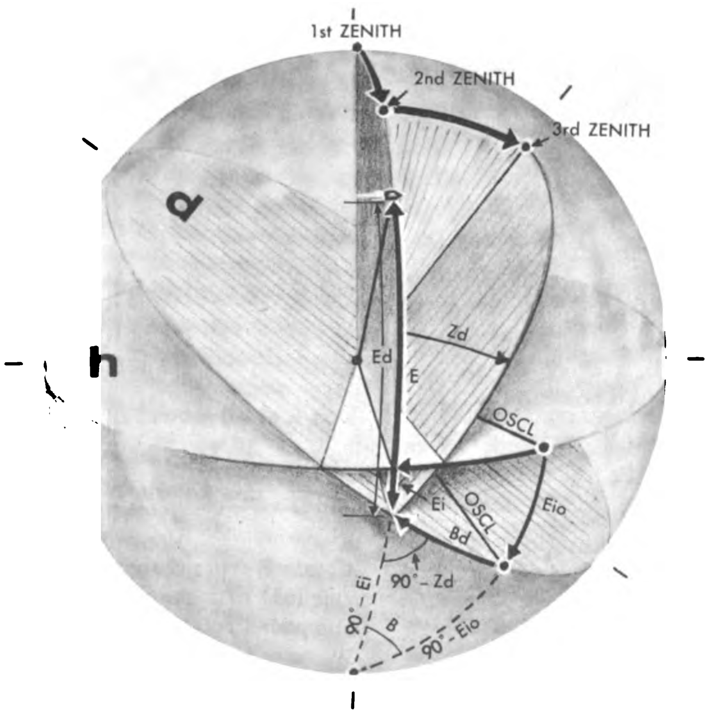
$$\cot B = \frac{\cos Ei - \sin Ei \tan Bd \sin Zd}{\tan Bd \cos Zd} \tag{38}$$

The value of E is obtained directly from:

$$E = Ed - Ei \tag{39}$$

Equations (38) and (39) are the equations of transformation from partially stabilized coordinates to fully stabilized coordinates.

We have transformed Director Train (Bd) and Director Elevation (Ed) (Stabilized Sights) to Relative Target Bearing (B) and Target Elevation (E).

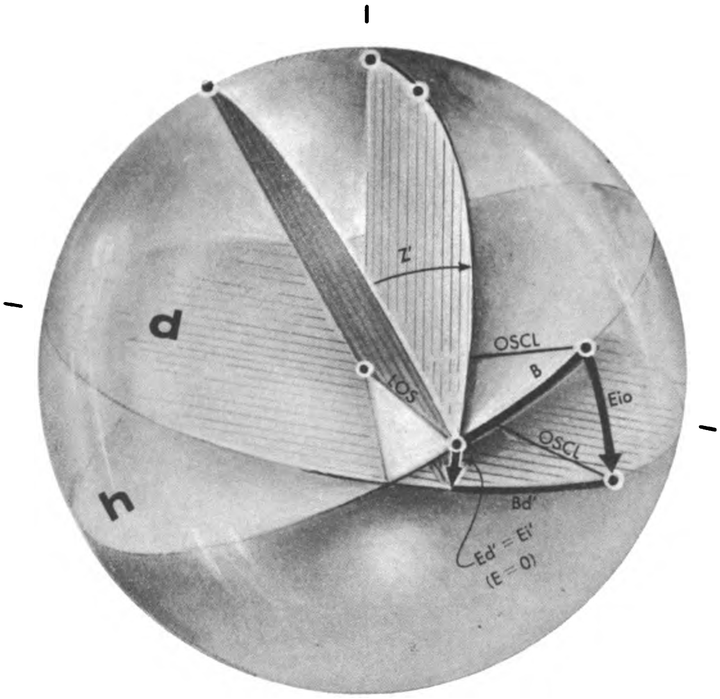


CROSS LEVEL AND LEVEL . . . (Z' and Ei')

We can also compute B and E, using cross level Z' and level Ei' (we use these quantities only with a horizontal line-of-sight.) Here, E=0

To compute B, proceed as follows:

Apply the cosine and sine rules to the two spherical triangles enclosed by arcs B, Eio, and q; and Bd', Ei', and q: (q is a great-circle arc).



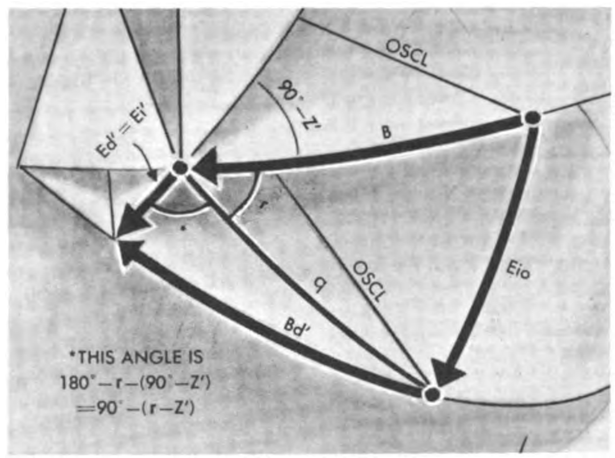
$$\cos q = \cos B \cos Eio = \cos Bd' \cos Ei' \tag{40}$$

$$\sin q = \frac{\sin Eio}{\sin r} = \frac{\sin Bd'}{\cos (r-Z')} \tag{41}$$

$$\cos B = \frac{\cos Bd' \cos Ei'}{\cos Eio} \tag{42}$$

where cos Eio can be obtained from sin Eio, which can be obtained by manipulation of equations (40) and (41).

We have transformed Director Train (Bd') and Director Elevation (Ed') — in this case equal to Level (Ei') — (Unstabilized Sights), to Relative Target Bearing (B) and Target Elevation (E).



SUMMARY

We have shown how the coordinates of a target measured from two different reference frames are different. We performed the transformation of coordinates from one frame to another when the frames are mutually displaced, and when they are mutually rotated. Displacement transformations are important in solving problems of parallax; rotation transformations are important in solving problems of stabilization. We gave special attention to rotation transformations, presenting specific examples in which one reference frame is in the tilting deck of a ship, and the other reference frame is horizontal.

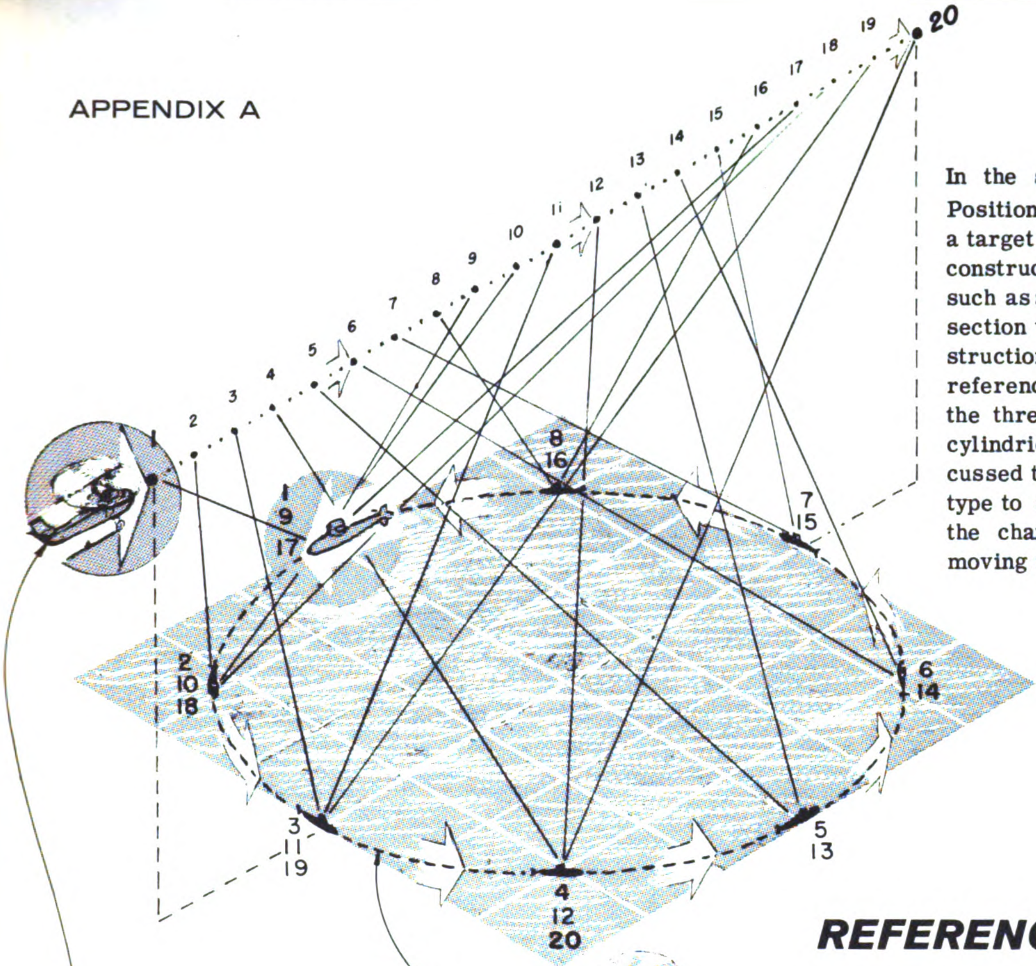
Using the spherical diagram, we described the deck tilt as a "compound rotation" in the following terms: pitch and roll, level Ei and cross level Zd, and cross level Z' and level Ei'. The example using roll and pitch proved cumbersome mathematically, and would have been difficult to solve if we had not converted the coordinates from spherical to rectangular. But this example was given for three reasons: (1) the concept of roll and pitch is easier to grasp, at first, than the concept of level and cross level, (2) the example shows how rectangular coordinates and coordinate conversions can be helpful, and (3) the use of roll and pitch reduces the number of measuring devices required. (All tilt measurements can then be made with the ship gyro compass, so a separate stable element is not needed.) The use of level and cross level has many applications in weapon control systems. The example using Ei and Zd gives a direct solution by spherical trigonometry. The example using Z' and Ei' also involves spherical trigonometry equations; these equations would not readily yield an "explicit" solution, but can be mechanized.

All three methods of resolving deck tilt lead to rather cumbersome solutions of the coordinate transformation problem. Therefore, in practice, exact solutions are usually not attempted. Empirical equations give sufficiently close approximations, are easier to mechanize, and enable us to obtain more rapid solutions. Often, the empirical equation gives the difference between B and Bd or Bd' (known as "deck tilt correction".) Our second example, using level and cross level with stabilized sights, particularly lends itself to this treatment, as the correction B-Bd is usually small.

SECTION 4

In the section "Reference Frames for Position" we showed that the position of a target can be defined only when we have constructed some kind of reference frame, such as a plane, line, and point. In another section we showed how to express the instructions (to get to the target from the reference point) as coordinates, stressing the three coordinate types: rectangular, cylindrical, and spherical. We also discussed the conversion from one coordinate type to another within a given frame, and the changes in coordinates caused by moving from one frame to another frame.

In this section we shall consider the subject of targets and frames in motion. Just as "where" needs precise definition, so do "how fast" and "in what direction" need to be precisely defined.

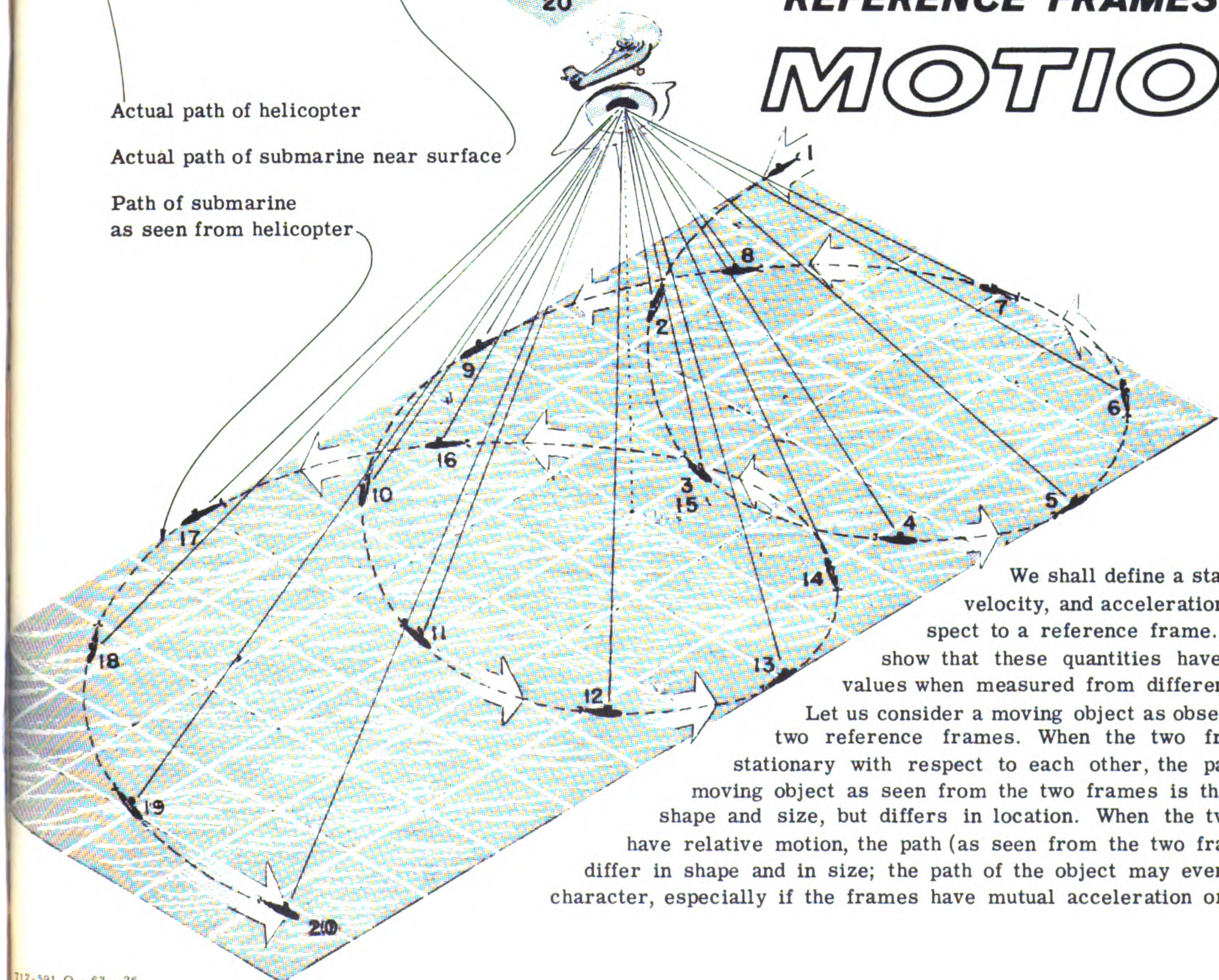


REFERENCE FRAMES FOR MOTION

Actual path of helicopter

Actual path of submarine near surface

Path of submarine as seen from helicopter



We shall define a state of rest, velocity, and acceleration, with respect to a reference frame. We shall show that these quantities have different values when measured from different frames. Let us consider a moving object as observed from two reference frames. When the two frames are stationary with respect to each other, the path of the moving object as seen from the two frames is the same in shape and size, but differs in location. When the two frames have relative motion, the path (as seen from the two frames) may differ in shape and in size; the path of the object may even differ in character, especially if the frames have mutual acceleration or rotation.

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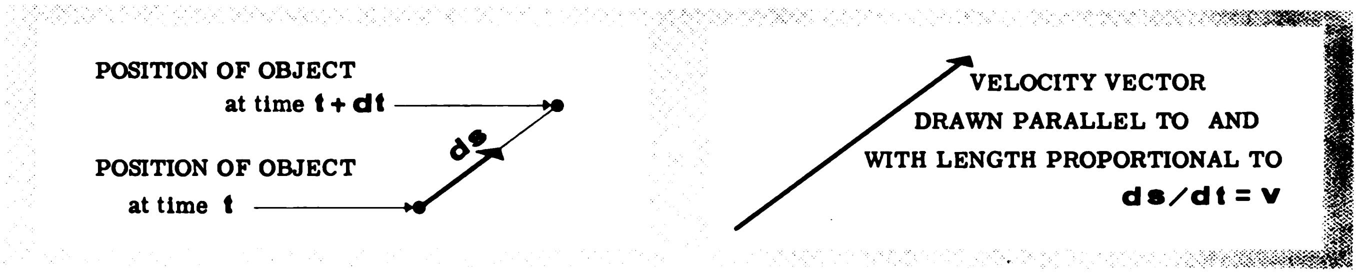
MOTION WITH RESPECT TO A FRAME

If we wish to apply reference frames and coordinate systems to moving objects, we must first define motion. Suppose that we observe the coordinates of an object with respect to a given reference frame, not once, but repeatedly. If the coordinates do not change, we say that the object is stationary with respect to that frame; if they change, we say that the object is moving with respect to that frame.

The rate of change of position with respect to time can be expressed as a vector . . .
. . . THE VELOCITY VECTOR

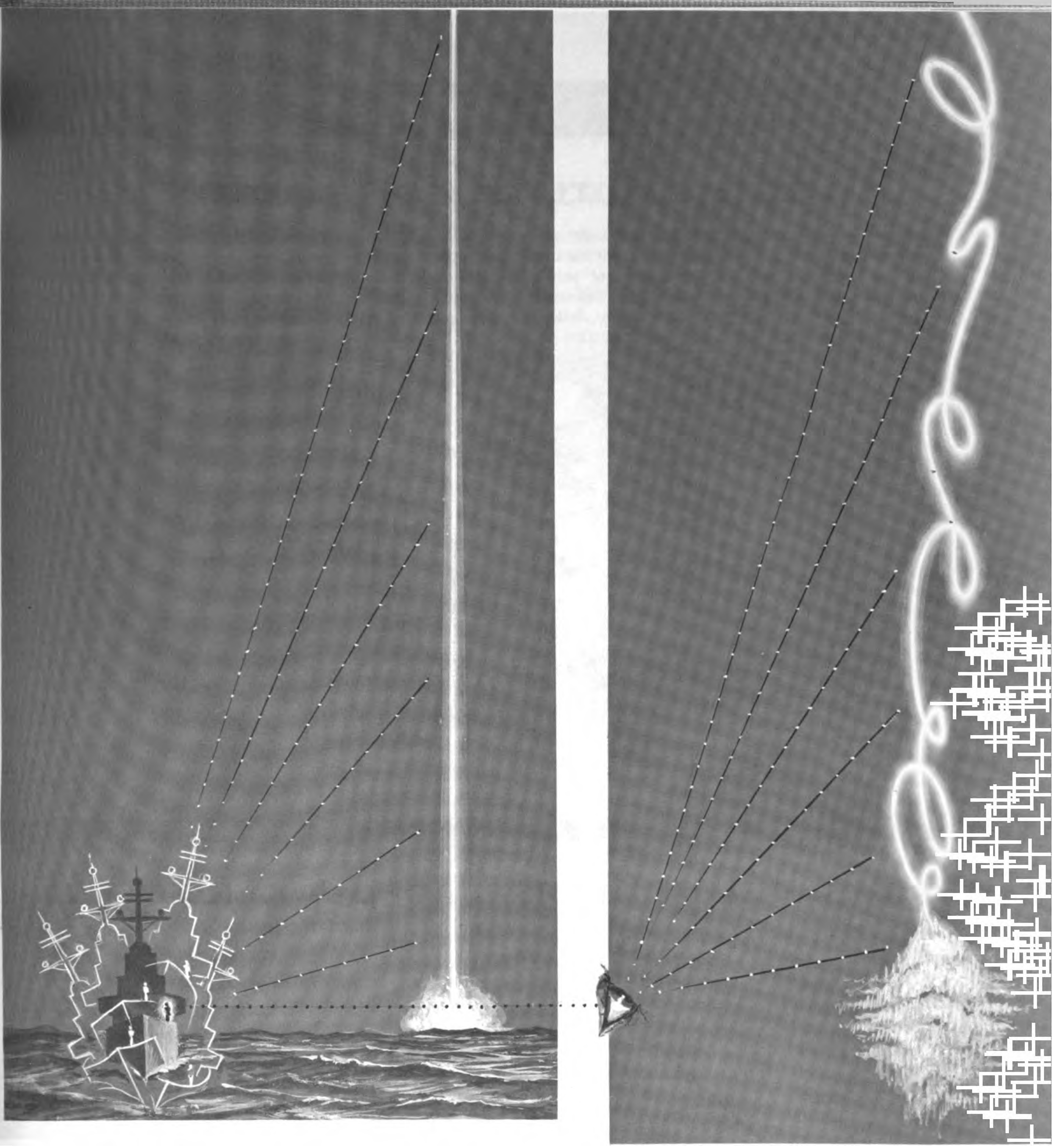


If the velocity vector itself does not change in magnitude or direction, we say that the body is moving with respect to that frame at constant velocity. If the velocity vector does change in magnitude or direction, we say that the body has acceleration with respect to that frame. We shall study the paths of a moving object as seen from different frames.

The motion of an object with respect to a frame may be defined as what its motion seems to be from the point of view of an observer in that frame who is located at the reference point. He may think, rightly or wrongly, that the frame is at rest with respect to Earth. If the frame is moving with respect to Earth, he is thinking wrongly, and his opinion of the object's motion will be influenced by the frame's motion.

For instance, a man in a standing railway coach looking out of a window at another train which is moving forward may think that his train is moving backward . . . . . .

until he looks beyond the other train and compares both trains with the platform; then he sees that his train is standing, and the other train is moving forward . . . .



Or, take the observations of a man on the deck of a rolling ship who does not know the ship is rolling. Observing a missile which is going straight up in the air, ... if he could not see the horizon he might think that the missile was pursuing a complicated path like the one shown above.

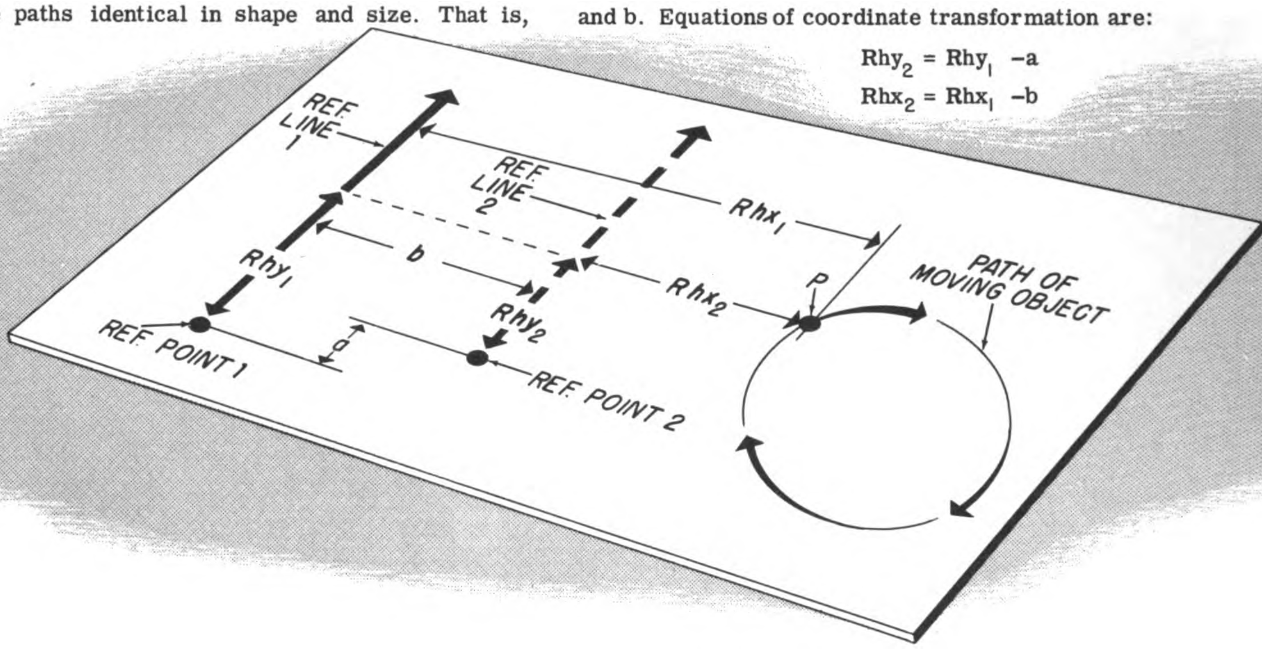
This instance may seem somewhat exaggerated, because, unless the roll is very slight, a sailor can tell when a ship is rolling by means of his "inner ear". However, if the position of the missile were measured continuously from the deck by unstabilized instruments, they would record it as moving in a complicated path. (Human beings carry their own built-in stabilizers.) But before investigating the paths of moving objects as seen from moving frames, let us consider the case of two frames at rest which are mutually displaced or rotated.

FRAMES MUTUALLY DISPLACED

Frames 1 and 2 and the path of moving object are coplanar. Frame 2 is displaced but not rotated with respect to frame 1. Coordinates of points along the path of the moving object, as measured from either reference point and line, when plotted, give paths identical in shape and size. That is, paths "seen" in either frame are the same in shape and size. But the coordinates of any point are different in the two frames. (Point P is a sample point on the path of the moving object.)

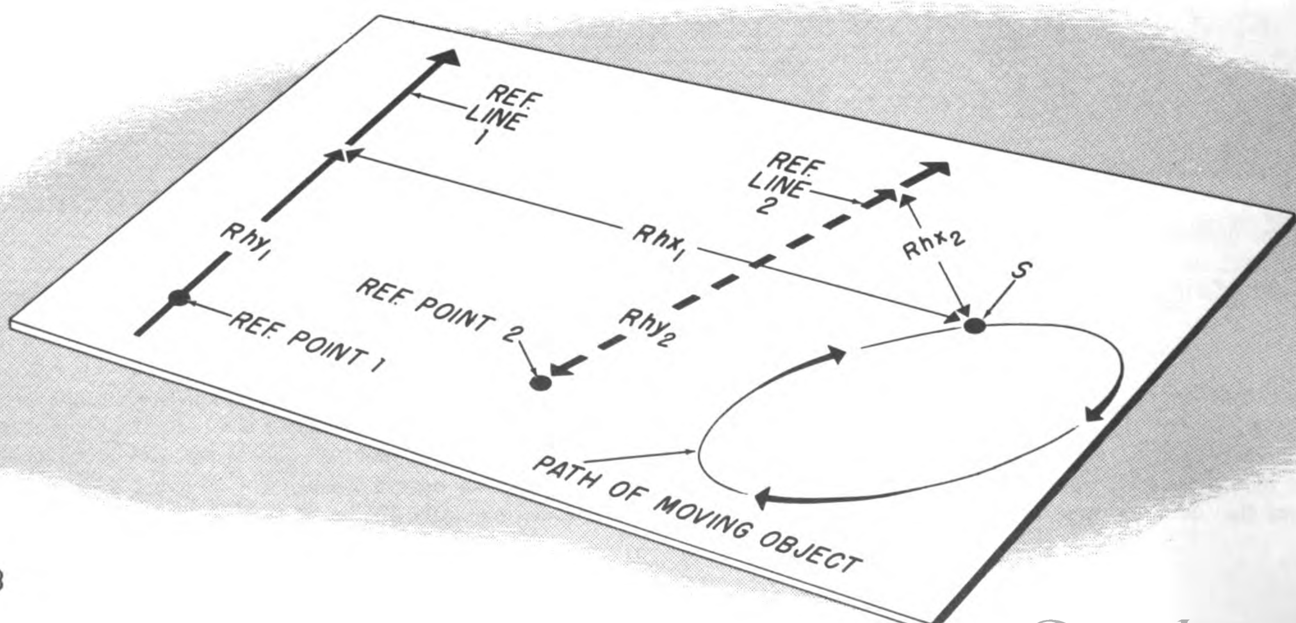
Components of displacement between frames are a and b. Equations of coordinate transformation are:

$$Rhy_2 = Rhy_1 - a$$
$$Rhx_2 = Rhx_1 - b$$



FRAMES MUTUALLY DISPLACED AND ROTATED

Frame 2 is rotated and displaced with respect to frame 1. Path of the moving object is the same, in shape and size, as viewed from both frames. (Point S is a sample point on the path of moving object.) Displacements, rotation angle, and the coordinates of transformation are not shown.



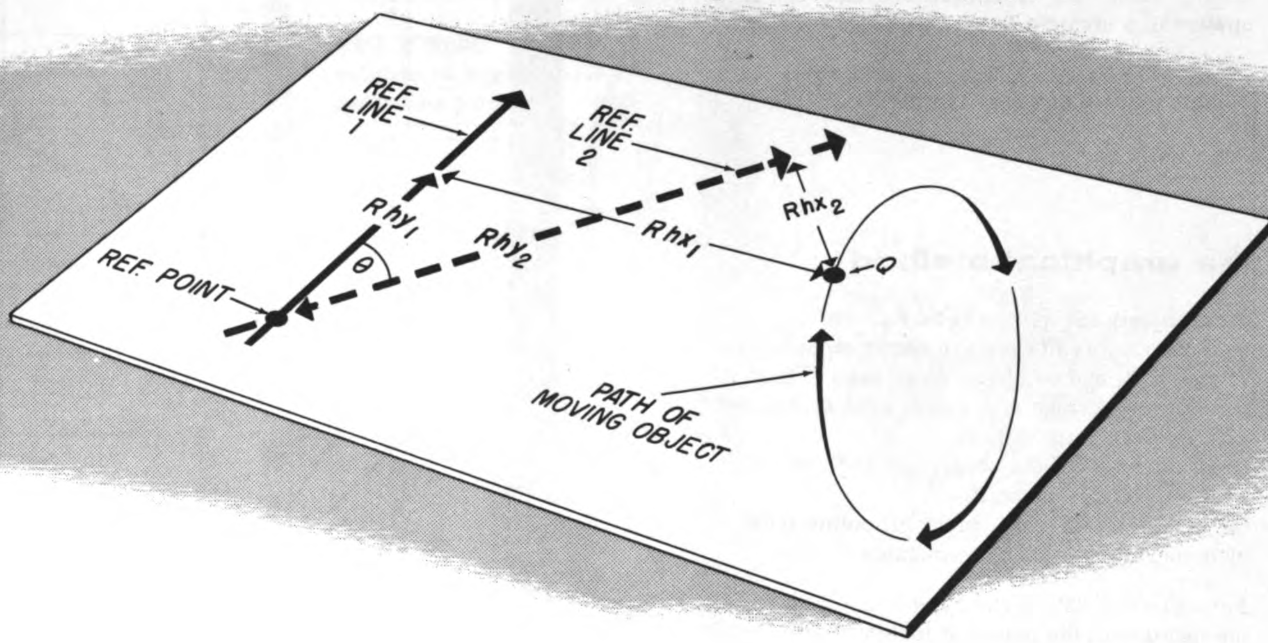
FRAMES MUTUALLY ROTATED

Frame 2 is rotated with respect to frame 1, about a common reference point. Path of moving object, as seen from both frames, is same in shape and size. (Point Q is a sample point on path of moving object.)

Angle of rotation is θ.
Equations of coordinate transformation are:

$$Rhy_2 = Rhy_1 \cos\theta + Rhx_1 \sin\theta$$

$$Rhx_2 = -Rhy_1 \sin\theta + Rhx_1 \cos\theta$$



FRAMES MUTUALLY DISPLACED AND ROTATED IN THREE DIMENSIONS

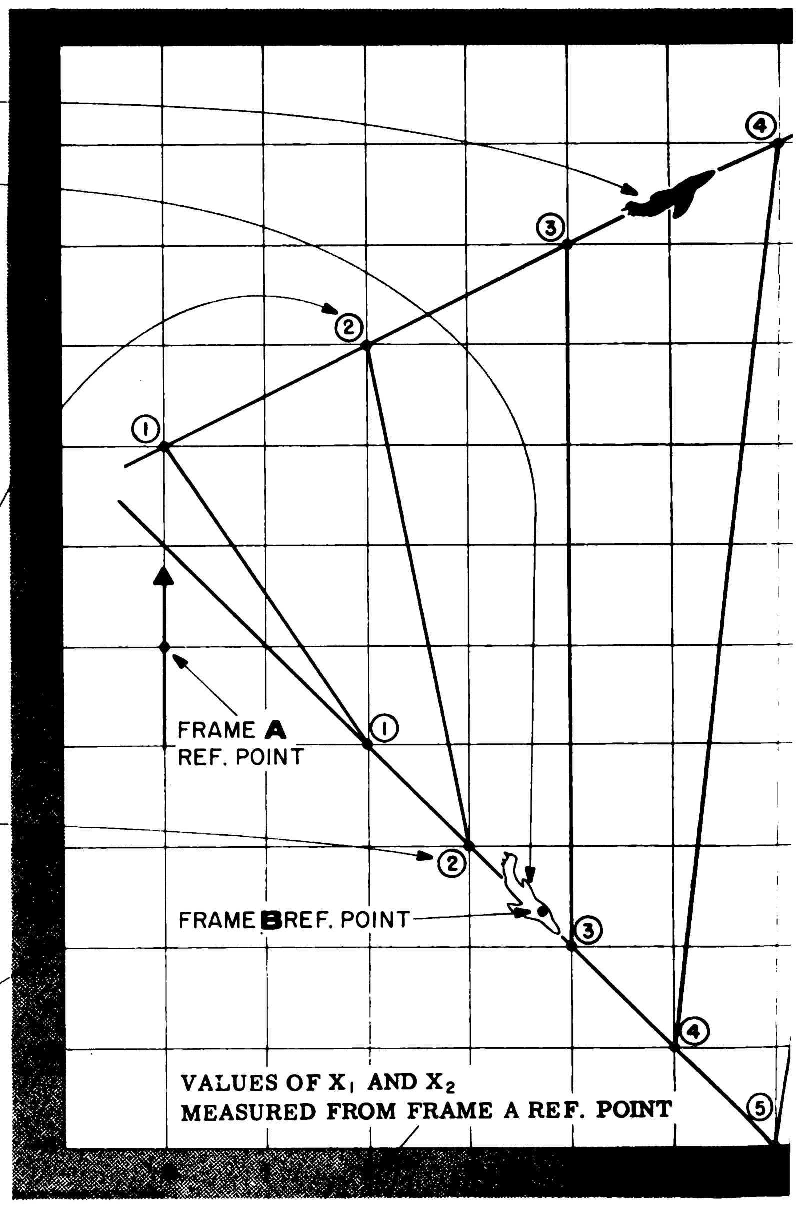
Two frames rotated and displaced in three dimensions were shown in "Reference Frames for Position". They will not be shown here because showing them with their respective coordinates of a moving object might be confusing. However, the two paths of the object, as seen from the two respective frames, are the same in shape and size.

FRAMES HAVING MUTUAL MOTION

FRAMES HAVING MUTUAL CONSTANT LINEAR VELOCITY.

Two methods of comparing the path of an object as seen in Frames A and B are illustrated in our first example. Frame A, hovering and stationary with respect to Earth, "sees" the object (an airplane) flying obliquely upward in a straight line.

Frame B has a reference point in another airplane flying obliquely downward in a straight line.



the graphical method

for translational motion is as follows:
Make two figures (They are shown completed at the right, and on the opposite page.) In the first figure, Frame A is considered stationary with respect to the paper.
Draw the paths of the object and of Frame B with respect to Frame A.
On each of these paths, mark off points labeled by numerals representing instants in time.

For example, "2" on the object path indicates the location of the object at instant 2.

"2" on Frame B path, indicates the location of Frame B reference point at instant 2.

If we join these two points, we have the instantaneous position of the line-of-sight from Frame B reference point to the moving object at instant 2.

Draw a series of these lines-of-sight at instants 1, 2, 3, 4, and 5.

VALUES OF Y_1 AND Y_2 MEASURED FROM FRAME A REF. POINT

the mathematical method

The mathematical method is as follows: Draw the same two figures without the lines-of-sight. Choose a fixed point in the first figure for Frame A reference point. Set up coordinates in both figures. If the frames have mutual translational motion without rotation, use rectangular coordinates x and y. Let the numerals along the paths be values of t, time (measured since some arbitrary time). Obtain x_1 and y_1, the coordinates of the object with respect to Frame A reference point, in terms of t.
Obtain x_2 and y_2, the coordinates of Frame B reference point with respect to Frame A reference point, in terms of t.

Path of object with respect to Frame A reference point:

$x_1 = 2(t-1) = 2t-2$

$y_1 = 2+1(t-1) = t+1$

$(t = y_1 - 1$

$x_1 = 2y_1 - 2 - 2$

$= 2y_1 - 4)$

Path of Frame B reference point with respect to Frame A reference point:

$x_2 = 1+t$

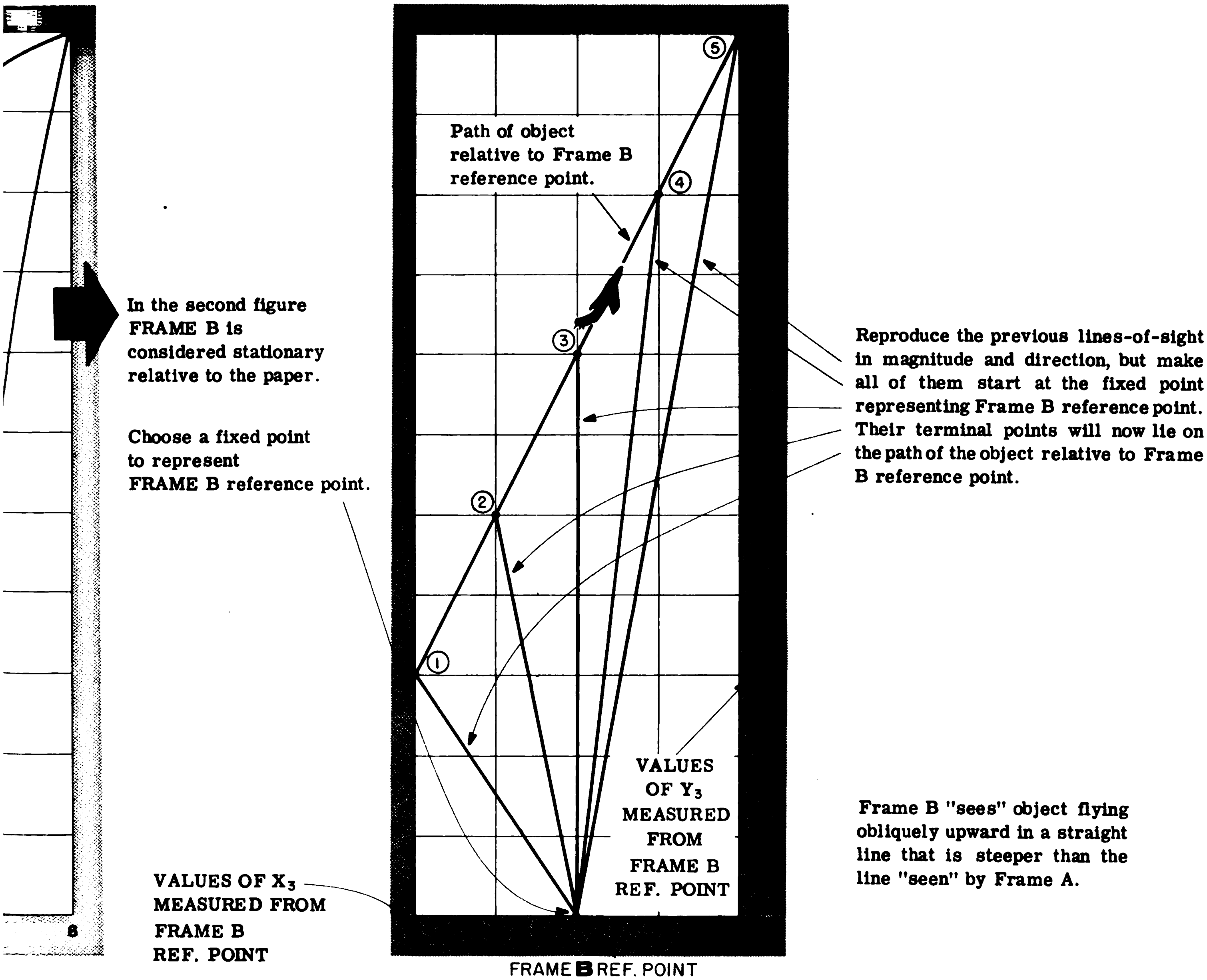
$y_2 = -t$

$(x_2 = 1 - y_2)$

GENERAL When a moving object is observed from two frames which are moving relative to each other, its path as seen from one frame differs from the path as seen from the other frame. In some cases, the paths will be different in character as well as in location.

In the examples given, we compare the path of a moving object relative to "Frame A" with the path relative to "Frame B", which is moving (in translation or rotation) relative to Frame A. This comparison of paths is made graphically and mathematically.

LINEAR PATH IN FRAME A



By subtraction, obtain $x_3 = x_1 - x_2$, and $y_3 = y_1 - y_2$. Then, x_3 and y_3 are the coordinates of the object with respect to Frame B reference point, in terms of t. If possible, eliminate t, and obtain an equation in x_3 and y_3. This will clearly reveal the nature of the new path. As a matter of interest in the examples given, we also give equations in x_1, y_1, and in x_2, y_2.

Path of object with respect to Frame B reference point:

$x_3 = x_1 - x_2 = 2t - 2 - 1 - t = t - 3$

$y_3 = y_1 - y_2 = t + 1 + t = 2t + 1$

$t = x_3 + 3$

$y_3 = 2x_3 + 6 + 1$

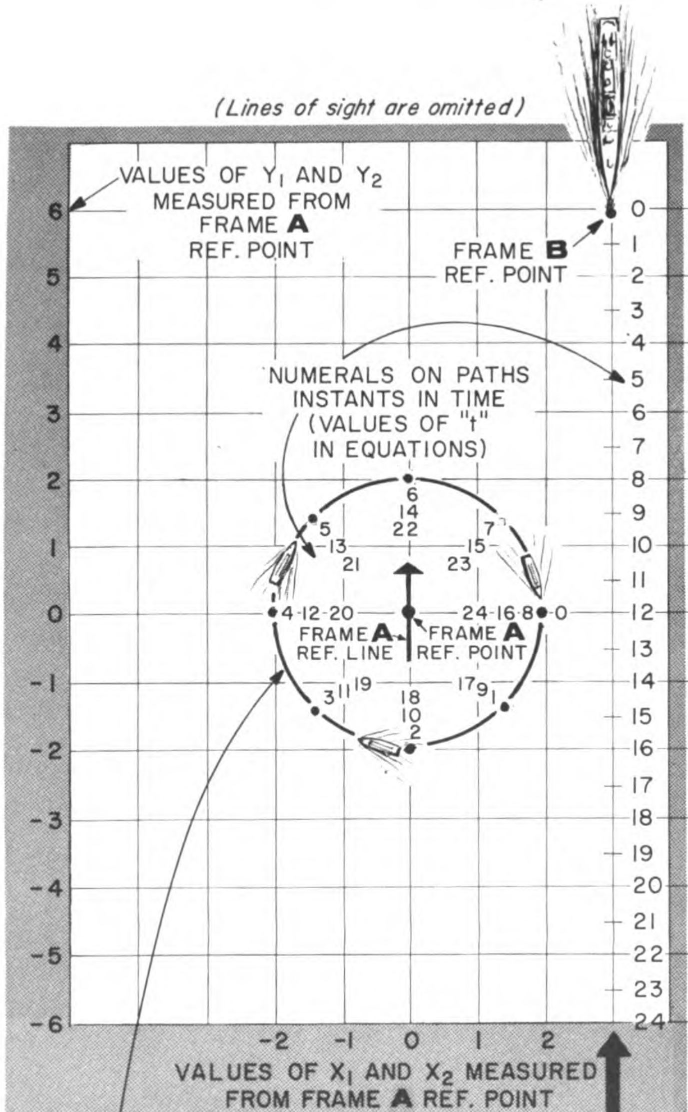
$= 2x_3 + 7$ (a straight line).

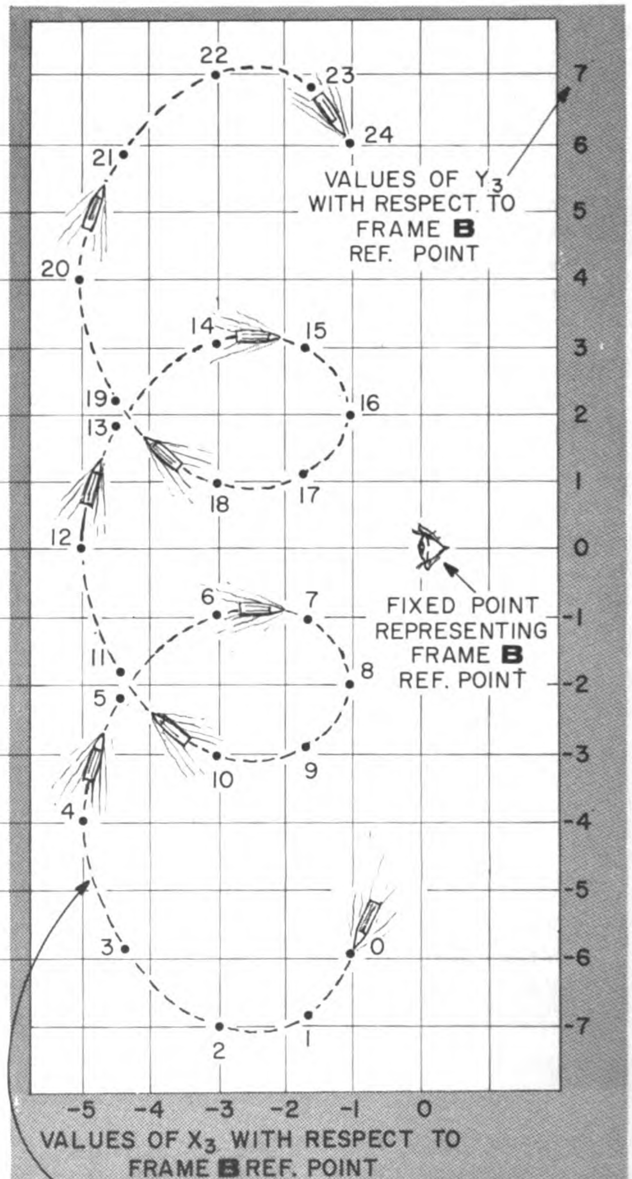
In the examples of mutual displacement motion, it is assumed that the reference lines of Frames A and B remain parallel, and also that Frame A is stationary relative to Earth. Reference lines need not be shown.

The graphical and mathematical methods for mutual rotational motion of the frames are simple; they will be demonstrated after the discussion of some additional examples of translational motion.

FRAMES HAVING MUTUAL CONSTANT LINEAR VELOCITY,
CIRCULAR PATH IN FRAME A

Frame A sees the moving object (a PT boat) describing a circular course at constant speed. Frame B is a cruiser steaming in a straight line at constant velocity. It sees the PT boat path describing a cycloid.





Path of object with respect to Frame A reference point:

$$x_1 = 2\cos\frac{\pi t}{4}$$

$$y_1 = -2\sin\frac{\pi t}{4}$$

$$(x_1^2 + y_1^2 = 4)$$

Path of Frame B reference point with respect to Frame A reference point:

$$x_2 = 3$$

$$y_2 = 6 - \frac{t}{2}$$

Path of object with respect to Frame B reference point:

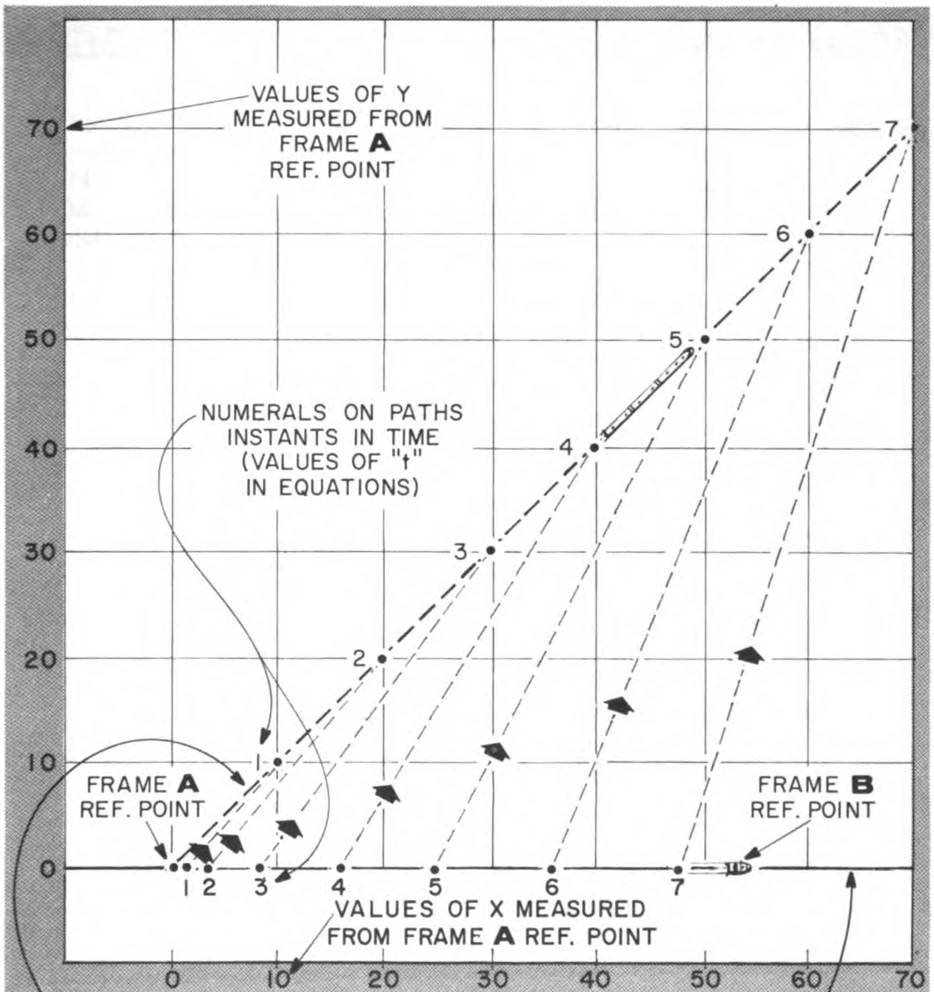
$$x_3 = x_1 - x_2 = 2\cos\frac{\pi t}{4} - 3$$

$$y_3 = y_1 - y_2 = -2\sin\frac{\pi t}{4} + \frac{t}{2} - 6$$

These are the parametric equations of a cycloid.

FRAMES HAVING MUTUAL CONSTANT LINEAR ACCELERATION

Throughout this example it is supposed that Frame A is stationary with regard to Earth. Frame A sees the moving object (a railroad train) traveling in a straight line at constant speed. Frame B is a rocket sled with a constant acceleration in a straight line. It sees the train path describing a parabola.



Path of object with respect to Frame A reference point:

$x_1 = 10t$

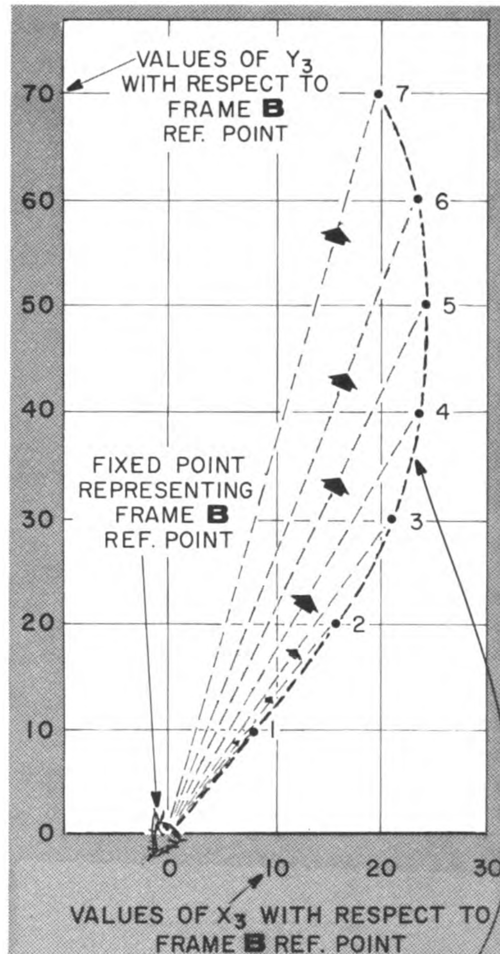
$y_1 = 10t$

$(x_1 = y_1)$

Path of Frame B reference point with respect to Frame A reference point:

$x_2 = t^2$

$y_2 = 0$



Path of object with respect to Frame B reference point:

$x_3 = x_1 - x_2 = 10t - t^2$

$y_3 = y_1 - y_2 = 10t$

$t = \frac{y_3}{10}$

$x_3 = y_3 - \frac{y_3^2}{100}$

This is the equation of a parabola.

FRAMES HAVING MUTUAL

GENERAL

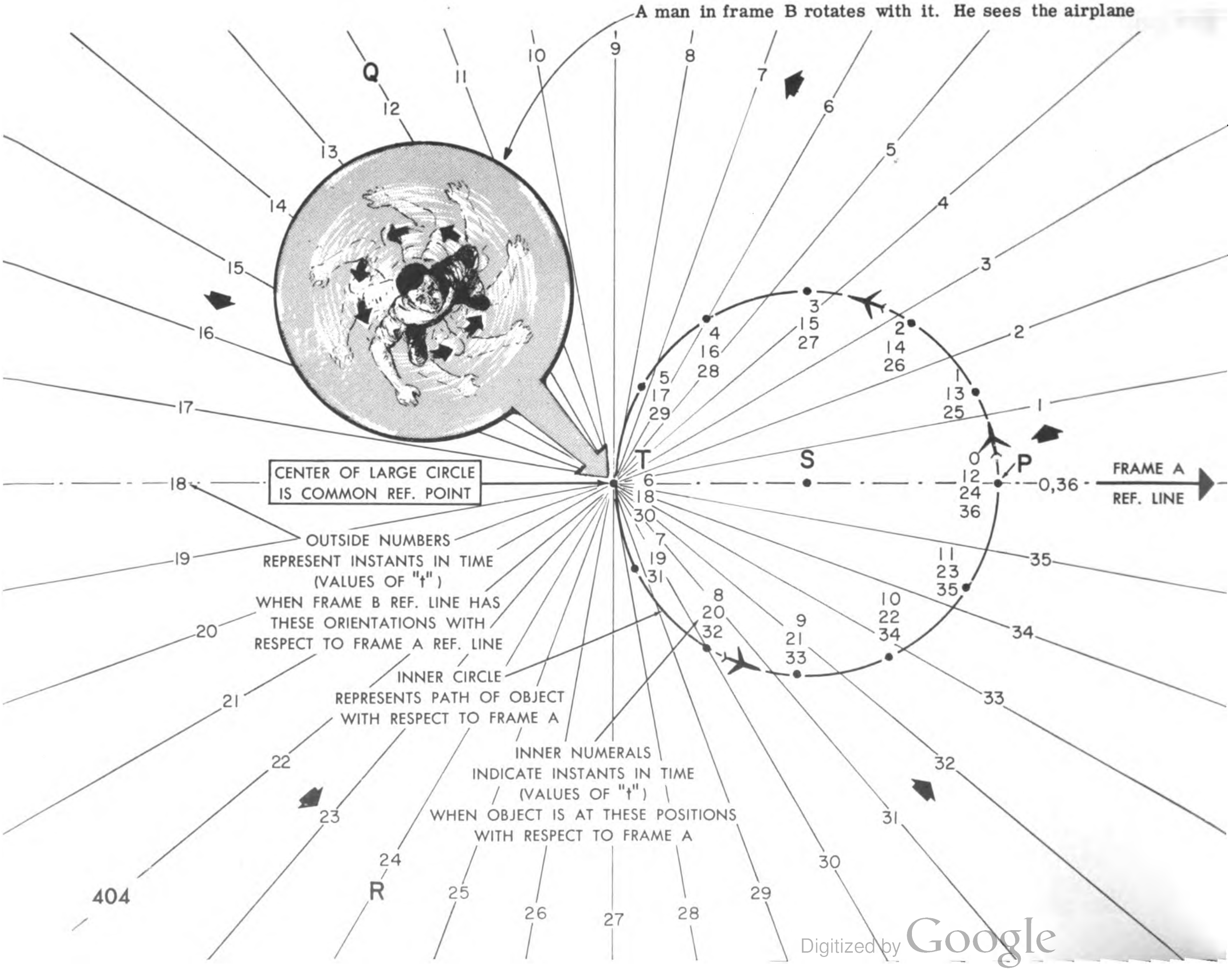
Throughout these examples it is supposed that frame A is stationary with respect to Earth.

Throughout the previous examples, which were concerned with purely translational velocities, we generally ignored reference LINES because the two reference lines remained parallel throughout. We were concerned only with the relative motion of the reference POINTS; i.e., the motion of each frame as a whole.

Now it is supposed that the two reference lines mutually rotate about the common reference point, so that we can not continue to ignore them. For brevity we shall say "with respect to frame A (or B)" when we mean "with respect to reference point and reference line A (or B)". The graphical method is as follows: Draw the path of the moving object with respect to frame A; along the path, number off the instants in time. Through the common reference point, draw a straight line representing reference line A, also other straight lines— similar to

CIRCULAR PATH IN FRAME A

Frame A sees the object (an airplane) moving in a circle at constant speed. Frame B is some stationary shipboard equipment making a slow circular scan; it is located under a point on the perimeter of the path of the airplane. Frame B rotates in the same direction as the airplane, but at one-third the angular velocity. A man in frame B rotates with it. He sees the airplane

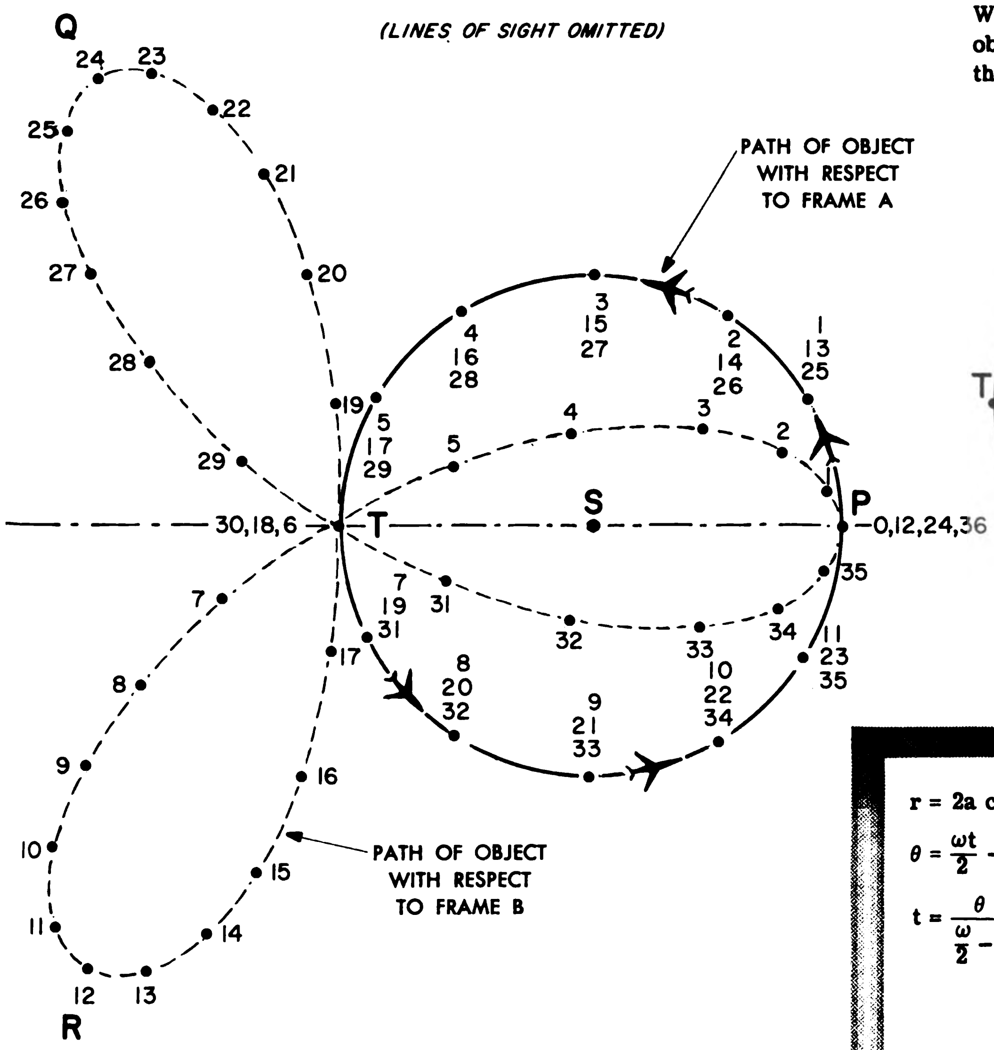


CONSTANT ANGULAR VELOCITY

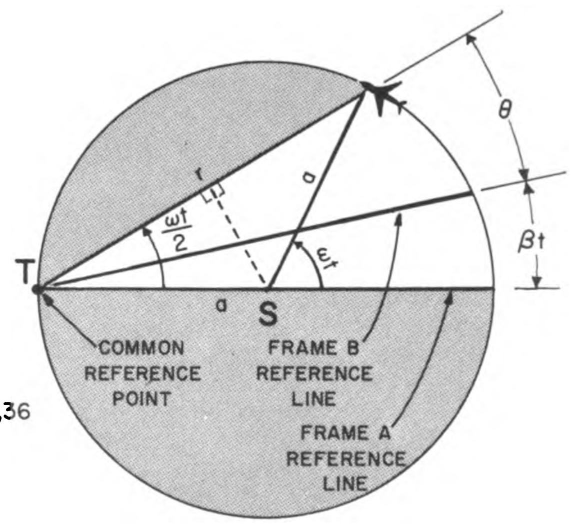
spokes of a wheel—(numbered), representing successive orientations of reference line B at these instants. Suppose that these orientations progress counterclockwise by 10-degree increments. Take each numbered point on the path of the moving object and, without altering its distance from the reference point, rotate it CLOCKWISE, each one 10 degrees more than the last. That will have the same effect as "seeing" it from a counterclockwise-turning position; this will become clear when we discuss the two given examples.

The mathematical method is simplified by the use of polar coordinates r and θ, and the fact that r is the same in both frames. Obtain equations involving r, θ, and t, and eliminate t. Details differ for different examples.
This example is simple enough to be easily solvable by mathematics. In other examples more directly related to weapons control, frame B could be the deck of a rolling ship, or the rotating Earth (frame A being outside Earth). Such examples are more complex, and do not lend themselves to simple mathematical solution.

describing a "3-leaf rose" path. (Changing the relative angular velocities of the airplane and frame B will change the curve seen from frame B into other paths, such as a 2-leaf rose, 8-leaf rose, a straight line, etc.).
Note that at instant "12", when the airplane has made one complete revolution, it is back at point P, where it started. But the man in frame B has swung counterclockwise until he is facing toward point Q. He thinks he is still facing toward P, so, as seen from his point of view, the airplane is located at point R.
As he sees it, the airplane has two rotations. It has reached point R by combining its counterclockwise rotation about point S with a CLOCKWISE rotation (at one-third the rate) about point T.



With the aid of the figure below, we can obtain r and θ, the polar coordinates of the airplane with respect to frame B.



ω = Angular velocity of plane
β = Angular velocity of Frame B

$r = 2a \cos \omega t/2$

$\theta = \frac{\omega t}{2} - \beta t = (\omega/2 - \beta)t$

$t = \frac{\theta}{\frac{\omega}{2} - \beta}$

$r = 2a \cos \frac{\omega/2}{(\omega/2 - \beta)} \theta$

$r = 2a \cos \frac{\omega}{(\omega - 2\beta)} \theta$

If $\beta = \frac{\omega}{3}$: $r = 2a \cos 3\theta$

This is the equation of a 3-leaf rose.

TARGET PREDICTION

In weapons control, one of the major problems is predicting the future position of a target. Before we attempt this, we must know its path. But the foregoing examples showed us that the path depends on what frame of reference it is viewed from, especially if the frames have mutual motion. In all the examples of frames having mutual motion, the path of a moving object as viewed from frame A was simpler than the path as viewed from frame B. Also, instants of time on frame A path (straight line or circle) were evenly spaced, whereas those on frame B path were not.

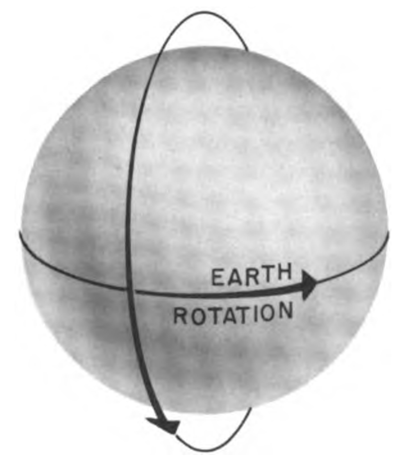
For instance, take our rotating frame example. Frame A path is a circle, where all time instants are evenly spaced. But frame B path is a "rose" (note how crowded together are instants 11, 12, and 13 compared with instants 7, 8 and 9). From these examples it is evident that position prediction would be easier from frame A than from frame B. Because we specified that frame A was stationary with respect to Earth, it appears that the Earth is the most practical frame from which to predict.

In many weapons control problems this is so, but there are exceptions. It must be remembered that, in all the foregoing examples, we chose cases where the path of the moving object was SIMPLE WITH RESPECT TO EARTH. However, some paths are not so simple.

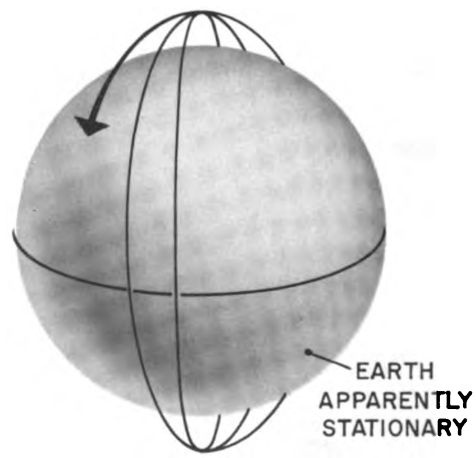
Here are two examples to illustrate this:

(1) A beetle is walking along a radius from the center to the edge of a rotating phonograph disk. With respect to a disk frame, the beetle describes a straight line; with respect to an Earth frame, it describes a spiral path.

(2) A polar satellite is describing a circular orbit around Earth. As seen from a point outside the Earth, this path is a circle. Because the satellite does not share in the rotation of the Earth, it appears to an Earth observer to describe a sort of 3-dimensional spiral path.



PATH OF POLAR SATELLITE
AS SEEN BY OBSERVER
NOT ROTATING WITH EARTH

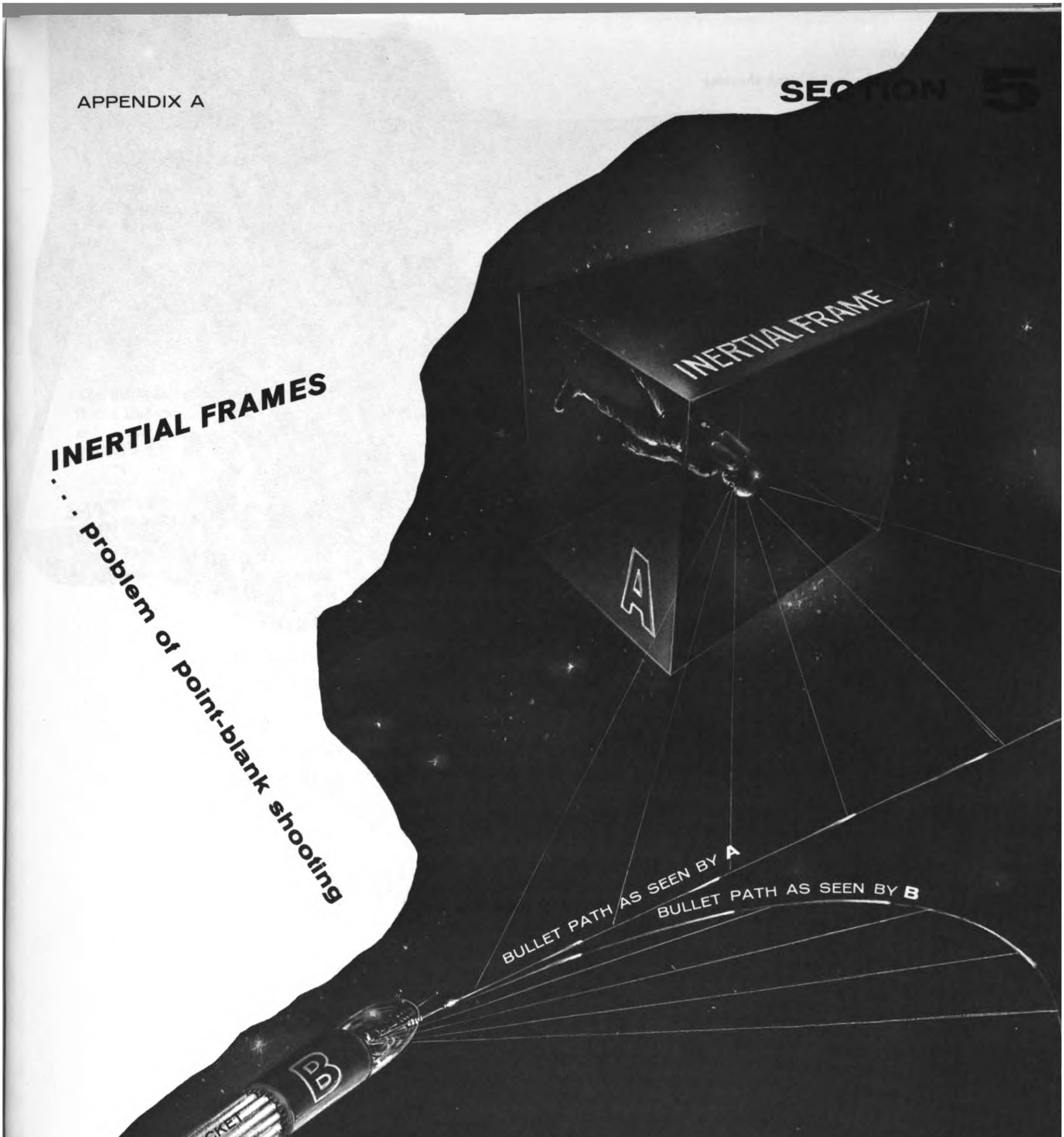


PATH OF POLAR SATELLITE
AS SEEN BY OBSERVER
ROTATING WITH EARTH

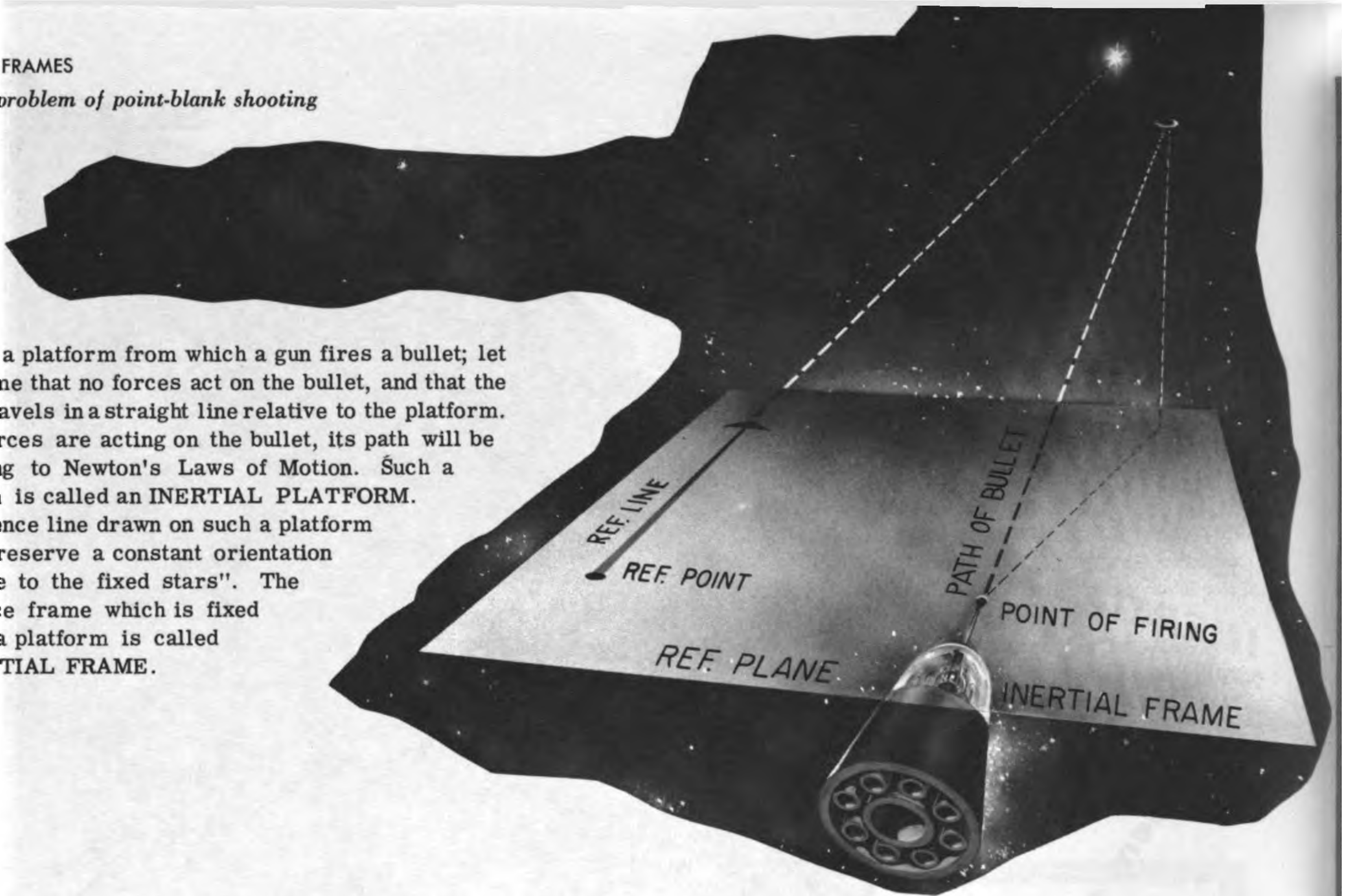
Example (1) is merely one of a large class of special cases--such as a man walking a chalk line across the deck of a rolling ship. His path is simple with respect to the deck, but is quite complicated with respect to Earth. But example (2) has a deeper significance. The "best frame from which to view" is evidently something MORE STABLE THAN EARTH. What is that "something"? In our search for the ideal frame from which to predict, we should examine this question. It approaches the subject of the "absolute reference". Remember, that in weapons control there are important objects (missiles and pieces of equipment) that behave like the polar satellite. An ICBM high above Earth's atmosphere does not share in Earth's rotation. A free gyro, for some time does not rotate with Earth, but remains pointing at the same point in the heavens. Has this outside Earth frame we are looking for something to do with "the fixed stars"? We shall look into this question in the following section.

INERTIAL FRAMES

. . . problem of point-blank shooting

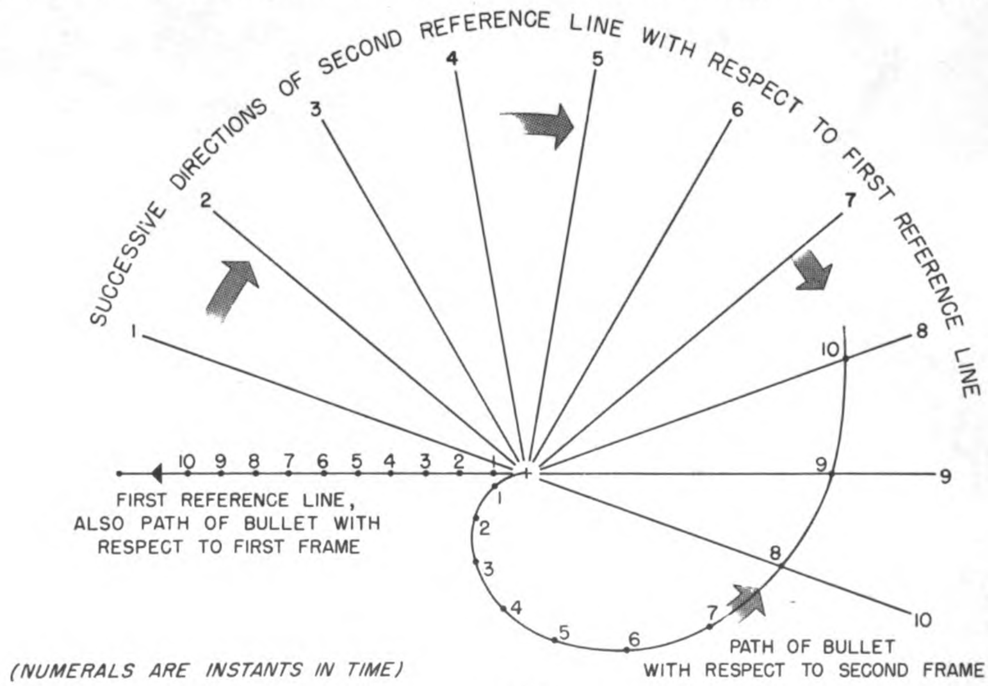


This section defines a special kind of reference frame called an inertial frame, as a frame in which Newton's Laws of Motion are obeyed. This is, generally speaking, the ideal frame in which to measure and predict the position and motion of a target. An inertial frame set up in outer space is used as a basis for studying the problem of hitting a target that is shot at point-blank. Non-rotation of the line-of-sight "in inertial space" is seen to be the condition for successful point-blank shooting. This study of point-blank shooting paves the way for the more general problem of shooting a target whose line-of-sight does rotate; such a target must be shot at, not point-blank, but with an offset. Throughout this section, the word "bullet" is used as a simplification applying to any type of projectile or missile fired or launched.



Imagine a platform from which a gun fires a bullet; let **us assume** that no forces act on the bullet, and that the **bullet travels** in a straight line relative to the platform. **When forces** are acting on the bullet, its path will be **according** to Newton's Laws of Motion. Such a **platform** is called an INERTIAL PLATFORM. **A reference** line drawn on such a platform **would preserve** a constant orientation **"relative** to the fixed stars". The **reference** frame which is fixed **in such a** platform is called **an INERTIAL FRAME**.

Let us observe the path of the bullet as seen from **another** frame. If the path of the bullet (with no forces **acting on** it) appears straight, the new frame is inertial. **Suppose** that the new frame is stationary with respect to **the first** frame. The path of the bullet appears straight, **so the new** frame is inertial.
Suppose that the new frame has a constant linear velocity with respect to the first frame. The path of the bullet appears straight, so the new frame is inertial. Suppose that the new frame is accelerating or rotating with respect to the first frame. The path of the bullet appears curved, so the new frame is not inertial. A rotation case is shown below. The cases of constant velocity and linear acceleration were shown in the preceding section.



note

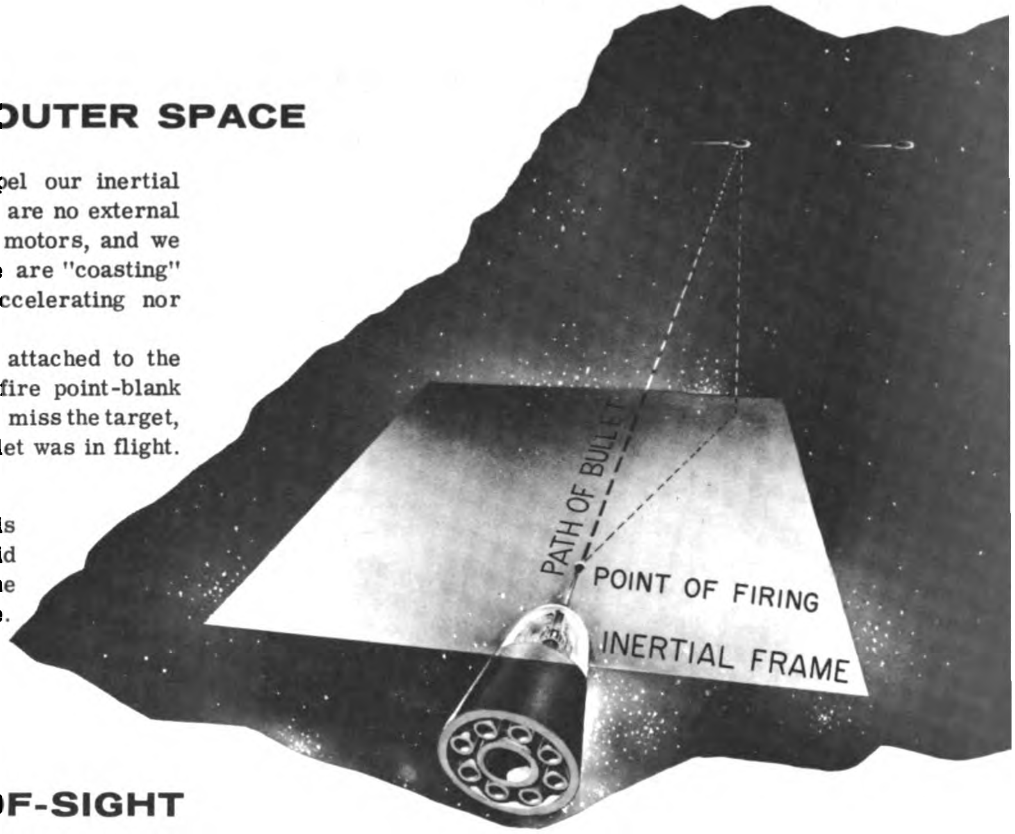
Throughout this section, we shall simplify the general problem in the following two ways:
(1) We shall ignore parallax; that is, any differences in position between the gun and the target sighter.
(2) We shall consider in detail cases in only two dimensions.

INERTIAL FRAME IN OUTER SPACE

For the sake of simplicity, let us propel our inertial frame out into distant space where there are no external forces. We have turned off our rocket motors, and we are far from any gravitational field. We are "coasting" at a constant linear velocity, neither accelerating nor rotating with respect to the fixed stars.

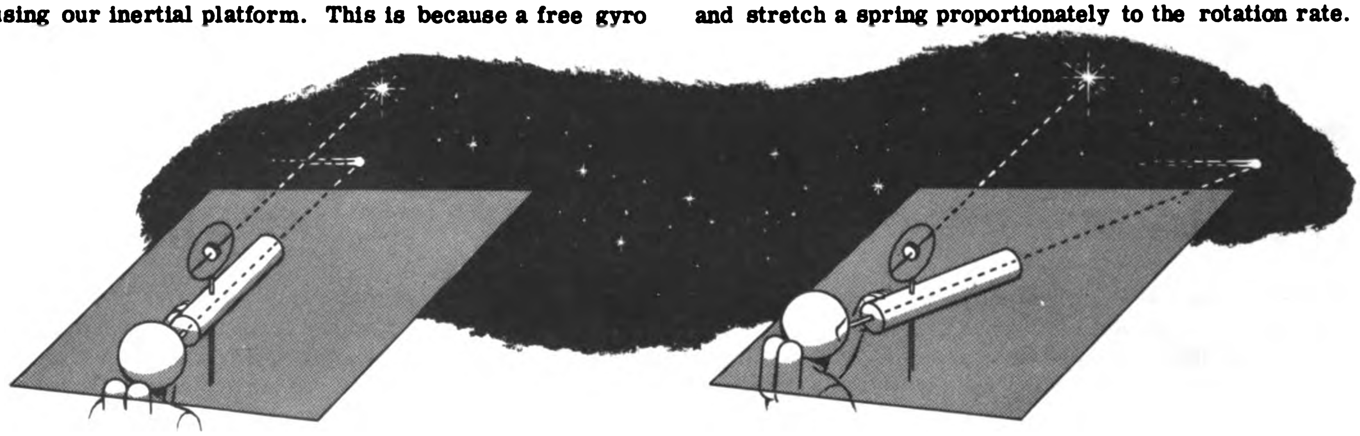
Now, if we fire point-blank at a target attached to the platform, we hit the target. But if we fire point-blank at a target moving across the platform, we miss the target, because it has moved away while the bullet was in flight.

The difference between these two cases is that a line-of-sight drawn to the target did not move in the first case, and that the line-of-sight did move in the second case.



ROTATION OF LINE-OF-SIGHT

We have shown that if the line-of-sight is stationary with respect to the platform, we can fire along the line-of-sight and hit; whereas, if it is rotating and we fire point-blank, we shall miss. How do we know it is stationary? We can tell this directly by using a gyro reference without even using our inertial platform. This is because a free gyro tends to keep its spin axis always pointing to the same point in the heavens. If the line-of-sight rotates with respect to a free gyro, that fact can be observed. Or, we can attach a rate gyro to the line-of-sight; if the line moves, the rate gyro will experience an applied rotation and stretch a spring proportionately to the rotation rate.



TARGET ACCELERATION

There is one other stipulation that we must make — if a point-blank shot is to hit the target, the target must not be accelerating relative to the bullet, as will be proved later.

ANALYSIS OF POINT-BLANK SHOOTING PROBLEM

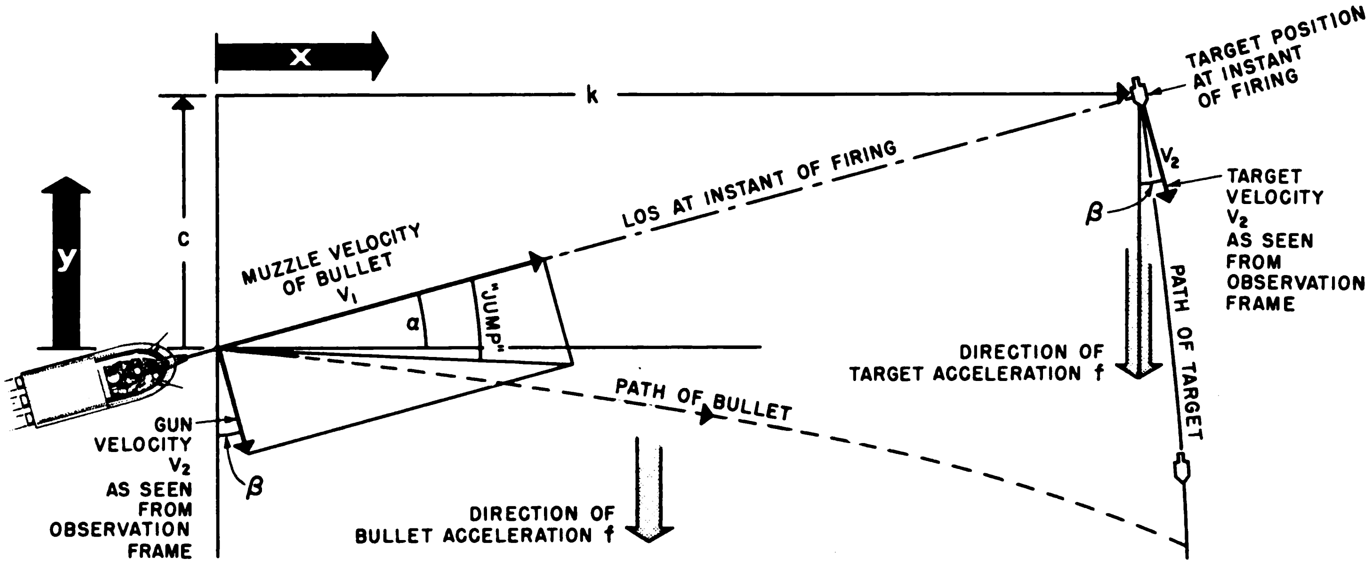
..... *PRELIMINARIES*

In the following analyses, we shall suppose that a gun fires a bullet at a target along a line-of-sight that is not rotating. For the sake of "generality" we shall (at first) observe what occurs from an outside observation frame relative to which the gun and target have a common velocity vector. (This velocity must be the same for the gun as for the target; otherwise, the line-of-sight will be rotating at the instant of firing.)

The behavior of the gun, prior to and after firing, does not concern us, provided that the line-of-sight is not rotating at the instant of firing. But the target's behavior during the flight of the bullet does concern us. We stipulate that apart from its initial velocity, the target shall have an acceleration which is equal to that of the bullet.

We shall now prove that a bullet fired under such conditions will hit the target.

ANALYSIS AS SEEN BY OUTSIDE OBSERVER



We set up a rectangular coordinate system as shown above, with x and y respectively perpendicular and parallel to the direction of acceleration. (The muzzle is the "origin" of the coordinates.)
At the instant of firing, the target coordinates are:

$x_T = k,\ y_T = c.$

The target has initial velocity v_2, and acceleration f.

At the instant of firing, the velocity of the bullet is the resultant of two velocities:

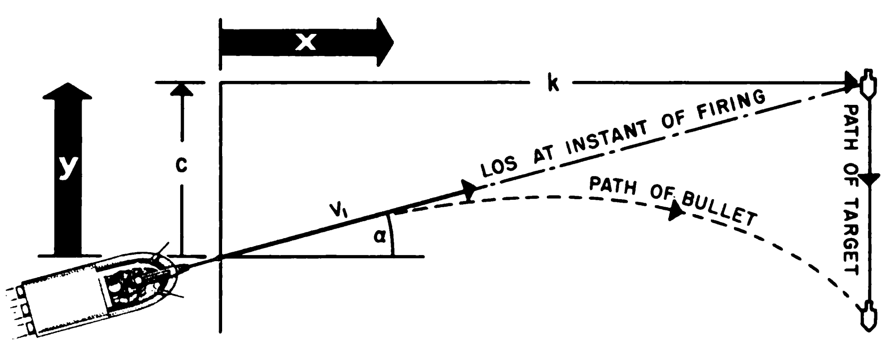
v_1 (muzzle velocity)

v_2 (gun velocity — same as target velocity)

(The inclination of the resultant to the gun axis is known as "jump".) Throughout the time of flight, the bullet also has acceleration f, equal in magnitude and direction to that of the target.

ANALYSIS AS SEEN FROM FRAME MOVING WITH

We have shown what occurs as seen from an outside observation frame. Let us observe the same occurrence from another frame which has a constant linear velocity vector equal to that of the gun and target at the instant of firing. In other words, a frame relative to which the gun and target are motionless at the instant of firing. We thus get rid of v_2 and β, and the analysis becomes simpler. Below, and to the right, we see simplified versions of the two previous figures, and a simplified analysis.



When $x_B = x_T$:

$$v_1\, t \cos\alpha = k$$

$$t = k/v_1 \cos\alpha$$

$$y_B - y_T = v_1 \sin\alpha \frac{k}{v_1 \cos\alpha} - c$$

$$= k \tan\alpha - c$$

$$= 0$$

So, the bullet hits the target.

Target position at any time t:

$$x_T = k$$

$$y_T = c - (1/2)ft^2$$

Bullet position at any time t:

$$x_B = v_1 t \cos\alpha$$

$$y_B = v_1 t \sin\alpha - (1/2)ft^2$$

$$y_B - y_T = v_1 t \sin\alpha - c$$

We have also got rid of jump. Its value depends on what observation frame we choose; the frame we have chosen reduces jump to zero.

At any time t after firing: Target position:

$$x_T = k + (v_2 \sin\beta)\,t$$

$$y_T = c - (v_2 \cos\beta)\,t - (1/2)f t^2$$

Bullet position:

$$x_B = (v_1 \cos\alpha)\,t + (v_2 \sin\beta)\,t$$

$$y_B = (v_1 \sin\alpha)\,t - (v_2 \cos\beta)\,t - (1/2)f t^2$$

$$y_B - y_T = (v_1 \sin\alpha)\,t - c \qquad \text{(Equation 1)}$$

Two terms drop out because "f" and "v_2" apply both to bullet and target. Let us see if, when $x_B = x_T$, $y_B - y_T = 0$. If this is so, the bullet will hit the target.

When $x_B = x_T$:

$$(v_1 \cos\alpha)\,t + (v_2 \sin\beta)\,t = k + (v_2 \sin\beta)\,t$$

$$(v_1 \cos\alpha)\,t = k$$

$$t = k/v_1 \cos\alpha$$

(Note that impact time is independant of v_2 and β, as it should be.)

Substitute this value for t in Equation 1:

$$y_B - y_T = v_1 \sin\alpha \frac{k}{v_1 \cos\alpha} - c$$

$$= k \tan\alpha - c$$

Since $\tan\alpha = c/k$:

$$y_B - y_T = c - c$$

$$= 0$$

$$y_B = y_T$$

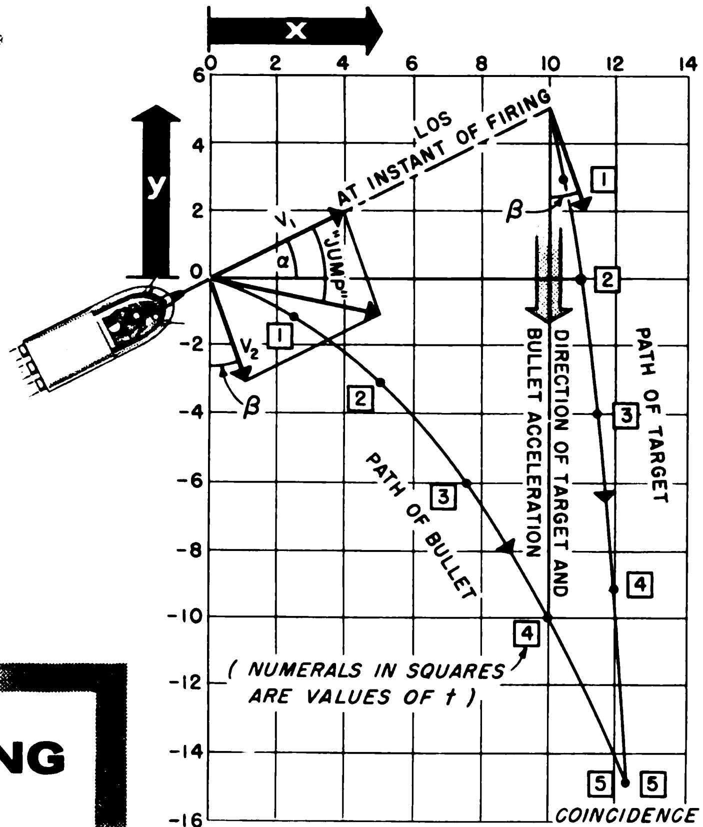
So, the bullet hits the target.

We have established the fact that the motion of the target does not cause a miss, provided that its velocity equals that of the gun at the instant of firing, and that its acceleration equals that of the bullet during flight.

numerical example

$k = 10$	$v_1 \sin\alpha = 1$
$c = 5$	$\tan\beta = 1/3$
$\tan\alpha = 1/2$	$v_2 \cos\beta = 3/2$
$v_1 \cos\alpha = 2$	$v_2 \sin\beta = 1/2$

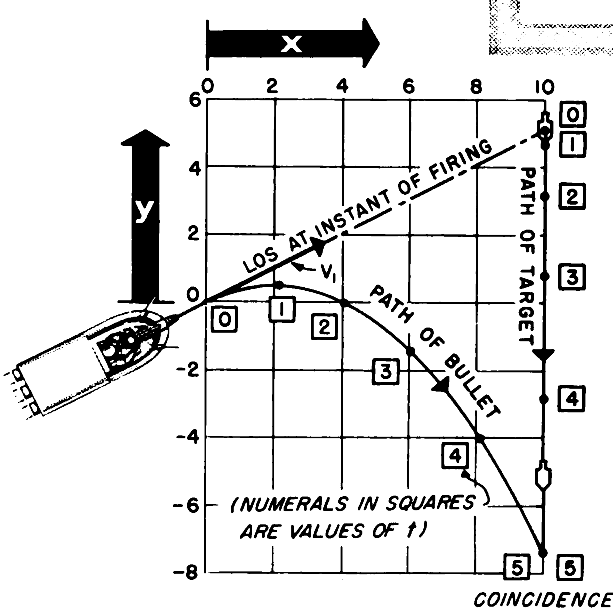
t=5 WHEN BULLET HITS TARGET



GUN AT INSTANT OF FIRING

numerical example

previous example
as seen from frame
with respect to which
gun and target
are stationary
at instant of firing



We have repeated our previous result, using a more convenient inertial frame in which to make our computation.

We shall make use of this new, useful frame when, as above, it is the most convenient. On other occasions it could be convenient or necessary for us to use other inertial frames.

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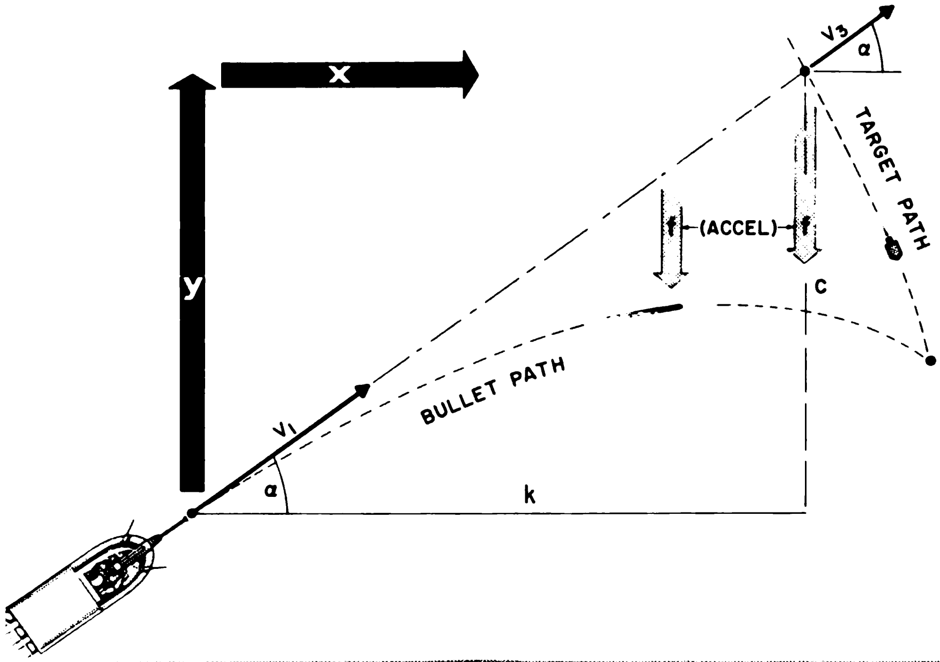
ANALYSIS WITH

Our analyses, however, are not as yet sufficiently general. It is obvious that target and bullet must have the same acceleration, f, so that the term "(1/2) f t^2" drops out of $y_T - y_B$, eventually giving us zero when $x_T = x_B$. But our method of making our line-of-sight stationary was too special. Using the old observation platform, we made target and gun both have the same velocity vector v_2 at the instant of firing. Using the new observation platform, we made both target and bullet be at rest at instant of firing (we gave them zero relative motion at that instant). However, a certain relative motion is allowed. RANGE RATE may be present; that is to say: a target motion, relative to the gun, which does not cause rotation of the line-of-sight at the instant of firing. And we shall still hit the target.

We shall now prove this statement, using our new convenient observation platform (i.e., frame of reference), with respect to which the gun is stationary at the instant of firing.

Below, we see the equivalents of the two previous figures with a velocity vector v_3 given to the target (v_3 is along the direction of the line-of-sight at the instant of firing):



Target position:

$$x_T = k + v_3 t \cos\alpha$$
$$y_T = c + v_3 t \sin\alpha - (1/2) f t^2$$

Bullet position:

$$x_B = v_1 t \cos\alpha$$
$$y_B = v_1 t \sin\alpha - (1/2) f t^2$$
$$y_B - y_T = v_1 t \sin\alpha - c - v_3 t \sin\alpha$$
$$= (v_1 - v_3)\, t \sin\alpha - c$$

When $x_T = x_B$:

$$k + v_3 t \cos\alpha = v_1 t \cos\alpha$$
$$(v_1 - v_3)\, t \cos\alpha = k$$
$$t = k/(v_1 - v_3)\cos\alpha$$
$$y_B - y_T = (v_1 - v_3) \sin\alpha \frac{k}{(v_1 - v_3)\cos\alpha} - c$$
$$= k \tan\alpha - c \quad (\tan\alpha = c/k)$$
$$= 0$$

SOME SPECIAL CONSIDERATIONS

So the target may have motion relative to the gun, if the line-of-sight does not rotate at the instant of firing.

A caution: though relative motion of gun and target is, for many purposes, enough for us to know, there are cases where even a relative motion of zero would cause our shot to miss:

(1) Rotation of the whole system. This would rotate the line-of-sight as sensed by a gyro, i.e., as observed in an inertial frame.

(2) Acceleration of target by rocket thrust. This type of acceleration would not be experienced by the bullet in flight, but would be experienced by the target.

(3) Acceleration in a non-homogeneous gravitational field. The bullet and target would not experience quite the same acceleration. The difference is slight, and this fact is responsible for the statement in some books that "frames in gravitational free fall are inertial".

Note that what matters is the gun velocity at the instant of firing, as this is imparted to the bullet. Gun acceleration is not imparted to the bullet; its sole importance is its previous effect in imparting a certain velocity to the gun. In the three proofs we have given, no reference was made to the behavior of the gun prior to firing or after firing. Suppose, for example, our gun and sighter (see right) were mounted on a train starting at rest, and traveling in a straight line with a constant acceleration f. From this frame we observed a target traveling parallel to us, with a constant velocity v. In general, the line-of-sight would be rotating. But at one instant – when the velocities were equal – its angular velocity would be zero. Up until then, rotation is clockwise, getting slower and slower. Afterward, rotation is counterclockwise, getting faster and faster. At the instant angular velocity "changes sign", it "crosses" the zero value. This may be sensed physically.

More formally:

$$\cot\beta = [v t - (1/2) f t^2]/k$$

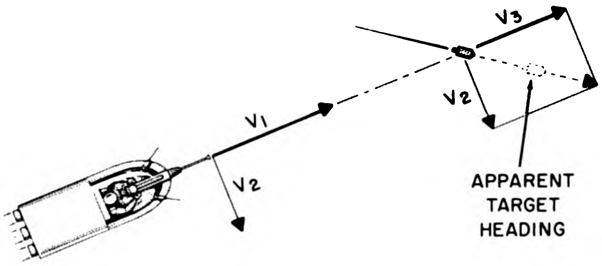
$$\frac{d\beta}{dt} = \frac{d\beta}{d(\cot\beta)} \cdot \frac{d(\cot\beta)}{dt} = (\sin^2\beta)(v - f t)/k$$

When velocities are equal, $v = f t$; therefore (since $\sin^2\beta$ can never be infinite) $\frac{d\beta}{dt} = 0$ at that instant.

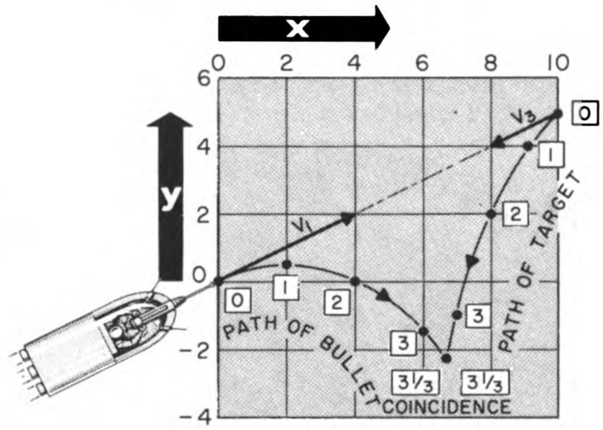
RANGE RATE

The bullet hits the target. But note that vector v_3 must be along LOS as seen by the gunner at the moment of firing, i.e., as viewed in this frame, and not as it was seen by the outside observer.

Our original outside observer would see the entire system, including the line-of-sight, having a translational velocity v_2. The target heading would not appear to be along the line-of-sight (although the target would still move along the line-of-sight, regardless of the frame).



numerical example



We have now proved that a bullet fired point-blank at a target will hit, provided that the following conditions exist:

(1) Line-of-sight is not rotating at the instant of firing.
(2) Target velocity relative to the gun during time of bullet flight is along the line-of-sight at the instant of firing.
(3) Target and bullet have the same acceleration (if any).

Note that although the bullet travels in the same path as before v_3 was introduced, and the target does not, the bullet still hits the target, but later—at time $t = k/(v_1 - v_3)\cos\alpha$ instead of at time $t = k/v_1 \cos\alpha$.

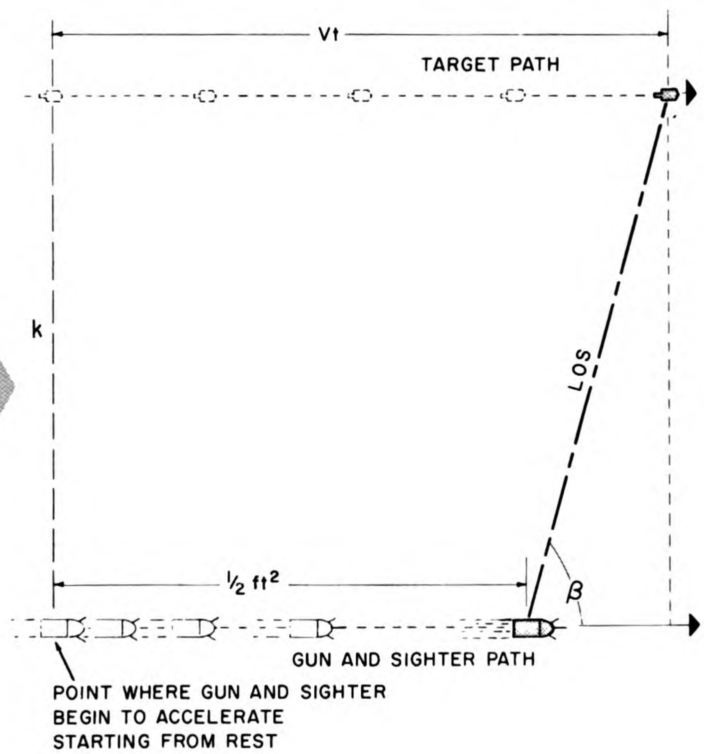
The numerical example shows what the previous numeral example becomes when a negative range rate, $v_3 = -(1/2)v_1$, is given to the target. Here the time of coincidence, $t = k/(v_1 - v_3)\cos\alpha$, becomes:

$$t = \frac{k}{[v_1 + (1/2)v_1]\cos\alpha}$$

$$= (2/3)\frac{k}{v_1 \cos\alpha}$$

Thrust is less than time of coincidence,

$$t = k/V_1 \cos\alpha$$

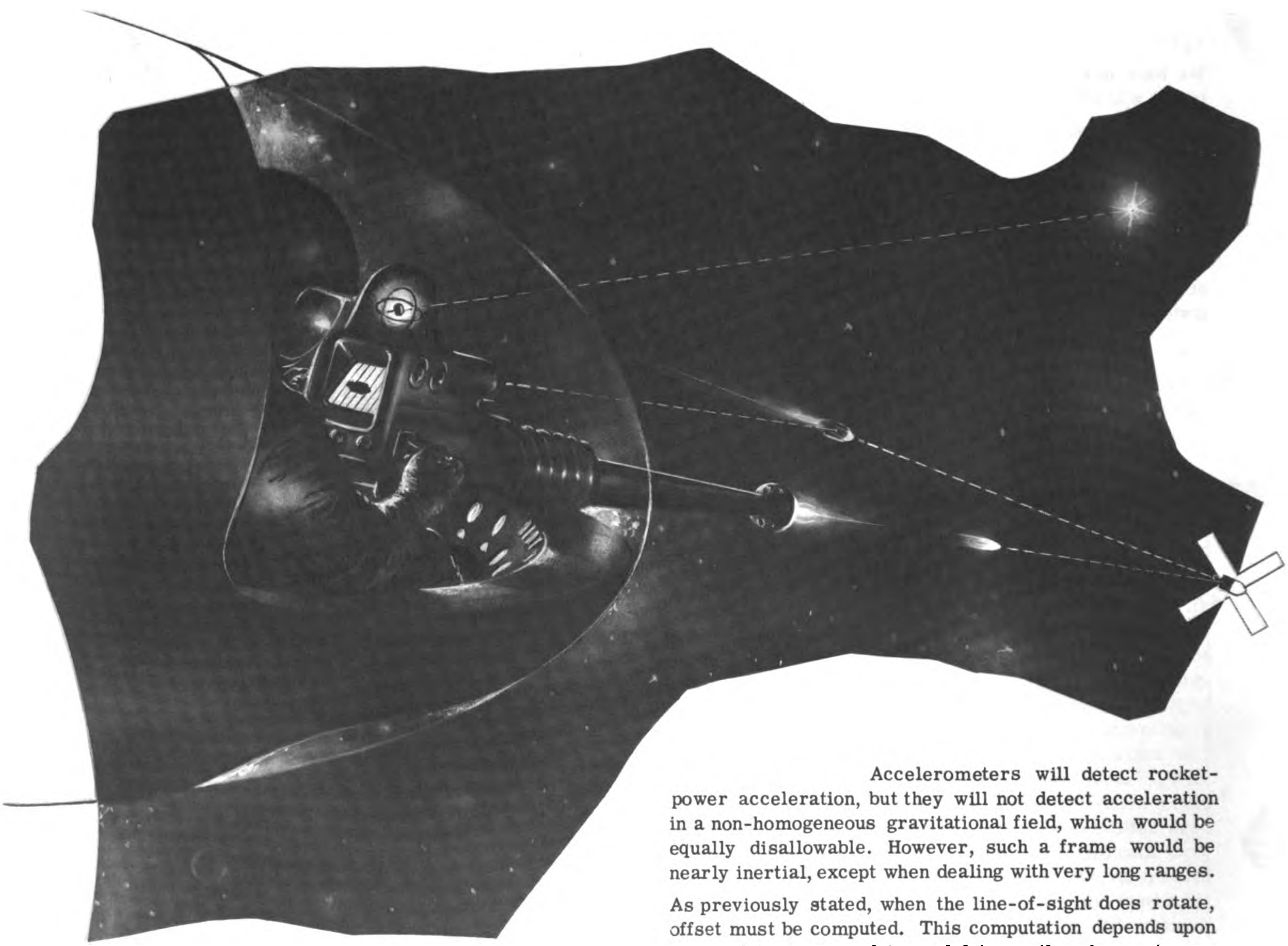


TARGET CONSTANT-VELOCITY ASSUMPTION

We have stated several times, and proved, that the target may accelerate, provided the bullet does so too by the same amount, and in the same direction – for point-blank shooting. In practice, we assume that the target does not accelerate. In other words, we assume that the target velocity vector will not have time to change perceptibly during the short time of travel of the bullet being fired, or the missile being launched.

A point-blank shot at a target will hit its mark if a gyro indicates that the line-of-sight is not rotating. (This non-rotation of the line-of-sight is often called "not rotating in INERTIAL SPACE".) If it is rotating, the gun must be aimed ahead of the line-of-sight by a calculated offset. If a system is provided with the required gyro, we can do without the platform. It may be inertial, or not inertial—it could be rotating, accelerating, or oscillating; we are not concerned, if a gyro is fixed to our line-of-sight to indicate whether or not the line-of-sight is rotating.

Many weapons control systems incorporate gyros. Some systems measure line-of-sight motion in relation to the platform. In such cases, we must examine the platform. If it is inertial, we can observe line-of-sight rotation or non-rotation in relation to the platform; this does not necessarily require the installation of a gyro — some instrument such as a tachometer will serve the purpose. To investigate the ability of a platform to measure the line-of-sight rotation (such as sensed by gyros), set up three mutually perpendicular gyros which will sense if the platform is an inertial frame or not. If all the gyros register zero, we know our platform is inertial, and will give us correct LOS rotation measurements. Otherwise, convert platform coordinate data to inertial frame data.



Accelerometers will detect rocket-power acceleration, but they will not detect acceleration in a non-homogeneous gravitational field, which would be equally disallowable. However, such a frame would be nearly inertial, except when dealing with very long ranges.

Acceleration of the platform does not matter so far as firing is concerned. However, if we wished to define an inertial frame that includes gun, bullet and target, it must have not only no rotation, but no acceleration except that which is given to all three components by a homogeneous gravitational field.

As previously stated, when the line-of-sight does rotate, offset must be computed. This computation depends upon target data, weapon data, and data on the circumstances in which a shot or missile launch is made. Just as an inertial frame is the most convenient in which to observe a target with a non-rotating line-of-sight, so it is the most convenient in which to set the general problem of weapons control. Eventually, we also consider problems associated with a bullet (or other missile) that has forces acting upon it, as is the case in actual weapons control problems.

WEAPONS CONTROL IN IDEAL INERTIAL FRAME

In the previous section, we discussed the conditions for point-blank shooting at a target to be successful. A primary condition was that the line-of-sight in inertial space does not rotate at the instant of firing. In this section, we study the problem of target shooting when the line-of-sight does rotate, so point-blank shooting is not successful.

To compensate for target motion, we must offset weapon line from line-of-sight by a LEAD ANGLE, so that the missile will intercept the target. That is, the weapon must be aimed at a point in space which the target will reach when the missile reaches that point. To compute this future target position we must: (1) obtain information on target position and motion (assuming the target will continue to move in the same way); (2) ascertain the missile's time of flight, and (3) compute how far the target will move during that time.

Target information is obtained for step (1) by a process of TRACKING. Steps (2) and (3) are accomplished by a process of PREDICTION which produces the lead angle. These steps will be discussed in this section.

To aid in understanding of the principles involved in hitting a moving target, the following simplifying assumptions are made:

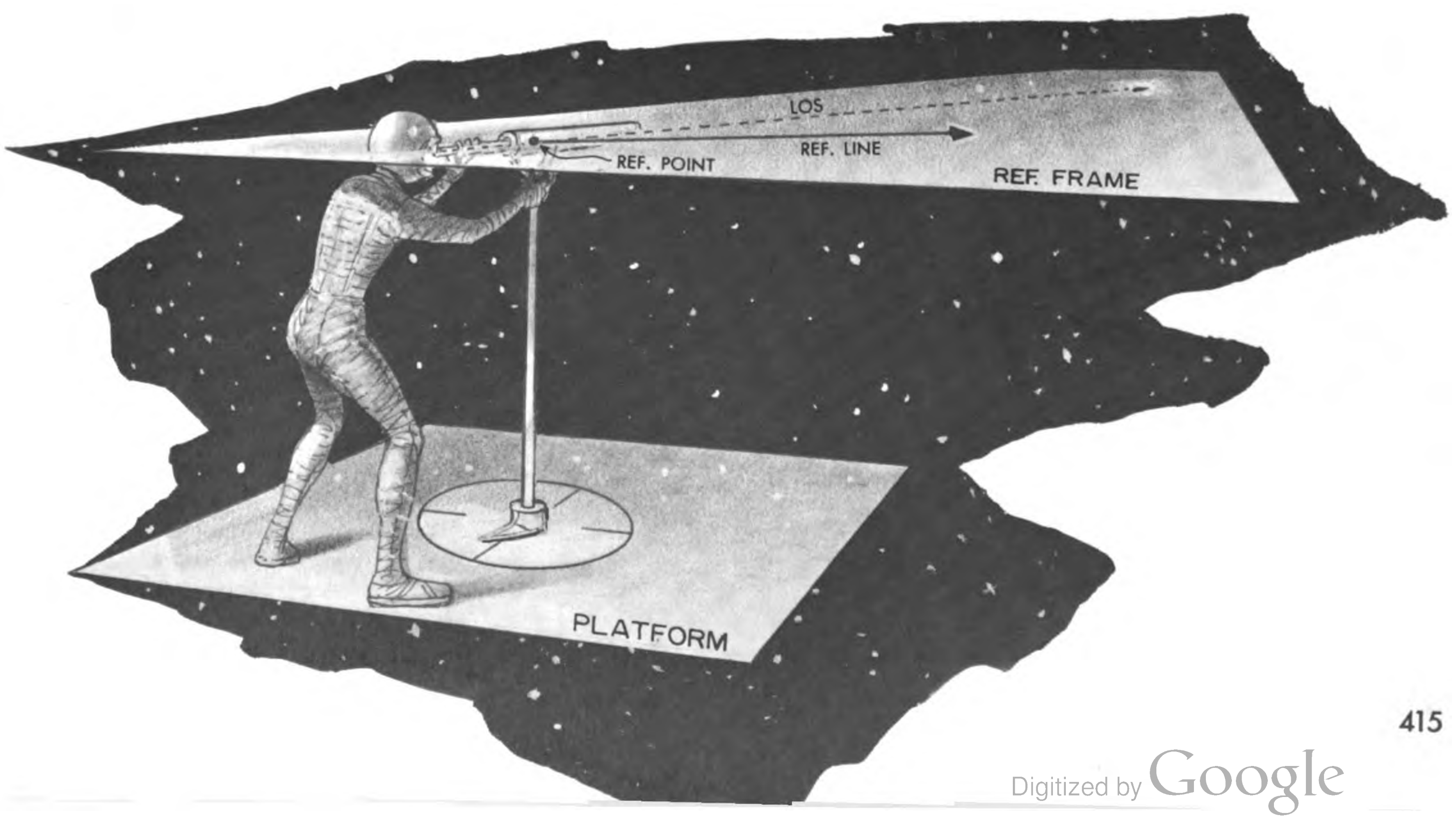
(1) We are operating in a reference frame which is inertial (having no accelerations or rotation), and is also removed from any force such as gravity and atmosphere. In other words, an ideal inertial frame in which a bullet travels in a straight line at constant speed.

(2) The target moves in only two dimensions.

(3) The target has constant speed and motion, in a circle or straight line, during tracking and bullet flight.

(4) The target is to be hit by one shot.

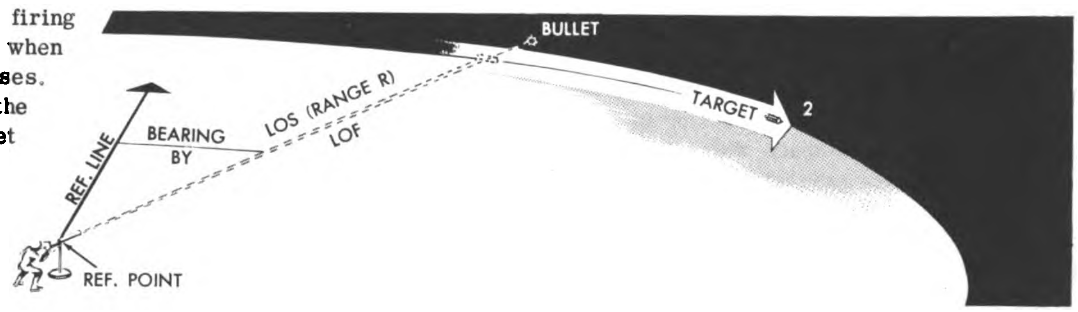
Suppose that in this airless, gravity-less inertial frame, the observer is standing on a steel platform aided by magnetized shoes. His basic measuring instruments consist of a telescope, to obtain the target's direction and a range measuring device. We show only a telescope, and assume it to have an optical range finder built into it. The weapon is a gun, firing a bullet in a straight line at constant speed. The reference point is the intersection of the axes of rotation of the telescope. We assume that the bullet is fired from this reference point (that is, we ignore parallax). The observer establishes a reference plane through the reference point, parallel to the platform. The target moves in the reference plane with constant motion. Let us now examine some cases of target motion, and determine what computations must be performed in order to aim the bullet so it will hit the target.



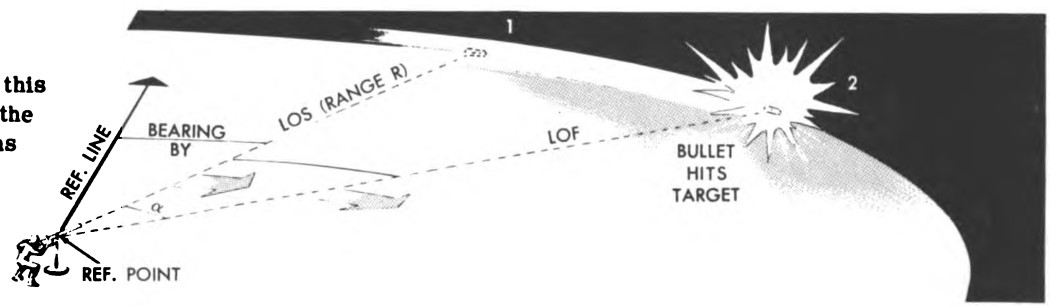
CIRCULAR TARGET COURSE

We begin by considering a simple case: a target that follows a circular path at constant speed with the reference point at its center. Thus, range and angular velocity are constant.

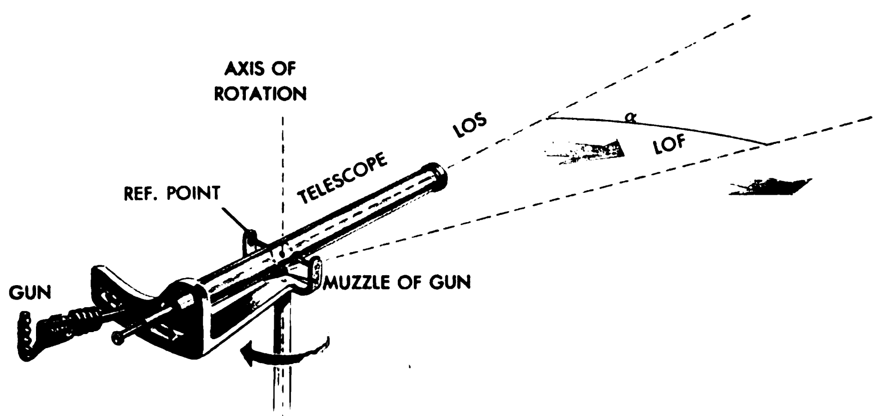
The observer starts by firing point-blank at the target when it is at point 1. He misses. This is because during the bullet's flight, the target has moved through an angle α from a first position, point 1, to a second position, point 2.



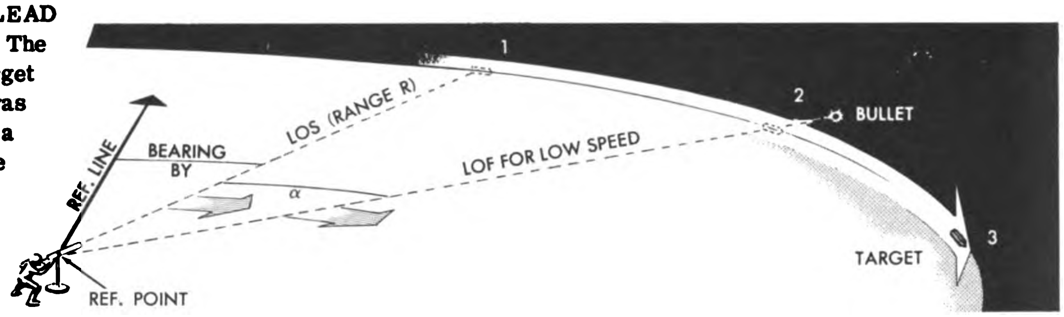
The observer sees how this happened, so the next time the target is at point 1 he aims ahead of it by an angle α; that is, he aims at point 2 — and scores a hit.



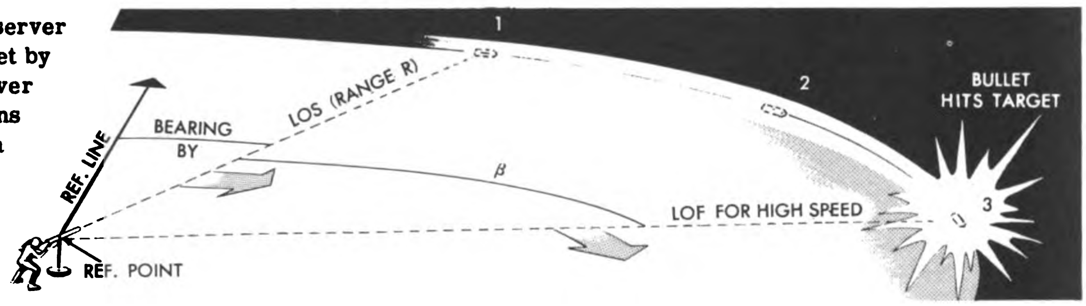
The observer desires to fire at the target at any time, however, and not just when the target is at point 1. So, he synchronizes his gun with his telescope, offsetting it by angle α; then, following the target with the telescope, he can fire at any time and hit the target.



Suppose, now, that the target gains a new, higher constant speed, and that the observer is not aware of this. When he again fires at his target, using his offset, or LEAD ANGLE, α, he misses. The bullet goes behind the target because offset angle α was based on the movement of a slower target. When he fired, the target was at point 1, but he aimed at point 2; when the bullet reached point 2, the target was at point 3.



To score a hit, the observer should have led the target by an angle β. The observer realizes this, but reasons that there should be a better way to hit the target than by trial and error. Here, his measuring instruments can help.



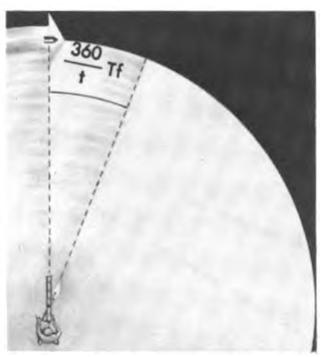
Using his range finder, the observer measures the range to the target and notes that it is constant. He divides the target range by the bullet speed, u, which he knows, and obtains the time of flight of the bullet to the target, Tf.

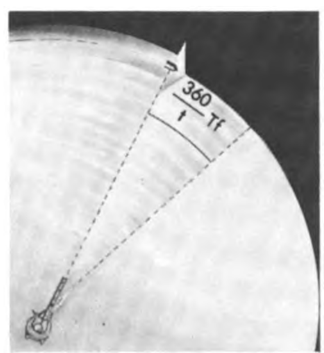
$$Tf = R/u \qquad (1)$$

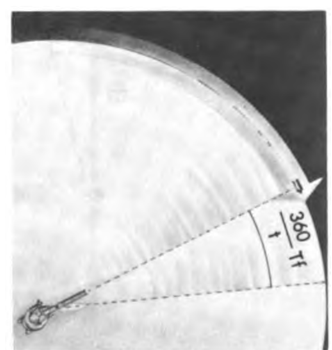
The observer recognizes that since range and bullet speed are constant, time of flight is also constant. He realizes that he must determine how far the target will travel during the time of flight of the bullet. He knows that to determine the distance traveled by the target during time Tf, first he must find the rate at which it is moving. Therefore, he times the target as it circles about him and notes that the time, t, for each revolution is the same. He notes that the number of degrees the LOS rotates through per unit time is 360/t, the bearing rate of the target, and decides to assume that it will remain constant during time of flight of the bullet. The observer now has sufficient information to compute how far the target will move during the time of the bullet's flight, and thus, the lead angle:

$$\text{Lead Angle} = 360/t \times Tf \text{ degrees}$$

The observer now follows the target with the telescope, synchronizes the gun with the telescope, offsets the gun axis by an angle 360/t x Tf from the telescope axis in the direction of rotation, fires and hits the target at any time.







If the target speed changes again, the observer must measure the new value of t and compute a new lead angle, 360/t x Tf. The observer notes that it is not necessary to measure the time t for a complete revolution, but that the time for any known part of a revolution is sufficient to permit him to calculate the bearing rate of the target, dBy/dt. He sees that he can measure bearing rate directly by tracking the target with the telescope and measuring the number of degrees the telescope traverses per unit time. This enables him to calculate lead angle with a known quantity (bullet speed) and measured quantities (bearing rate and range.)

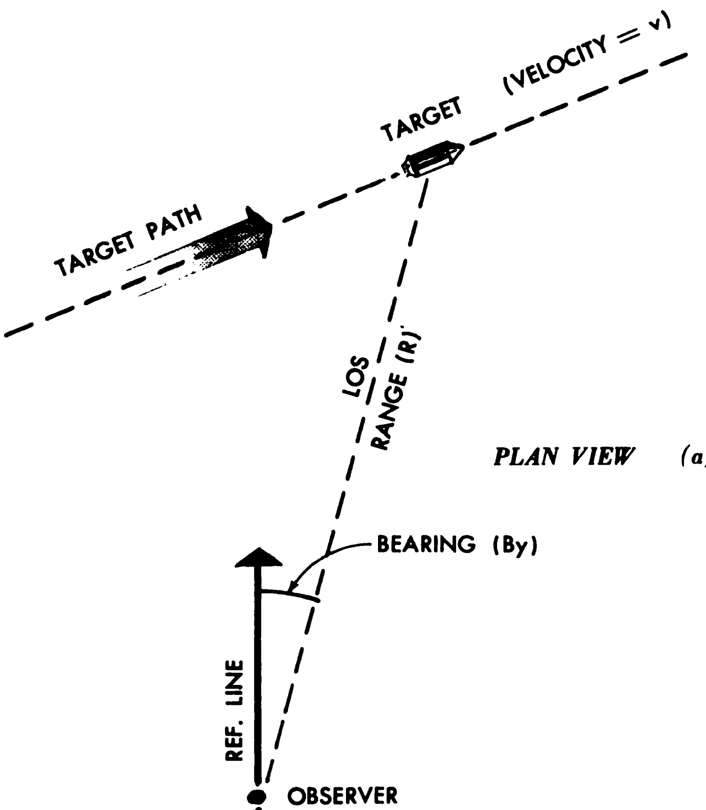
$$\text{Lead Angle} = dBy/dt \times Tf \qquad (2)$$

$$= dBy/dt \times R/u \qquad (3)$$

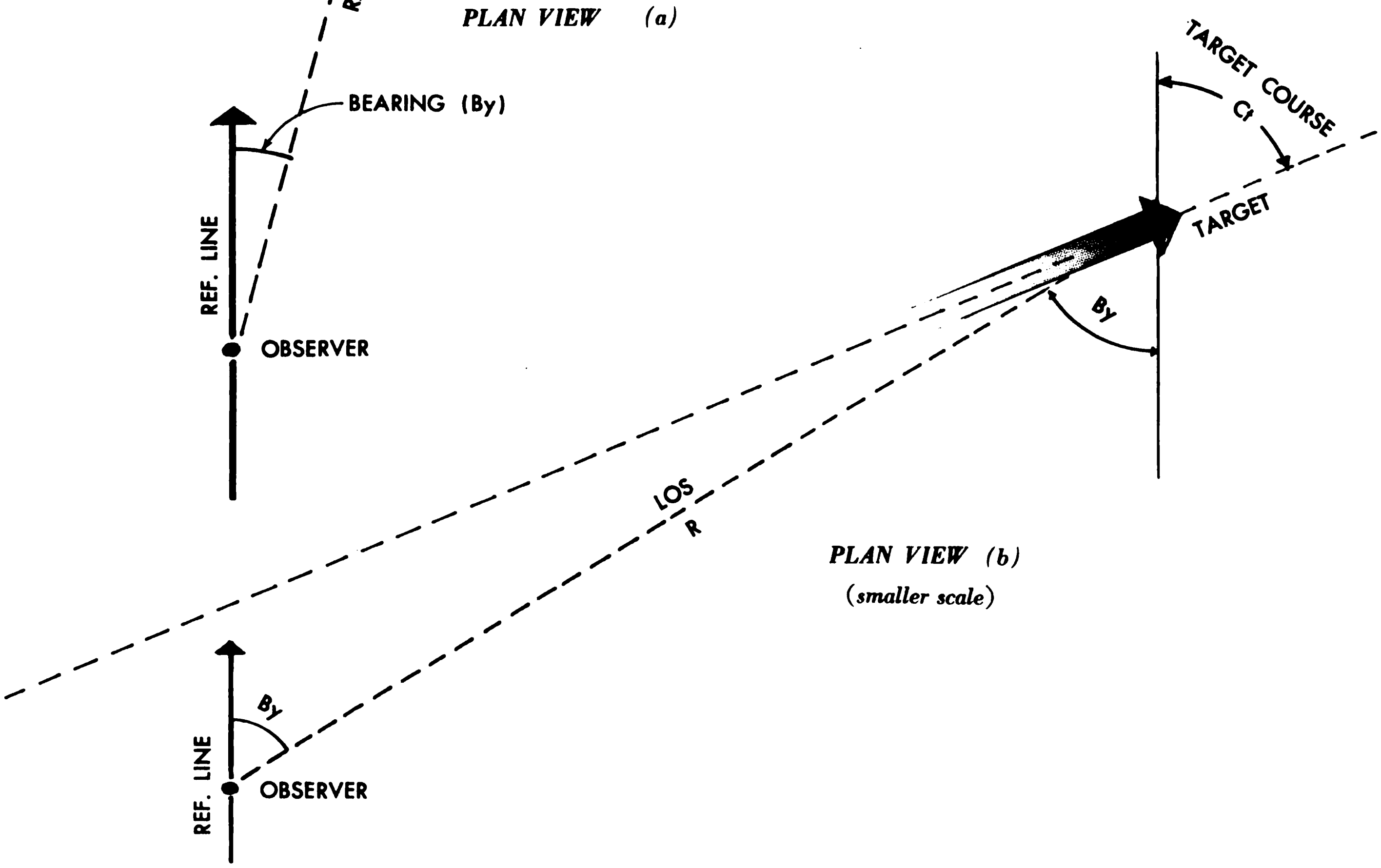
In his attempts to hit the target, the observer learned how to obtain target information by TRACKING: that is, by keeping the telescope axis (a line he controls) coincident with the line-of-sight, he tracked the target and was able to measure target position (range and bearing) and motion (bearing rate) in this two-dimensional case. With the tracking data, assuming constant target motion and knowledge of bullet speed, the observer could PREDICT the future target position, and thus compute the lead angle necessary to score a hit.

OBLIQUE LINEAR TARGET COURSE

Let us consider the case of a target traveling at constant speed in a straight line, in a direction inclined to the line of sight. The observer measures bearing rate and range. From range he computes flight time and then computes lead angle, as before, by multiplying time of flight by the bearing rate. He shoots, and misses. This is because range and bearing rate are not constants, as they were in the case of the target traveling in a circle; both bearing rate and range have changed by the time the bullet intercepts the target's path. This is shown from the two diagrams below. Plan view *(b)* shows what plan view *(a)* becomes after some time has passed. Note that *(b)* is drawn to a smaller scale than *(a)*. As the target goes further away, bearing By increases more and more slowly, and approaches closer and closer to target course Ct. This shows that bearing rate dBy/dt decreases as target range increases. As the target gets farther and farther away, range rate dR/dt increases, approaching target velocity, v.

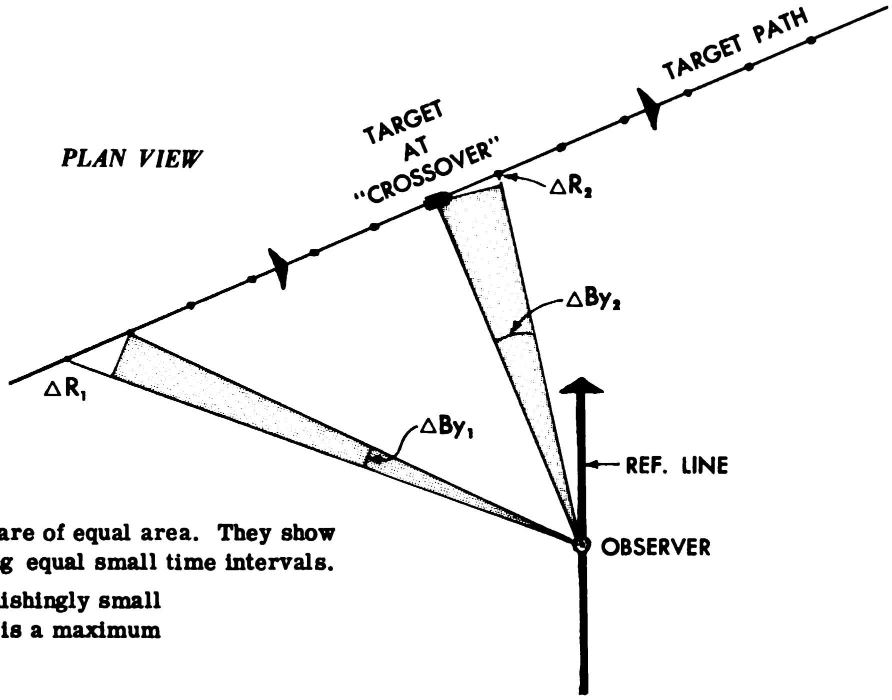


PLAN VIEW (a)



PLAN VIEW (b)
(smaller scale)

Note that at "crossover" (when the line of sight is perpendicular to the target path), dR/dt is zero and dBy/dt is a maximum. When the target is close, and moving very fast, its high bearing rate near crossover creates a special problem.
This non-constancy of bearing rate, range, and range rate makes the prediction problem more difficult than when we considered the case of the circular target course.

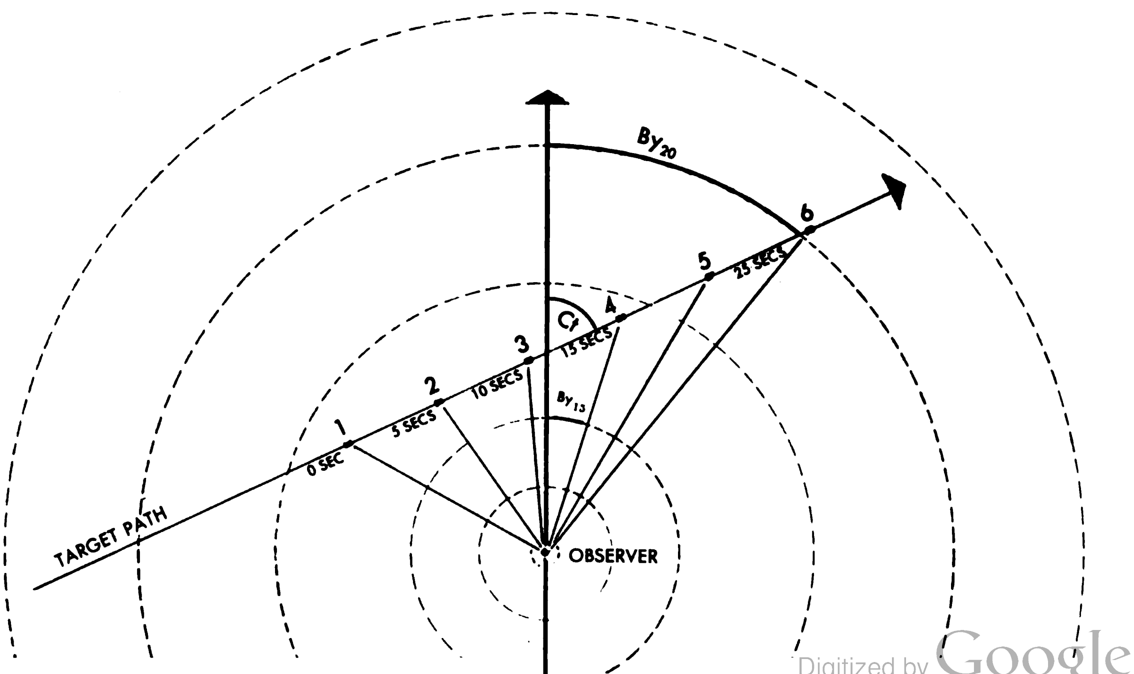


The two shaded triangles are of equal area. They show changes in R and By during equal small time intervals.

ΔR_2 is vanishingly small
ΔBy_2 is a maximum

Recognizing that this problem is more difficult than the previous one, the observer measures range and bearing and plots the target path on graph paper, noting the time of each observation. He sees that the graph is a straight line, and that points reached by the target at equal time intervals are equal distances apart. Dividing a distance increment by a time interval, he computes target speed v. He measures target course Ct from the graph. The observer notices that these values are constant and assumes that they will remain so during the time of the bullet's flight. Assuming constant target course and speed at any time, the observer can predict the position of the target at any future time by simply extrapolating the graph.

Suppose he has plotted its position at a starting time (zero second) and after intervals of 5, 10, and 15 seconds (points 1, 2, 3, and 4). The position of the target 20 seconds after starting will be point 5, further along the line, at the same distance from 4 that 4 is from 3.
The observer decides to try to hit the target when it reaches point 5. He aims at that point which has a bearing of By_{20}. He knows the range to 5; from this he computes Tf, which he ascertains to be 3/4 second. He must fire at 20-3/4 = 19 1/4 seconds from the starting time, aiming at bearing By_{20}. Thus, the observer has solved a problem of how to hit an obliquely moving target. One difficulty with this procedure, however, is that the solution is unique. Using bearing By_{20}, he must fire at the instant t = 19 1/4 seconds, if he is to score a hit. For each target location, he must compute a new Tf and a new By. Moreover, plotting and prediction in this manner with a fast moving target are not very efficient.



OBLIQUE LINEAR TARGET COURSE

The observer decides that he needs a faster way to compute the lead angle, so that he can fire at any time he chooses. He finds this faster way as a result of the following analysis: Let target velocity be v and bullet velocity be u. (The observer knows u but, as yet, does not know v.) Let Tf be time of flight. During the time of flight the target travels distance AB = vTf, while the bullet travels OB = uTf. If BE is drawn perpendicular to the line of sight OA extended, then:

$$BE = uTf \sin L$$

$$\text{also,}\quad BE = vTf \sin \gamma$$

$$\text{So,}\ uTf \sin L = uTf \sin \gamma \qquad (4)$$

$$u \sin L = v \sin \gamma$$

$$\sin L = v \sin \gamma / u \qquad (5)$$

The inset vector diagram shows that v sin γ is the component of v perpendicular to the line of sight at the instant of firing, or target cross-velocity. Similarly, u sin L is the bullet cross-velocity. So, equation (4) states that, at the instant of firing:

$$\text{Bullet cross-velocity} = \text{target cross-velocity} \qquad (6)$$

This is necessary for scoring a hit.

The observer has obtained an equation for lead angle L - (5) - but he is not satisfied; he cannot measure the quantities v and γ directly. He decides to express the quantity v sin γ in another way. In order to do this, he draws a diagram that shows the infinitesimal displacement of the target during infinitesimal time, dt, broken up into two components, along the LOS and perpendicular to the LOS. (In this diagram, all angles are in radians.) From this diagram he sees that:

$$v \sin\gamma = R\frac{dBy}{dt} \qquad (7)$$

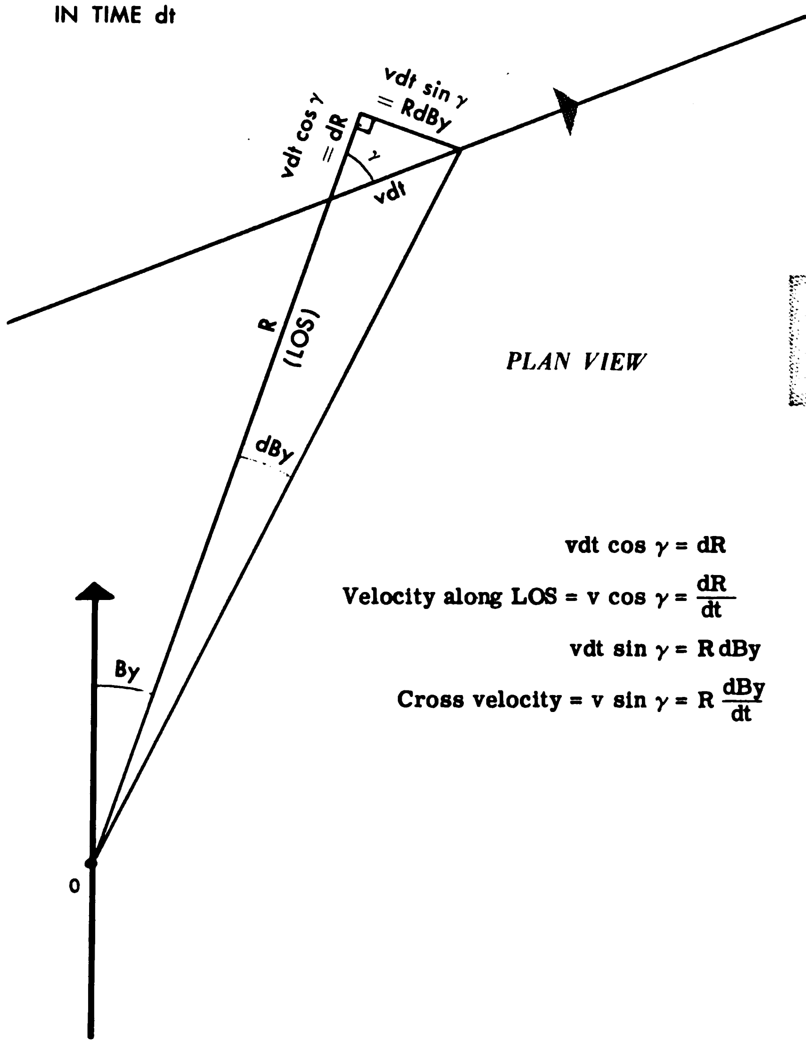
$$v \cos\gamma = \frac{dR}{dt} \qquad (8)$$

where $\frac{dBy}{dt}$ is in radians per second. At present, he is concerned only with equation (7). Substituting this value for v sin γ in equation (5) gives:

$$\sin L = R\frac{dBy}{dt}/u \qquad (9)$$

So the observer can now compute the lead angle from the measured quantities, bearing rate and range, and the known quantity, bullet velocity. From equation (7) the observer sees that linear cross-velocity $R\frac{dBy}{dt}$ is a maximum at crossover —— where $\gamma = 90°$ — being equal to v at that point. Note that, although he must measure bearing rate, he need not measure range rate.

INFINITESIMAL DISPLACEMENTS IN TIME dt

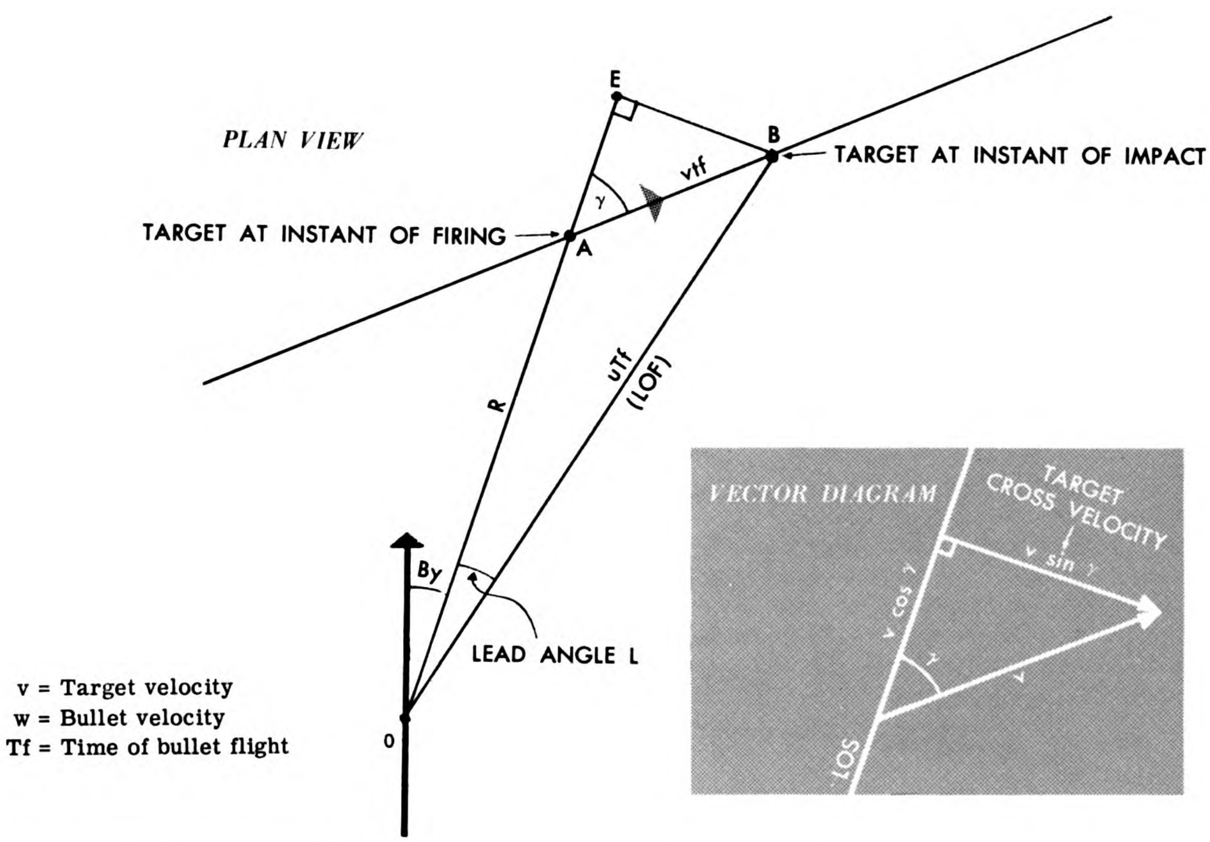


$$vdt \cos \gamma = dR$$

$$\text{Velocity along LOS} = v \cos \gamma = \frac{dR}{dt}$$

$$vdt \sin \gamma = R\,dBy$$

$$\text{Cross velocity} = v \sin \gamma = R\frac{dBy}{dt}$$



Equation (5) may be rewritten in the following form:

$$\sin L = \frac{\text{target cross-velocity at instant of firing}}{\text{bullet velocity}} \qquad (10)$$

This tells him that (under these ideal conditions) for a given bullet velocity, the lead angle depends only on target cross-velocity at the instant of firing.

The observer wonders whether there is any link between the equations for L in the cases of circular and linear target paths. He devises an artificial quantity: Tf' — the time of flight the bullet would have if the target were hit at the point it occupies at the instant of firing. Then he rewrites equation (9) thus:

$$\sin L = \frac{(R)}{(u)} \frac{dBy}{dt} \qquad (11)$$

But $\frac{R}{u}$ = Tf' — the new quantity. Substituting for $\frac{R}{u}$ in equation (11) gives:

$$\sin L = Tf' \frac{dBy}{dt} \qquad (12)$$

Equation (12) closely resembles equation (2) which was computed for the circular target path.

$$L = Tf \frac{dBy}{dt} \qquad (2)$$

For small values of L, these two equations approach identity. For small angles, the sine of the angle approximates the angle in radians. But, in practice, lead angles may be large.

For large angles, linear target motion presents a situation completely different from the case of circular motion. In the case of circular motion, once the observer has computed L, it will remain valid as long as the target maintains its speed and distance. But in the case of linear motion, L changes continually because dBy/dt and Tf change continually. Thus, in order to hit the target, the observer must measure bearing rate and range, compute L, aim the gun and fire it -- all in an instant. A new solution is needed at every instant. He realizes that he needs some device that will measure these quantities continuously, compute a correct solution at every instant, and aim the gun accordingly.

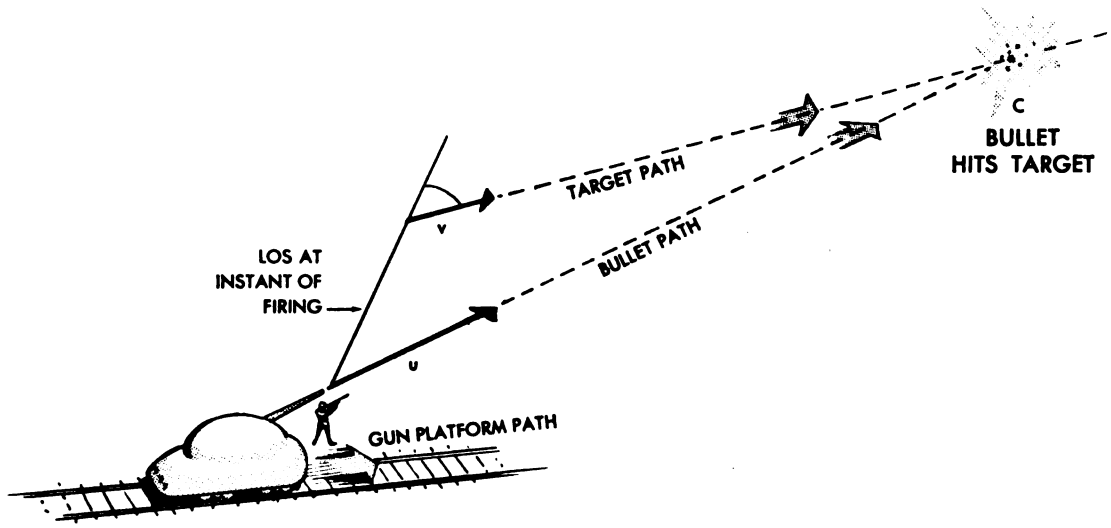
summary

In solving the problem of hitting a target moving in a straight line at constant velocity across an airless, gravity-less, inertial platform, the observer found that he must offset the gun from the line-of-sight by a lead angle $L = \frac{\text{target cross-velocity}}{u}$; where u is the bullet velocity, and cross velocity is measured at the instant of firing.

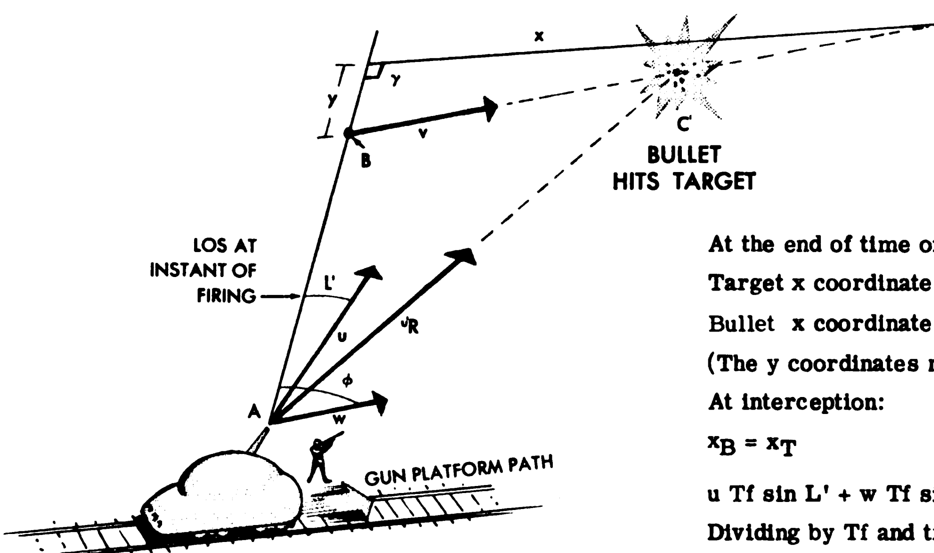
VELOCITY JUMP

Suppose that the observer has just scored a hit on a target at point C, having offset the gun by lead angle:

$$L = \frac{\text{Target cross velocity}}{u}$$



The observer realizes he has a new problem. After some experimenting, he decides to adopt the method of analytical geometry. This leads him to a simple solution. He takes B (position of target at instant of firing) as the origin of a rectangular coordinate system. He measures coordinate y along the line of sight at the instant of firing, and measures coordinate x perpendicular to that line-of-sight. Time between firing and interception (time of flight) is Tf. Let L' be the new, correct, value of the lead angle.



At the end of time of flight:

Target x coordinate = x_T = v Tf sin γ

Bullet x coordinate = x_B = u Tf sin L' + w Tf sin ϕ

(The y coordinates need not be used.)

At interception:

$x_B = x_T$

u Tf sin L' + w Tf sin ϕ = v Tf sin γ

Dividing by Tf and transposing:

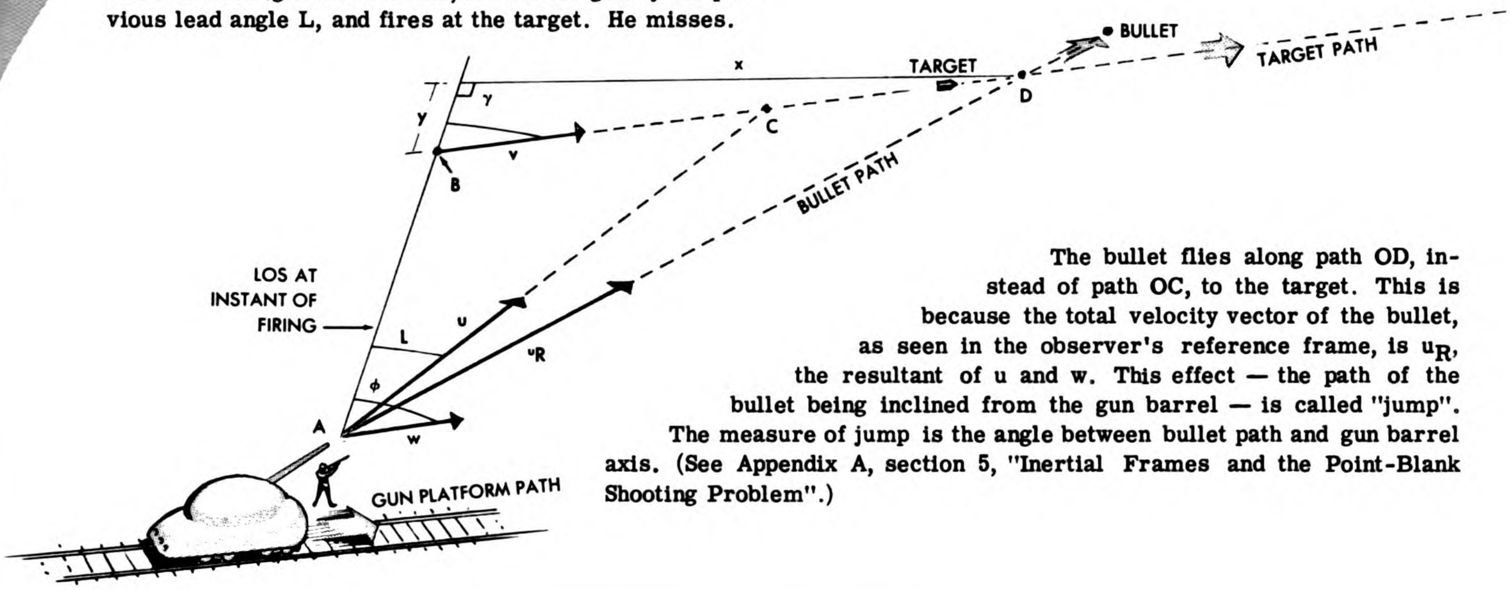
sin L' = (v sin γ - w sin ϕ)/u

That is to say:

$$\sin L' = \frac{\text{Target cross velocity - gun cross-velocity}}{\text{bullet velocity}}$$

The observer computes the new value for lead angle L', offsets his gun by this amount, fires, and hits the target.

Now, suppose that the gun itself is set in motion. It is given a velocity w, relative to the reference frame at the instant of firing, inclined at angle ϕ to the line-of-sight. The observer ignores this fact, offsets the gun by the previous lead angle L, and fires at the target. He misses.



The bullet flies along path OD, instead of path OC, to the target. This is because the total velocity vector of the bullet, as seen in the observer's reference frame, is u_R, the resultant of u and w. This effect — the path of the bullet being inclined from the gun barrel — is called "jump". The measure of jump is the angle between bullet path and gun barrel axis. (See Appendix A, section 5, "Inertial Frames and the Point-Blank Shooting Problem".)

The observer discovers that the same result could be achieved by using a different reference frame; namely, a frame that moves with the gun. That is, it has velocity w with respect to the old frame. Looking from the new frame, the observer measures target bearing rate and range, and computes new target cross-velocity. Dividing by bullet velocity u, he obtains the sine of the new lead angle L". He offsets the gun from the LOS by angle L" fires, and hits the target.

He may be surprised to notice that the lead angle in this case is identical with the one he computed using the old frame, when he corrected for jump.

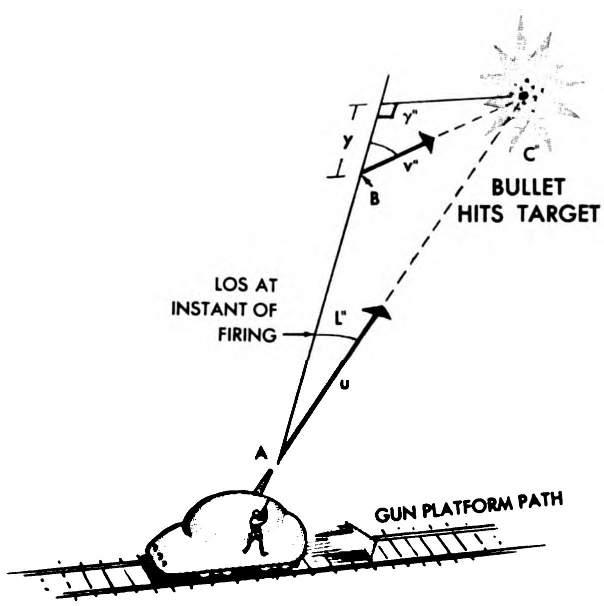
The observer recalls that the velocity of an object depends upon the frame in which it is computed. (See Appendix A, section 4, "Reference Frames for Motion".) Thus, target velocity computed in this new frame should have a new magnitude v", and a new direction γ". And, new target cross-velocity should equal old target cross-velocity, minus gun cross-velocity.

$$v'' \sin \gamma'' = v \sin \gamma - w \sin \phi$$

$$\sin L'' = \frac{v'' \sin \gamma''}{u} = \frac{v \sin \gamma - w \sin \phi}{u} = \sin L'$$

Therefore:

$$L'' = L'$$



From his observations the observer concludes that the magnitude of the lead angle is independent of the inertial frame in which it is computed. He also finds that his computations for lead angle can be simplified by selecting a reference frame that moves with the gun.

VELOCITY JUMP

The observer decides to verify his conclusions about velocity jump with some numerical examples illustrated by diagrams drawn to scale.

Target speed $v = 10$

Target path inclination γ

$$= \sin^{-1} \frac{4}{5} = \cos^{-1} \frac{3}{5}$$

Target cross velocity $= v \sin \gamma = 8$

Bullet velocity $u = 16$

Gun velocity $w = \sqrt{29}$

Gun path inclination $\phi = \sin^{-1} \frac{5}{\sqrt{29}}$

If there were no jump (that is, if w were zero):

$$\begin{aligned} \sin L &= v \sin \gamma / u \\ &= 8/16 \\ &= 1/2 \\ L &= 30° \end{aligned}$$

Bullet cross velocity $= u \sin L = 16 \times 1/2 = 8$

$= $ target cross velocity

Bullet intercepts target at C, and scores a hit.

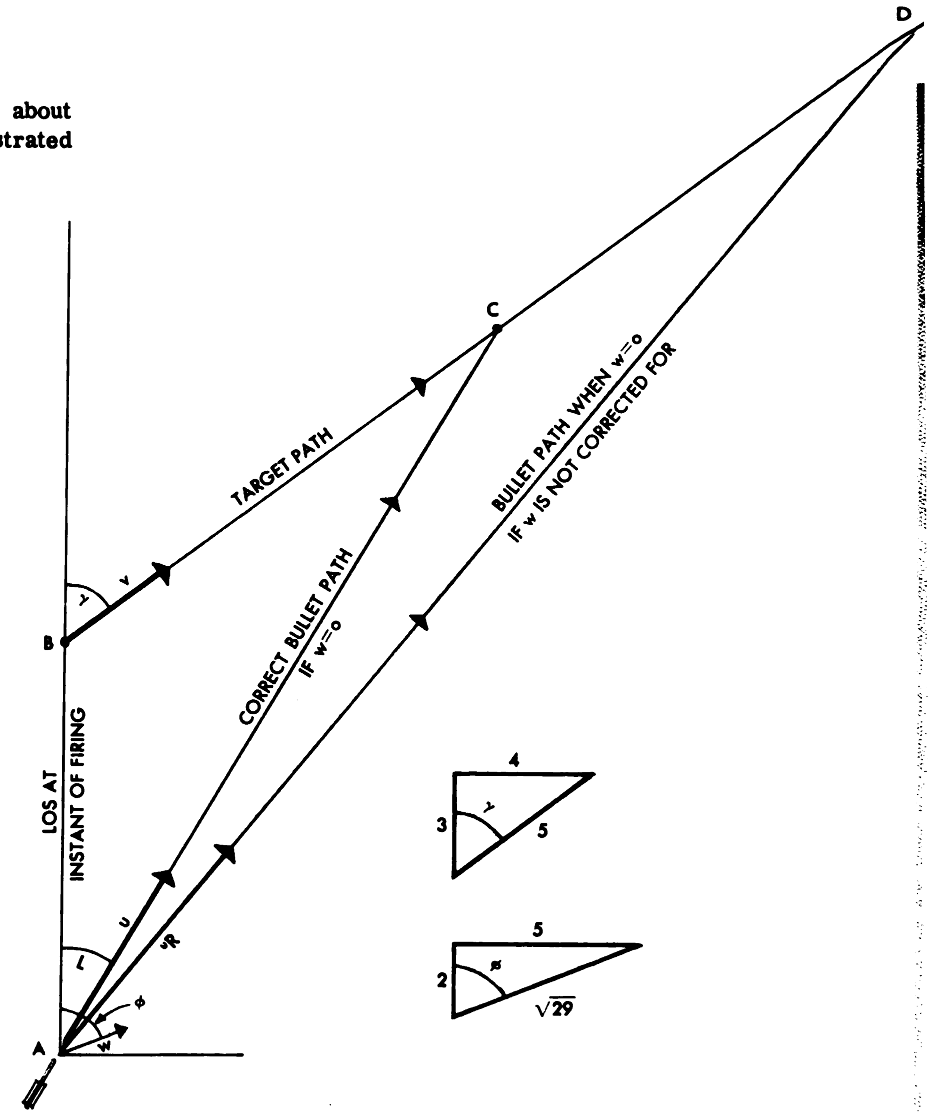
When there is jump, and it is not corrected for:

$w \sin \phi = 5$

$w \cos \phi = 2$

Resultant velocity vector of bullet is u_R (the resultant of u and w).

Bullet crosses target path at D and misses target.



When there is jump, and it is corrected for:

L' is the corrected lead angle

$$\sin L' = \frac{v \sin \gamma - w \sin \phi}{u}$$

$$= \frac{8-5}{16}$$

$$\sin L' = \frac{3}{16}$$

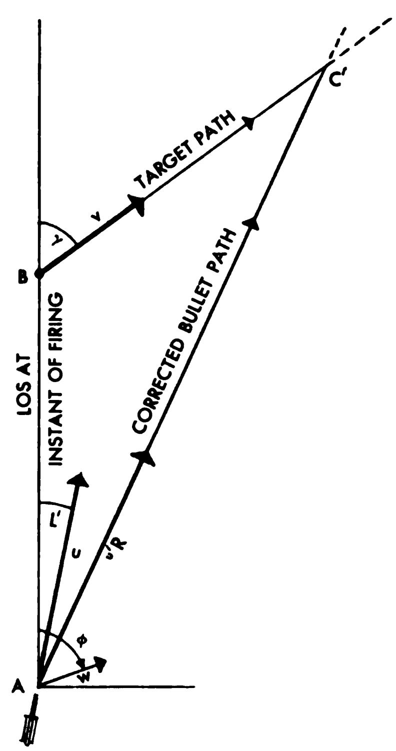
$$= 0.1875$$

$$L' = 10° 48'$$

Resultant velocity vector of bullet is u'_R (the resultant of u in its new direction, and w).

$$\begin{aligned} \text{Bullet cross velocity} &= \text{cross component of } u'_R \\ &= \text{cross component of } u \\ &\quad + \text{cross component of } w \\ &= u \sin L' + w \sin \phi \\ &= 3 + 5 \\ &= 8 \\ &= \text{target cross velocity} \end{aligned}$$

Bullet intercepts target at C' and scores a hit.



When target is tracked in a frame that moves with gun:

Target velocity vector v'', relative to new frame

= vector v - vector w

Components of v'':

Cross component: $v'' \sin\gamma'' = v \sin\gamma - w \sin\phi$

$$= \left(10 \times \frac{4}{5}\right) - \left(\sqrt{29} \times \frac{5}{\sqrt{29}}\right)$$

$$= 3$$

LOS component: $v'' \cos\gamma'' = v \cos\gamma - w \cos\phi$

$$= \left(10 \times \frac{3}{5}\right) - \left(\sqrt{29} \times \frac{2}{\sqrt{29}}\right)$$

$$= 4$$

$$v'' = \sqrt{3^2 + 4^2}$$

$$= 5$$

$$\gamma'' = \tan^{-1} \frac{3}{4} = 36°52'$$

Bullet velocity relative to new frame = u

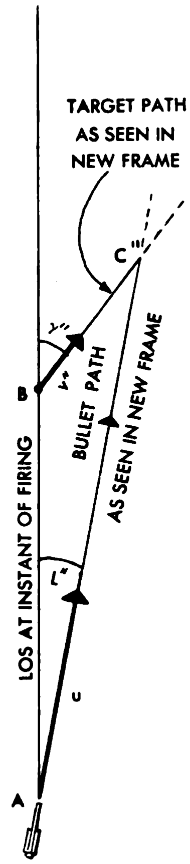
$$\sin L'' = \frac{v'' \sin\gamma''}{u}$$

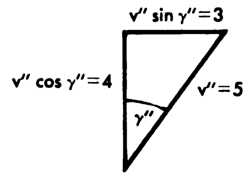
$$= \frac{5 \times 3/5}{16}$$

$$= 3/16$$

$$= 0.1875$$

$$= 10°48'$$





Note that L'' = L'

Cross velocity of target = $v'' \sin\gamma'' = 3$

Cross velocity of bullet = $u \sin L'' = 16 \times \frac{3}{16} = 3$

Bullet intercepts target at C'' and scores a hit.

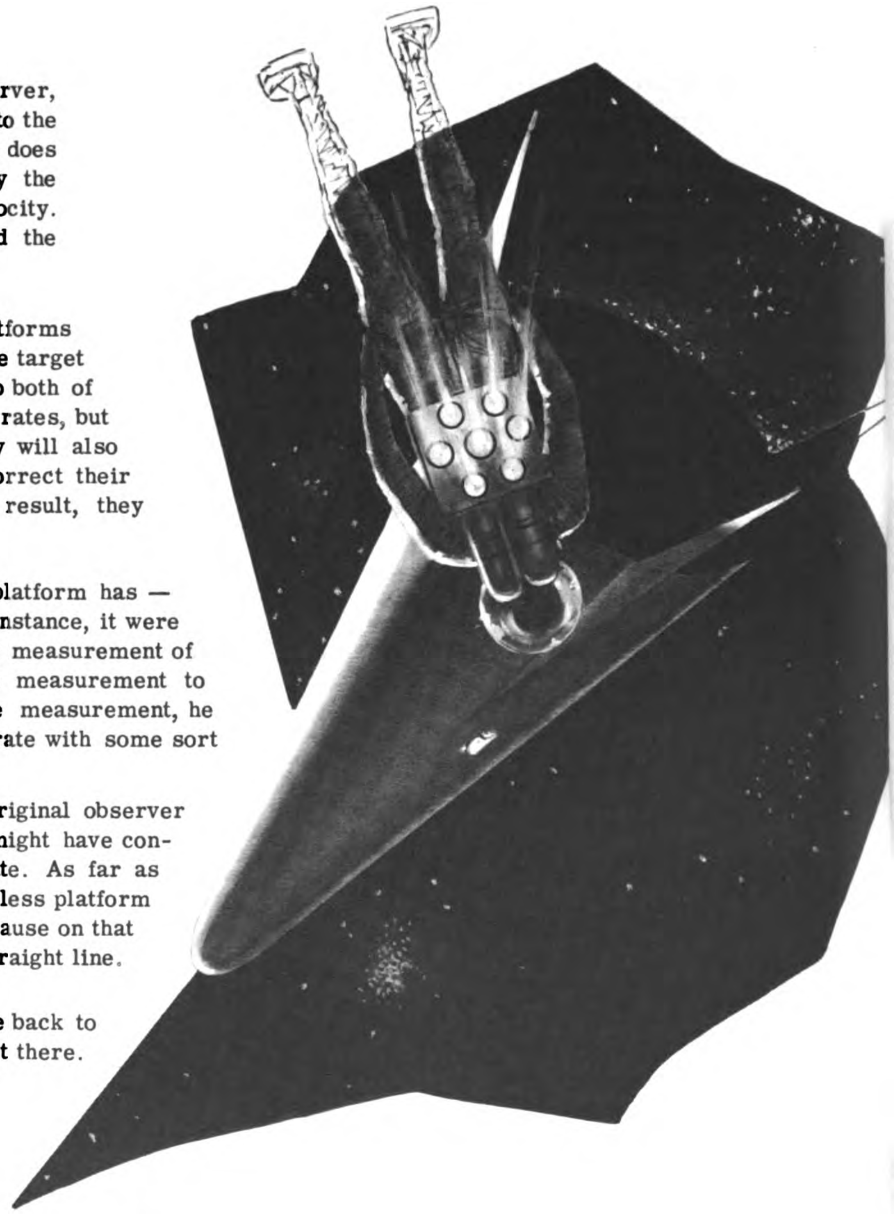
SUMMARY

and some considerations

An observer on an airless, gravity-less inertial platform learned that in order to hit a target whose line of sight is rotating, he must aim a gun ahead of the target by an offset angle known as lead angle. To compute this angle, he had to be able to predict target future position. To do this, he had to obtain data on target position and motion. He obtained the target data by tracking the target; that is, by placing the telescope axis (which is a controlled line) so that it lies along the line-of-sight.

The observer made measurements on a target moving in two dimensions. He obtained equations for lead angle in terms of bearing rate and range at the instant of firing. In the case of linear target motion, these quantities are changing continually, so he deduced that instantaneous and continuous computation of lead angle would be necessary. The problem was further complicated by "jump" when the gun had a velocity of its own in some arbitrary direction. The bullet then had an actual velocity which was, in general, not along the weapon line. This necessitated a correction in the lead angle computation. The observer discovered that the corrected lead angle was independent of the inertial reference frame in which measurements were made; also, that the computation was simplified by selecting a reference frame that moved with the gun at the instant of firing.

SUMMARY *and some considerations*



The invariance of the lead angle puzzled the observer, until he realized that target cross velocity relative to the gun is what determines lead angle. His own motion does not affect this cross velocity — it affects only the measurements from which he computes cross velocity. His last doubts were resolved after he pursued the following train of reasoning.

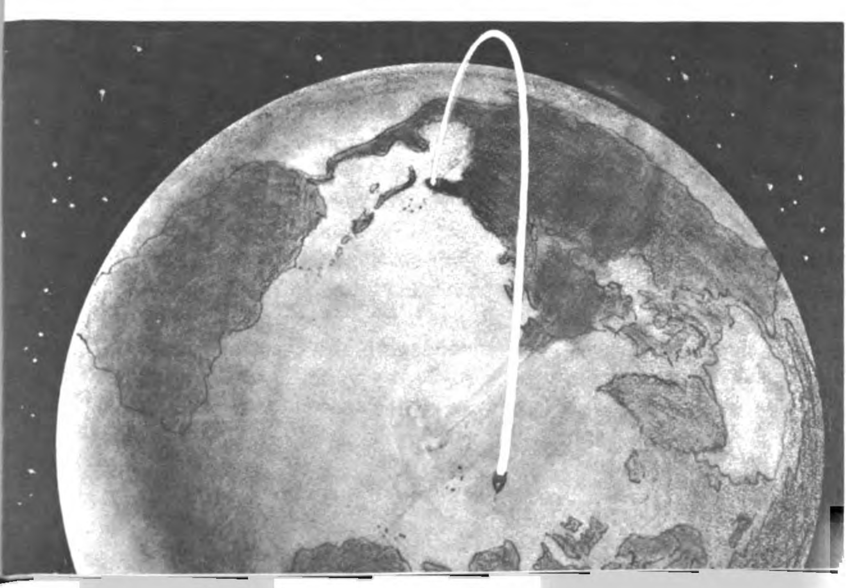
Suppose that two observers on two inertial platforms having different constant velocities observe the same target and the same gun (which is moving with respect to both of them.) They will measure different target bearing rates, but this will be compensated for by the fact that they will also measure different gun platform velocities, and correct their target motion measurements accordingly. As a result, they will compute the same lead angle.

An observer need not know what motion his own platform has — PROVIDED that it is an inertial platform. (If, for instance, it were rotating in inertial space, he would obtain a wrong measurement of target bearing rate, and there would be no jump measurement to compensate for this. To correct the bearing rate measurement, he would have to measure his own platform rotation rate with some sort of gyroscopic device.)

In obtaining data for all of his computations, our original observer had to measure only bearing rate and range. He might have concluded that he did not need to measure range rate. As far as tracking and prediction on this gravity-less and airless platform was concerned, he would have been quite right, because on that platform a bullet flies with constant velocity in a straight line.

Let us transfer our observer from his ideal frame back to Earth, and consider shooting conditions as they exist there.

When we take leave of the airless, gravity-less inertial platform and, so to speak, "come back to Earth," we find very different conditions. Bullets, or any other missiles in ballistic flight, do not fly in straight lines or at constant velocity. This is because they are acted on by external forces, such as gravity and aerodynamic forces. It is true that we can control certain types of missiles and cause them to travel in straight lines at constant velocity.



In fact, the method of computing the lead angle in an ideal inertial frame works very well with torpedoes, here on Earth, but this method will not work with bullets.

In general we need to know range rate, as well as range, bearing, and bearing rate.

Furthermore, on the Earth, targets may move in three dimensions. This introduces another quantity - target elevation, whose magnitude and rate must both be ascertained. However, this necessity does not change the nature of the essential problem.

The conclusions our observer drew as to velocity jump are, generally, applicable on Earth. It is advantageous to track targets in a reference frame that moves with the gun. When this is not practicable, jump must (theoretically) be corrected for. However, in many short range problems, jump is practically negligible and can be ignored.

In the case of long trajectories, the prediction problem is further complicated, by the fact that the Earth has curvature and rotation. The tracking and prediction problems as they exist on Earth, with some actual means employed in their solution, are discussed in the main portion of this volume.

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